Bio-inspired computation, communication topologies, and computational complexity

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Outline

- The first and second machine classes
- Examples of parallel computing models
- Membrane systems
- Complexity theory of membrane systems
- Communication topologies and their role
- A research project

The first machine class

Definition (van Emde Boas). A machine model is first class iff it simulates and is simulated by a Turing machine in polynomial time

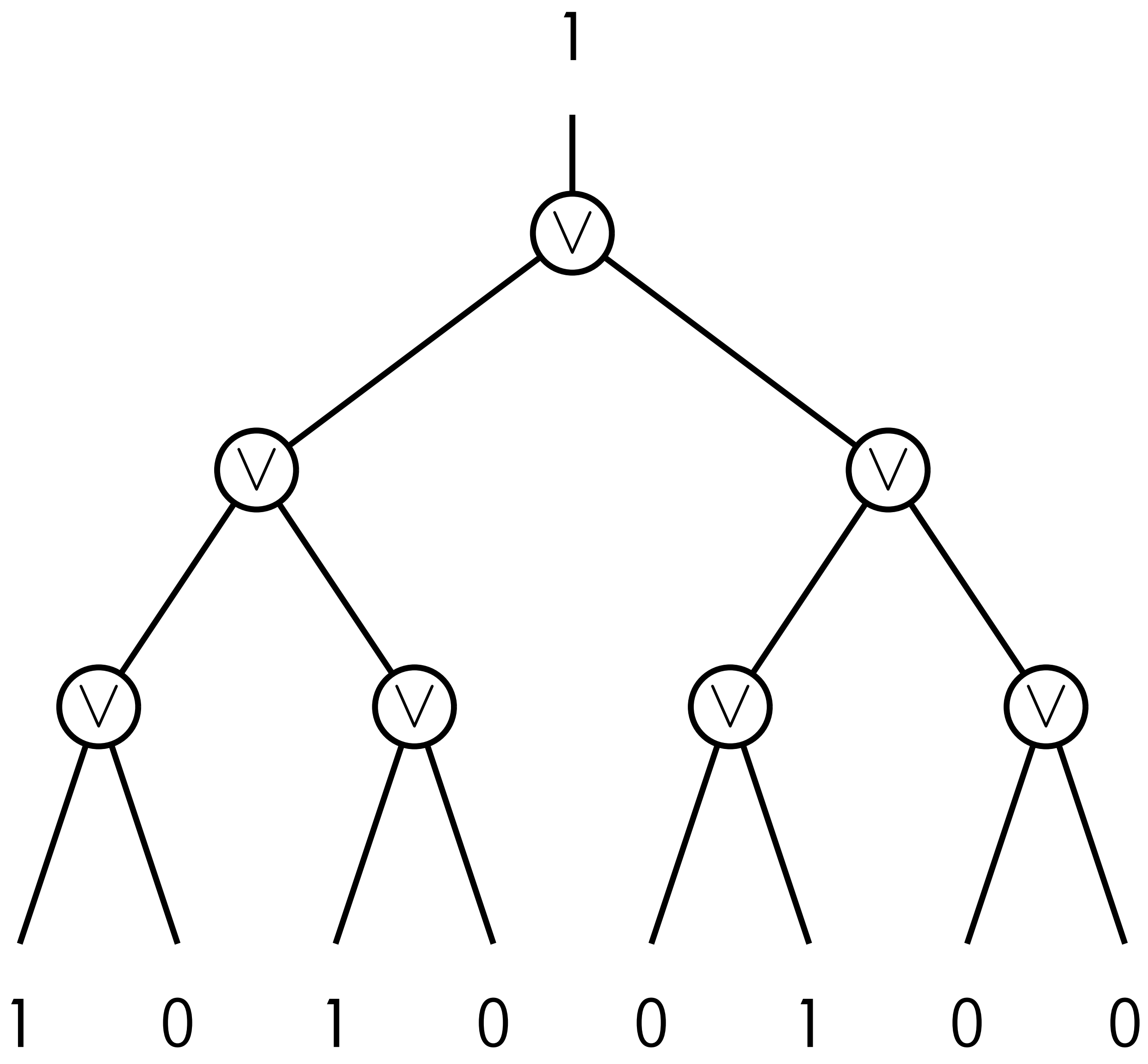
- Turing machines
- Random access machines with $+$ and $-$
- Cellular automata with finite initial configuration

The second machine class

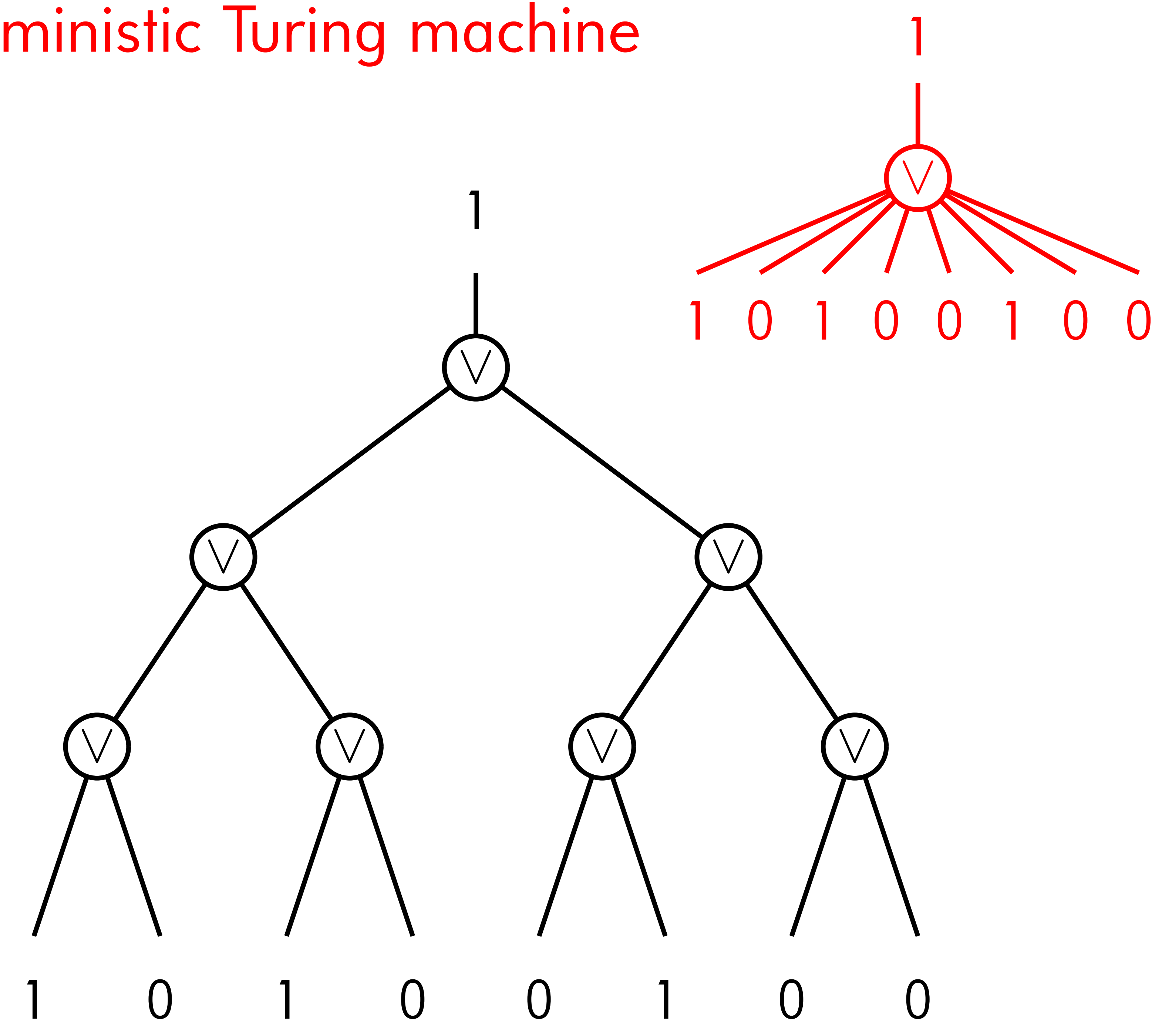
Definition (van Emde Boas). A machine model is second class iff it characterises **PSPACE** in polynomial time (deterministically and nondeterministically)

- Alternating Turing machines
- Vector machines
- Random access machines with $+ - \times \div$
- Parallel processes with `fork()` with unbounded number of processors
- Cellular automata over hyperbolic grids

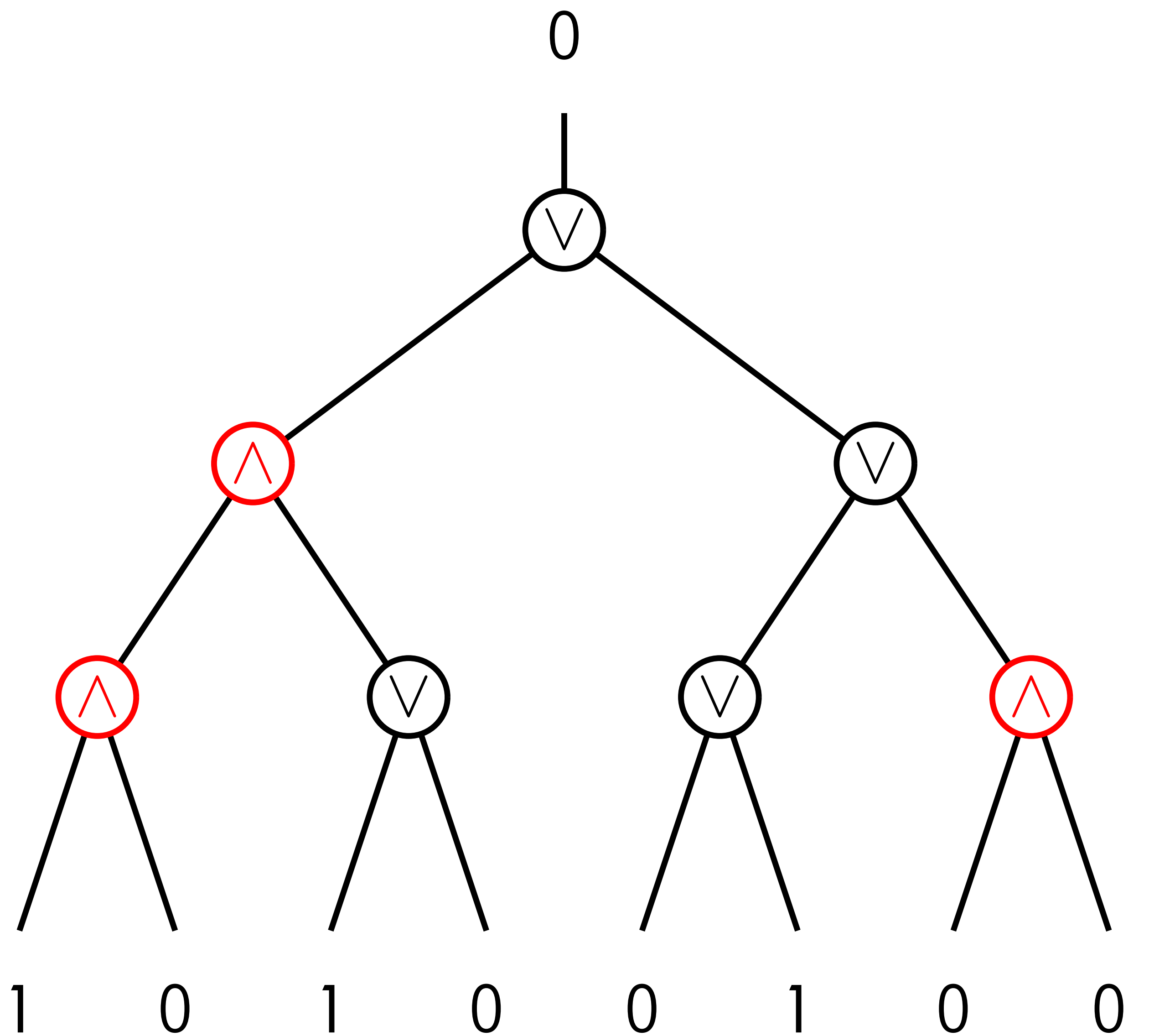
Nondeterministic Turing machine



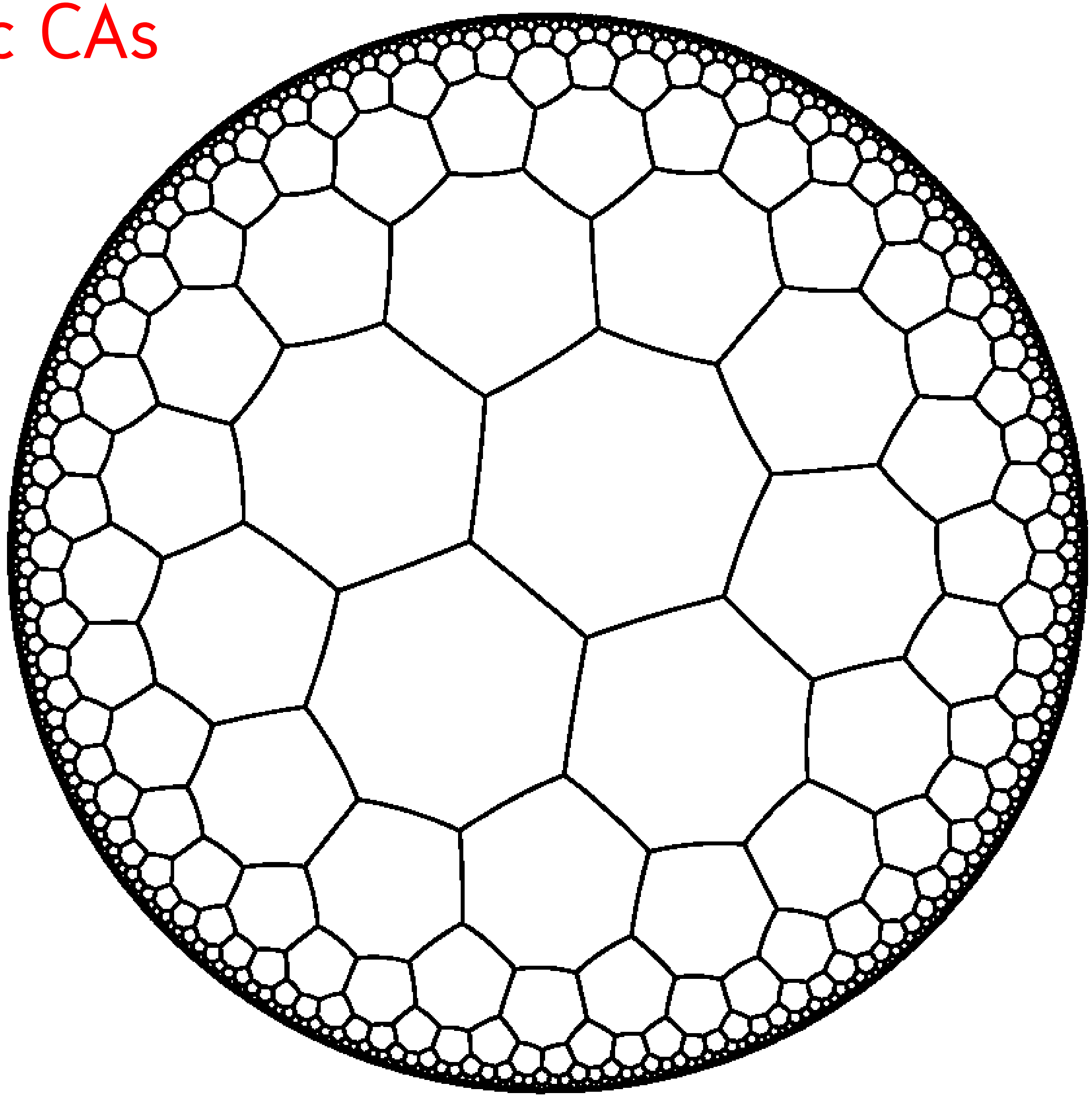
Nondeterministic Turing machine



Alternating Turing machine



Hyperbolic CAs



Hyperbolic tiling by Theon, used under CC BY-SA 3.0
`https://en.wikipedia.org/wiki/File:PavageHypPoincare2.svg`

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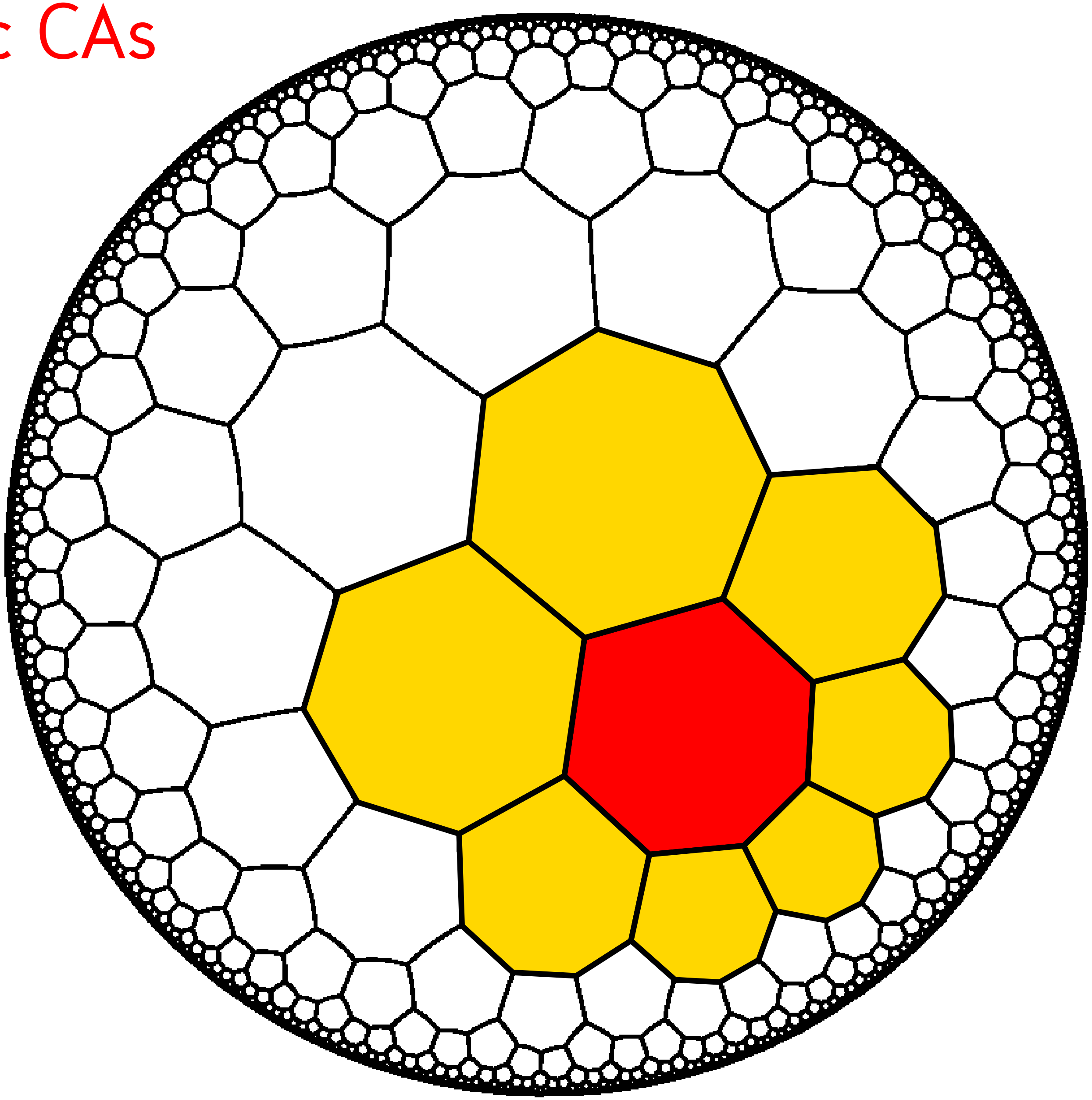

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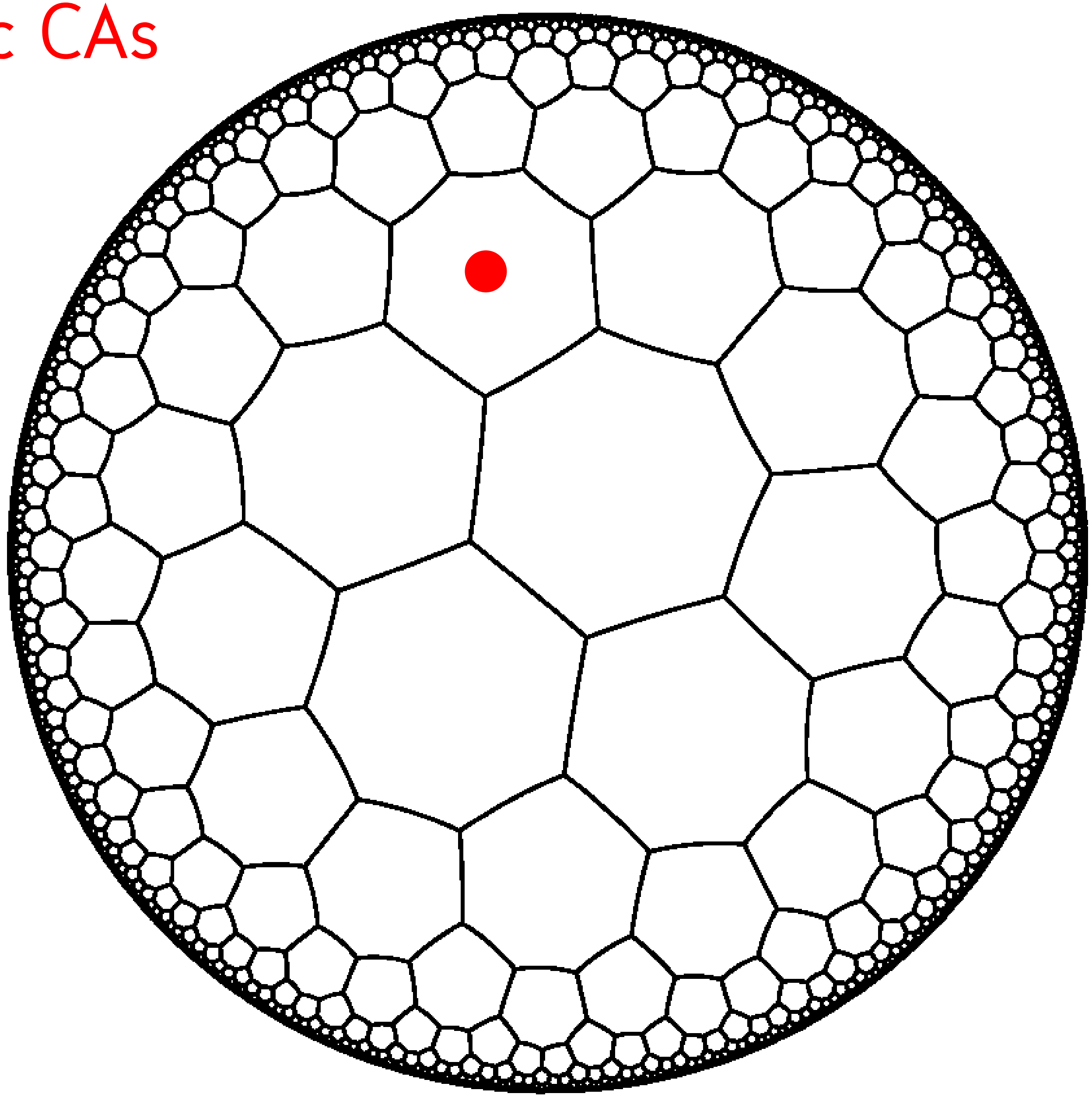

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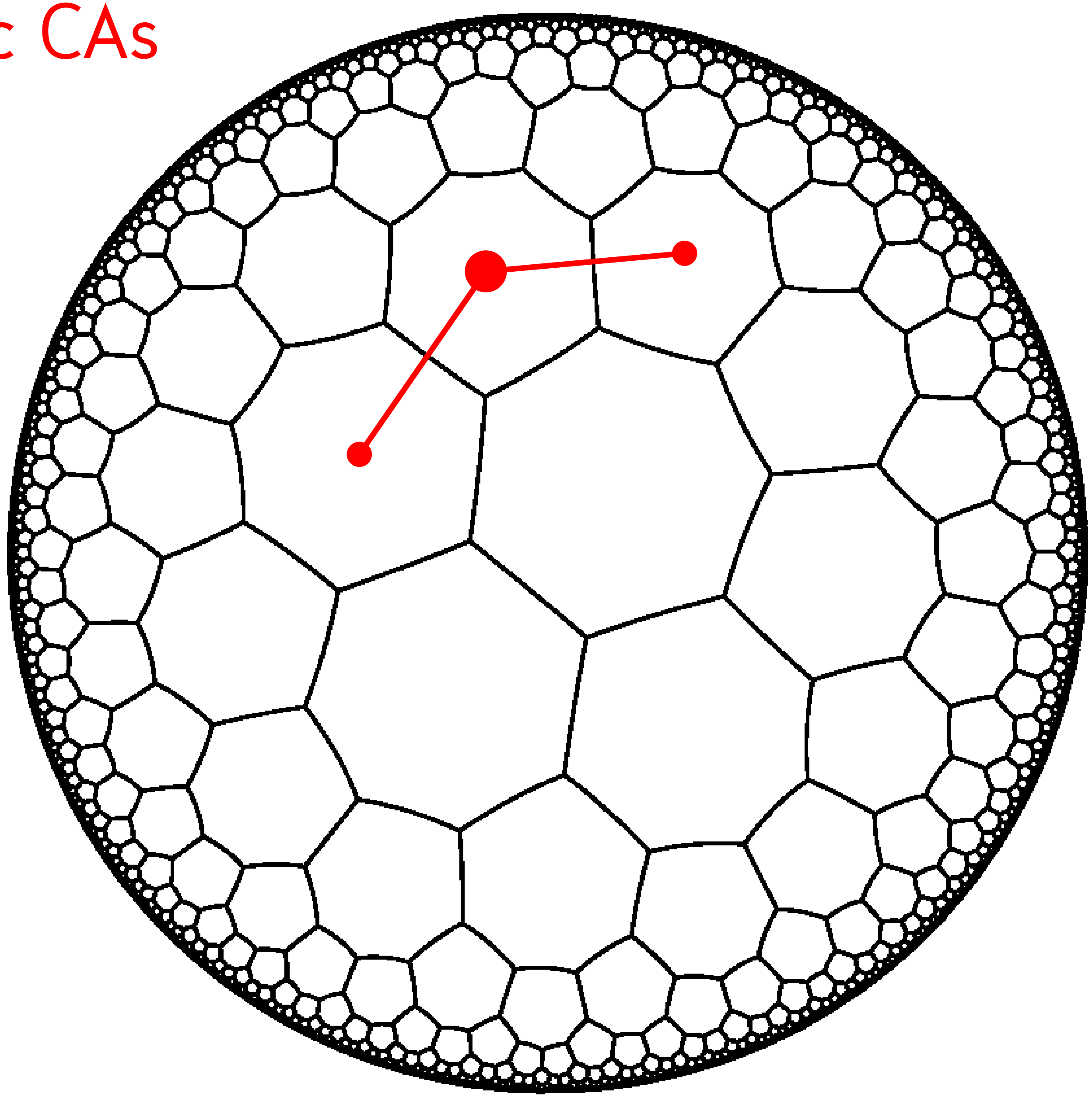

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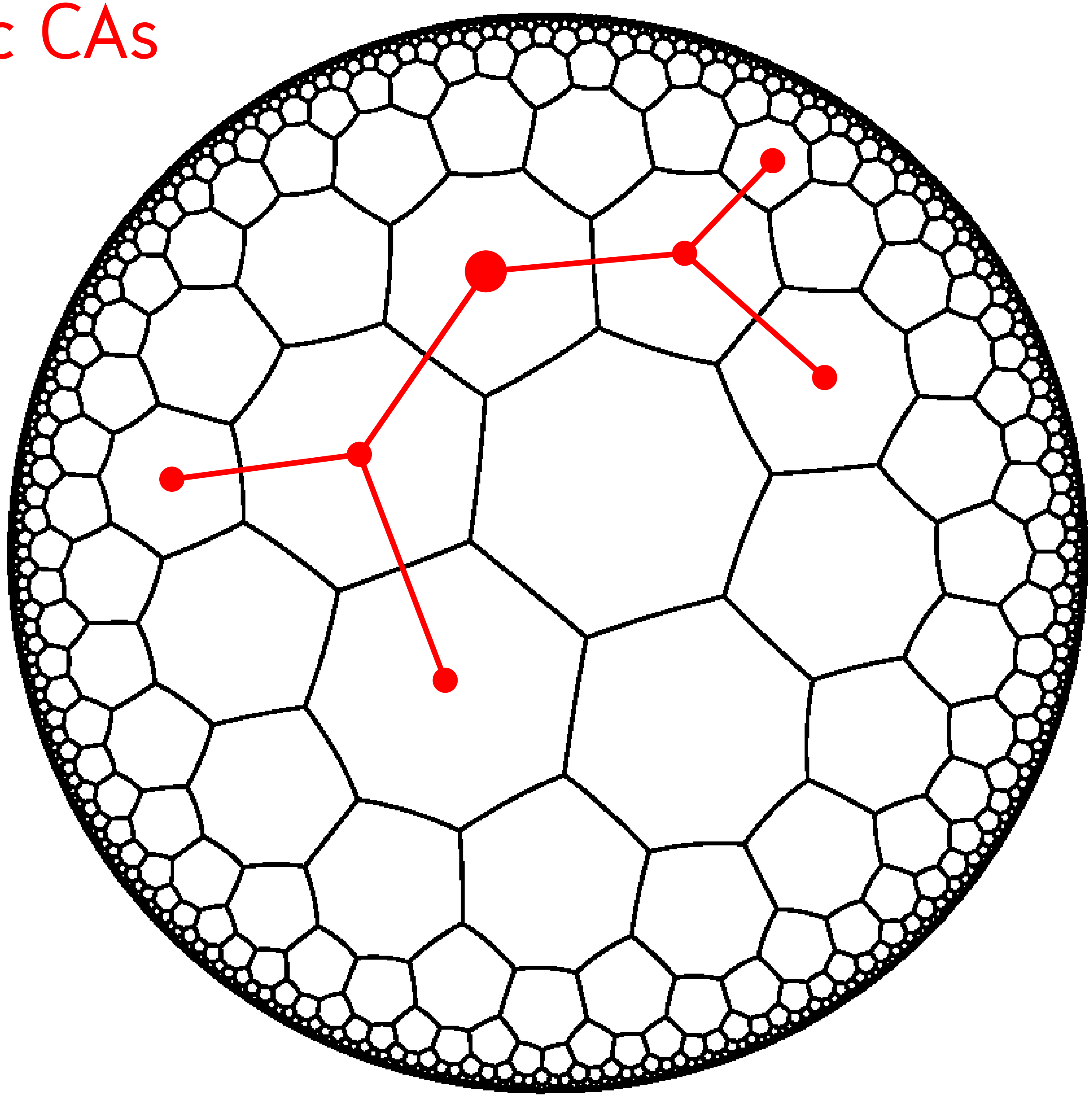

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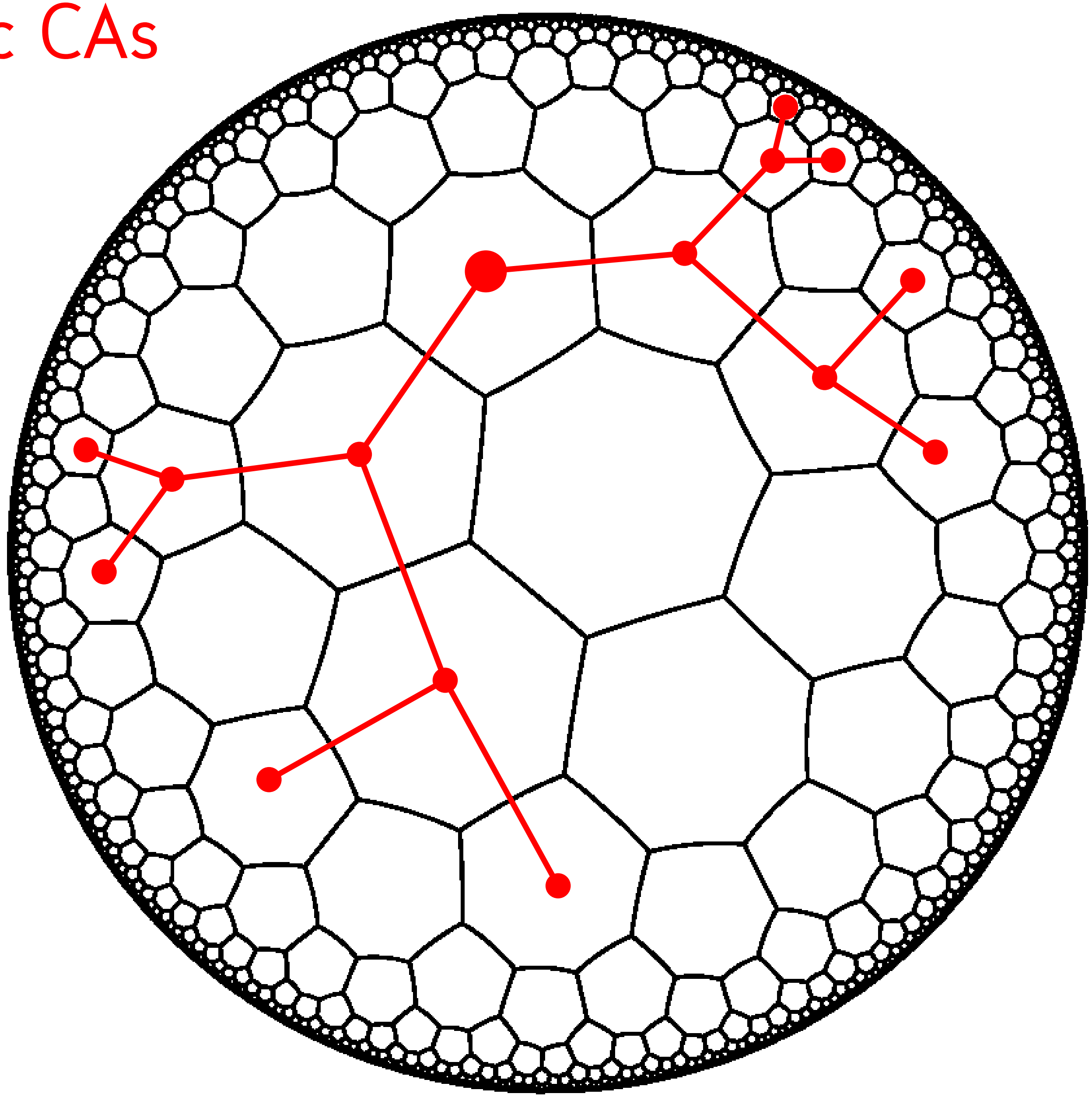

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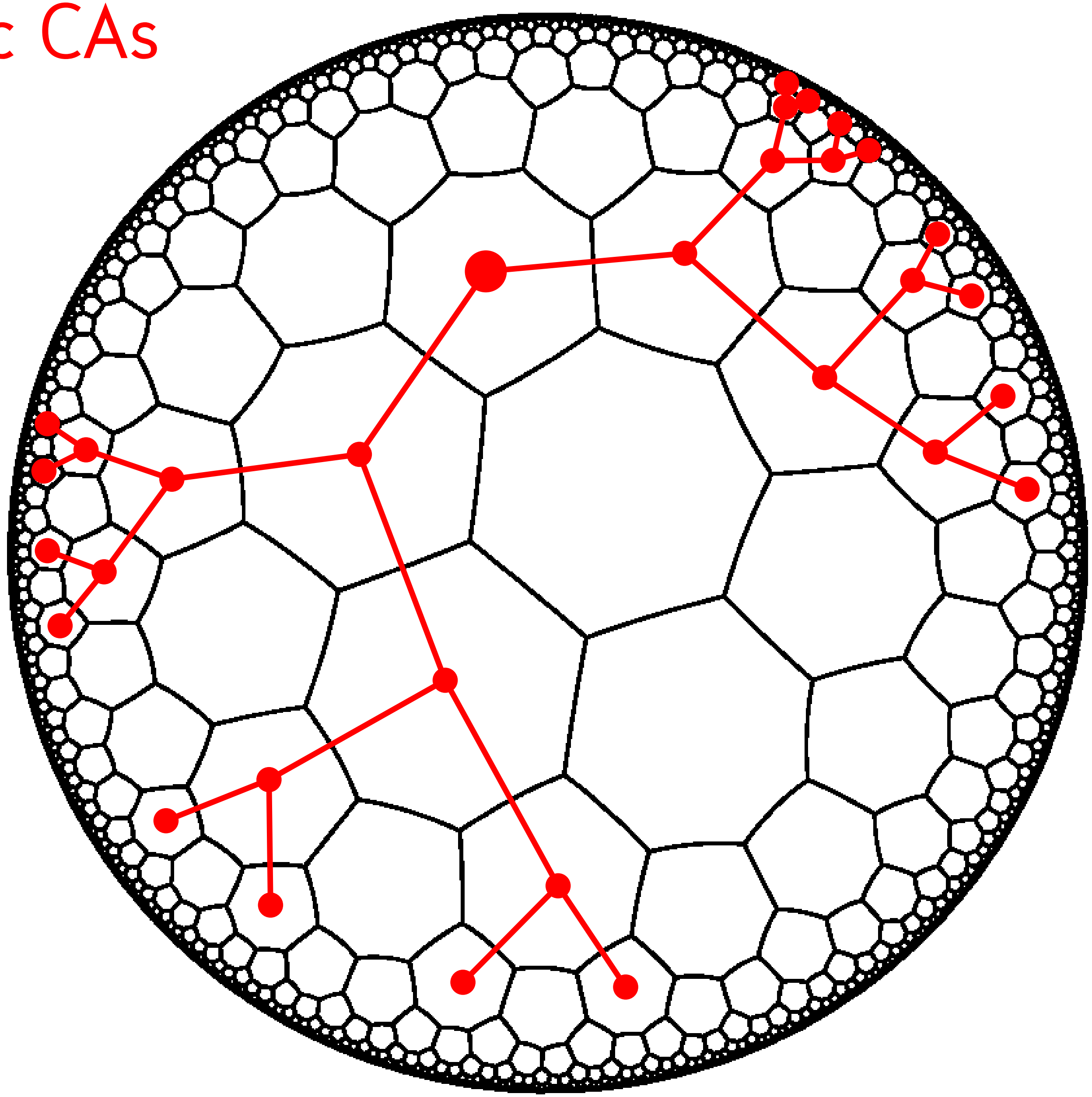

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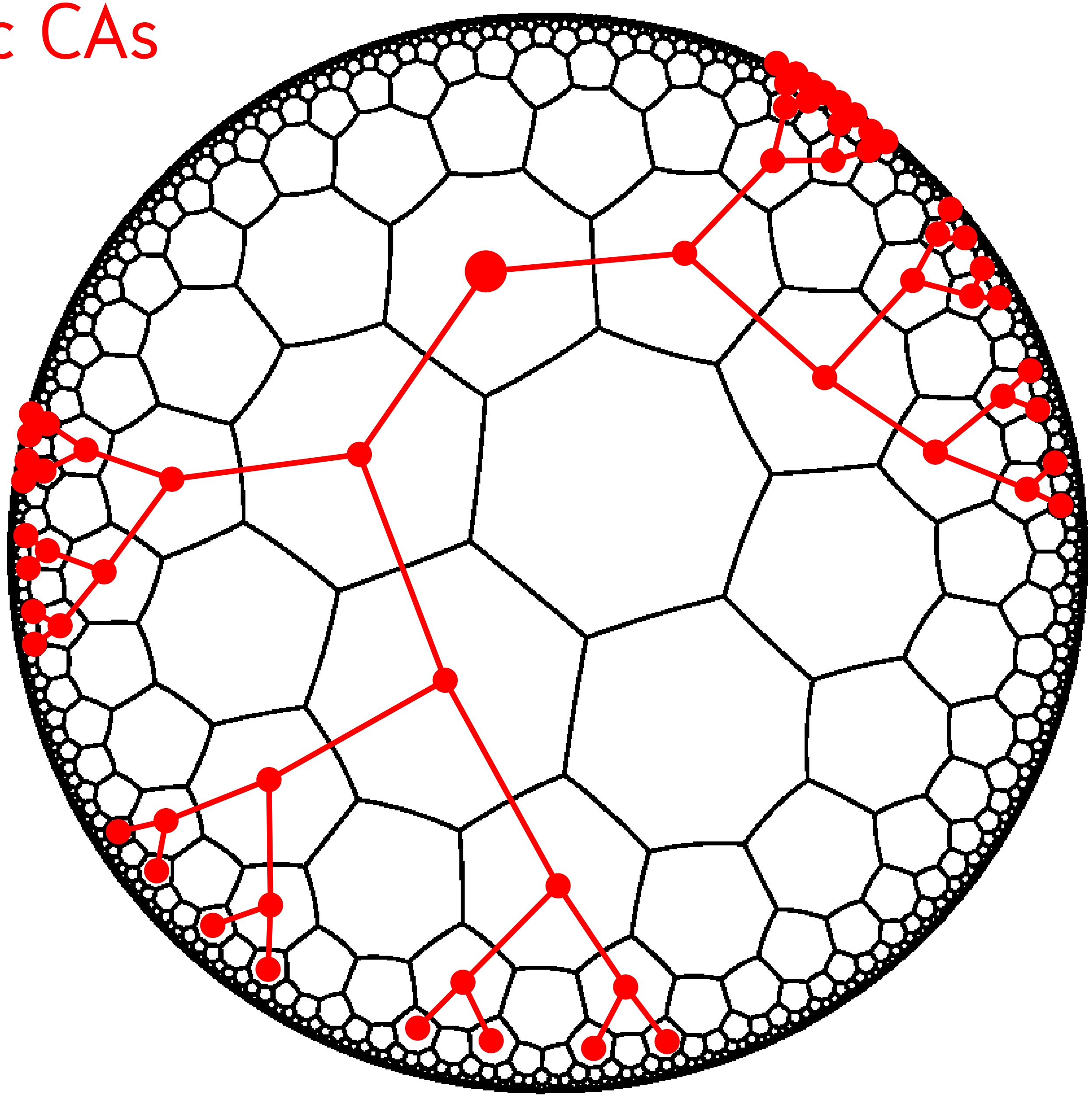

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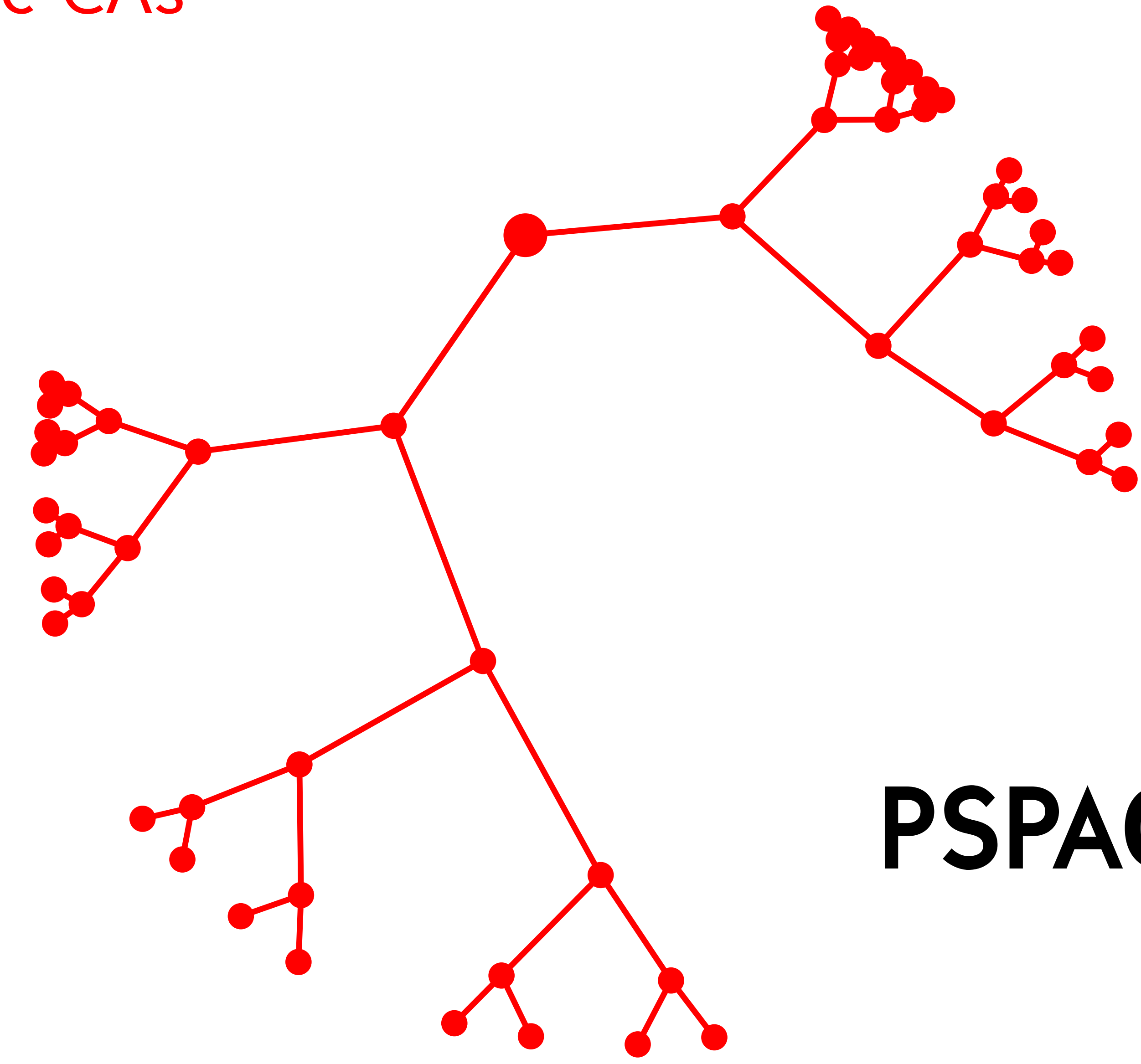

PSPACE

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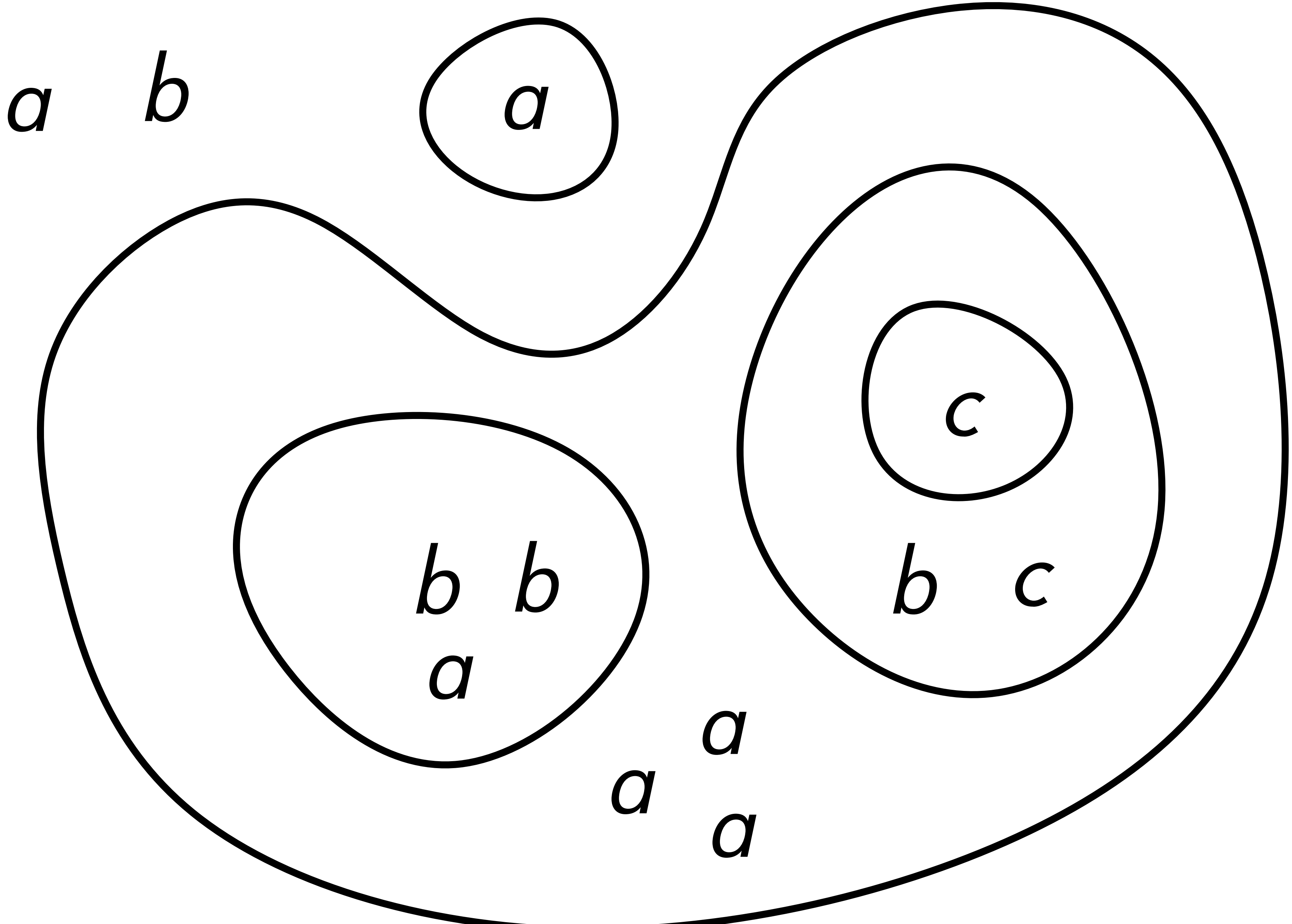
First and second machine class

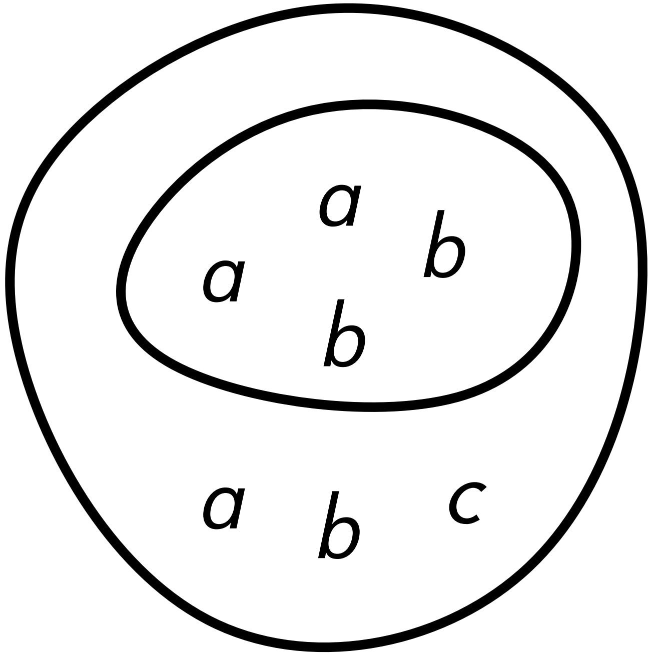
Many “concrete” sequential and parallel computing models are either first or second class machines

Membrane systems



Simple chemical reactions

$$ab \to cd$$



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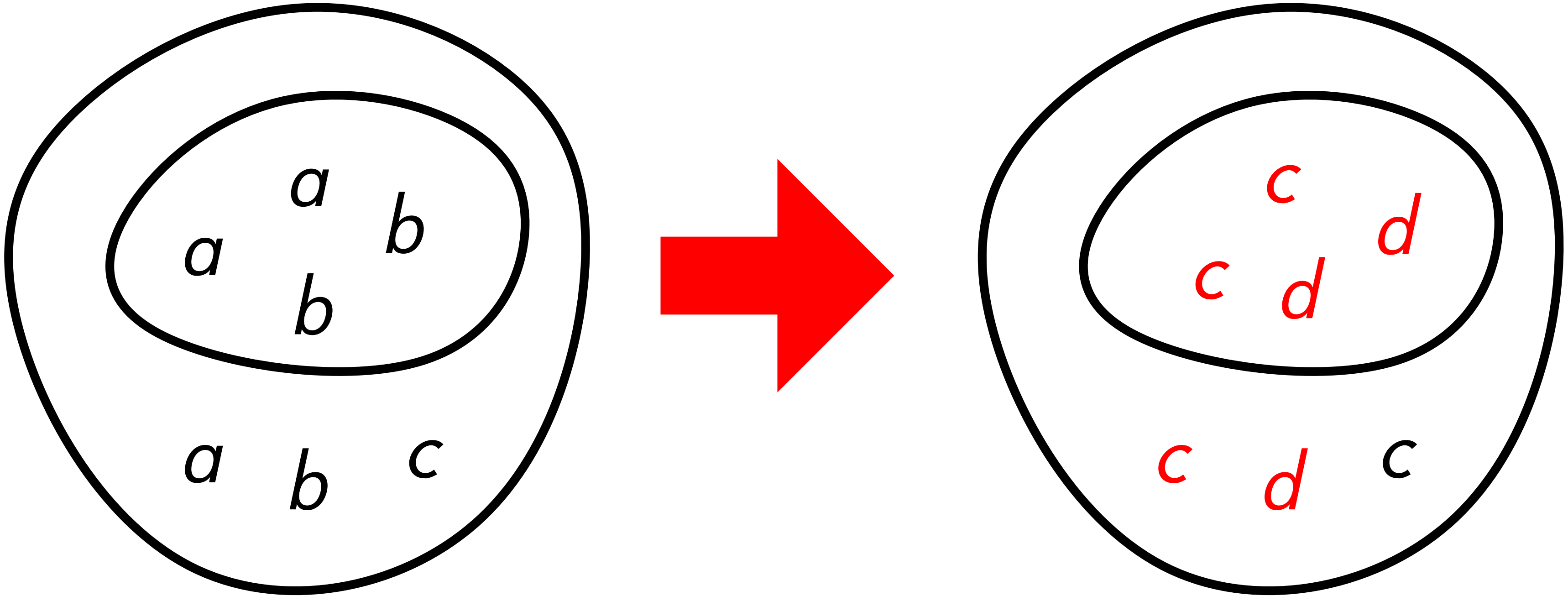
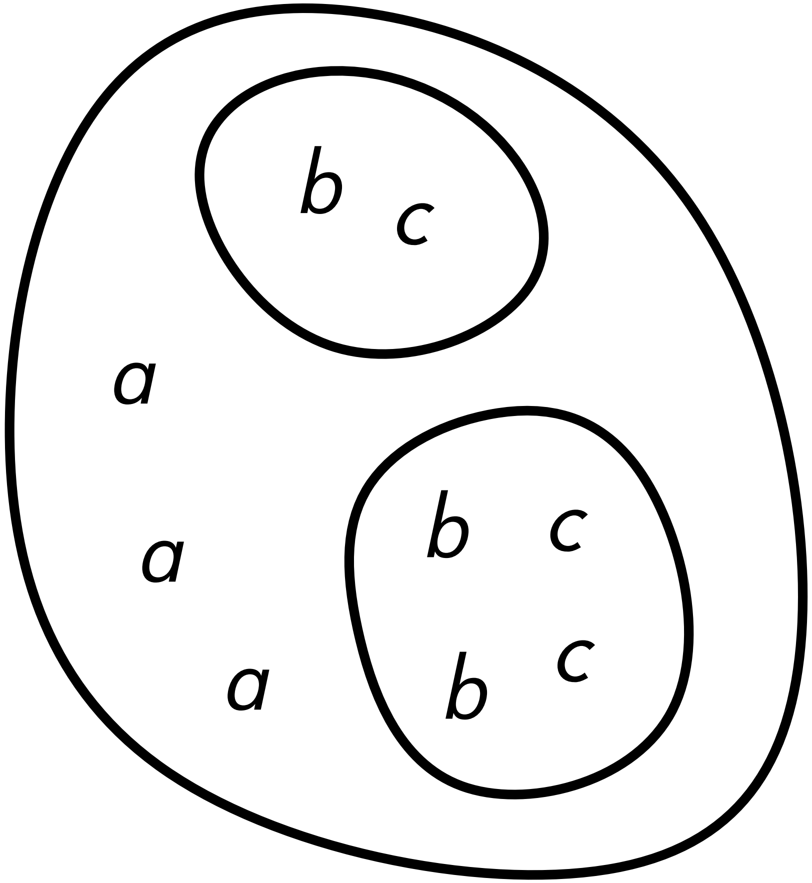


Communication between regions

$$a\,[bc] \rightarrow [d]\,e$$



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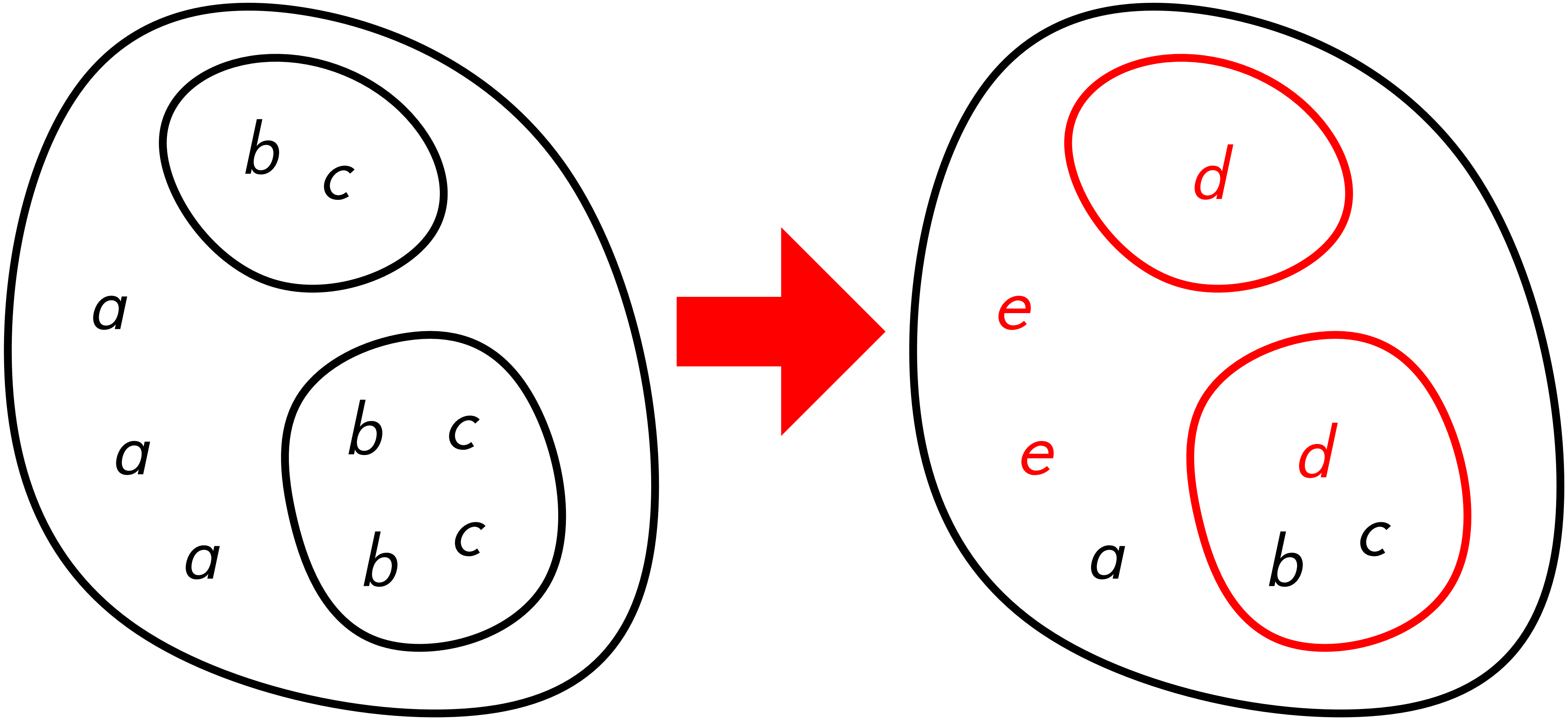
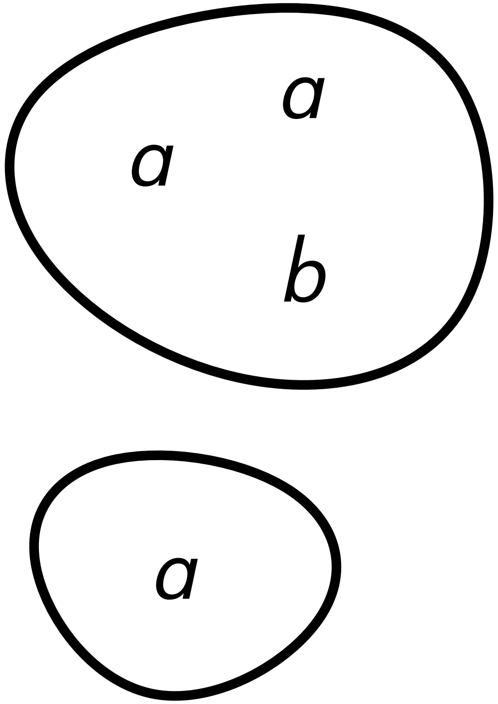


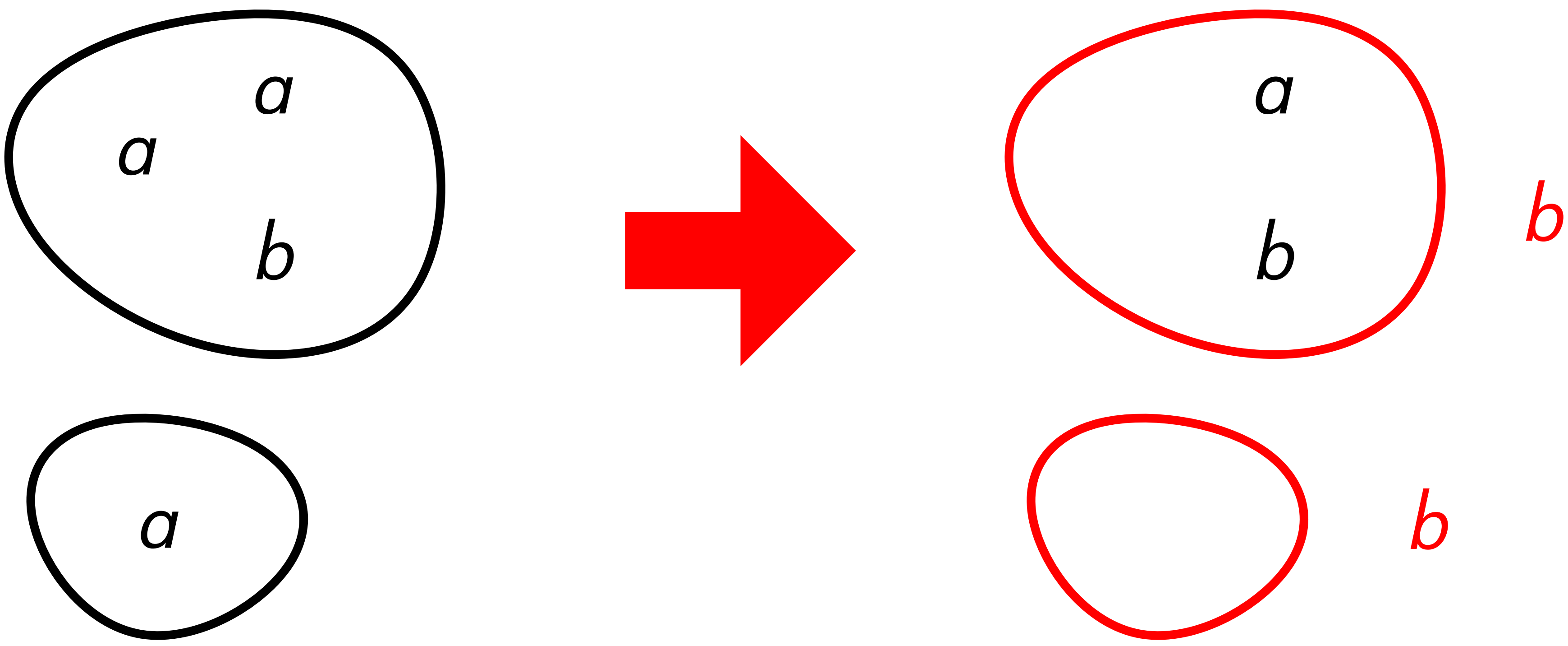
Monodirectional communication between regions

$$[a] \rightarrow [\ ]\, b$$



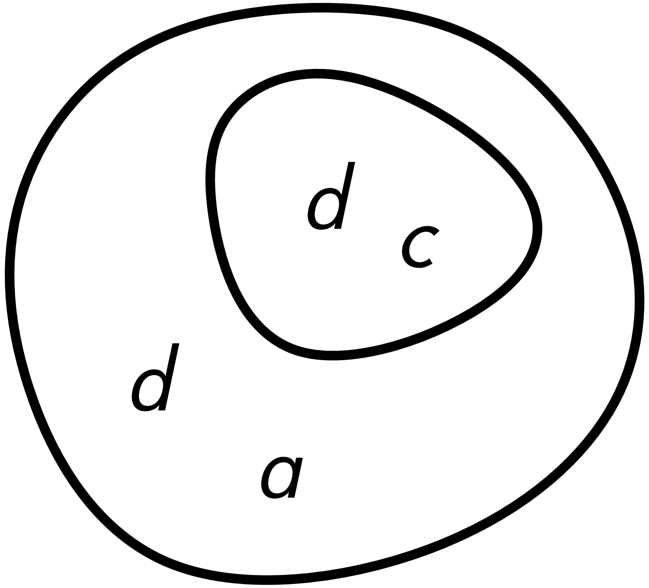
Monodirectional communication between regions

$$[a] \to [\ ]\, b$$



Membrane division

$$[a] \to [b]\ [c]$$



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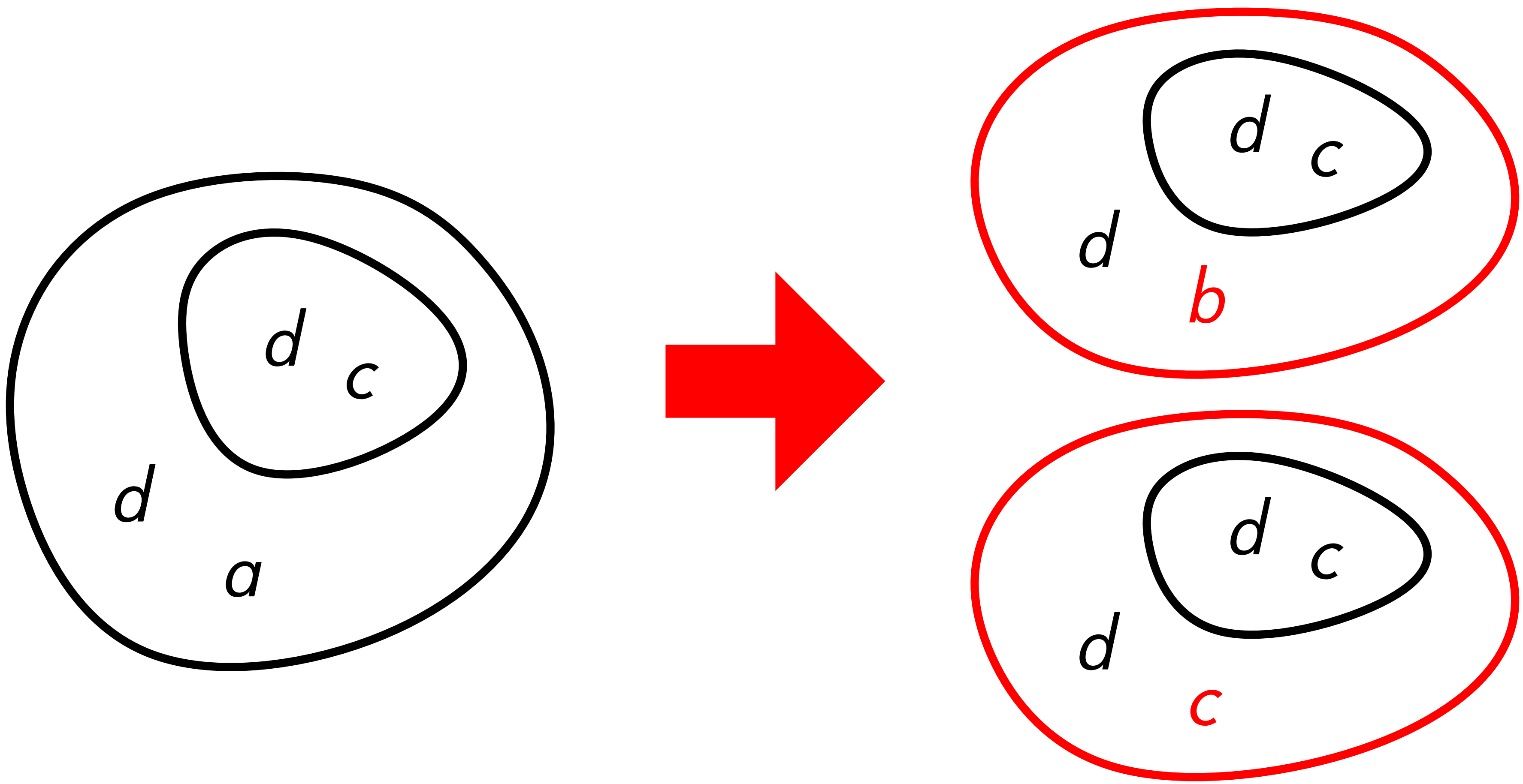
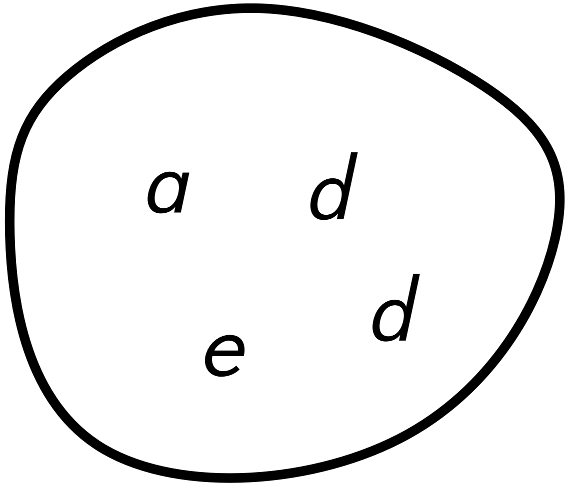


Elementary membrane division

$$[a] \to [b]\ [c]$$



Elementary membrane division

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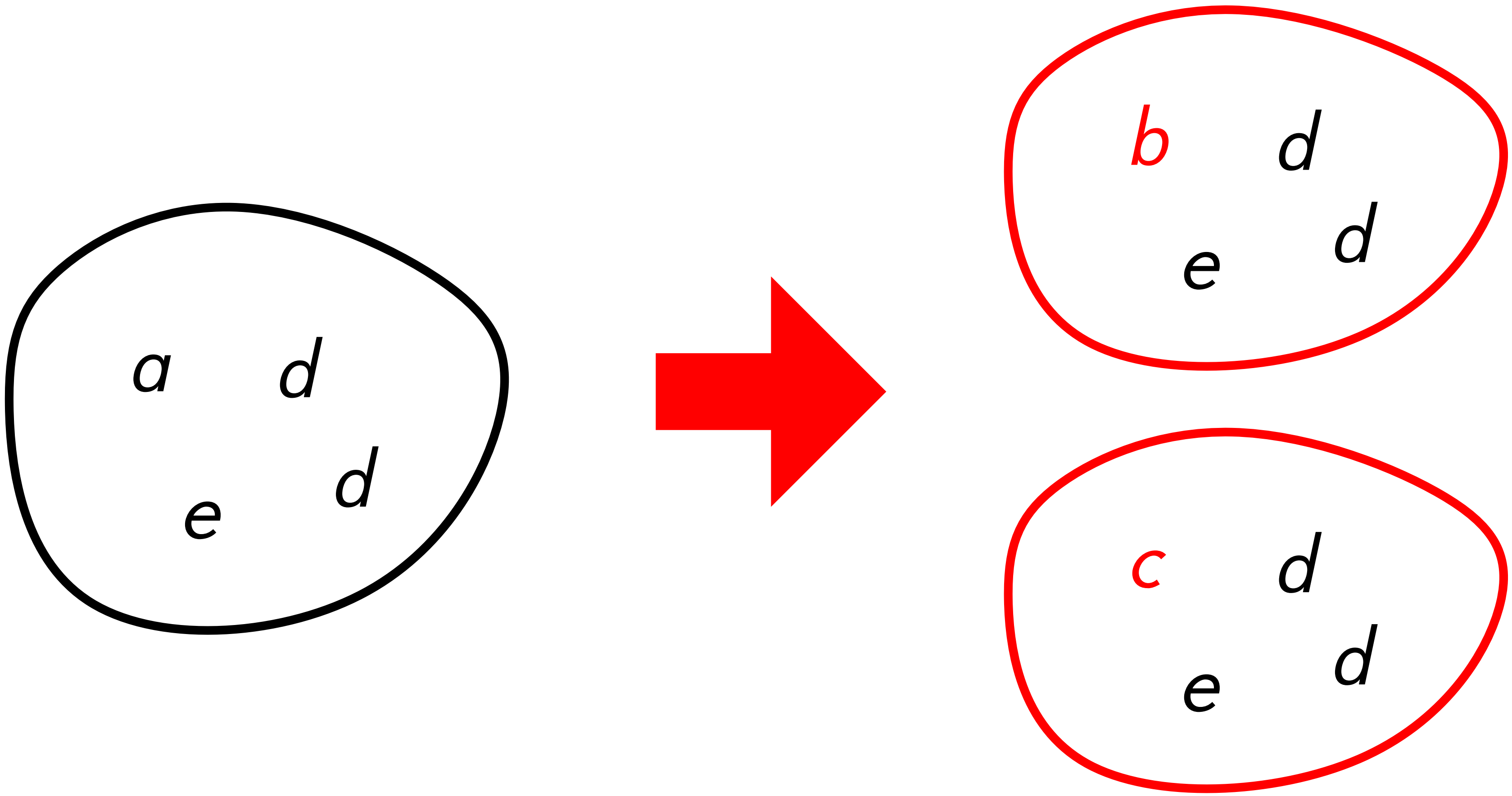

Register/counter machines

1: dec x, 2, 4
2: inc y, 3
3: inc y, 1
4: halt

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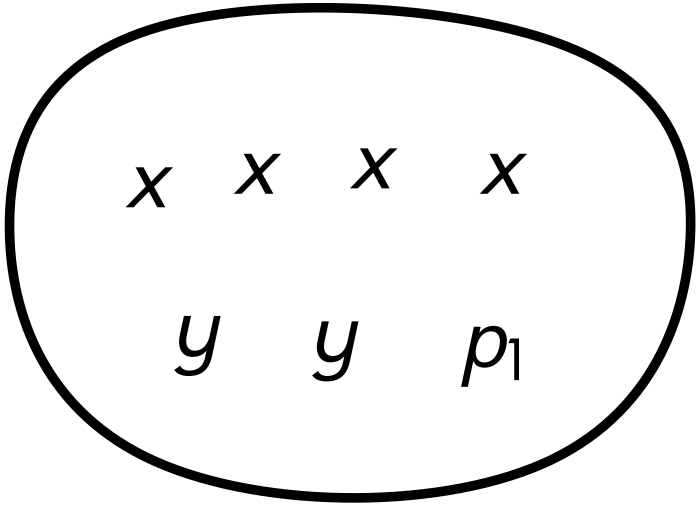

Register/counter machines

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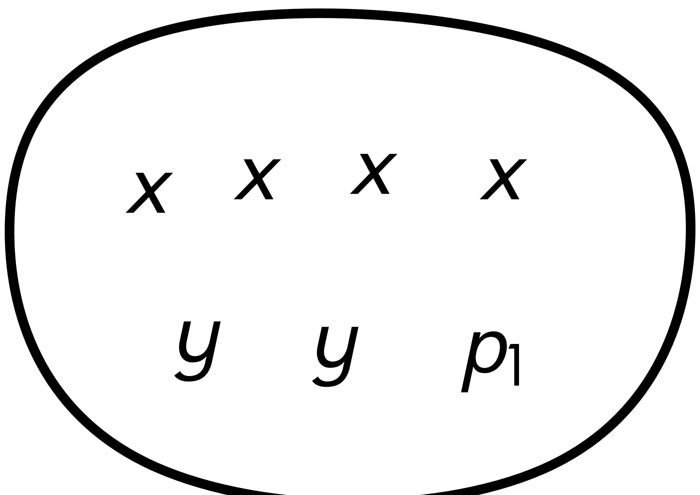
$$p_2 \to y\ p_3$$



Register/counter machines

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$$p_2 \rightarrow y\ p_3$$

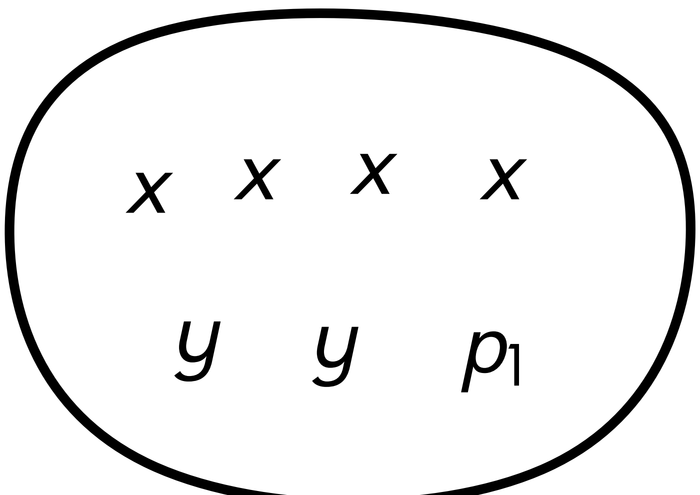
$$p_1 \rightarrow p_1'\ \overline{x}$$

$$p_1' \rightarrow p_1''$$

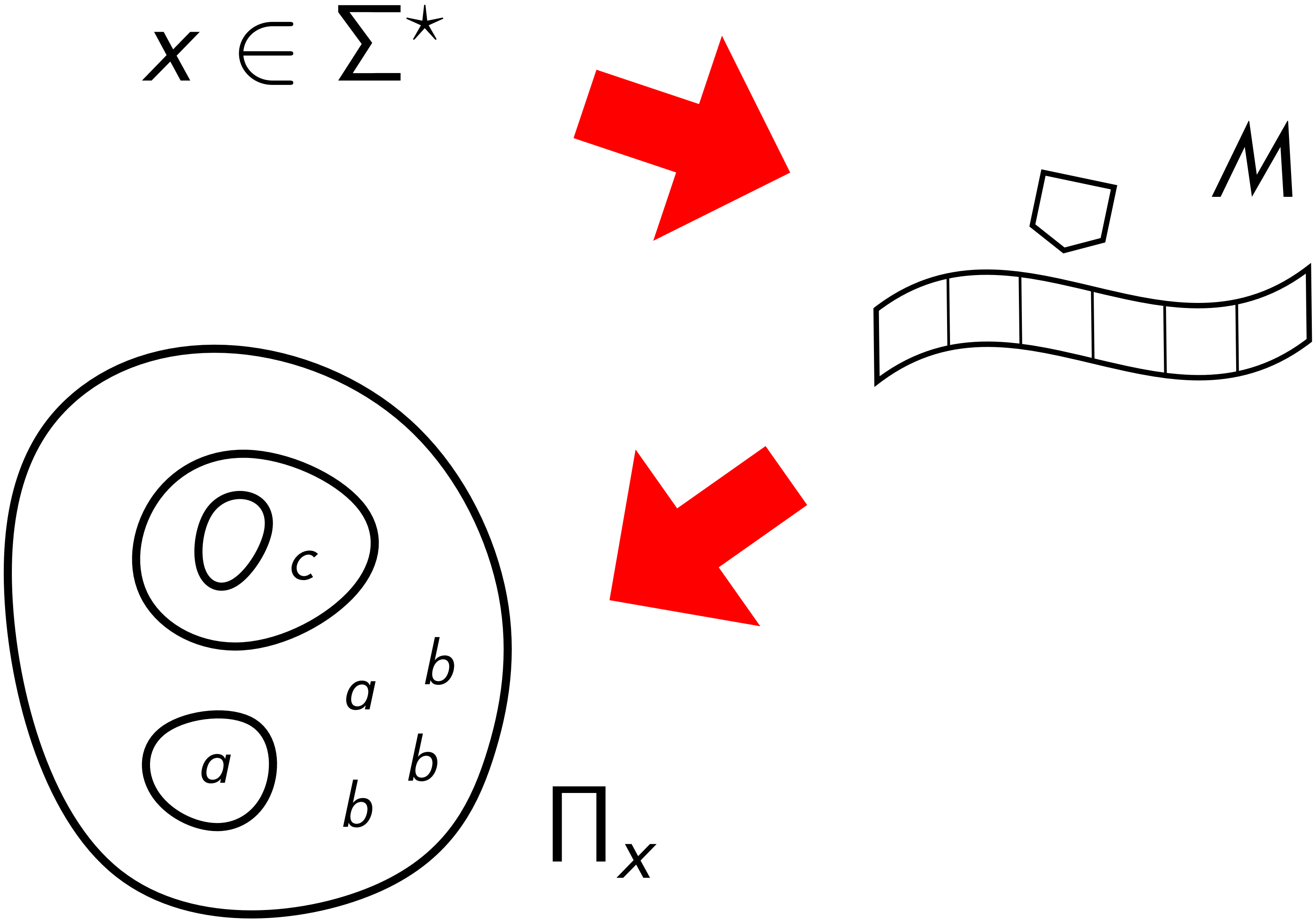
$$x\ \overline{x} \rightarrow x'$$

$$p_1''\ x' \rightarrow p_2$$

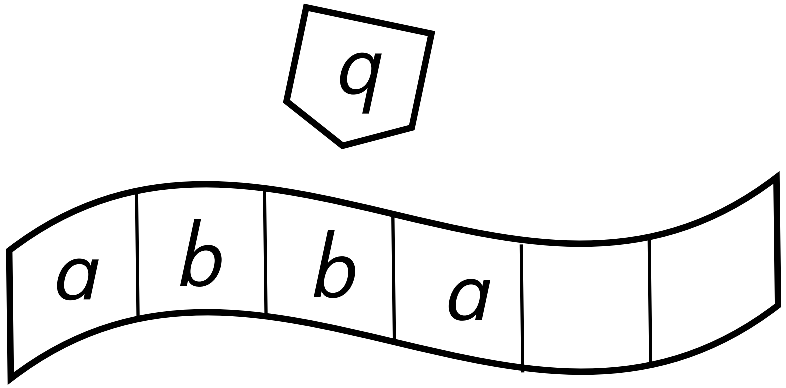
$$p_1''\ \overline{x} \rightarrow p_4$$



(Semi-)uniform families of membrane systems



Simulating Turing machines efficiently



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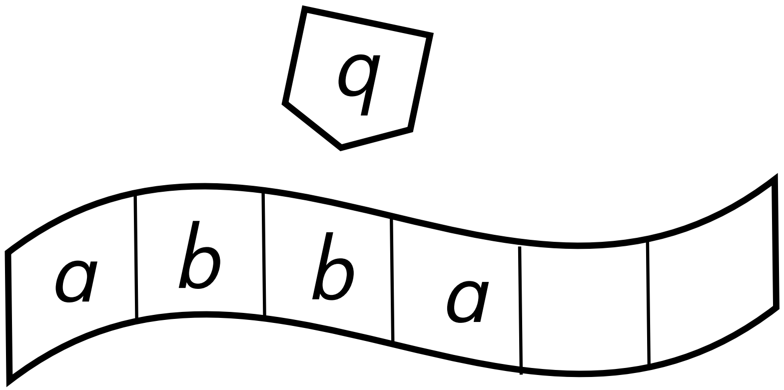

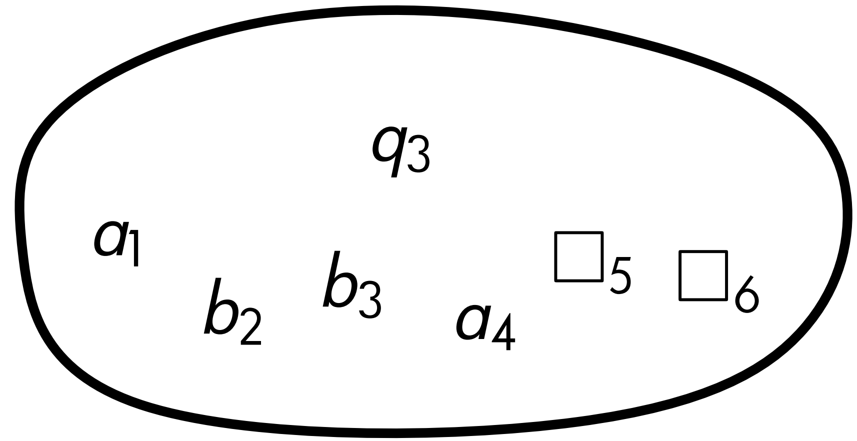

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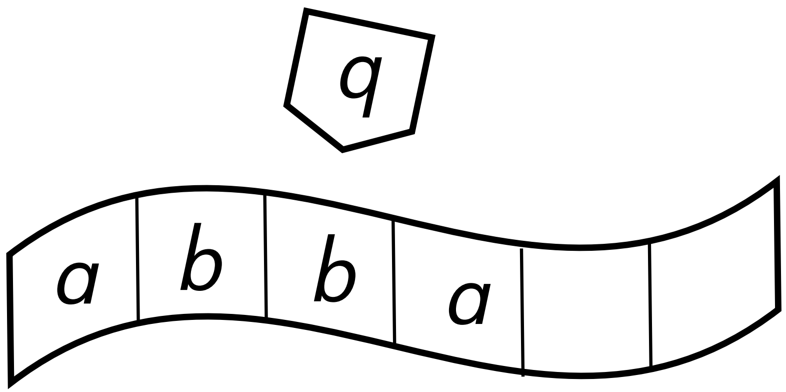
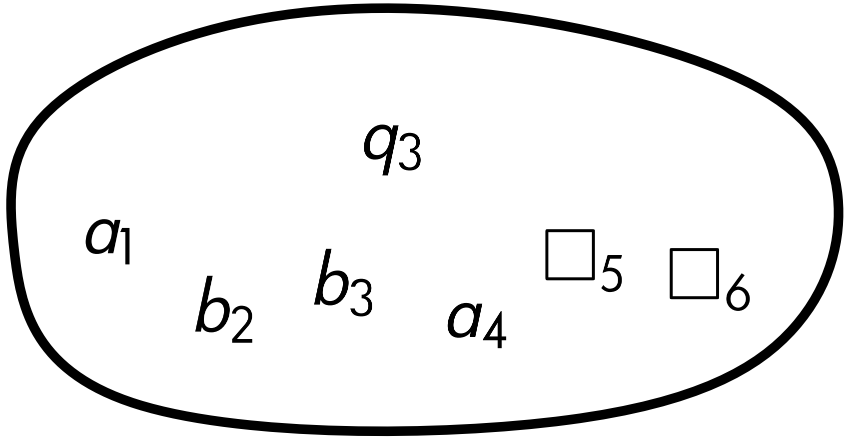
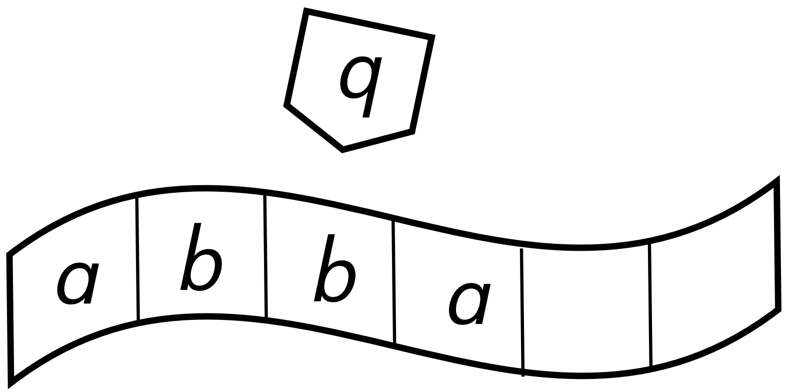
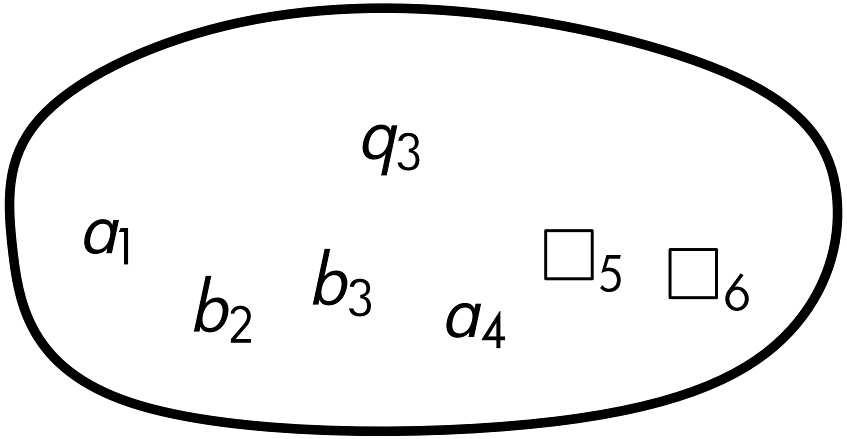




$$\delta(q, b) = (q', a, +1)$$

Simulating Turing machines efficiently





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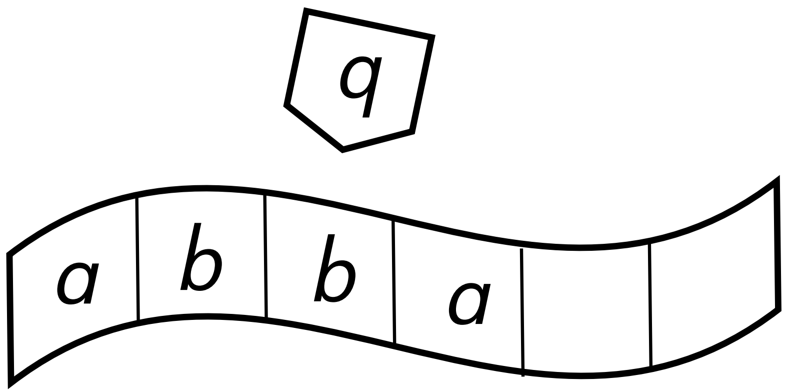
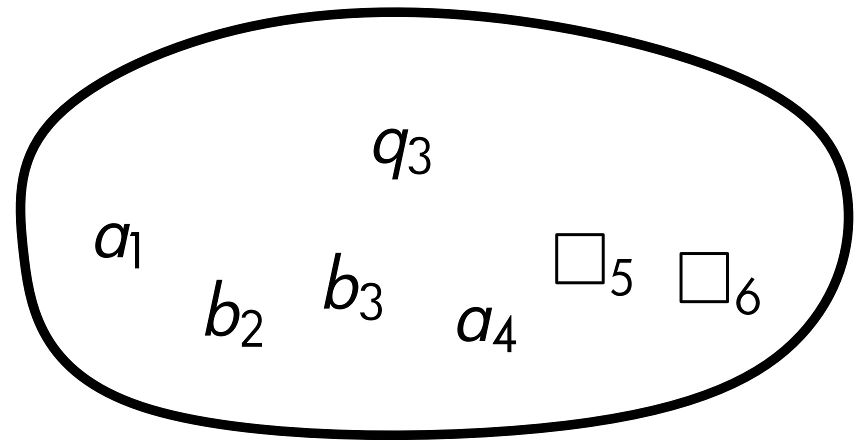
$$q_i\ b_i \rightarrow q'_{i+1}\ a_i \qquad 1 \leq i \leq n$$

Simulating Turing machines efficiently





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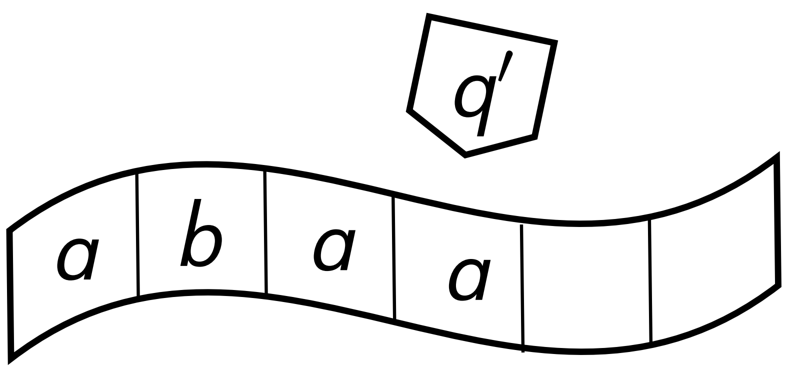
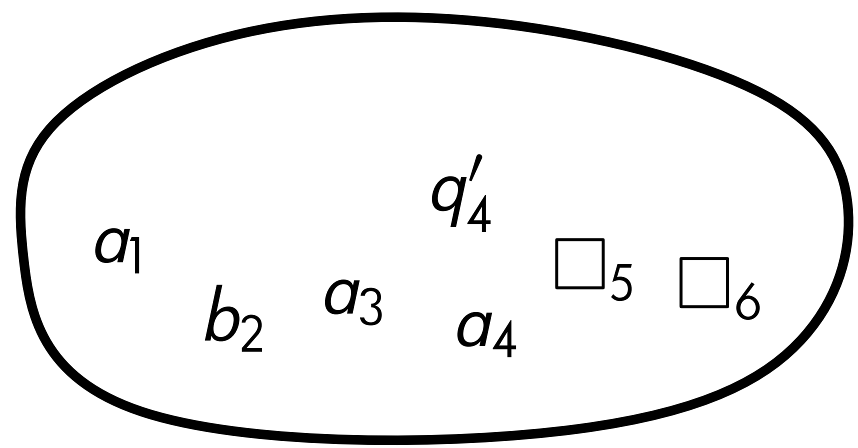
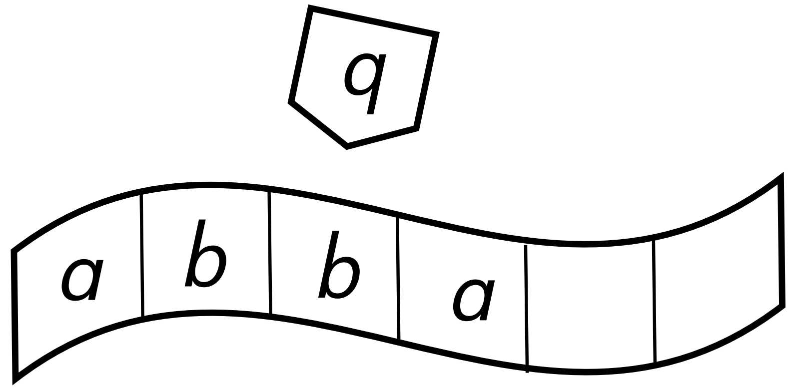


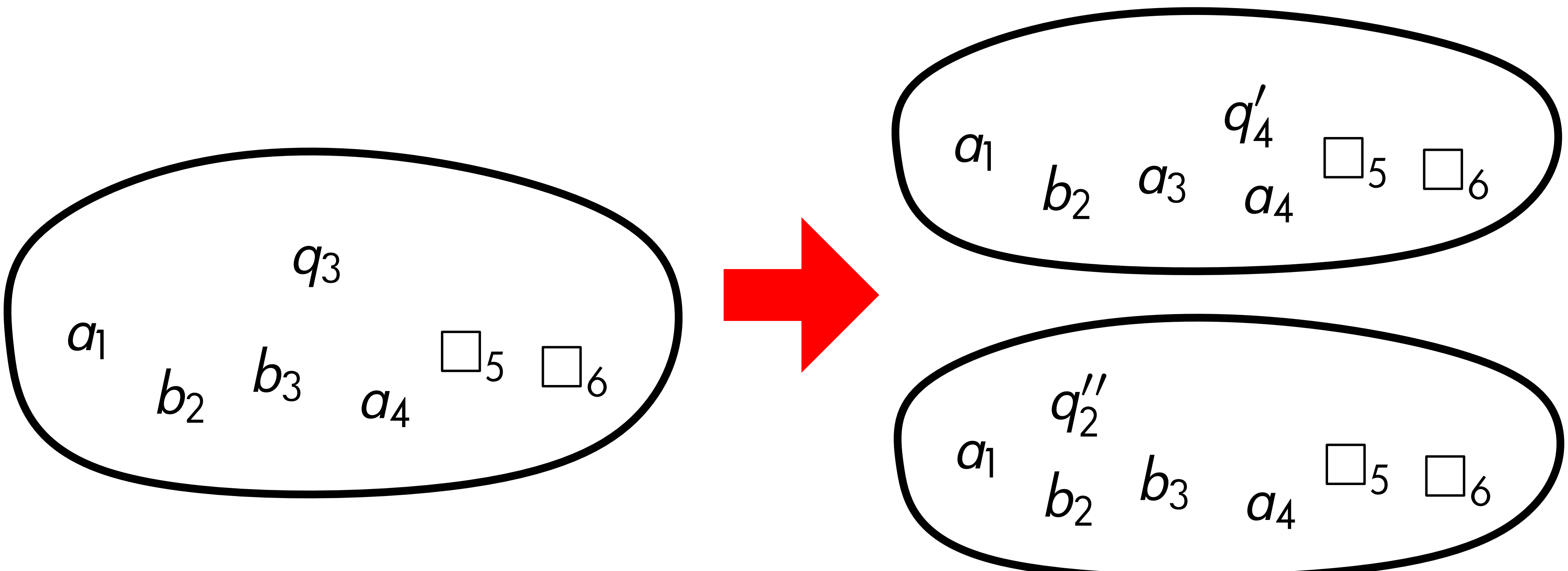


Simulating nondeterministic Turing machines



$$\delta(q, b) = \begin{cases} (q', a, +1) \\ (q'', b, -1) \end{cases}$$

$$[q_i \; b_i] \to [q'_{i+1} \; a_i] \; [q''_{i-1} \; b_i]$$



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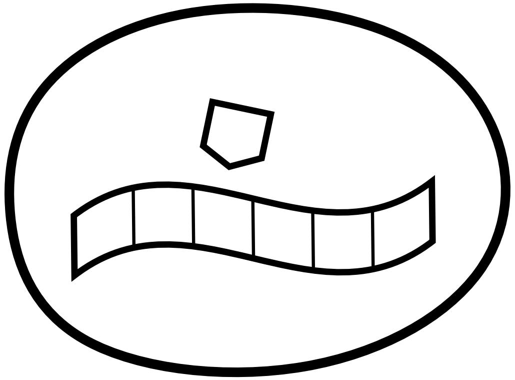

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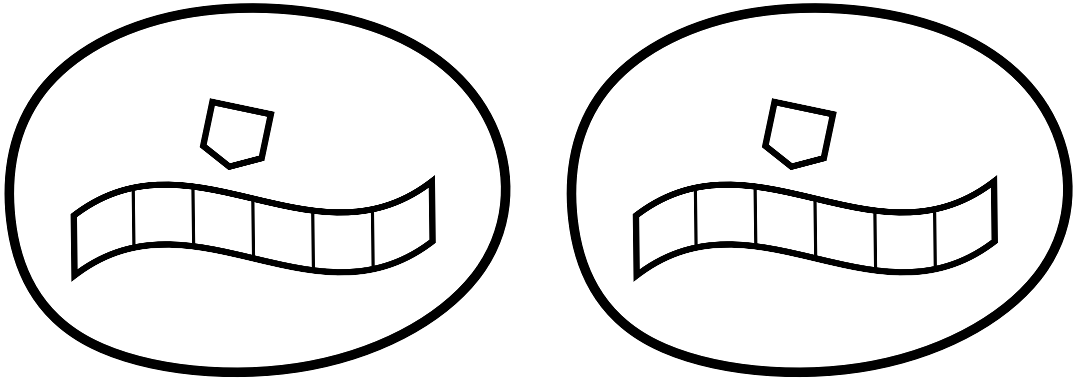

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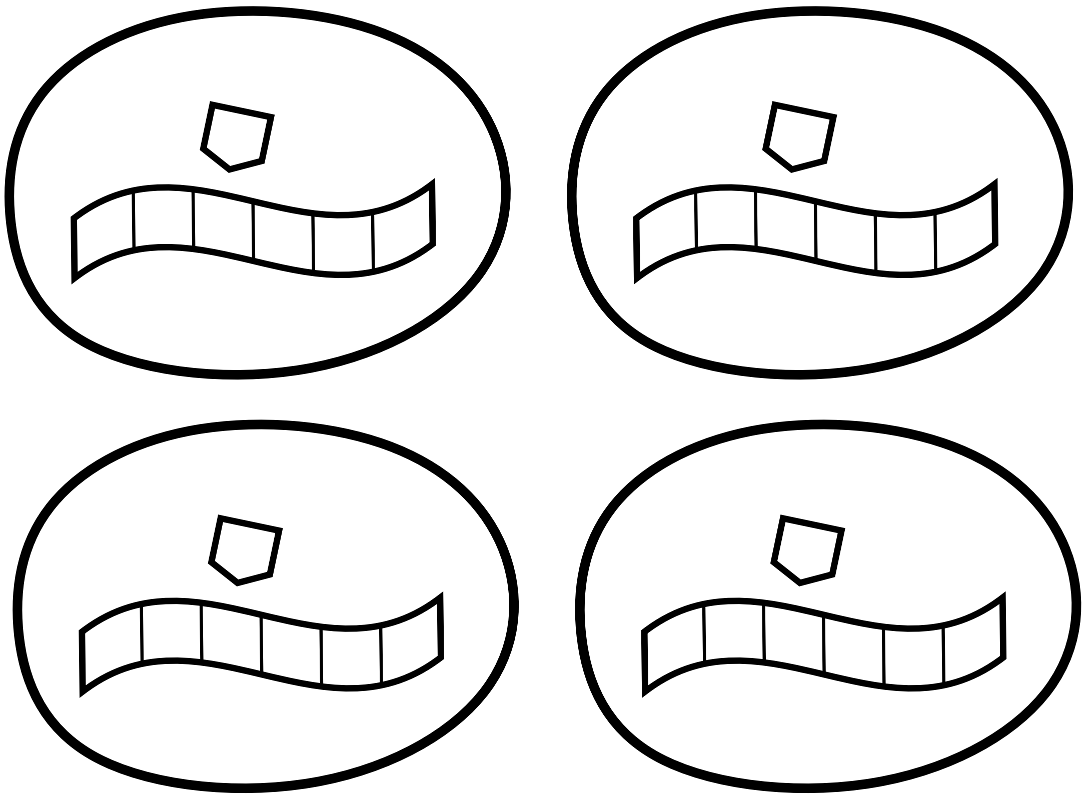

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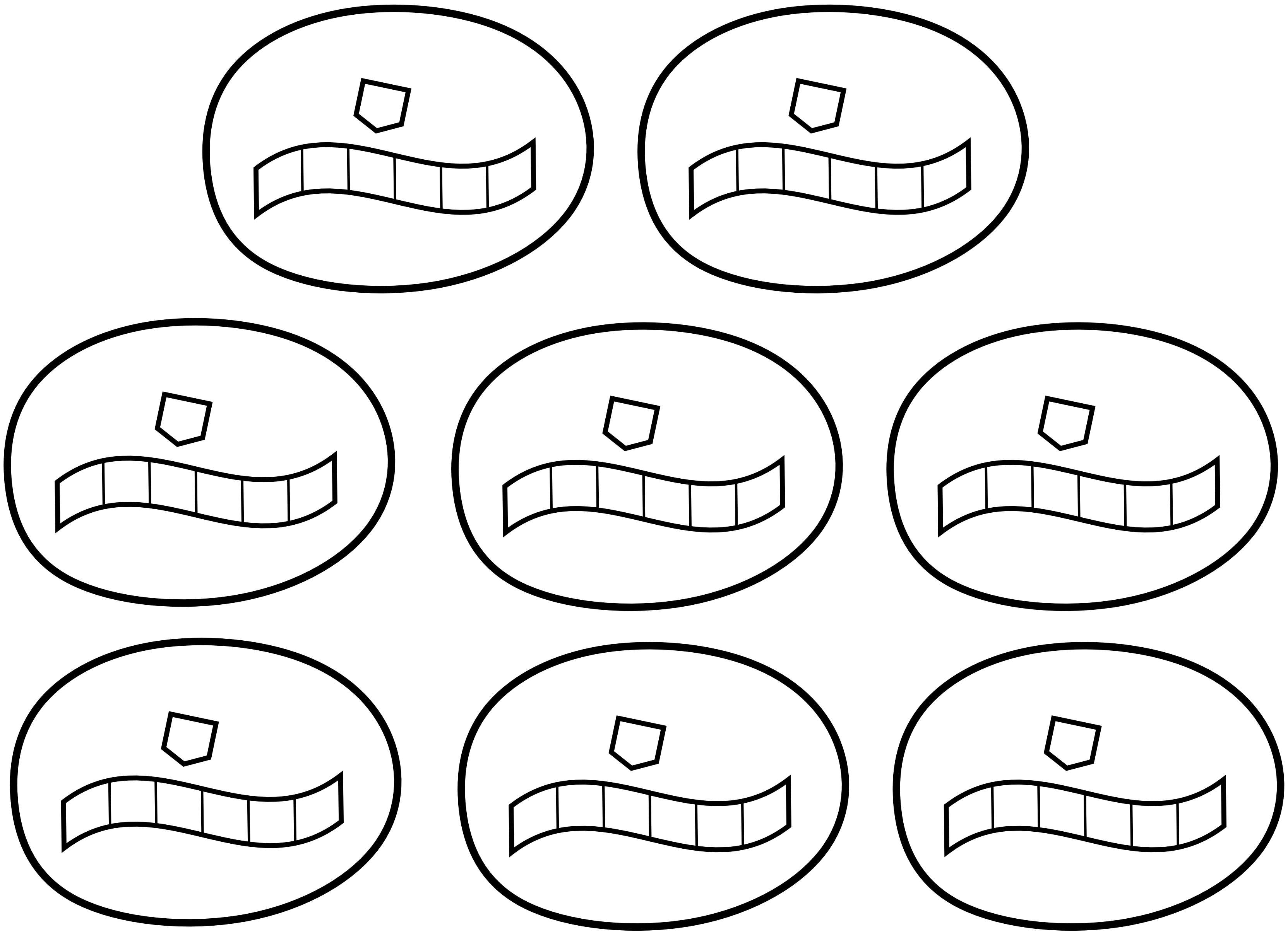

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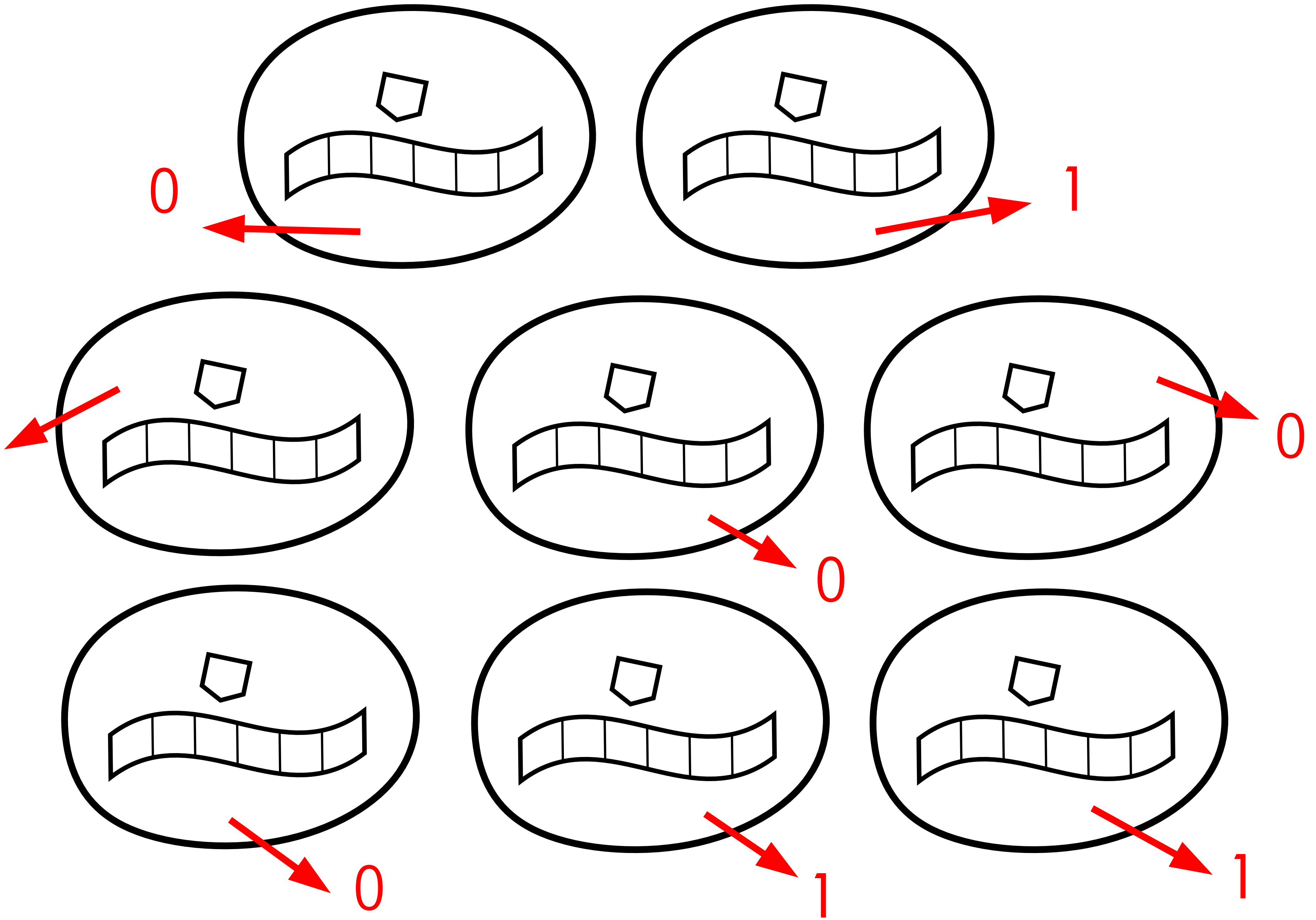
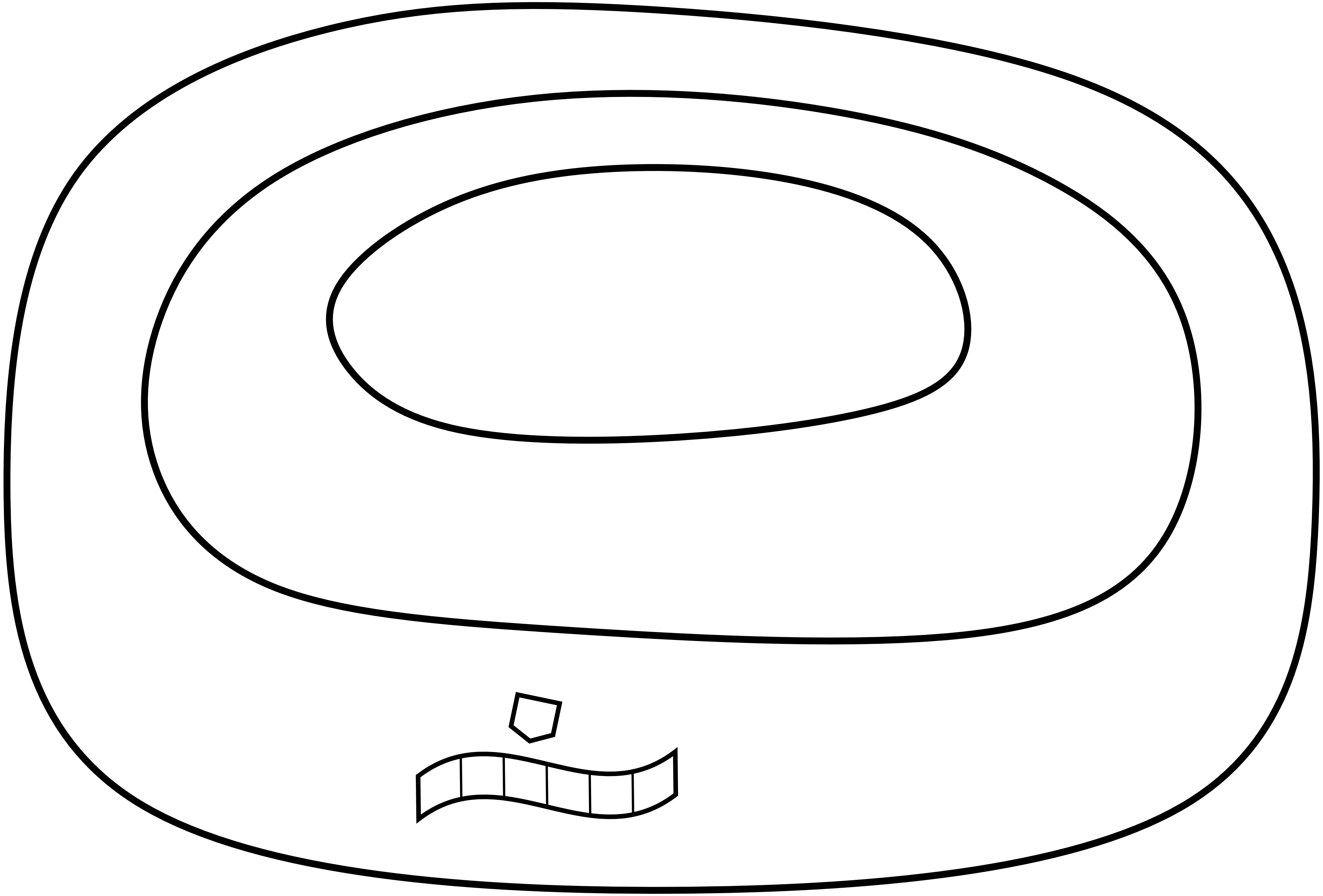


Simulating alternating Turing machines



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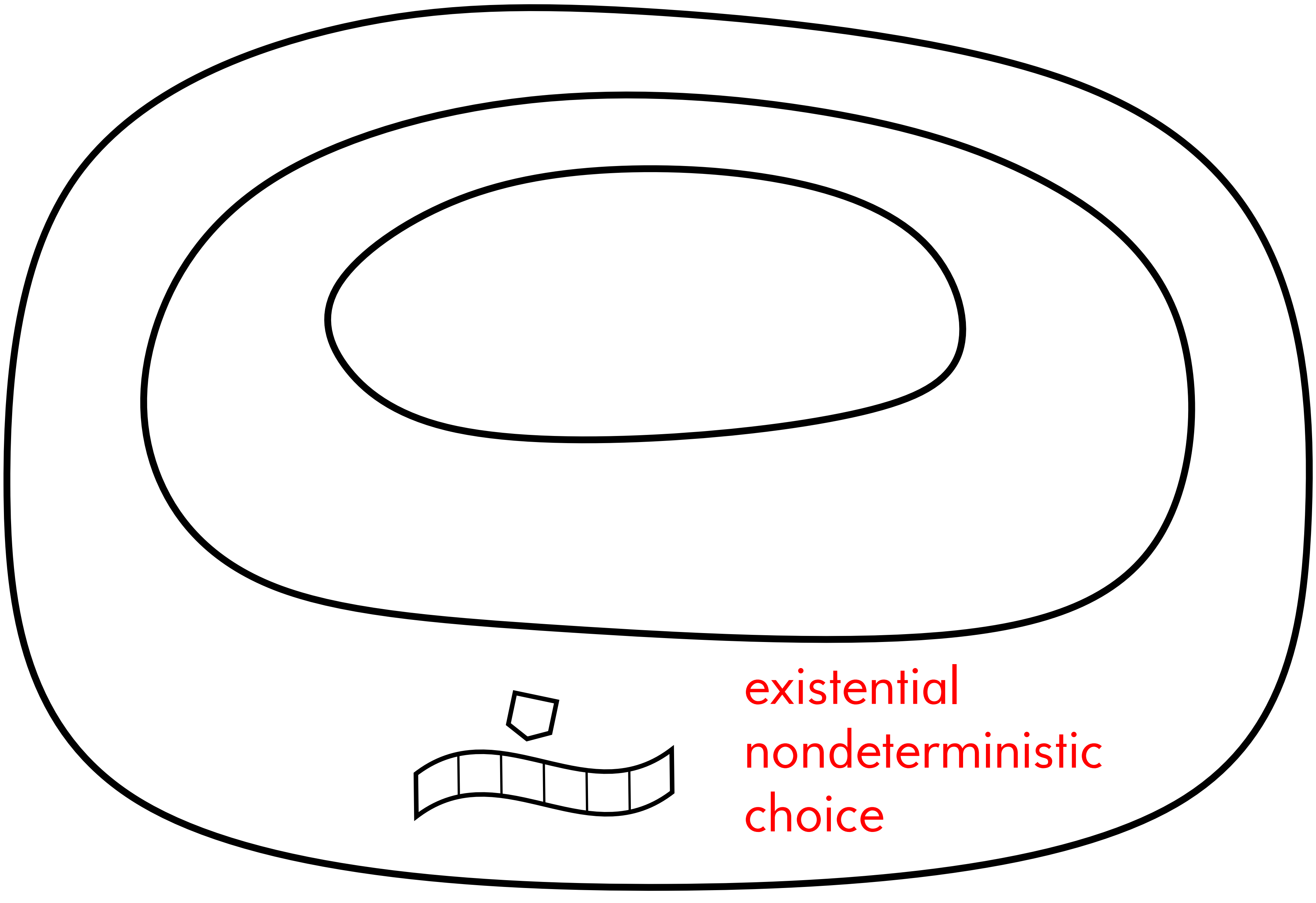

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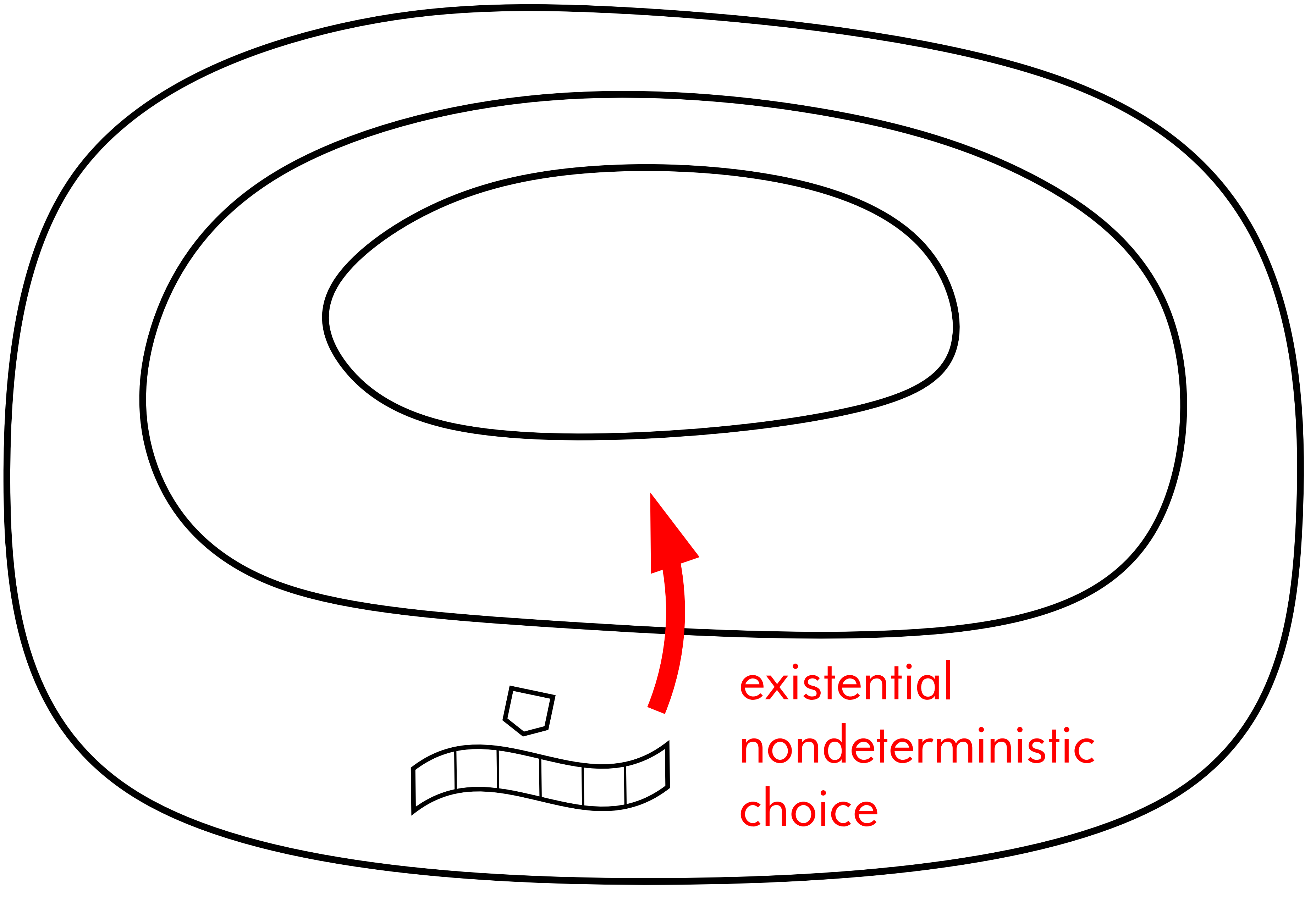

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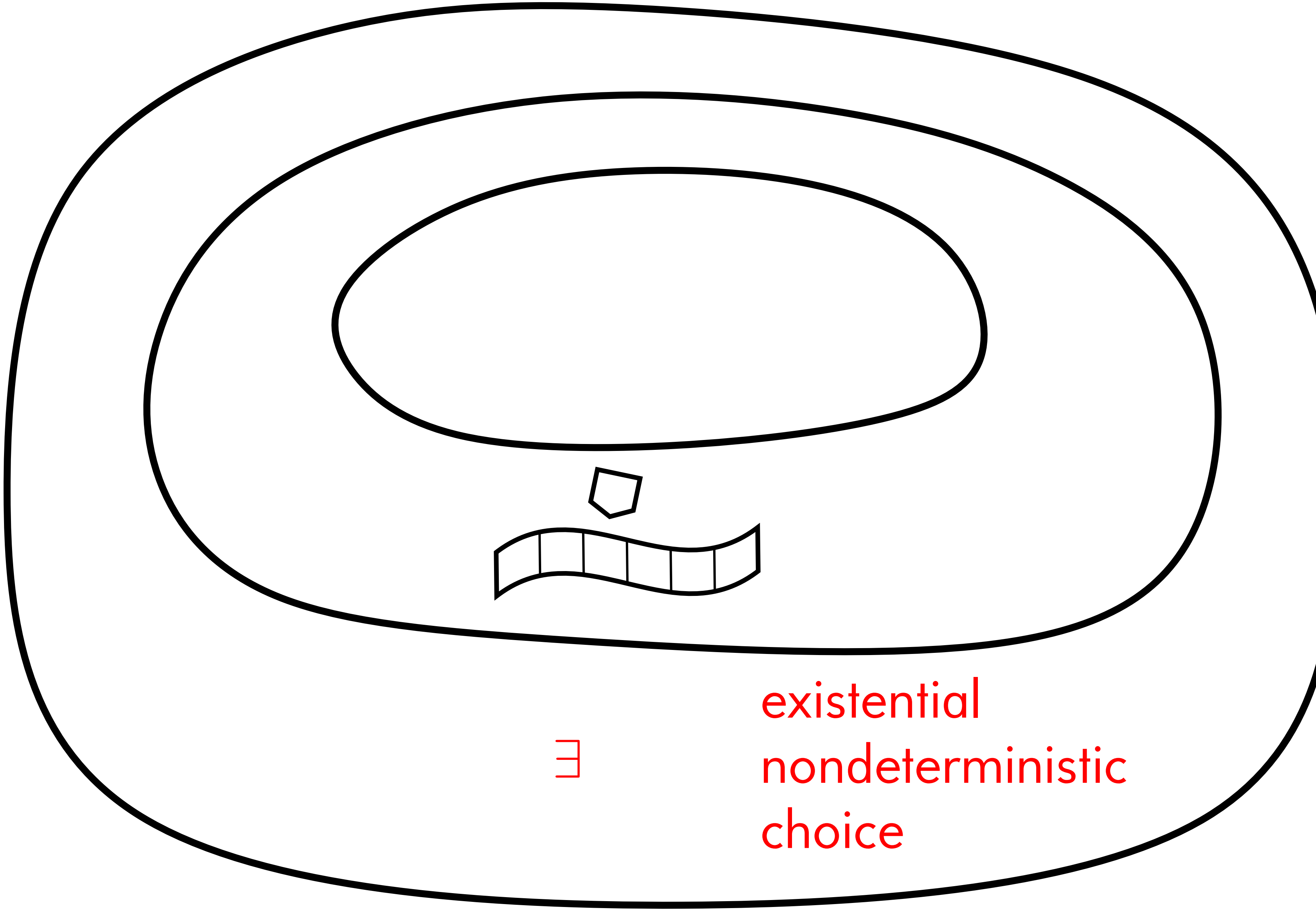

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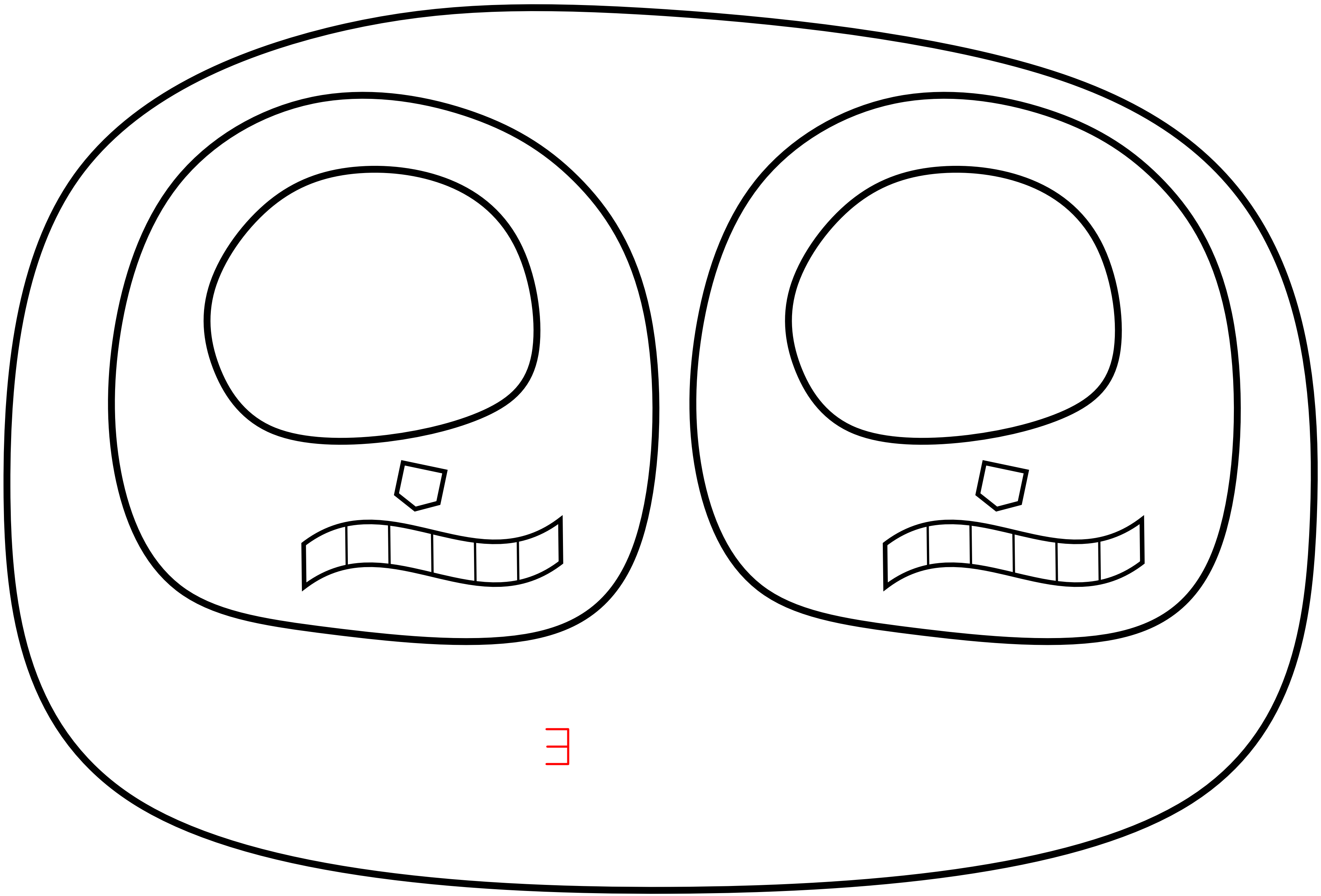

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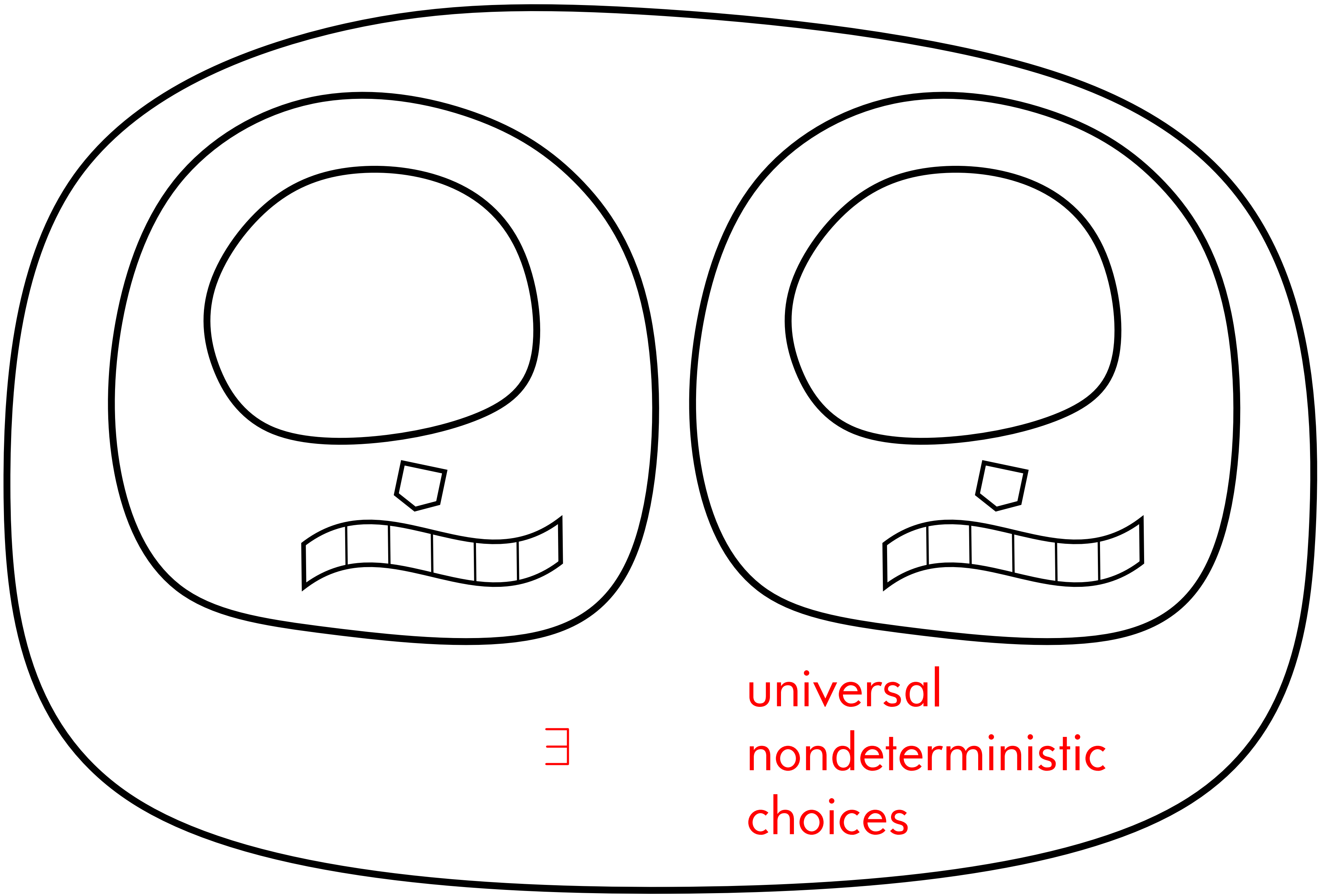

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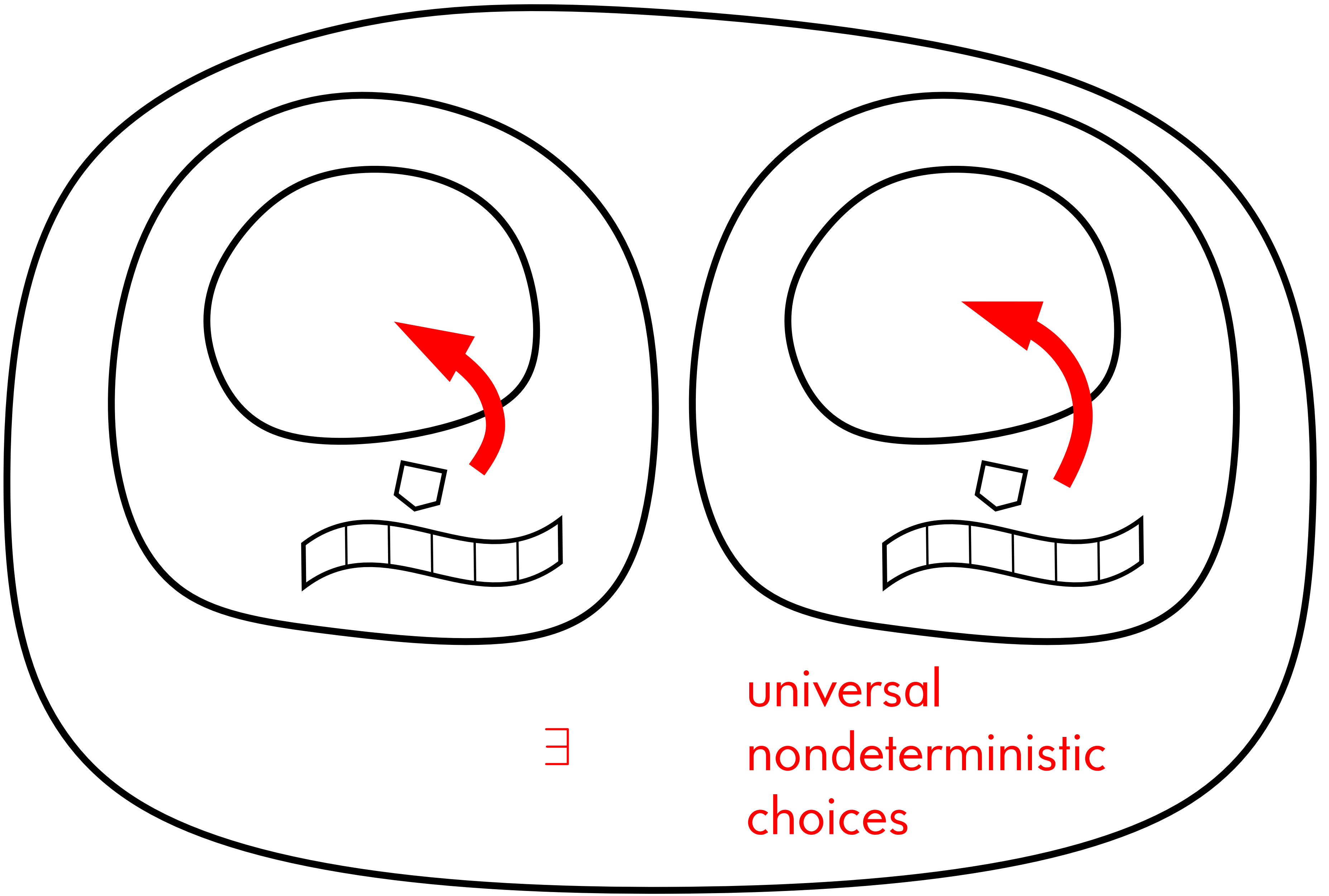

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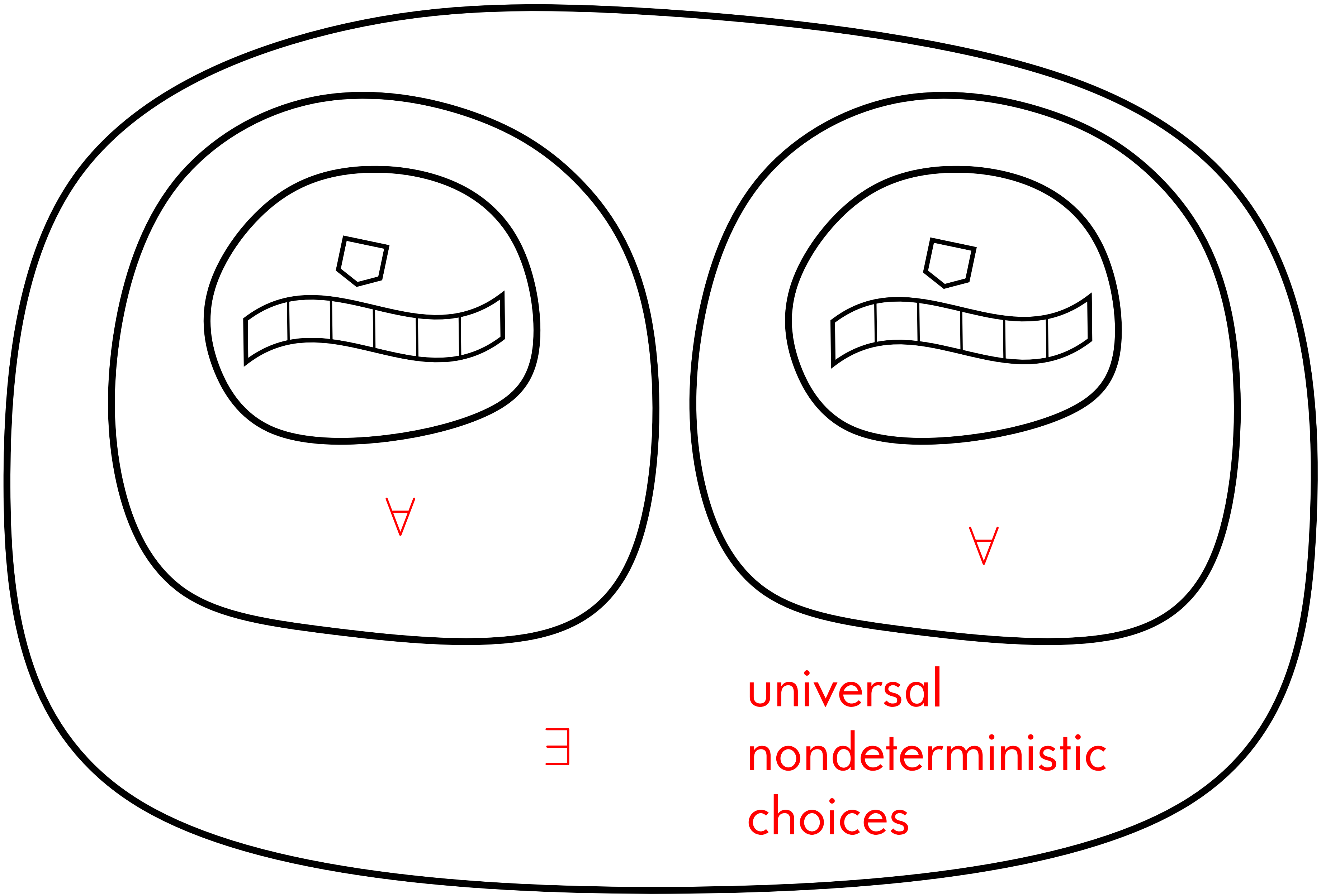

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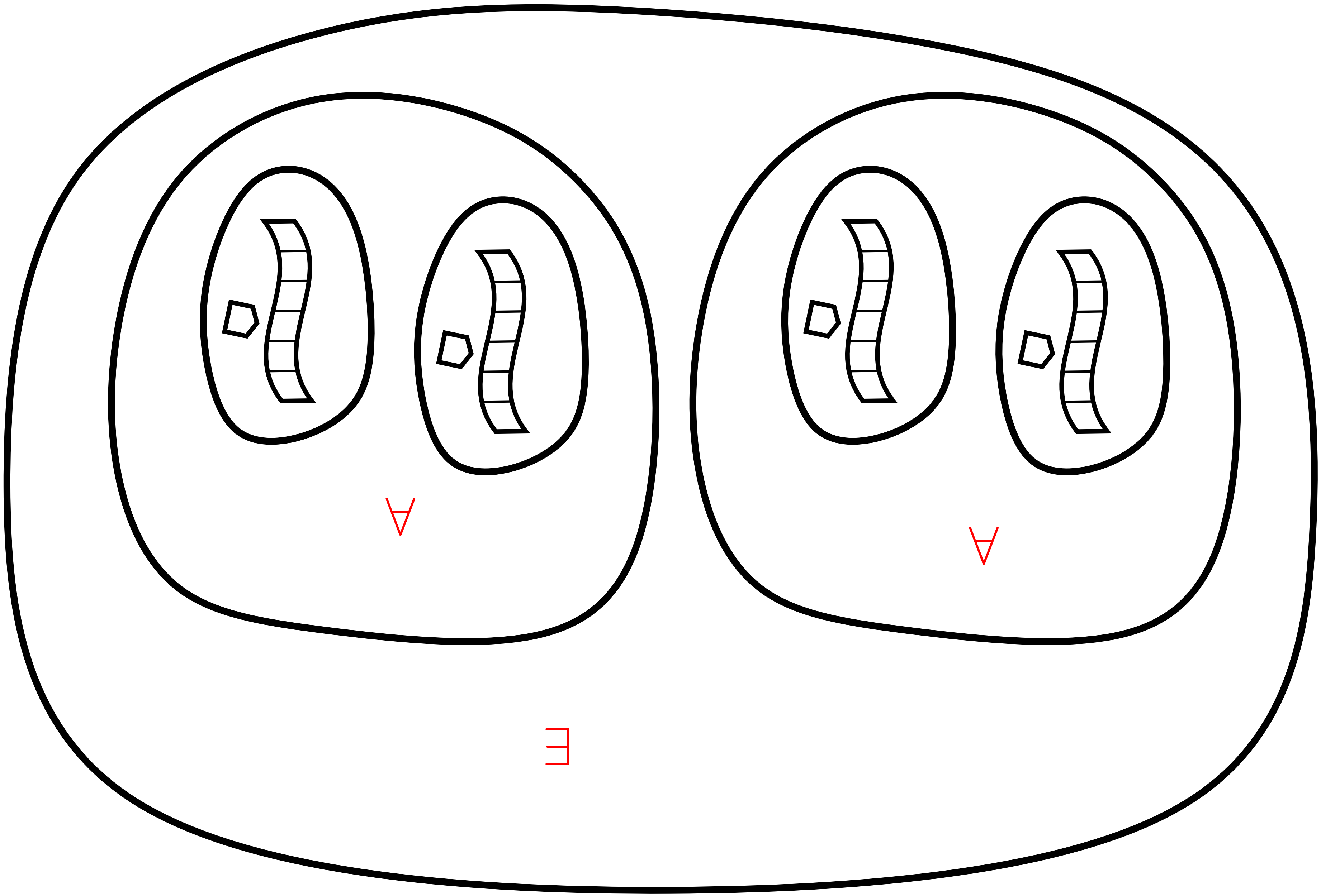

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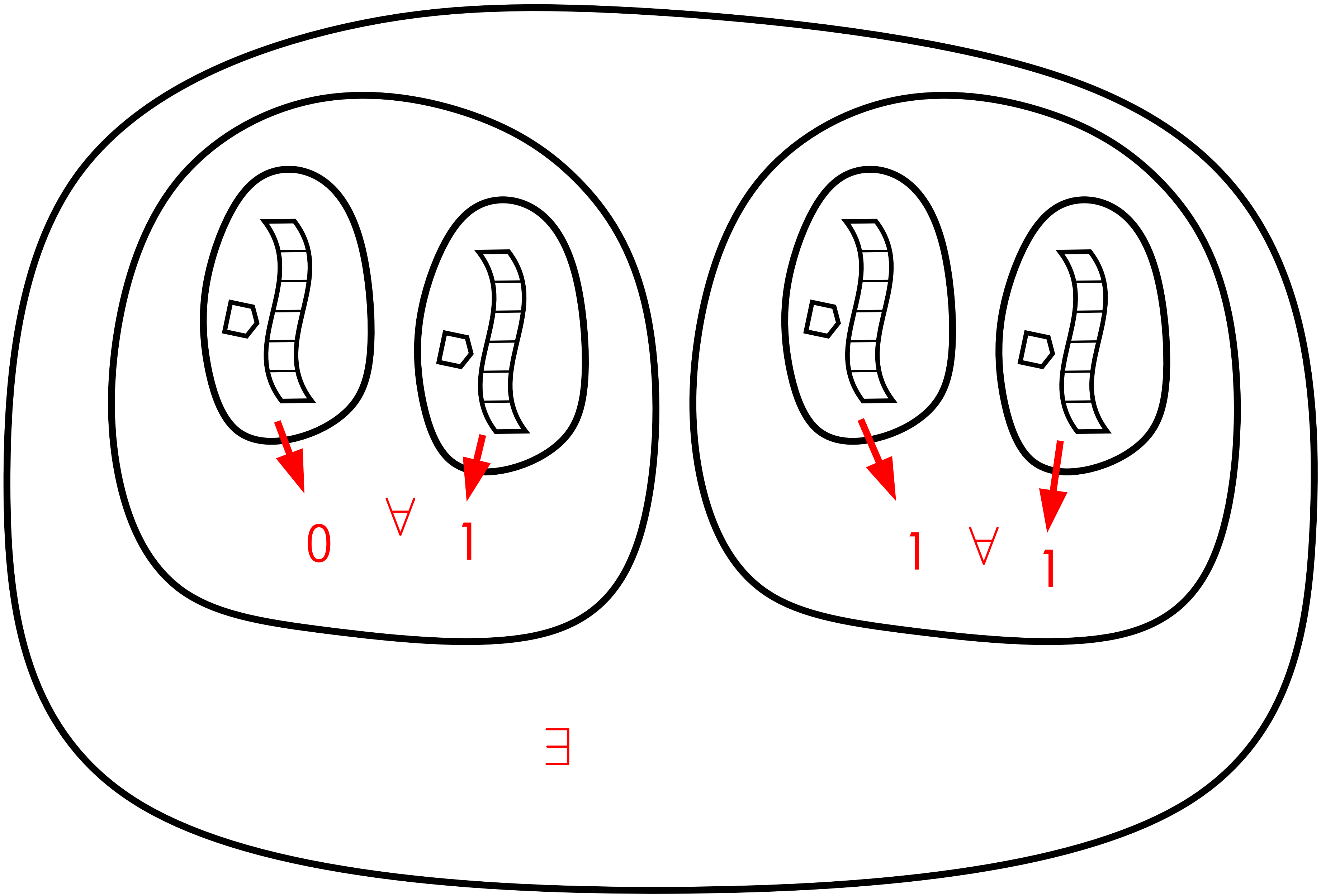

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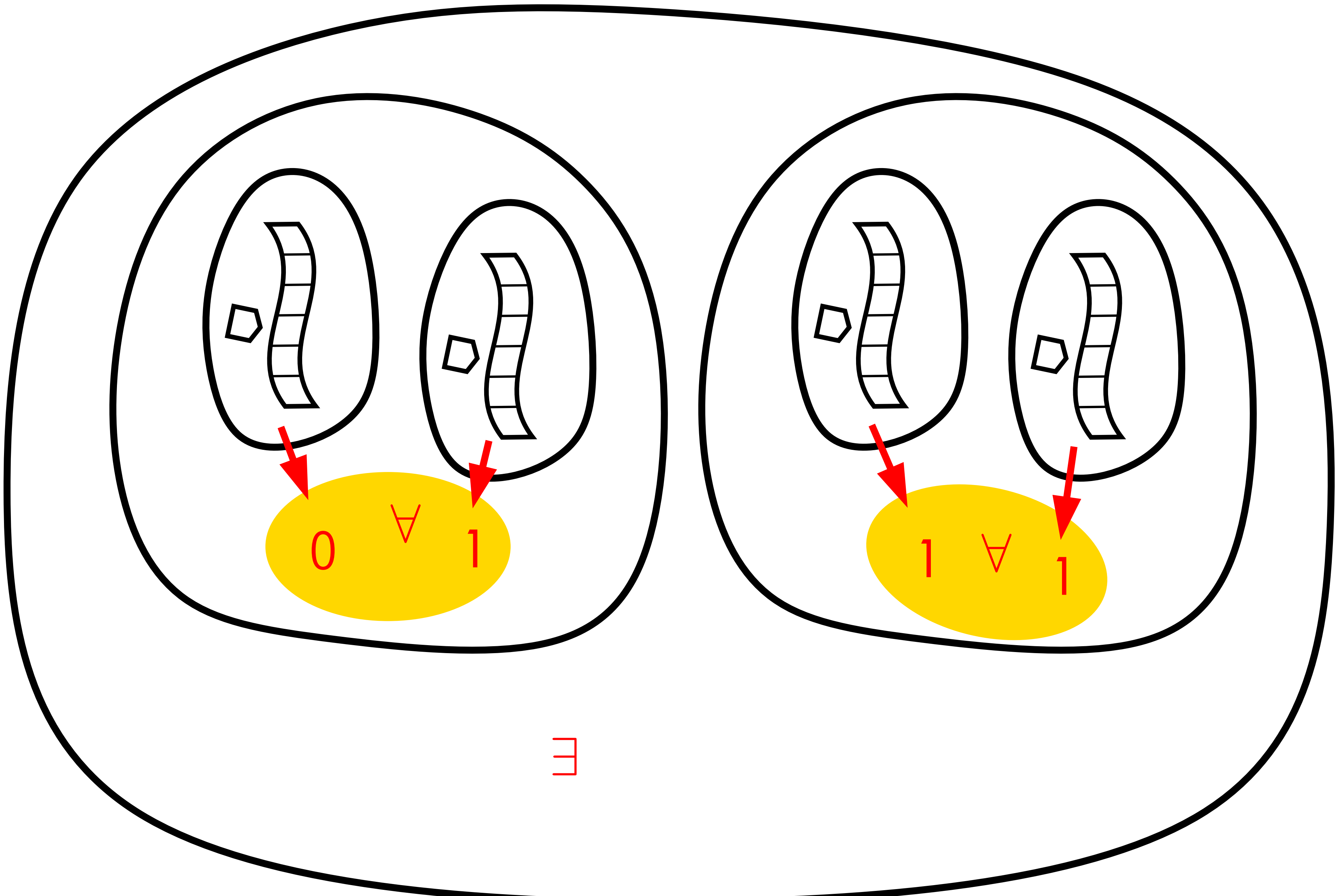

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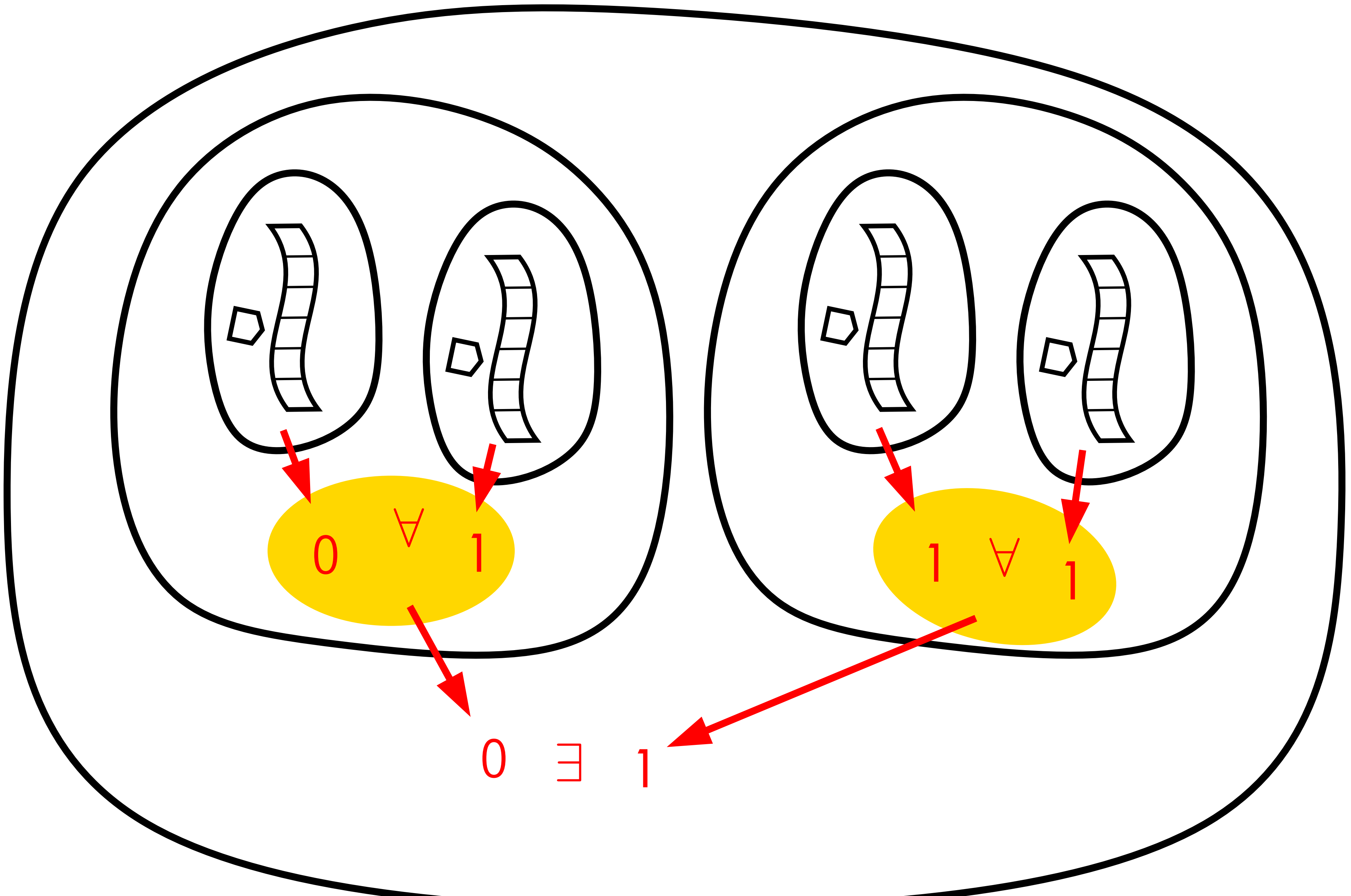

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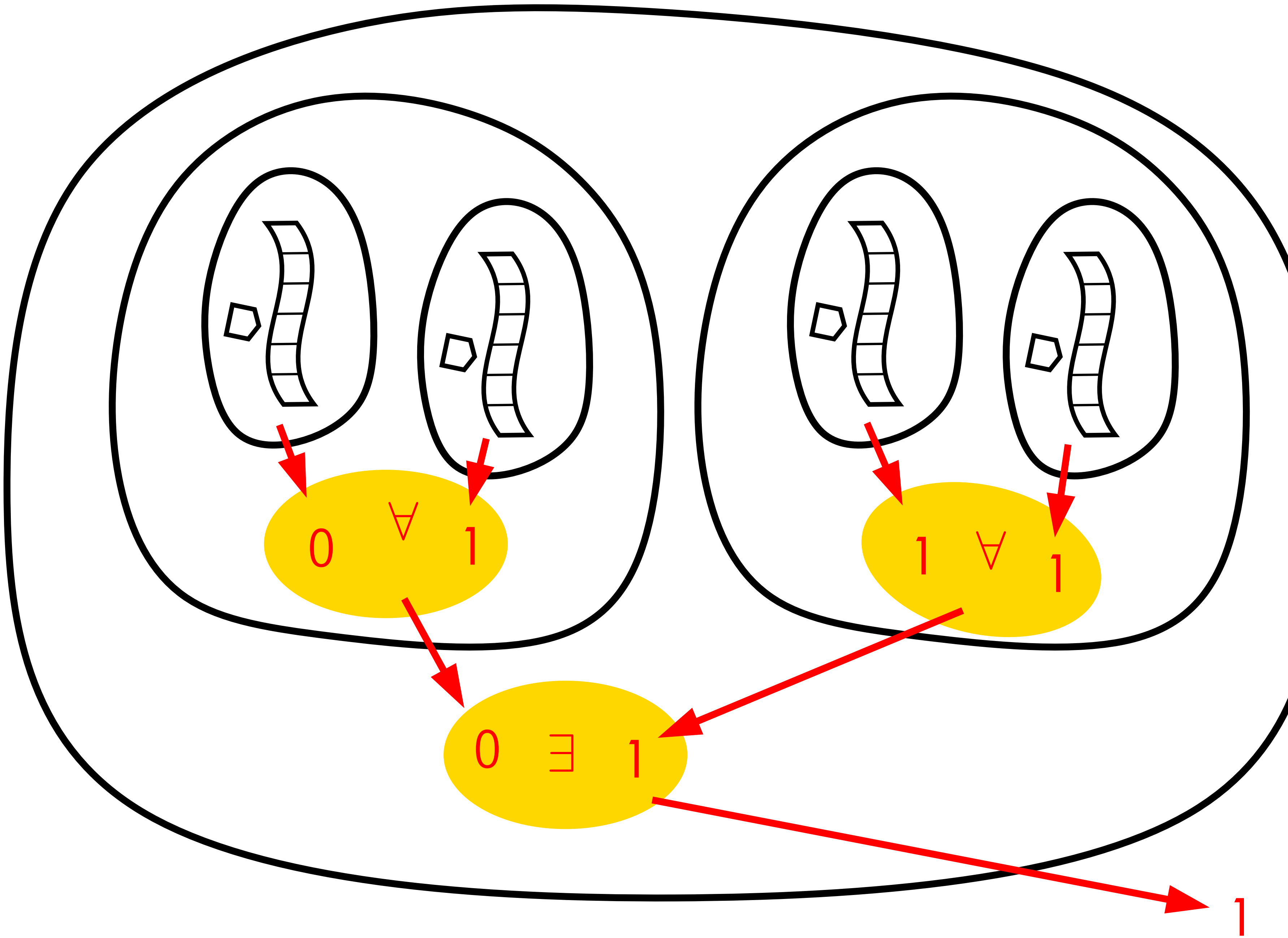
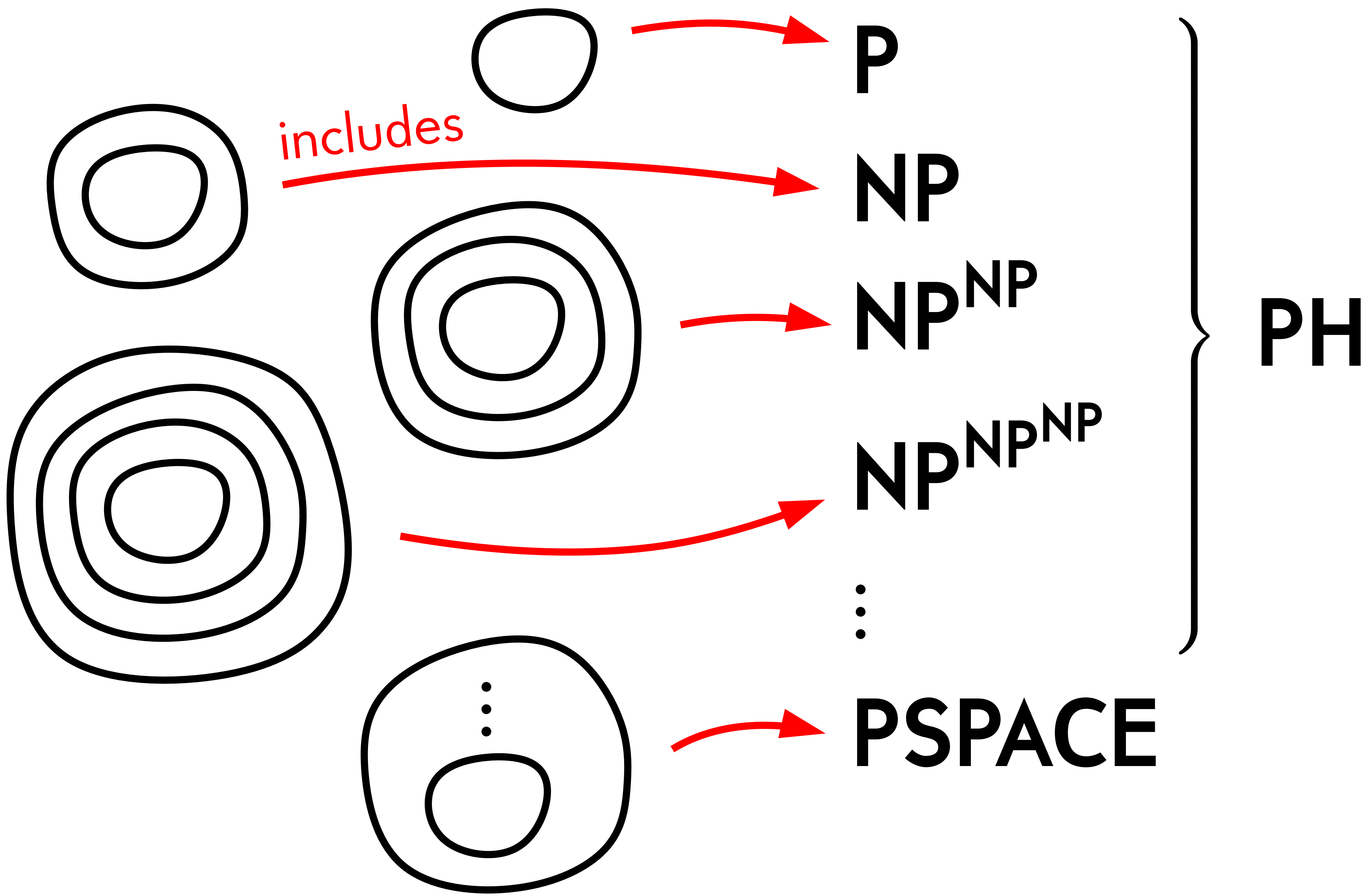
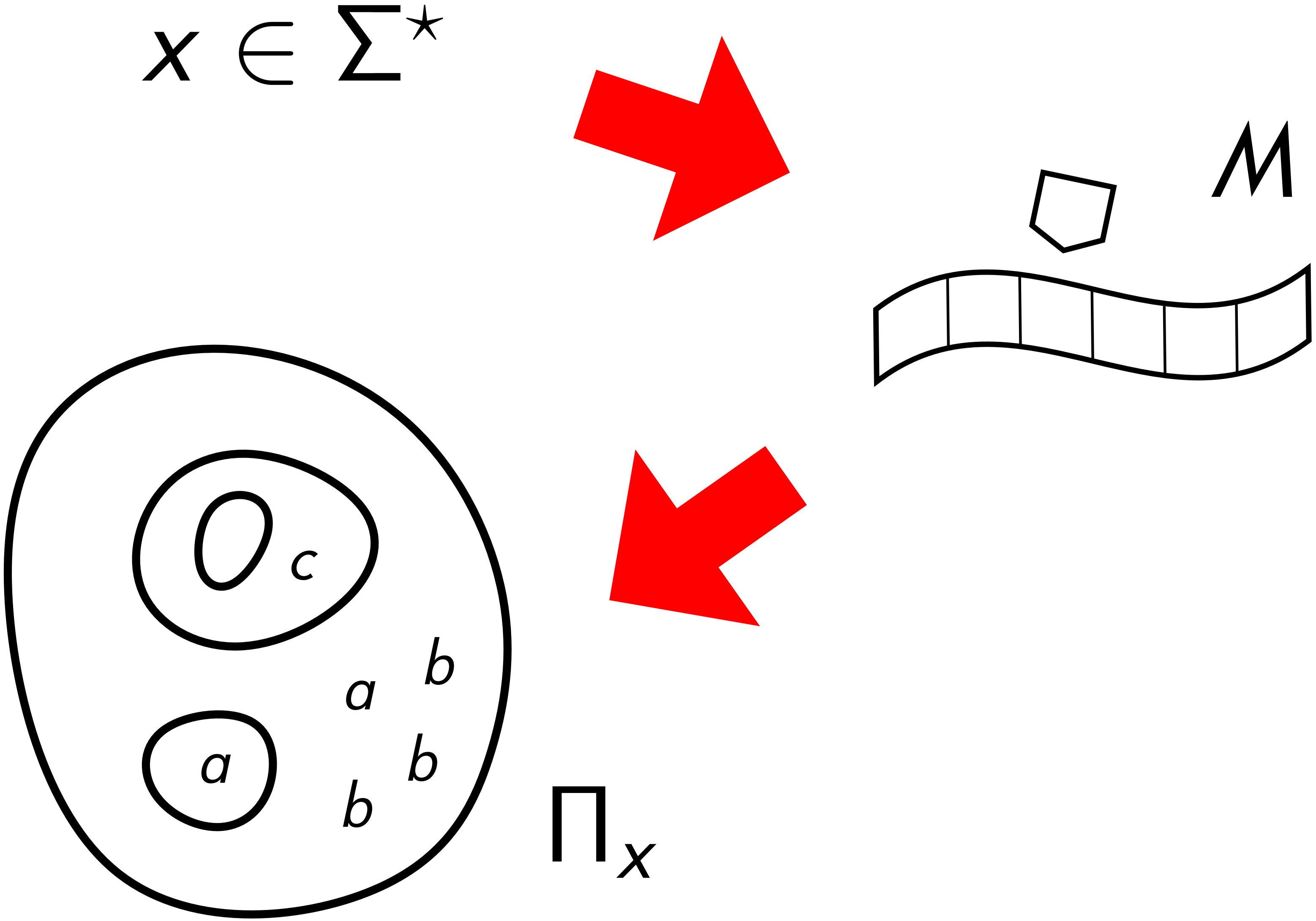


The polynomial hierarchy



(Semi-)uniform families of membrane systems



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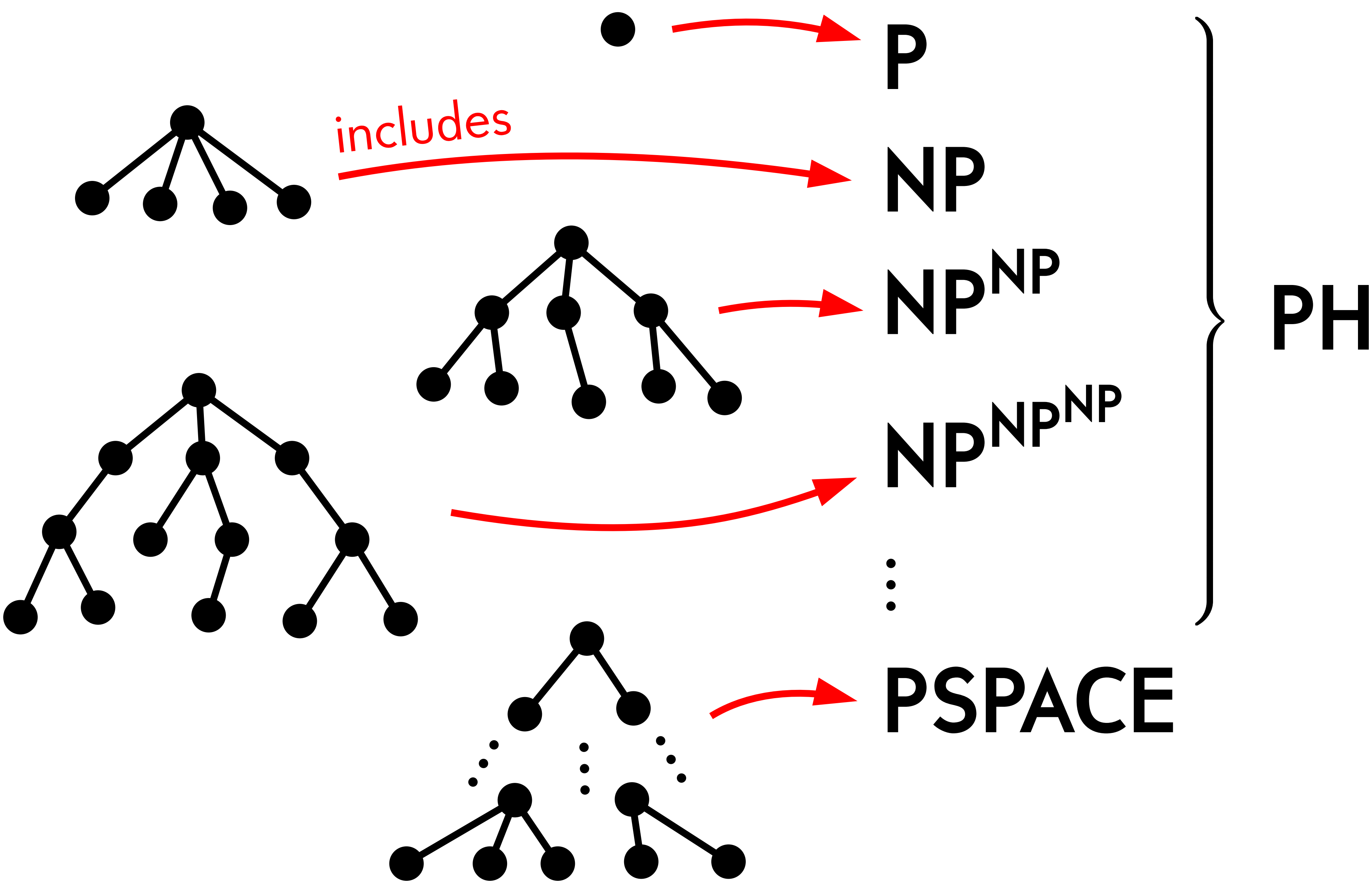
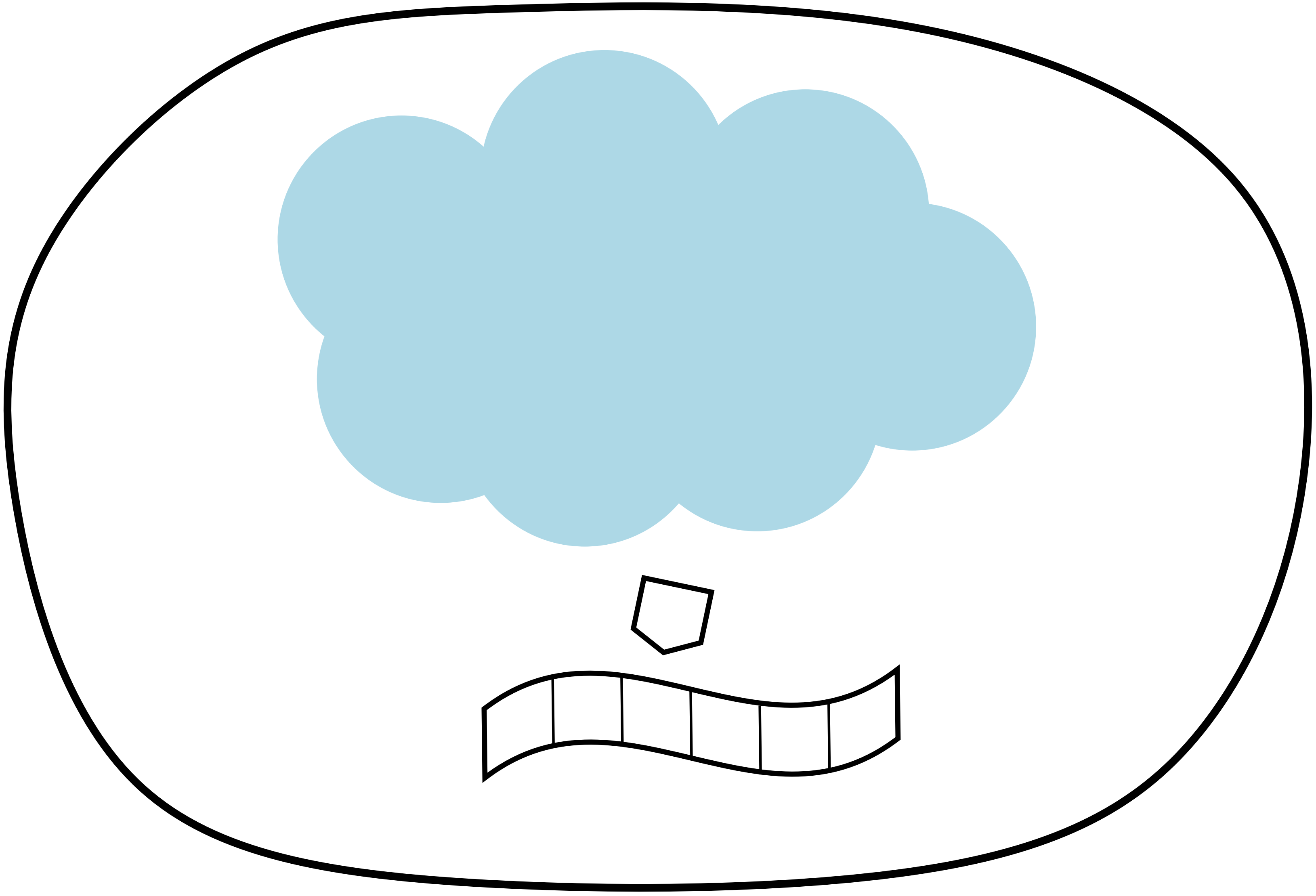
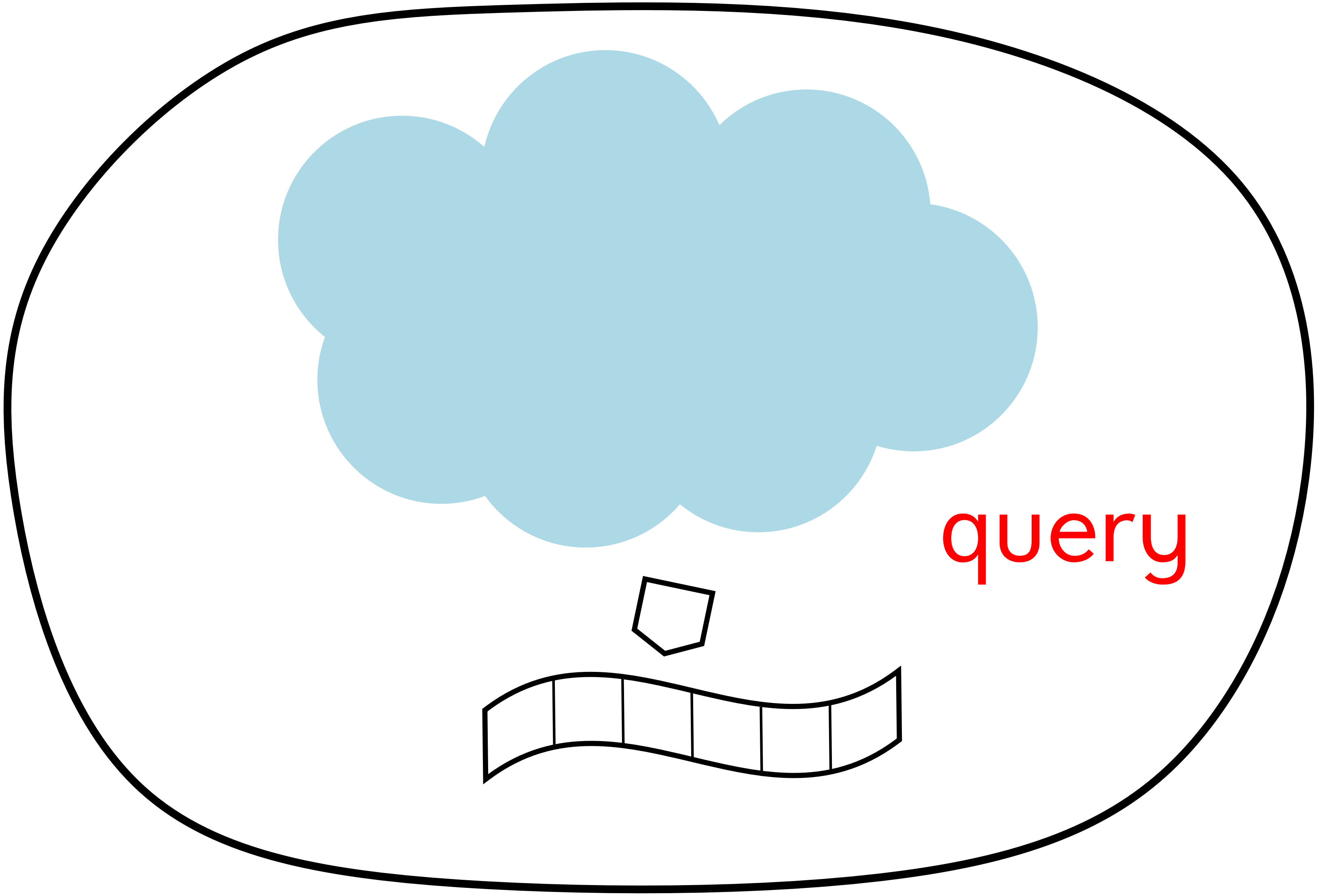


Simulating Turing machines with oracles



Simulating Turing machines with oracles



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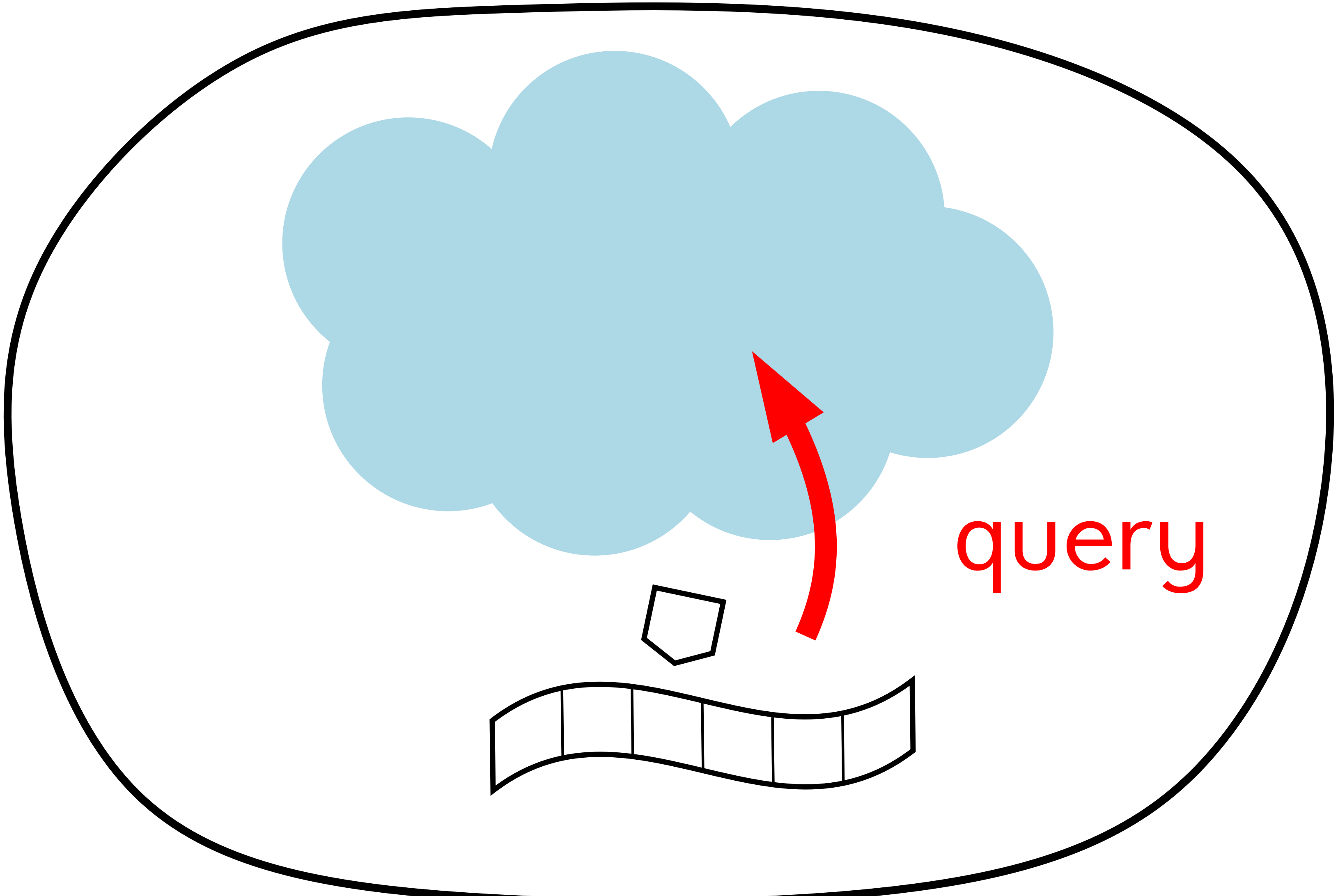

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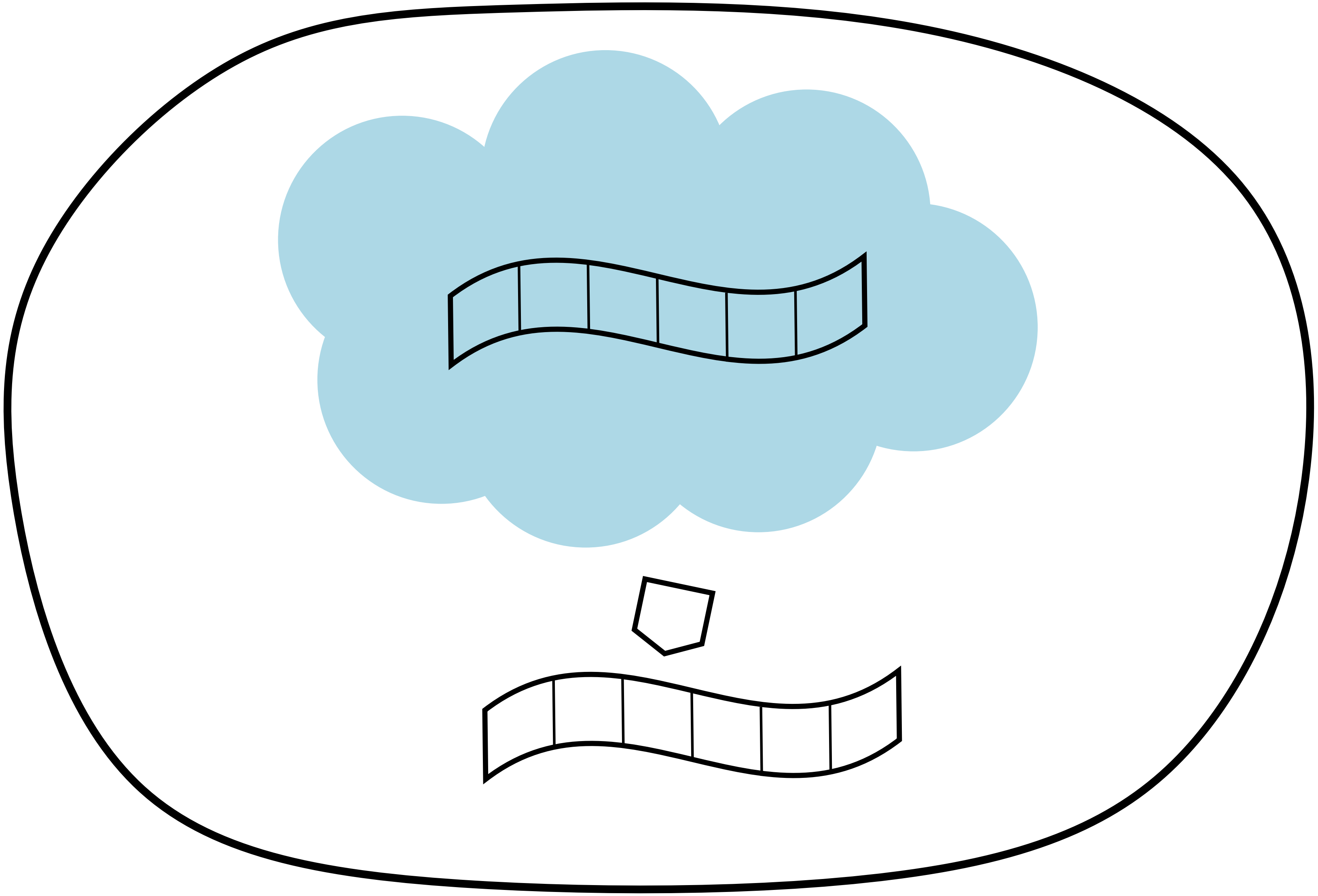

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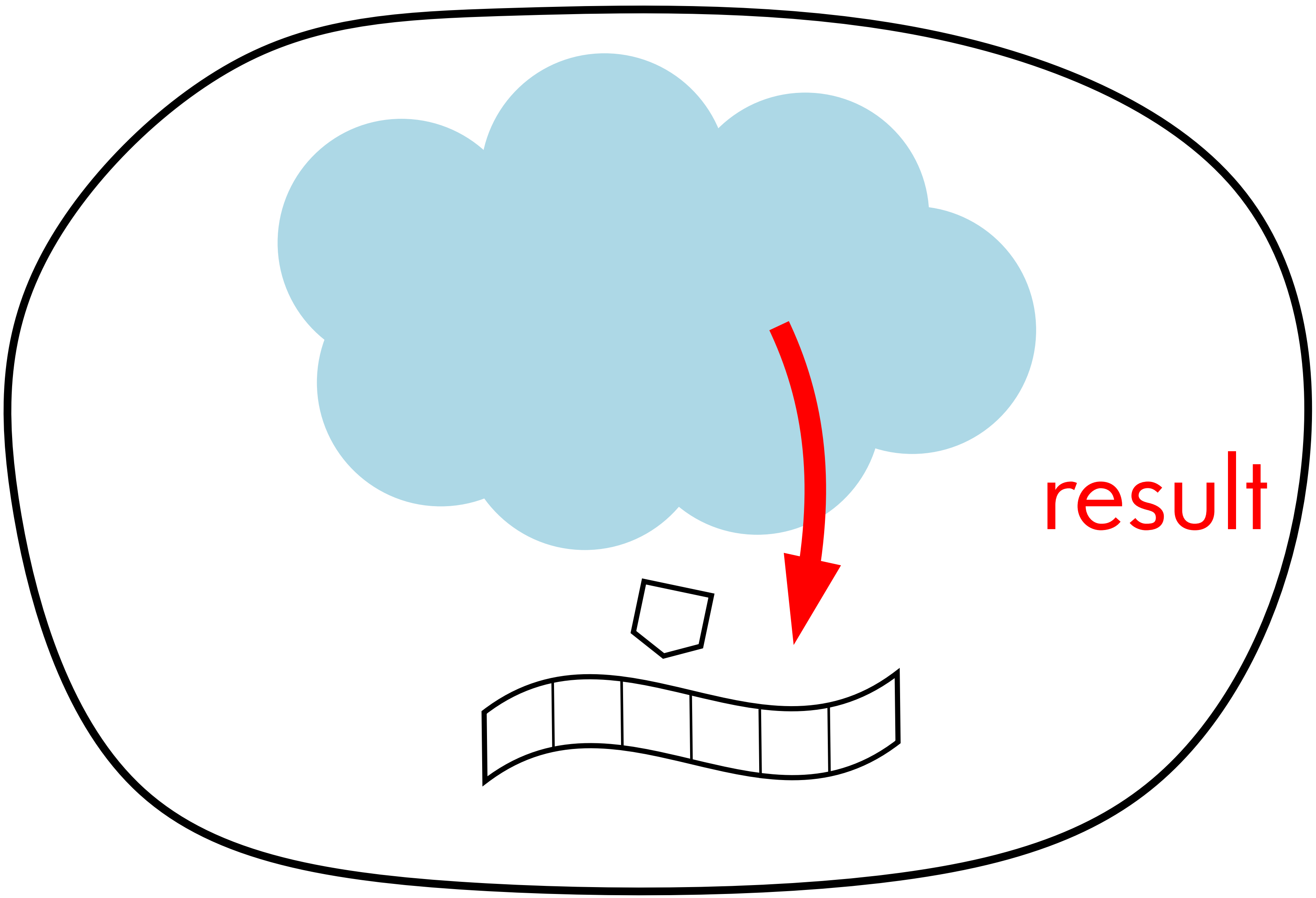

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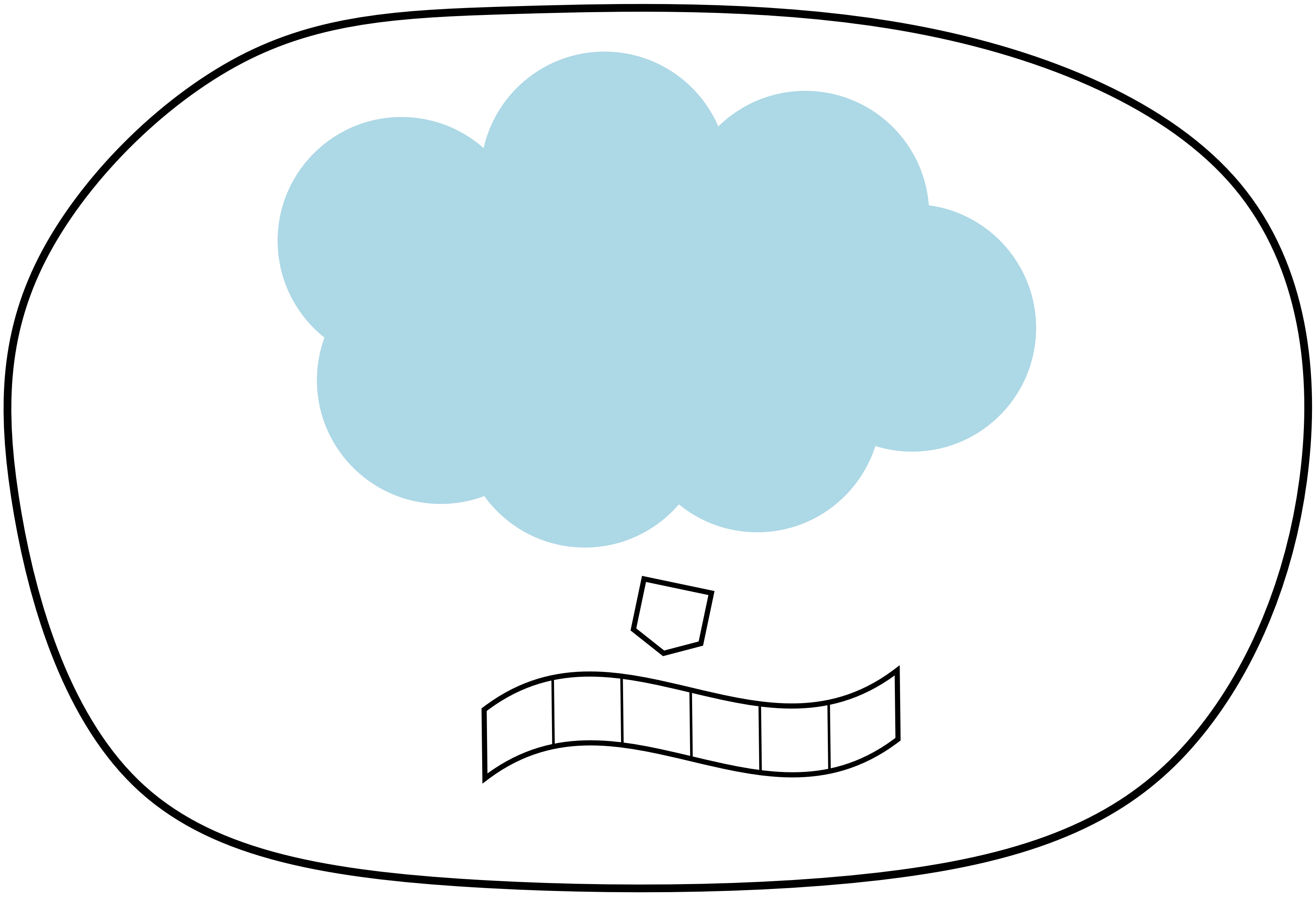
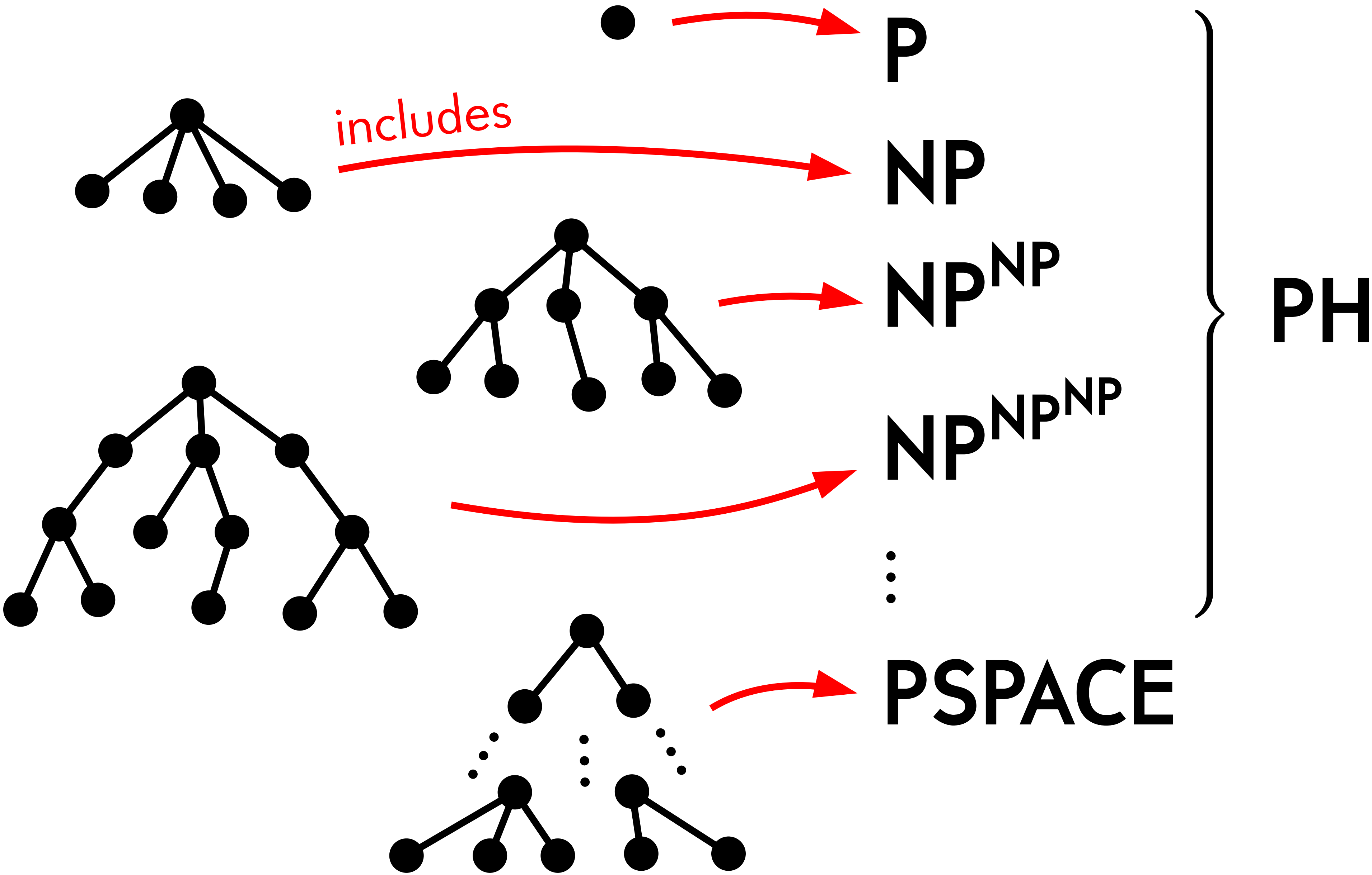


The polynomial hierarchy



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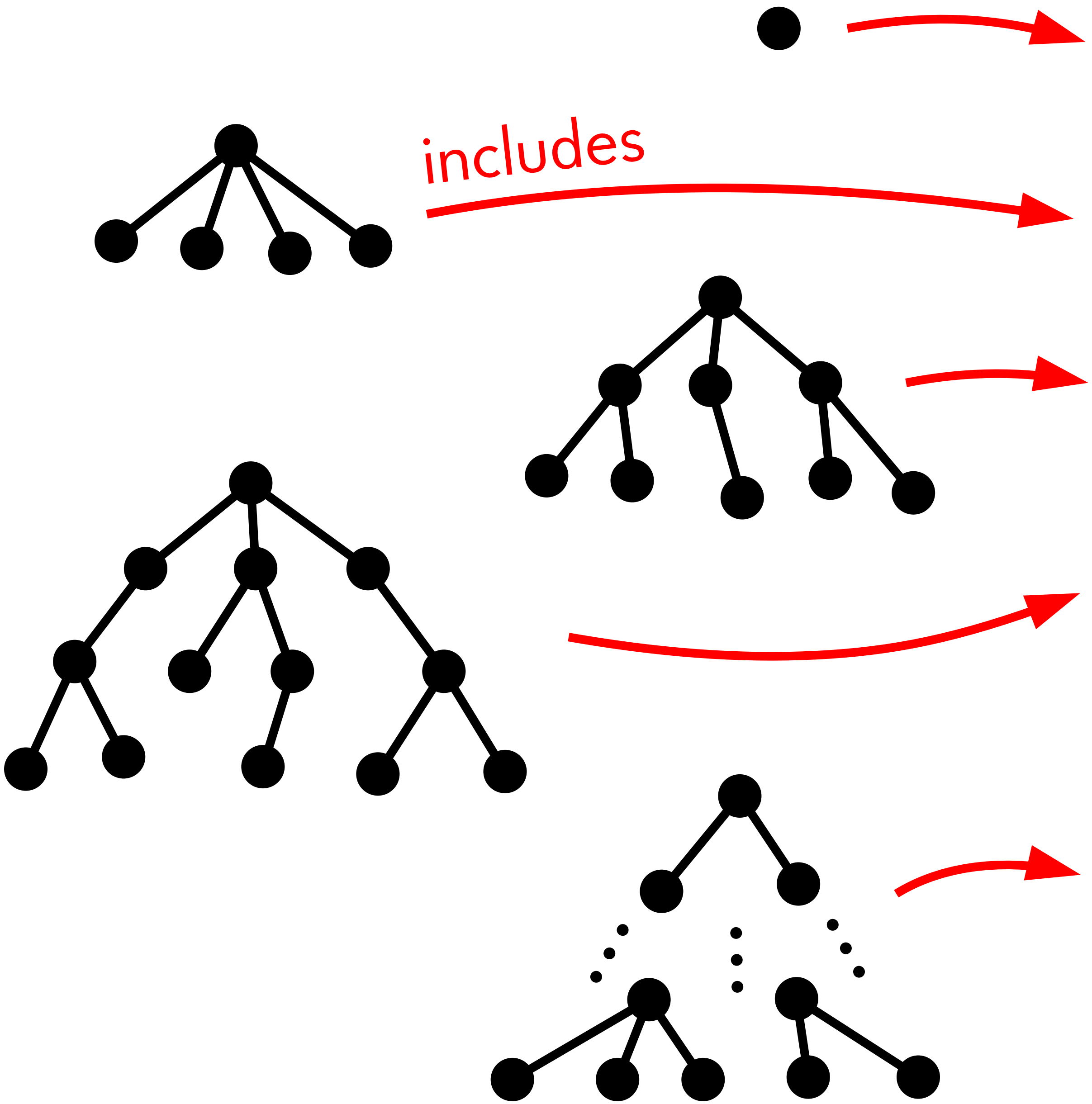


$$P^{P} = P$$

$$P^{NP} \supseteq NP$$

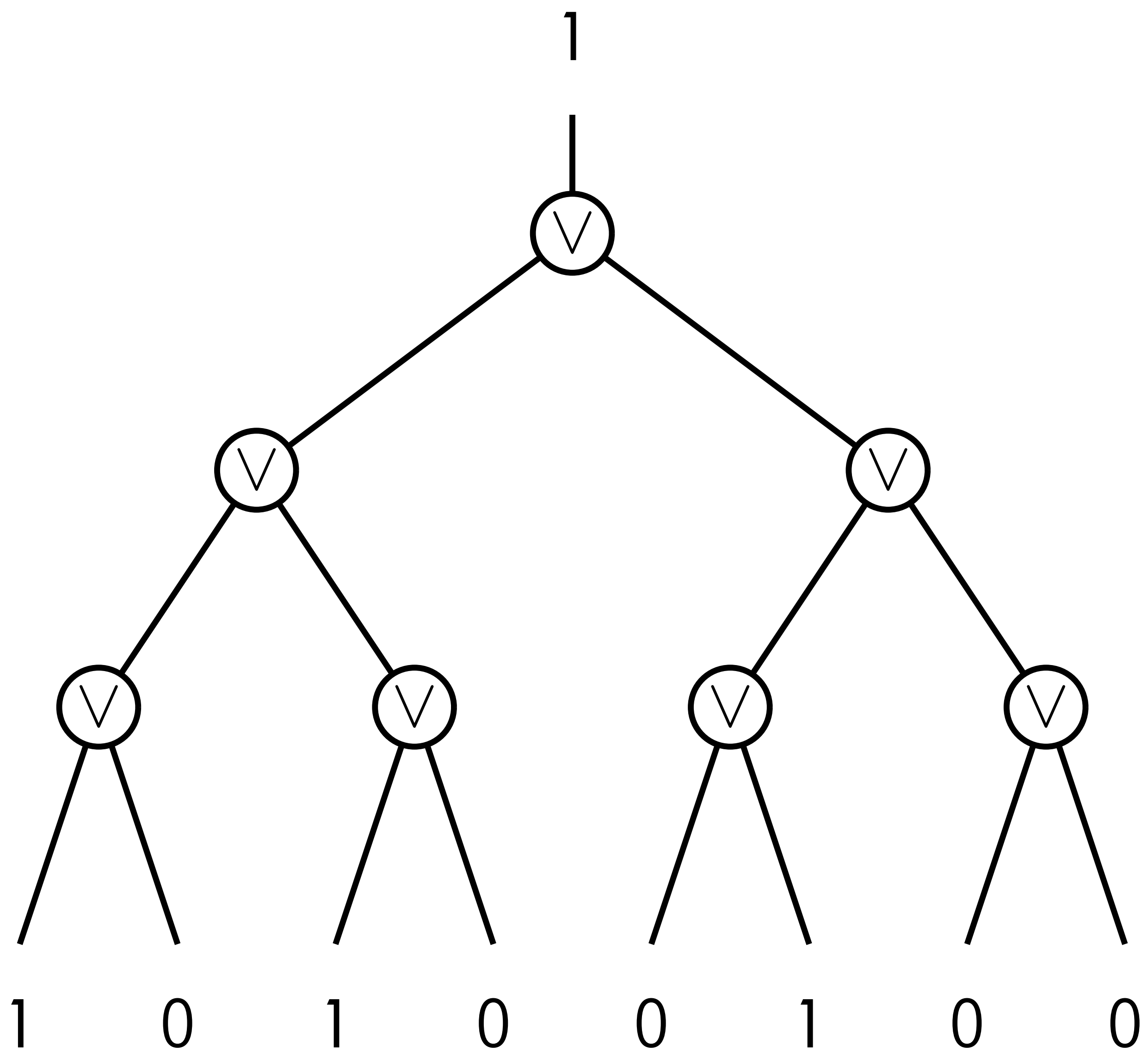
$$P^{NP^{NP}} \supseteq NP^{NP}$$

$$P^{NP^{NP^{NP}}} \supseteq NP^{NP^{NP}}$$

$$\vdots$$

$$P^{PSPACE} = PSPACE$$

Nondeterministic Turing machine



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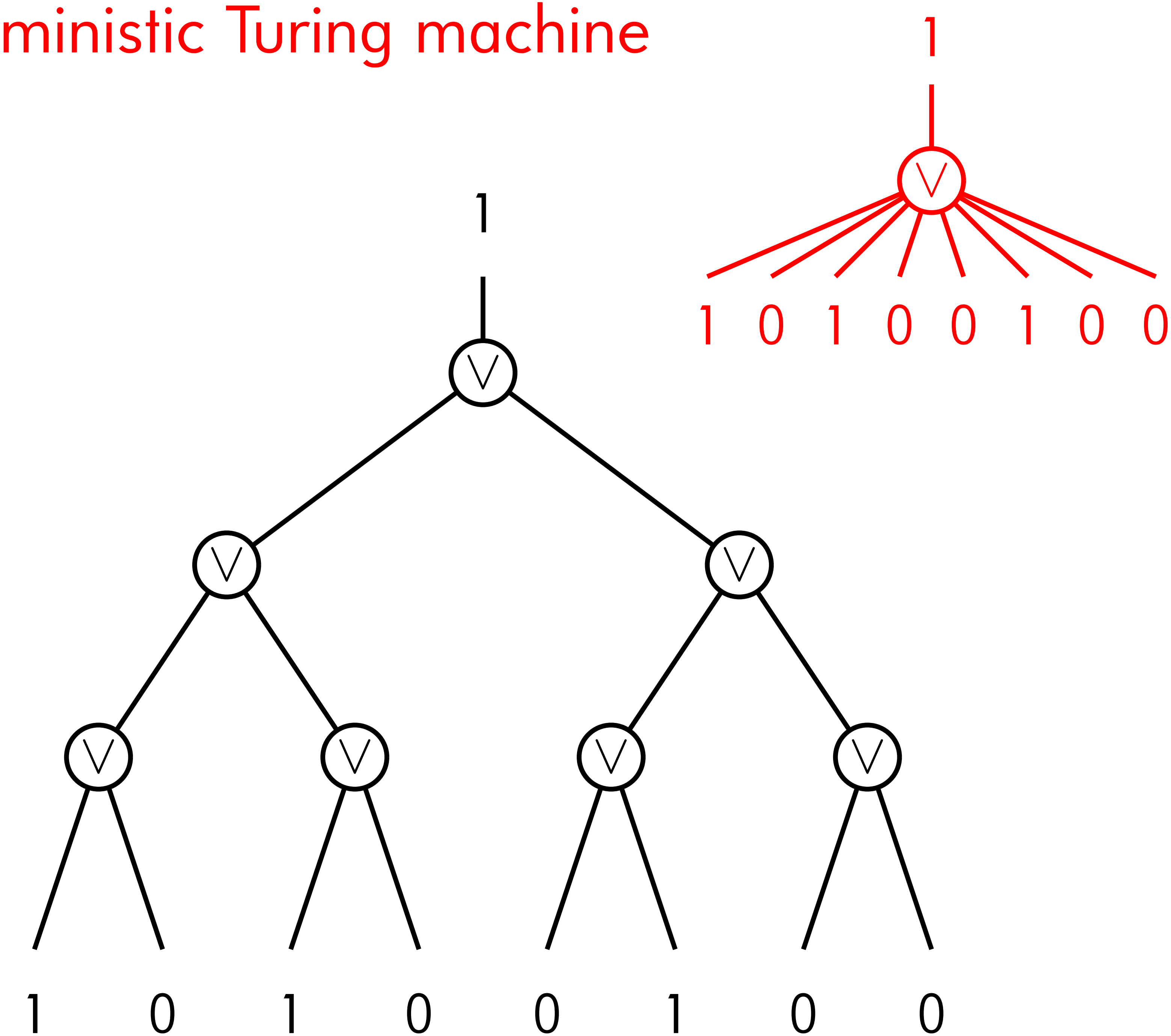
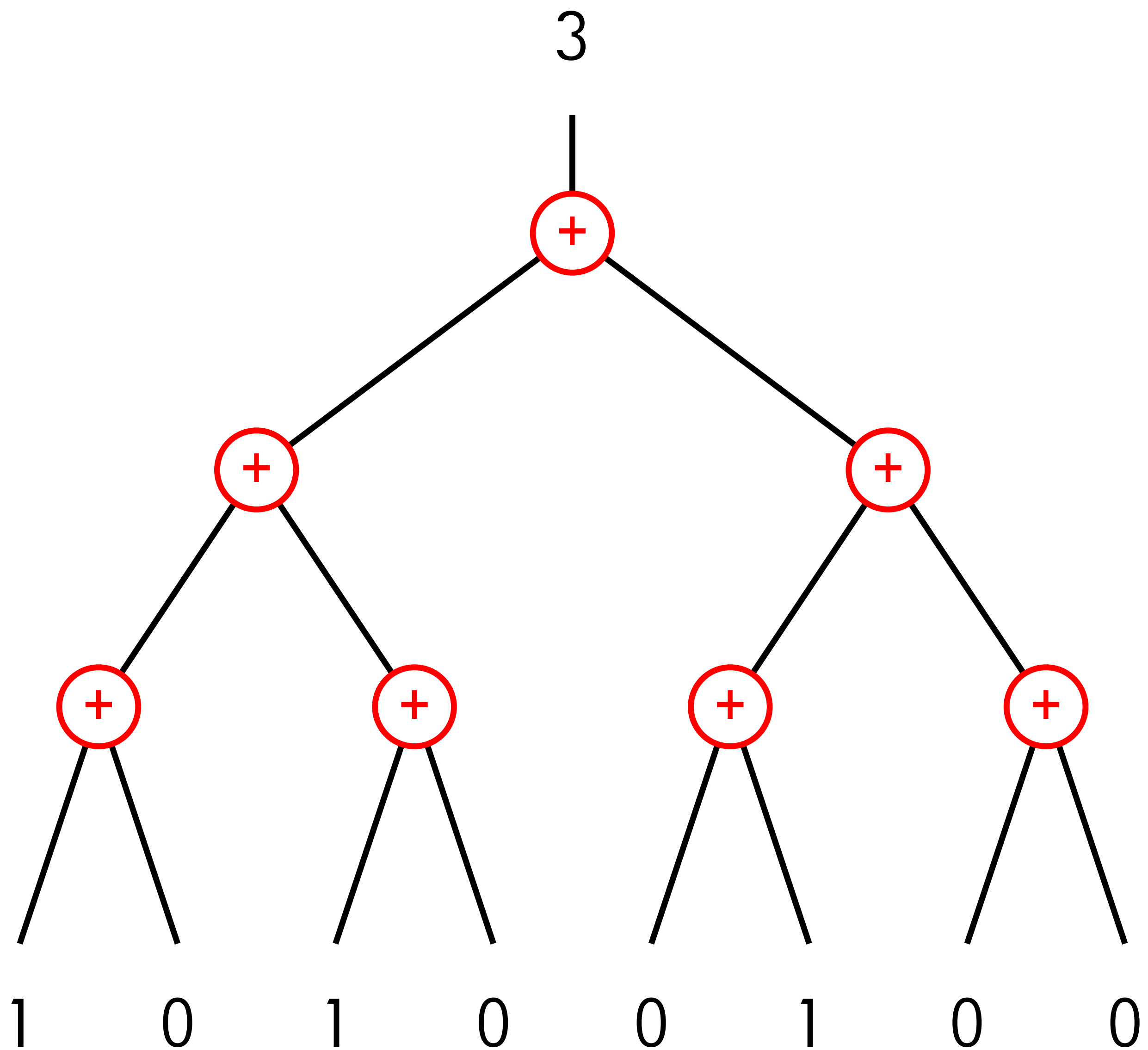
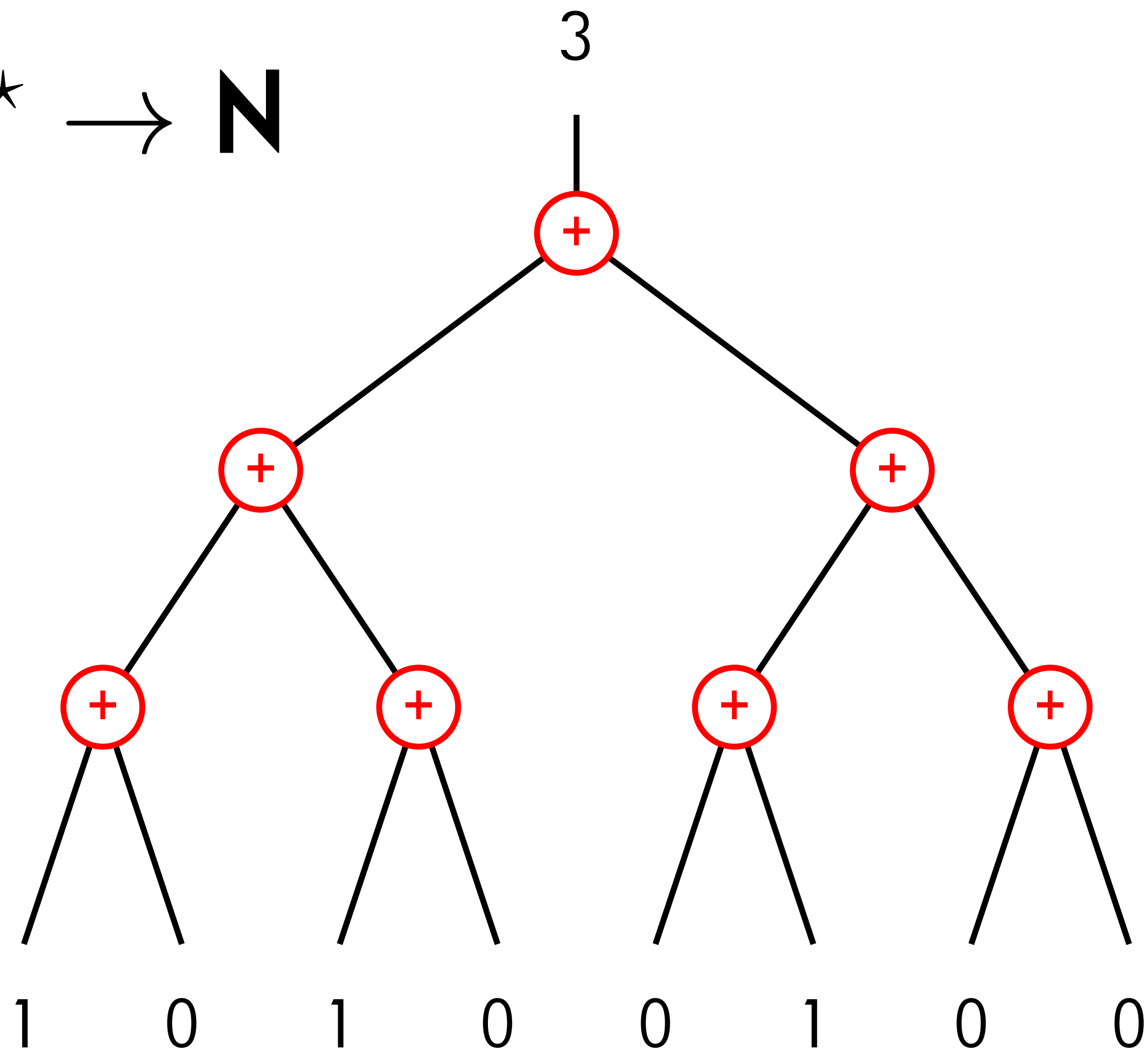


Counting Turing machine



Counting Turing machine

$$f : \Sigma^{\star} \to \mathbf{N}$$



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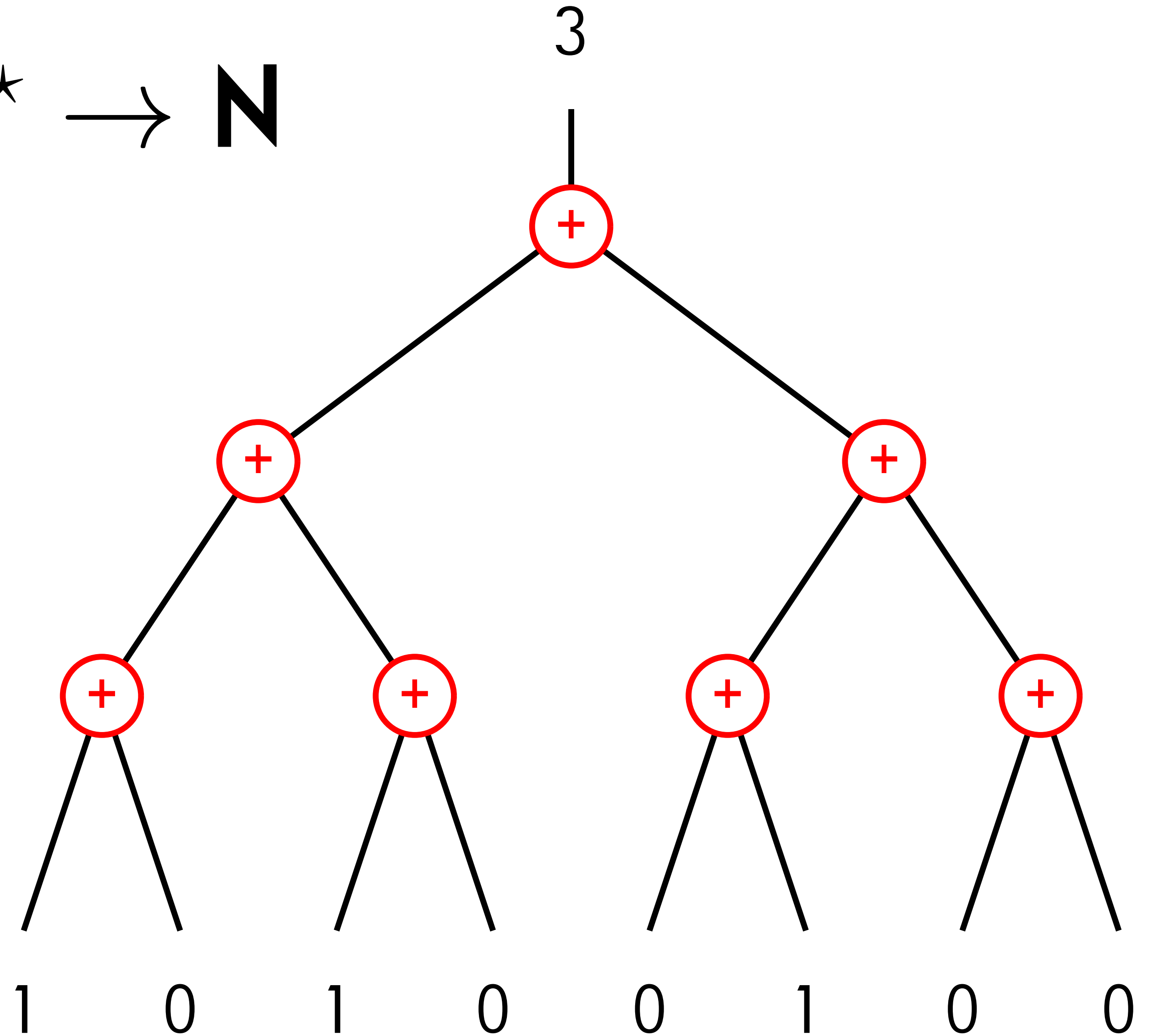

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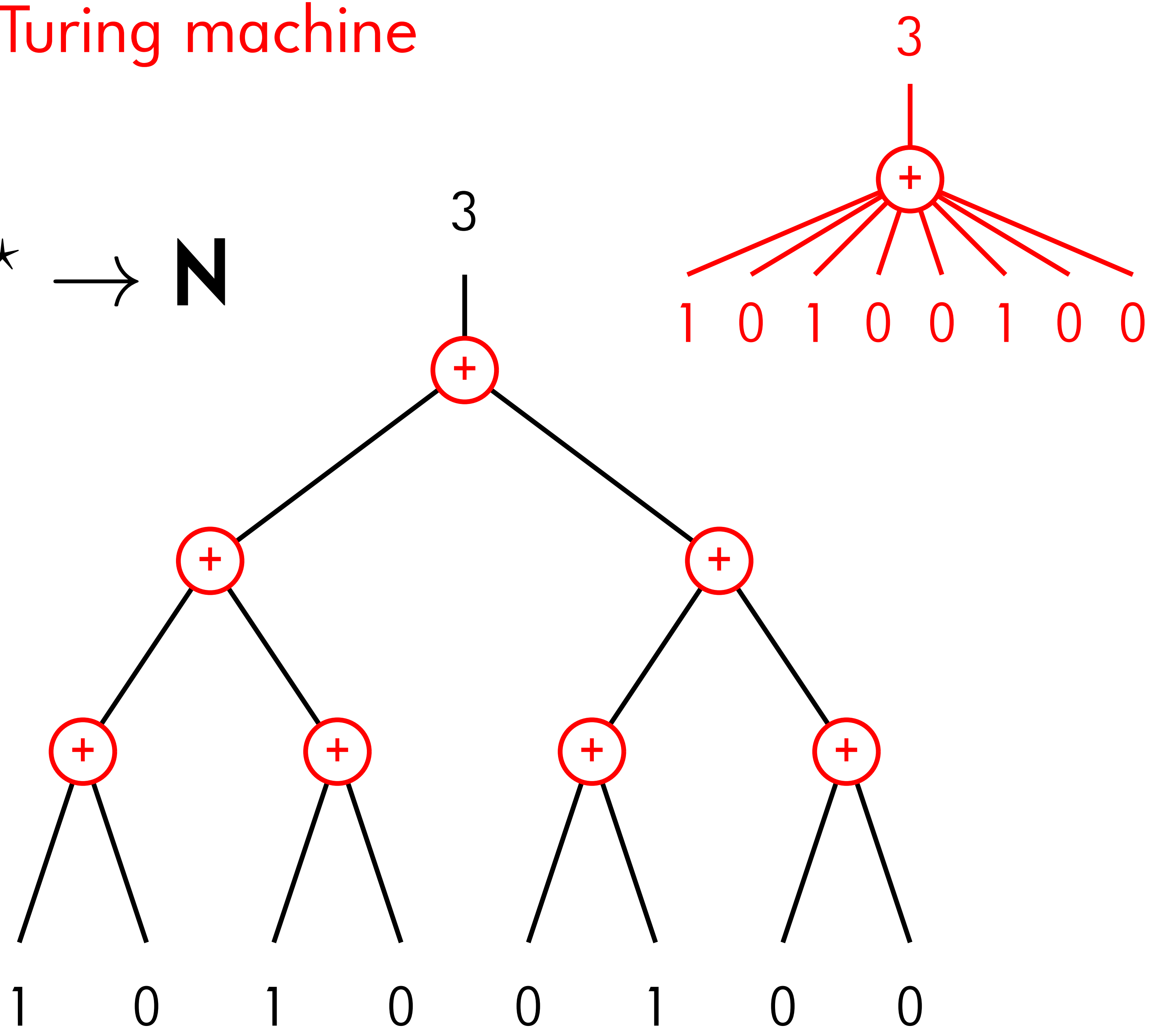
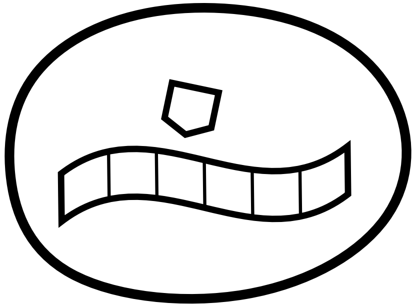


Simulating counting Turing machines



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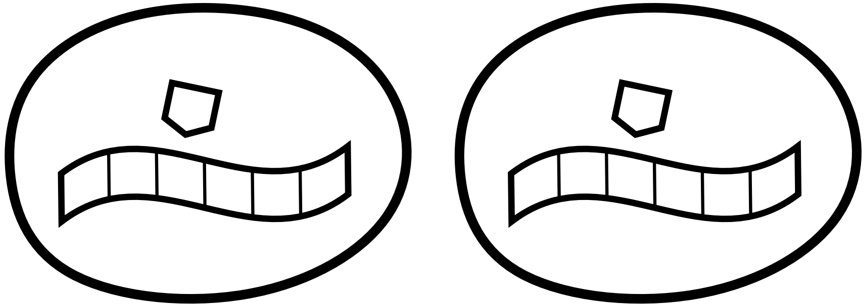

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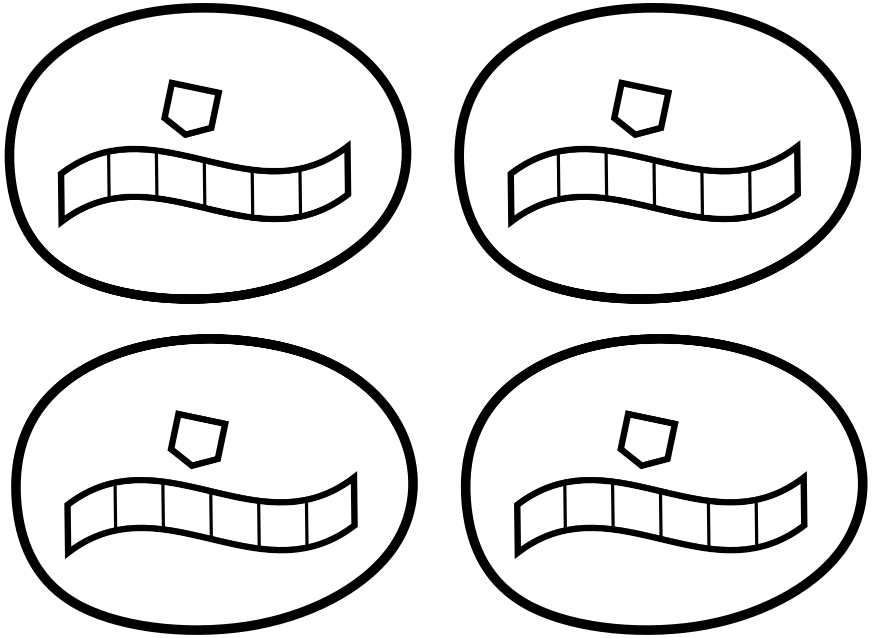

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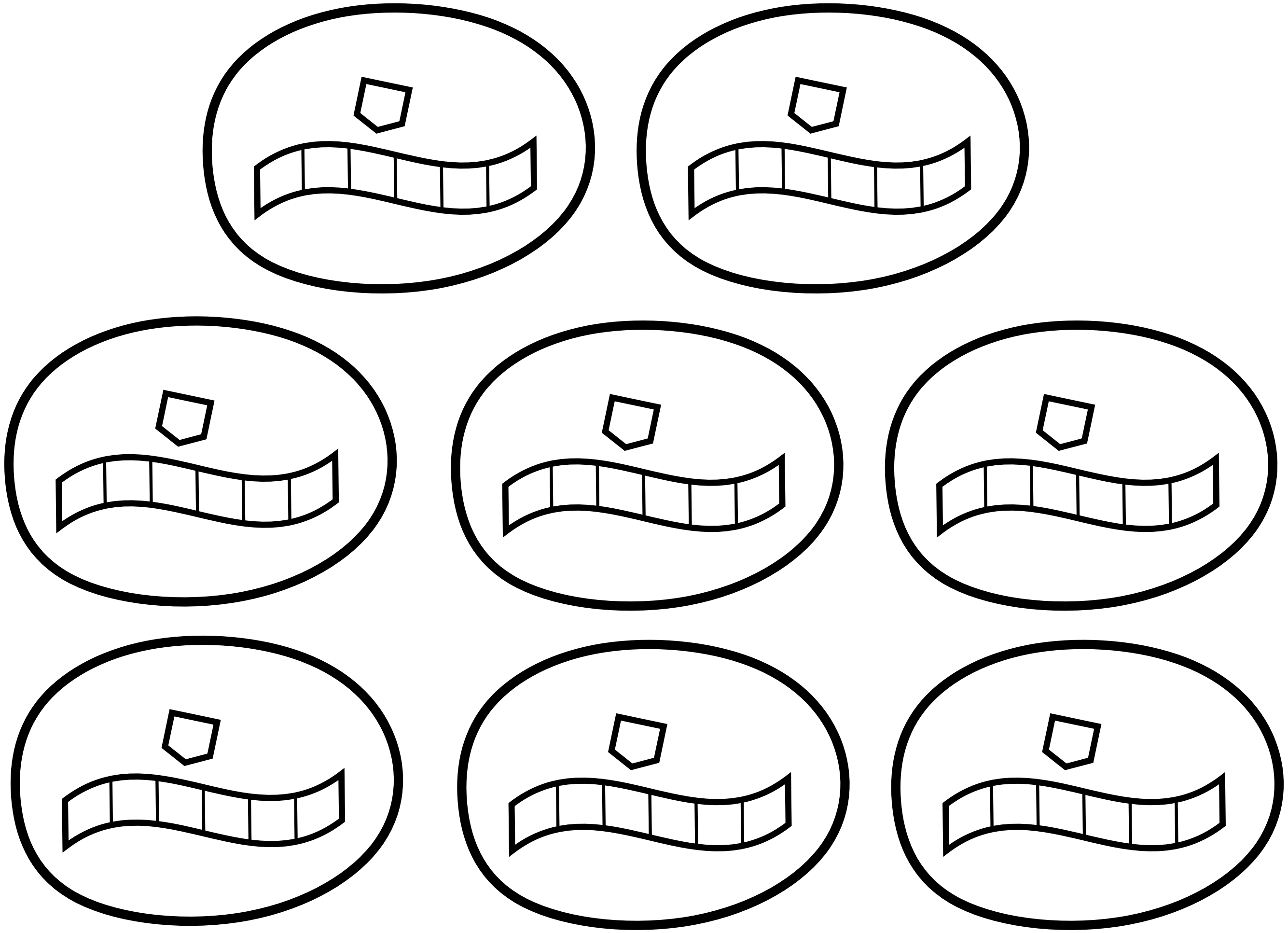

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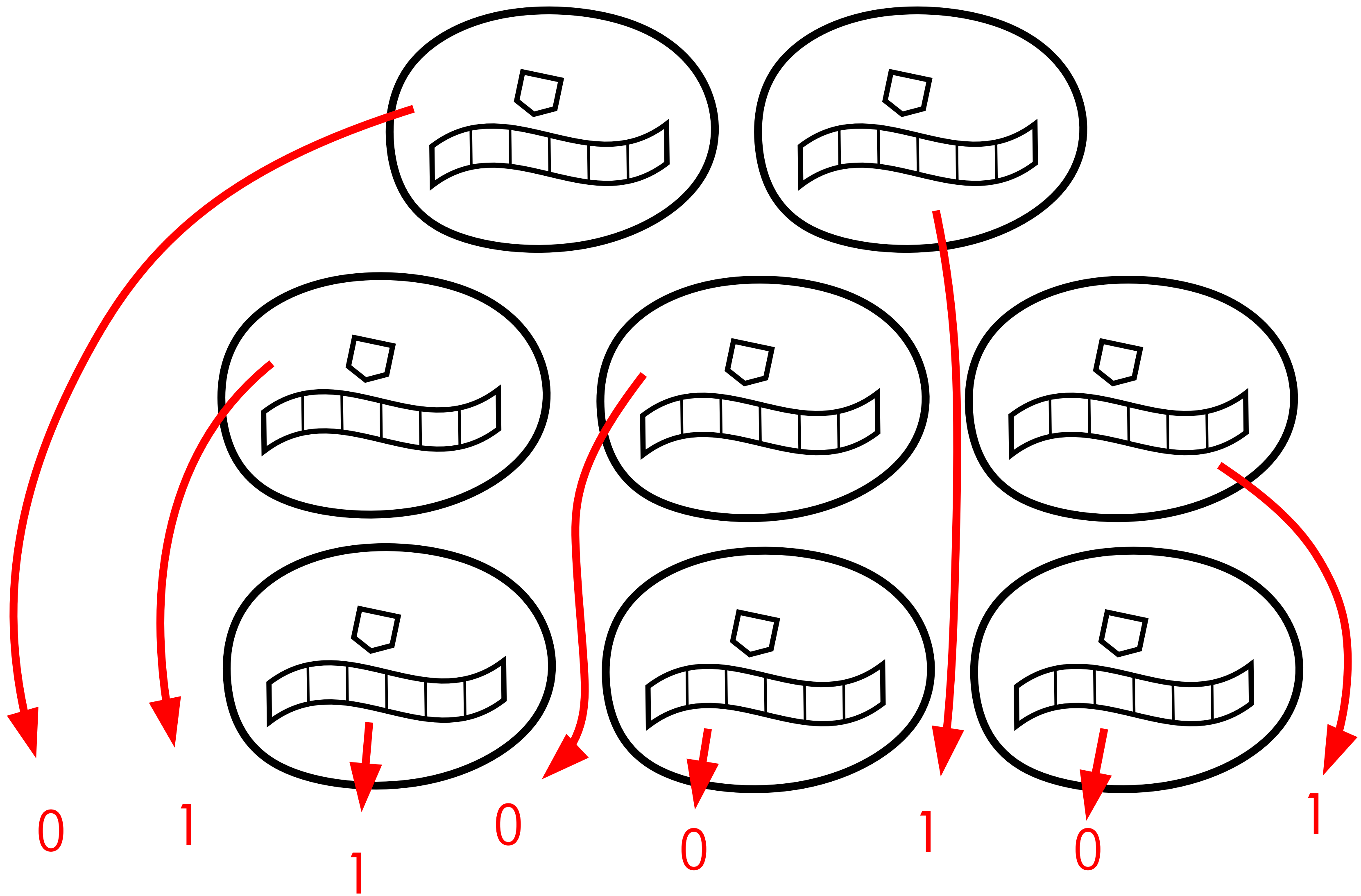

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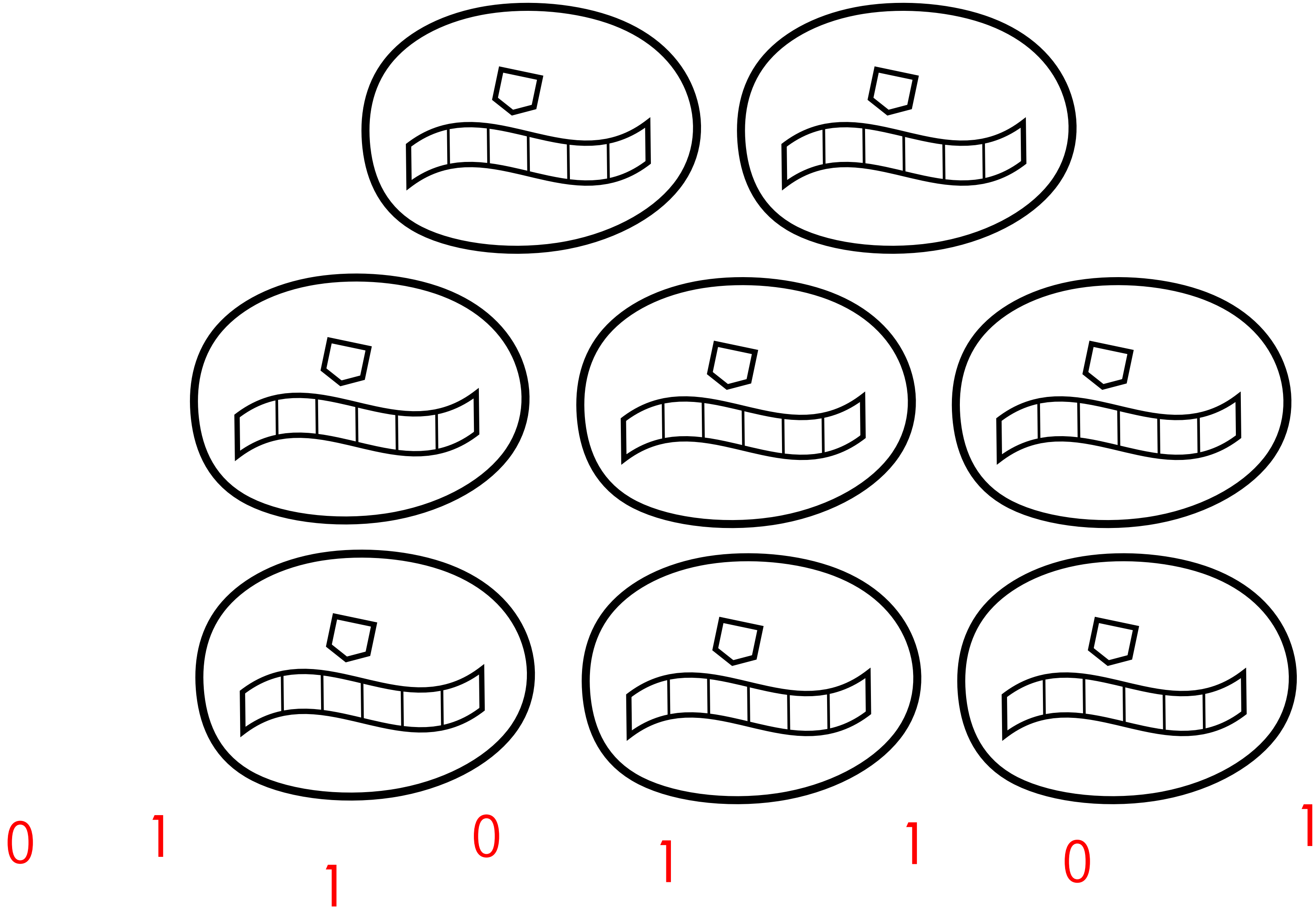

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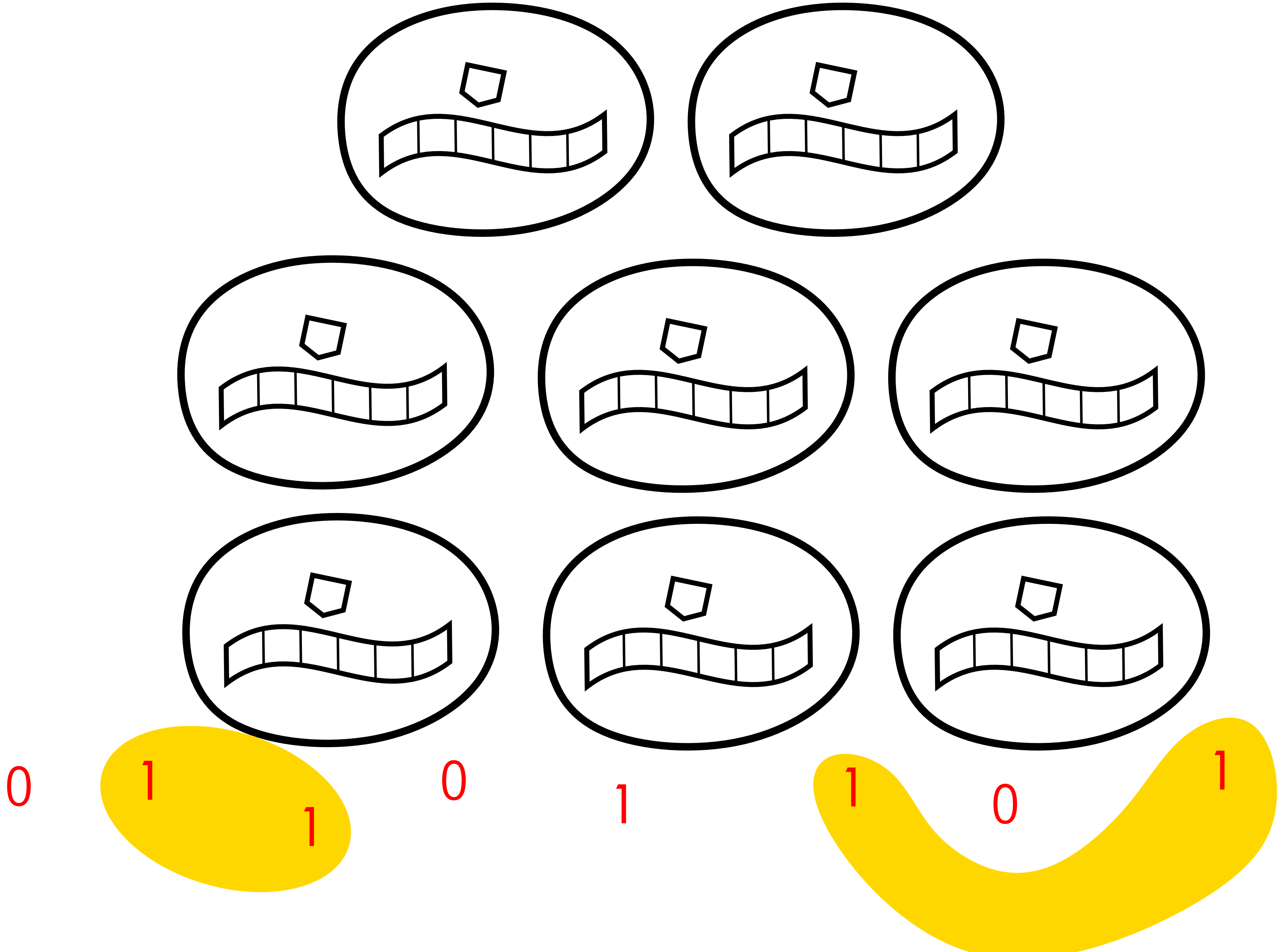

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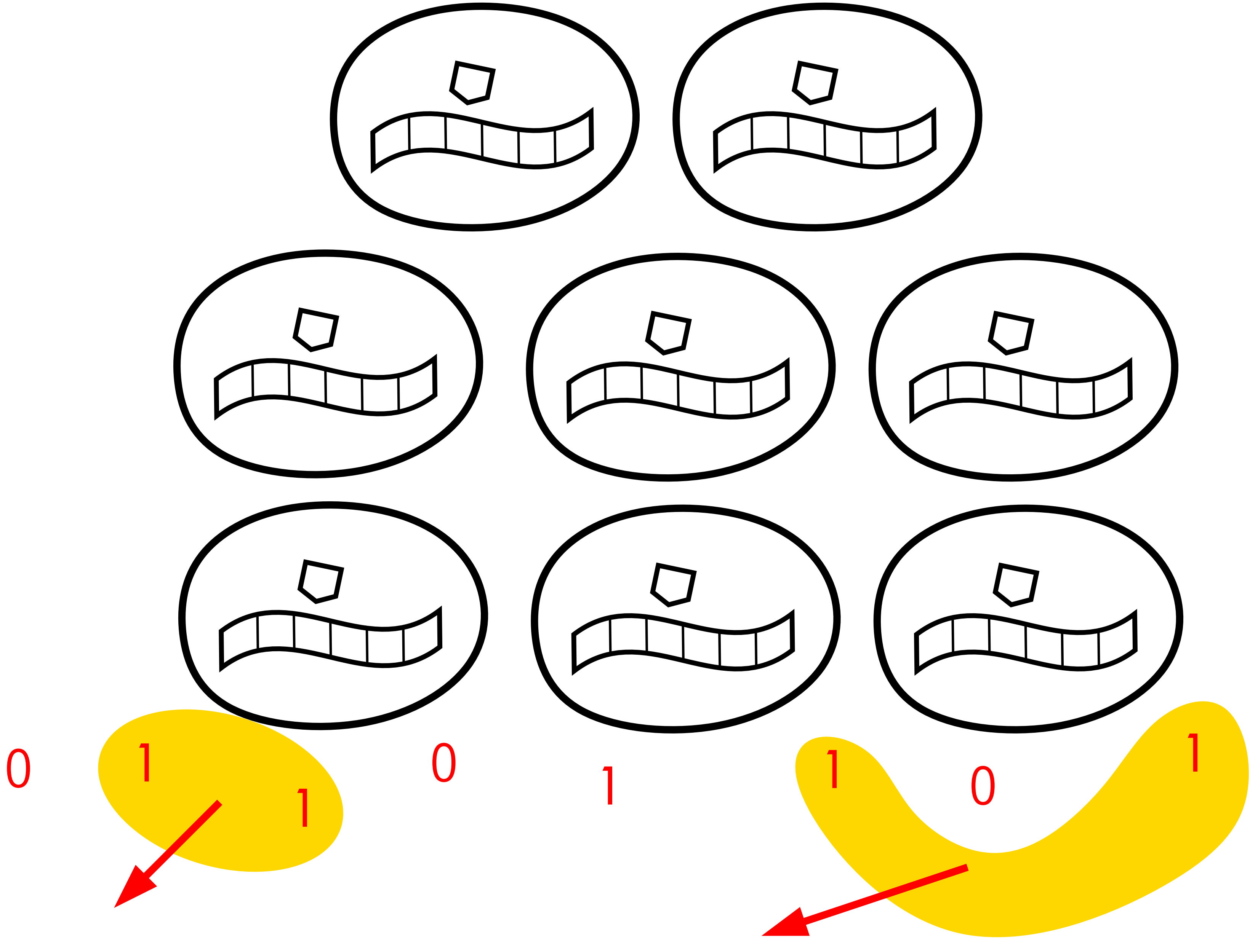

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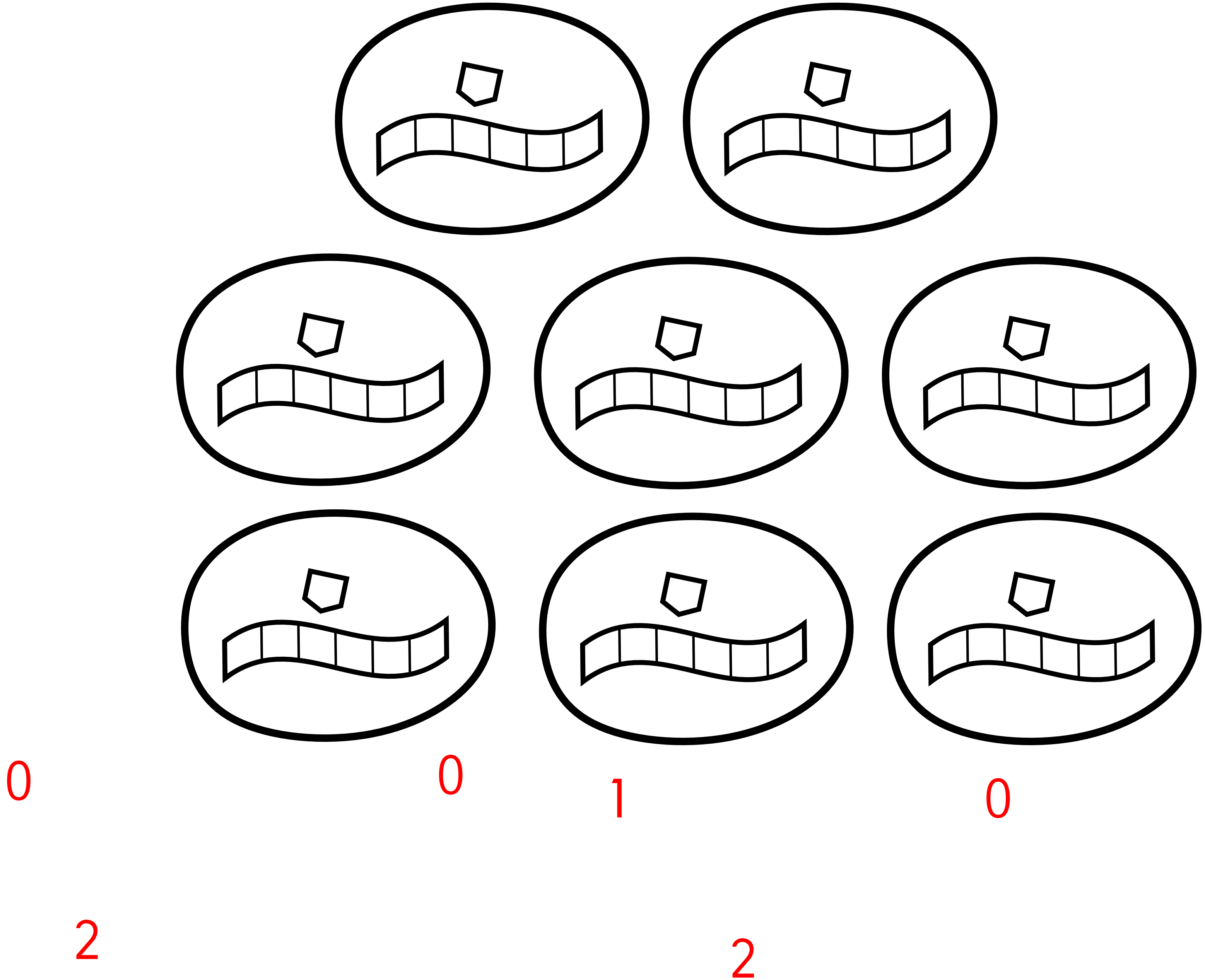

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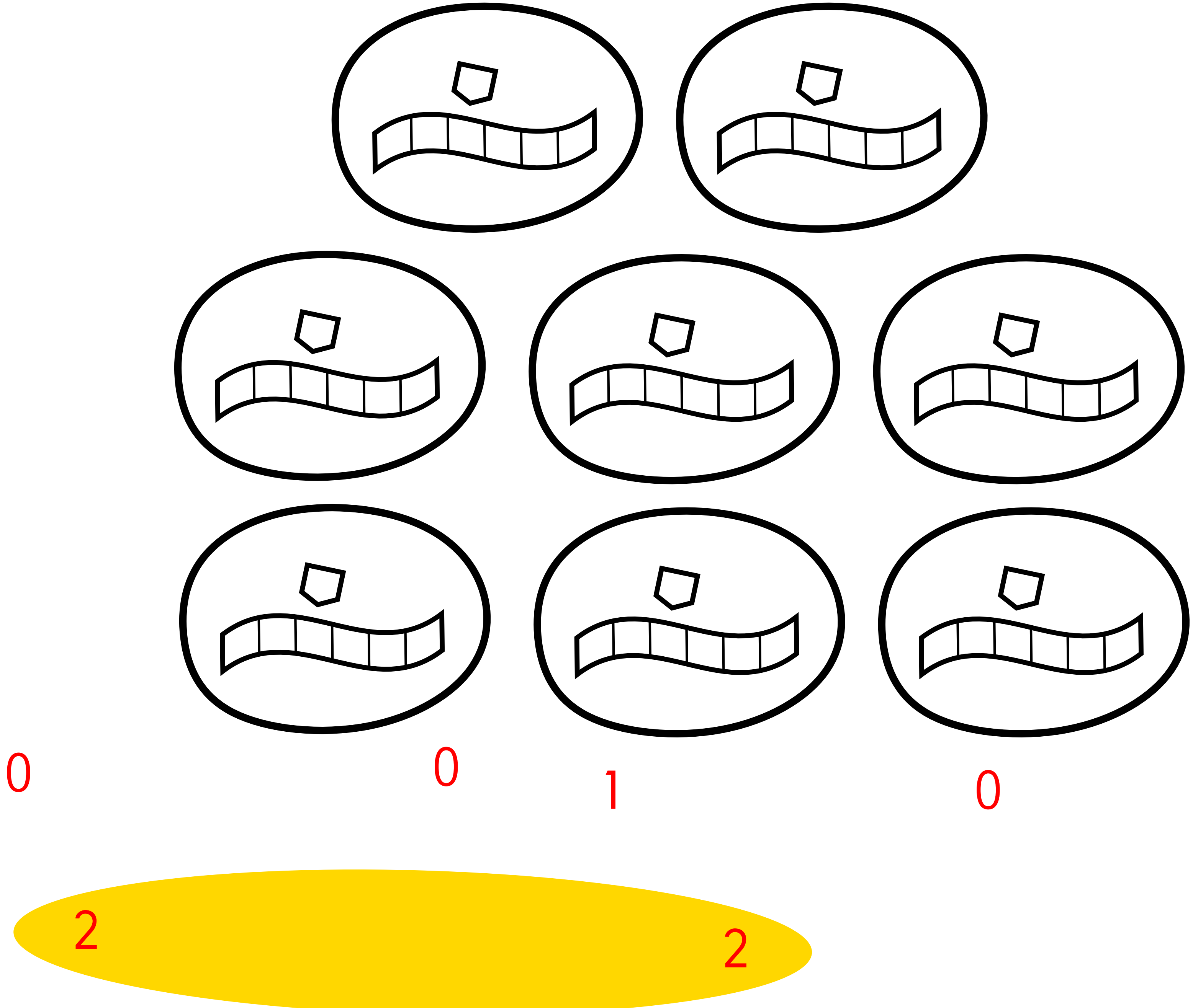

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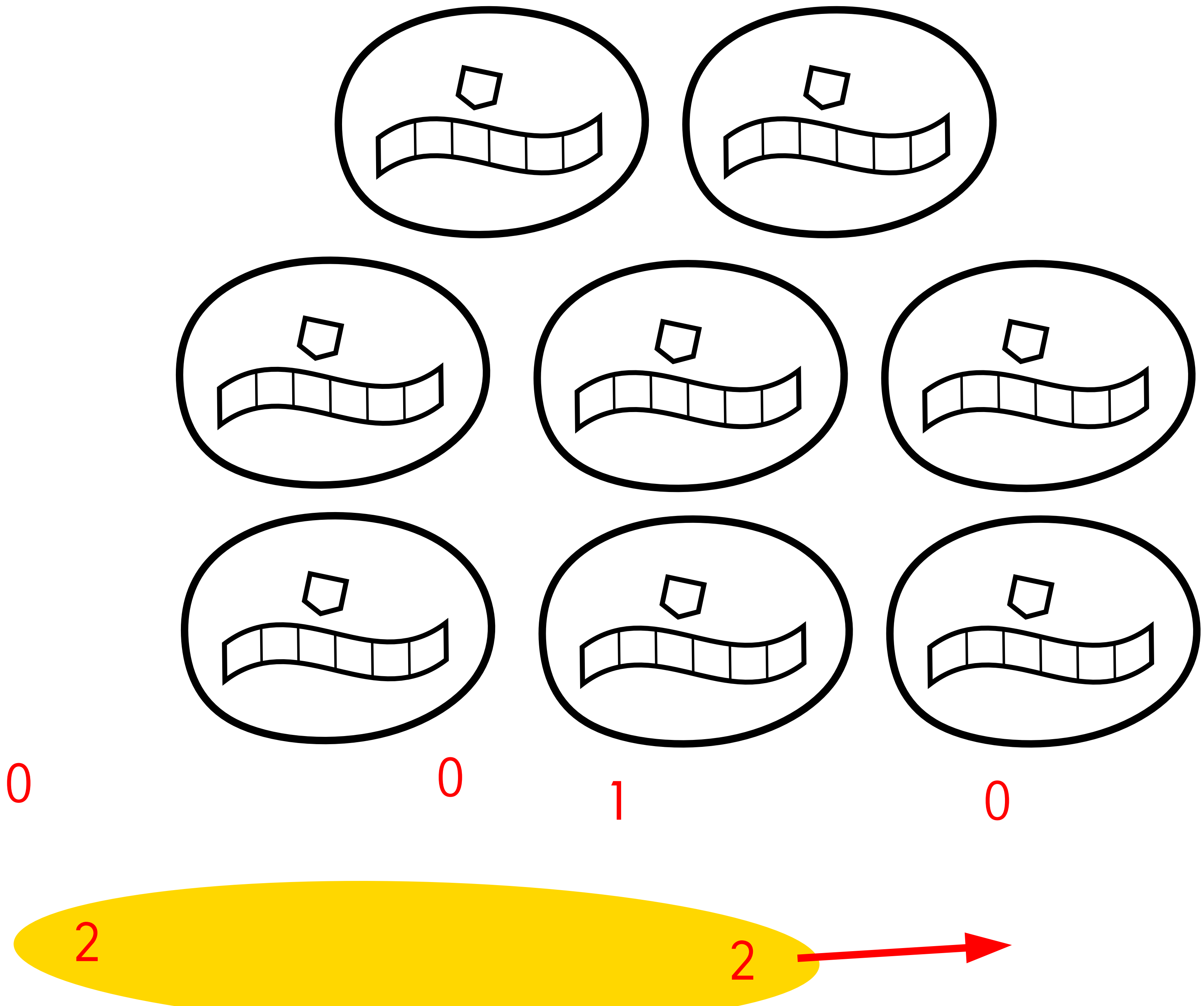

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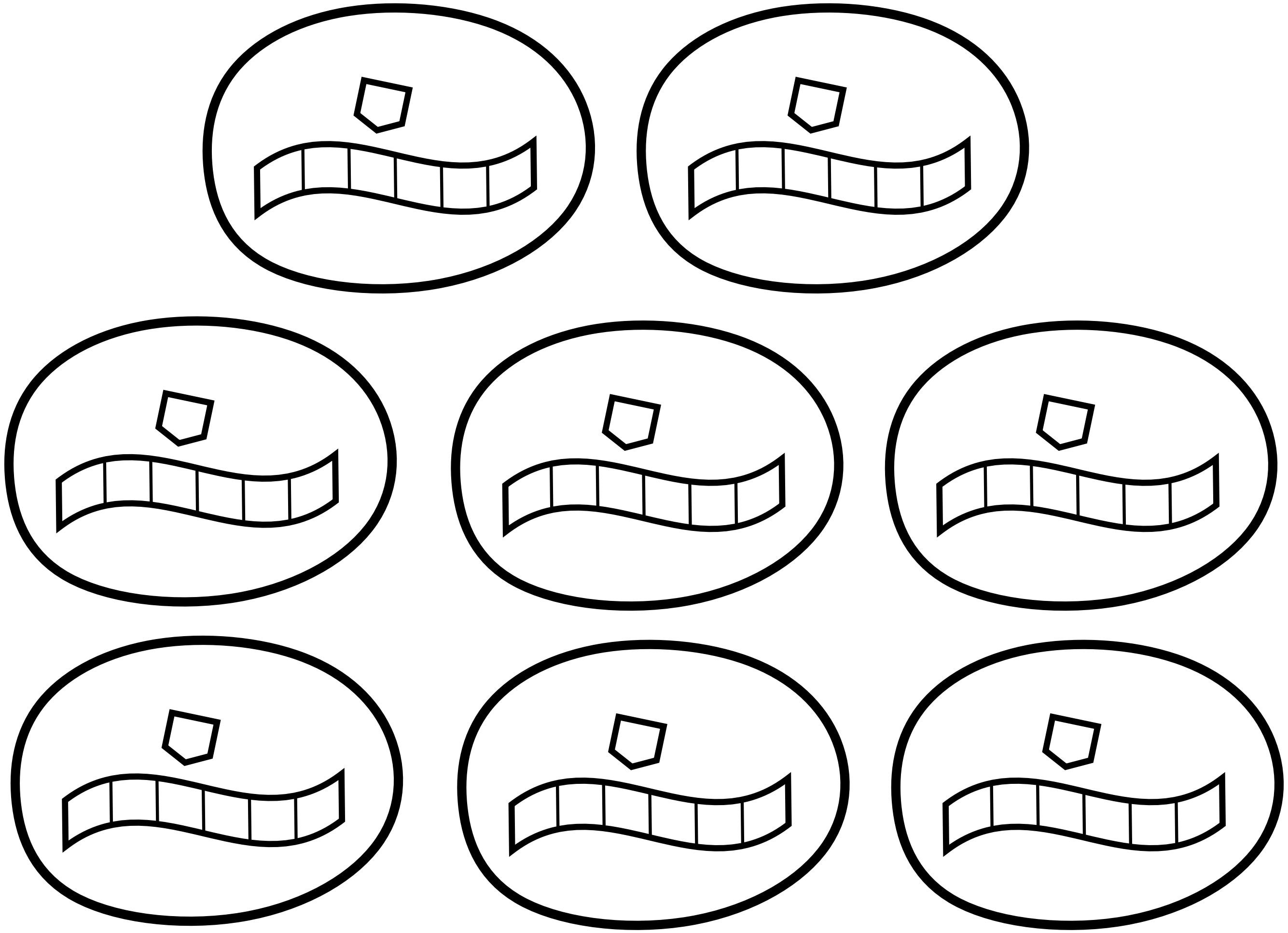

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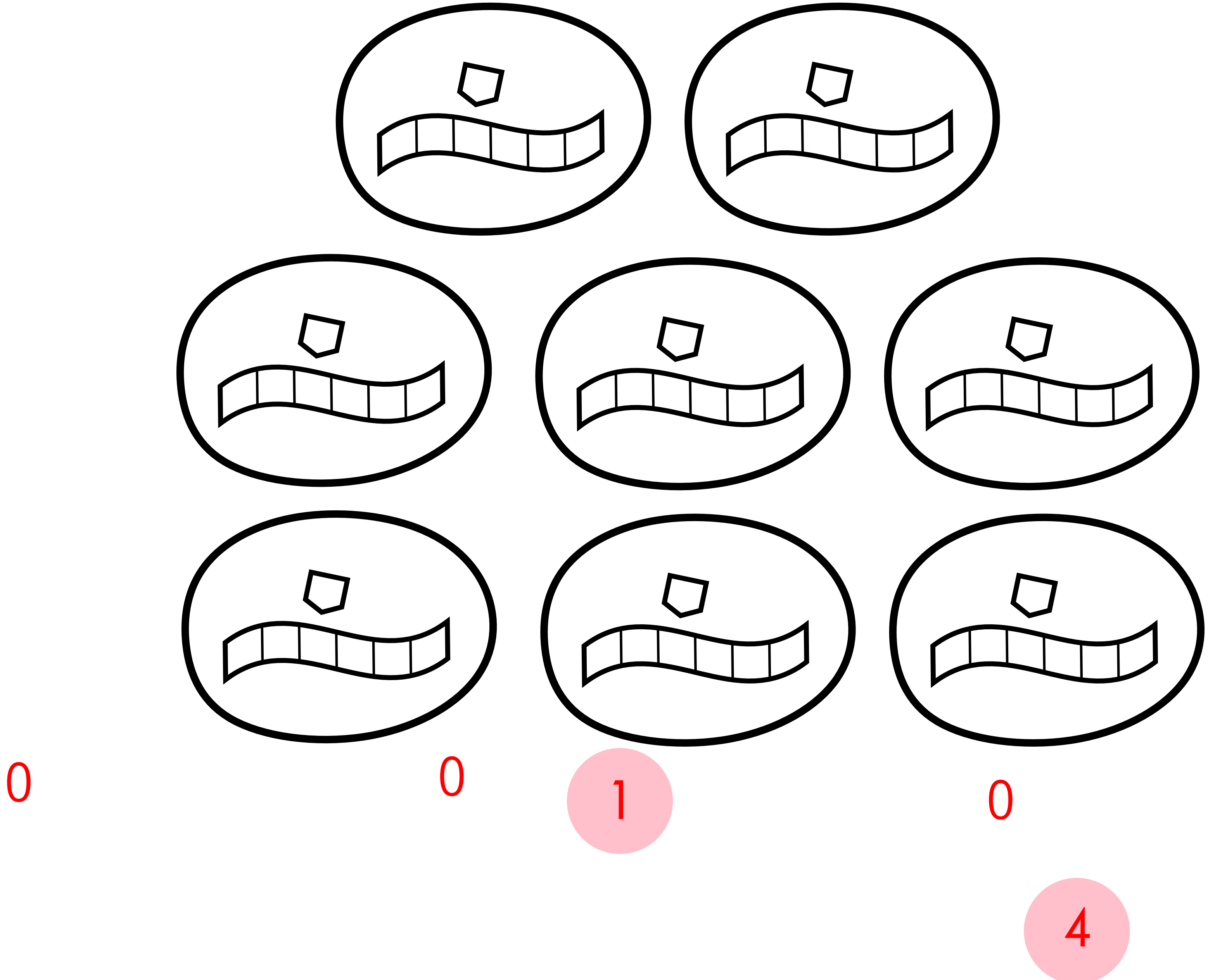
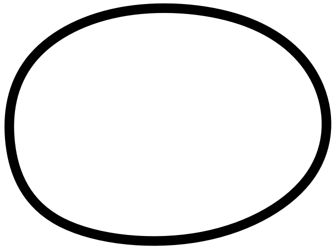
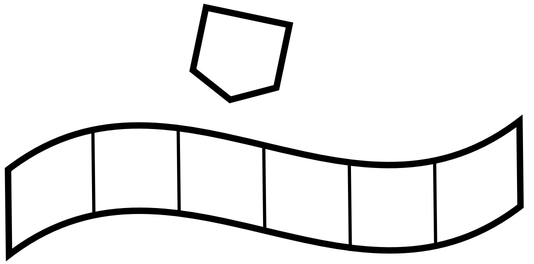
Simulating counting Turing machines



Simulating Turing machines with #P oracles





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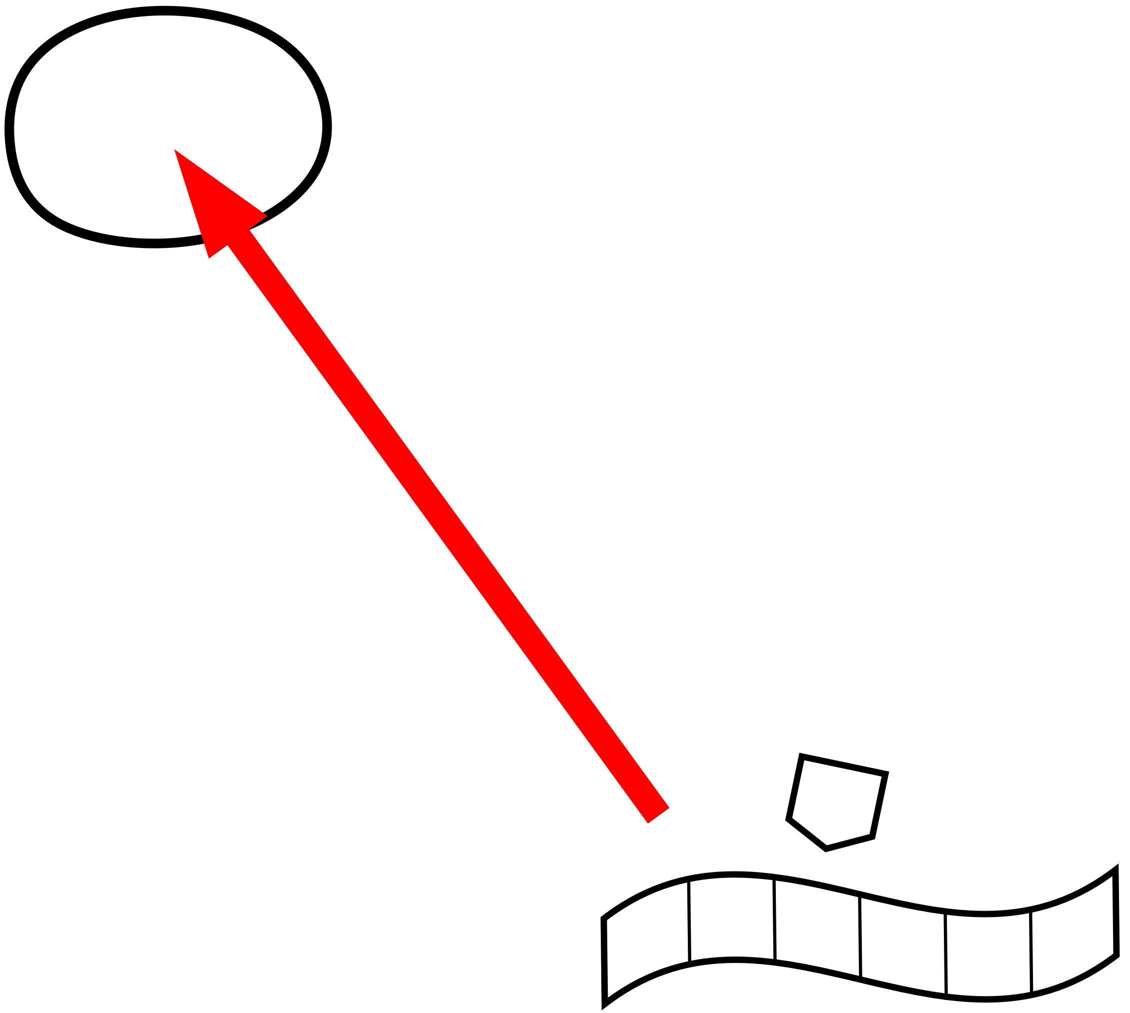

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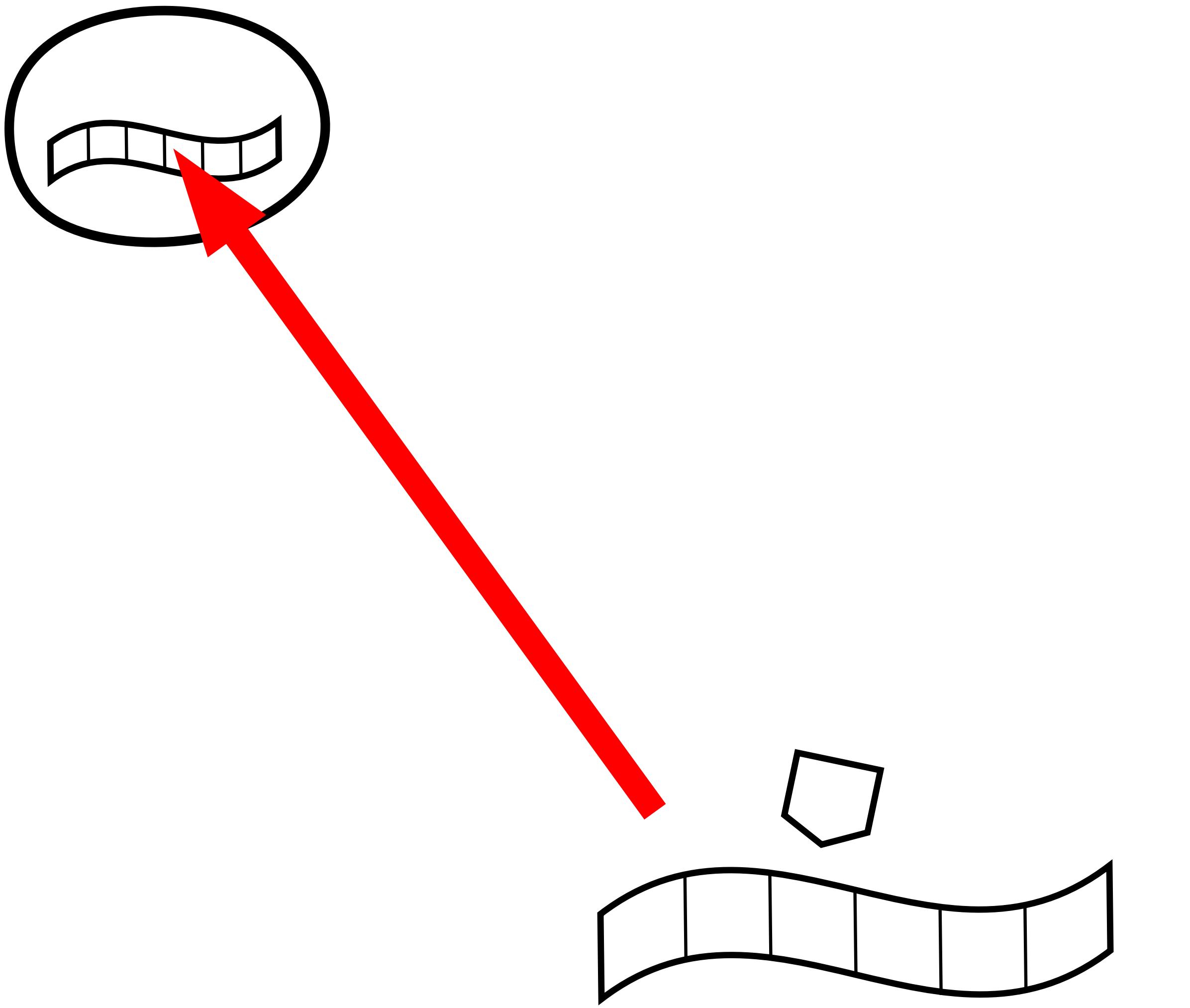

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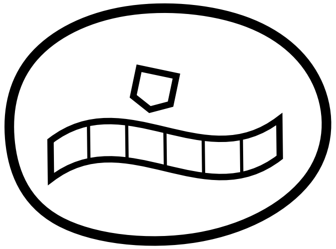

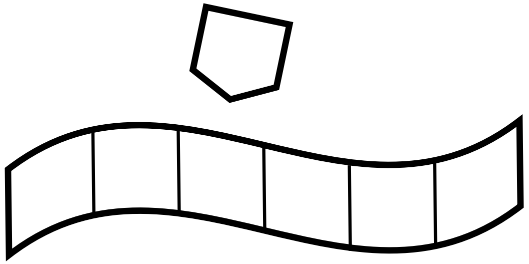

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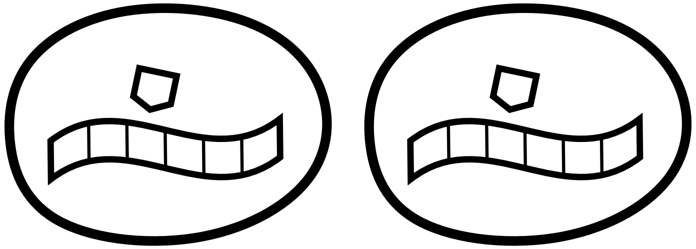

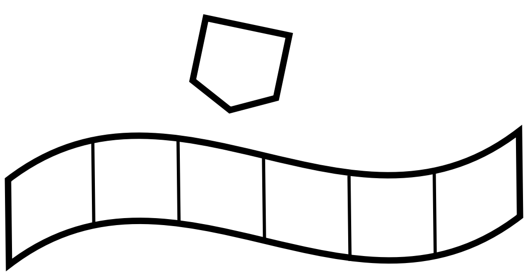

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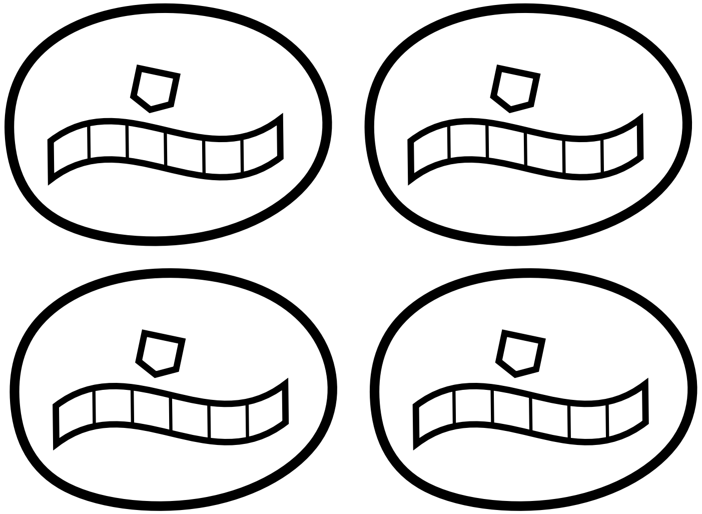

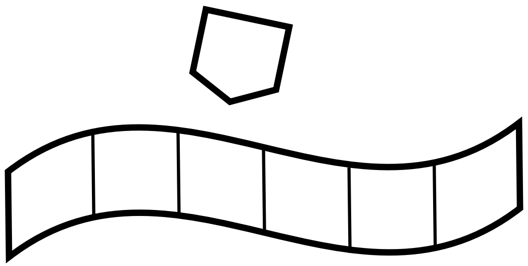

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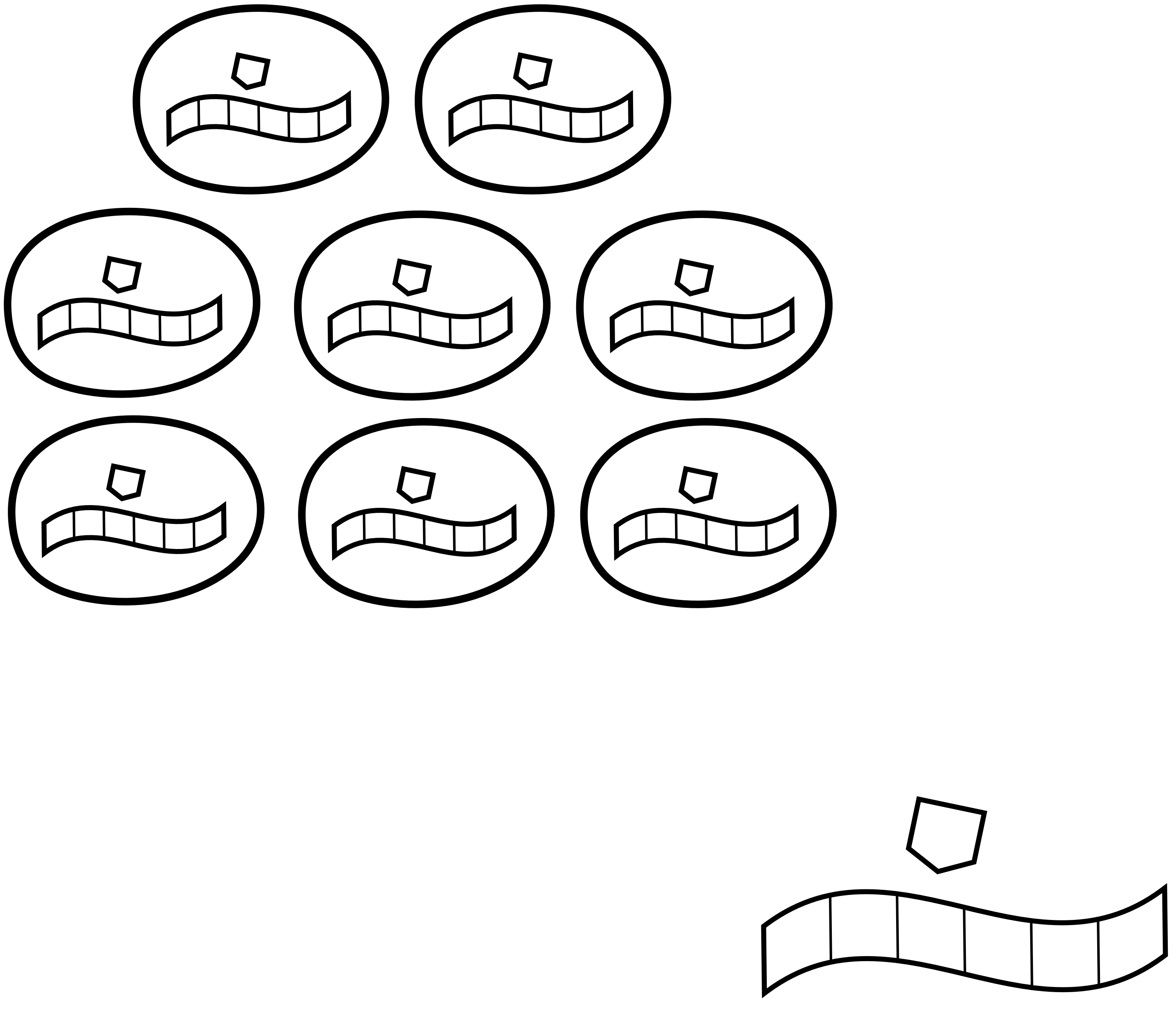

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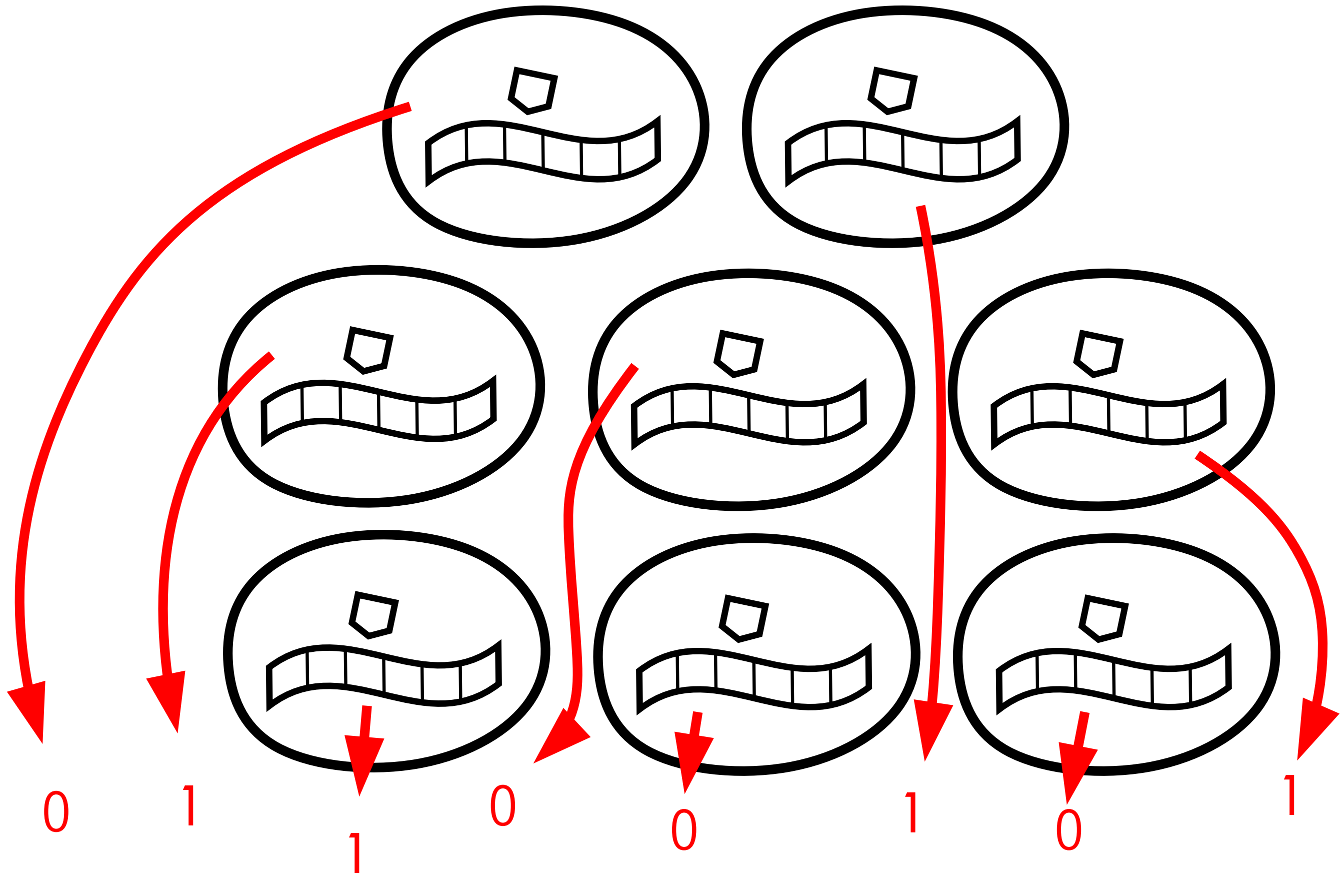

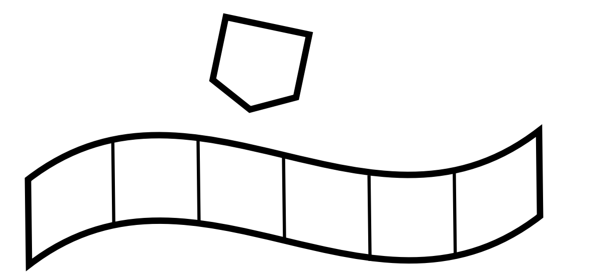

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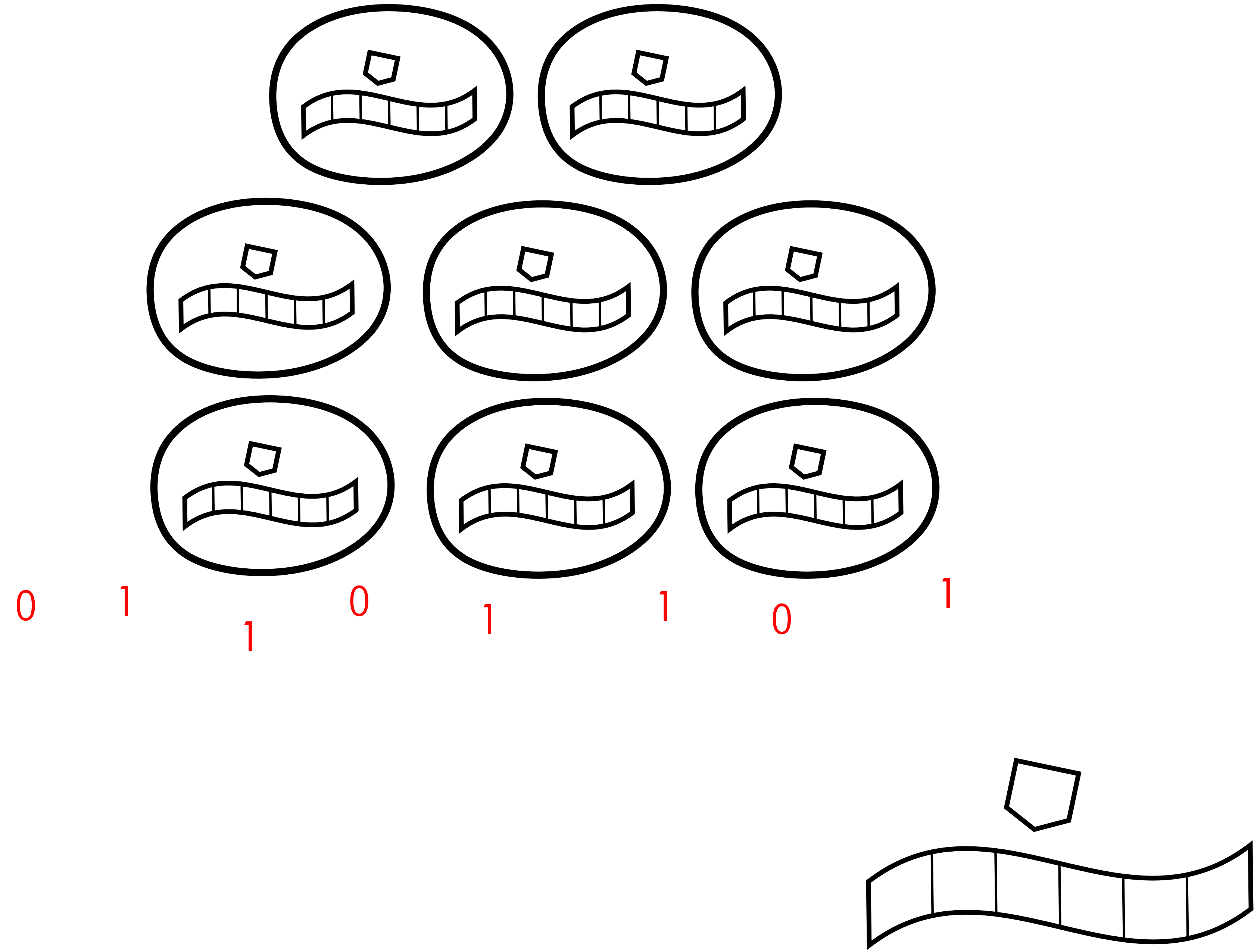

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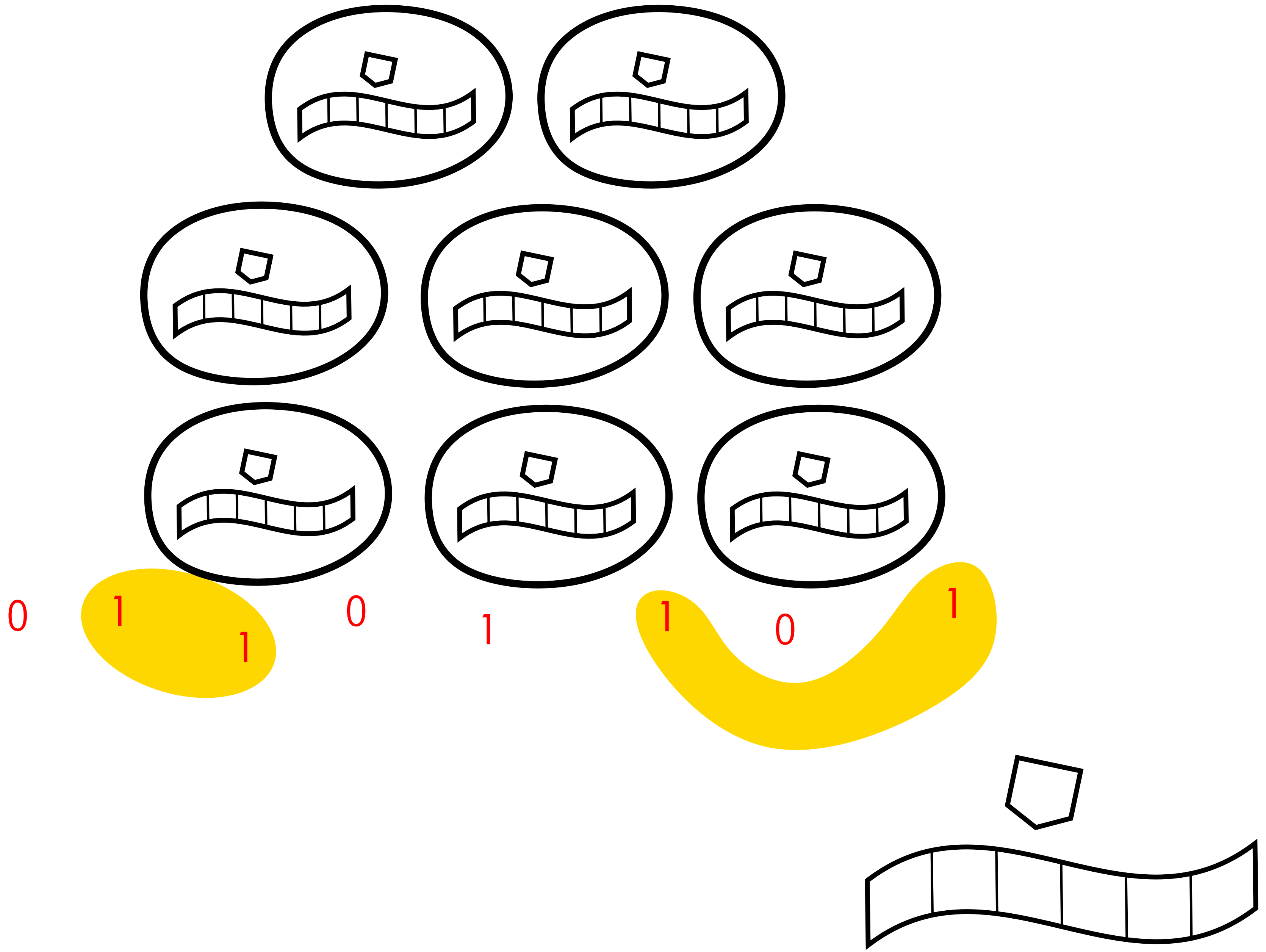

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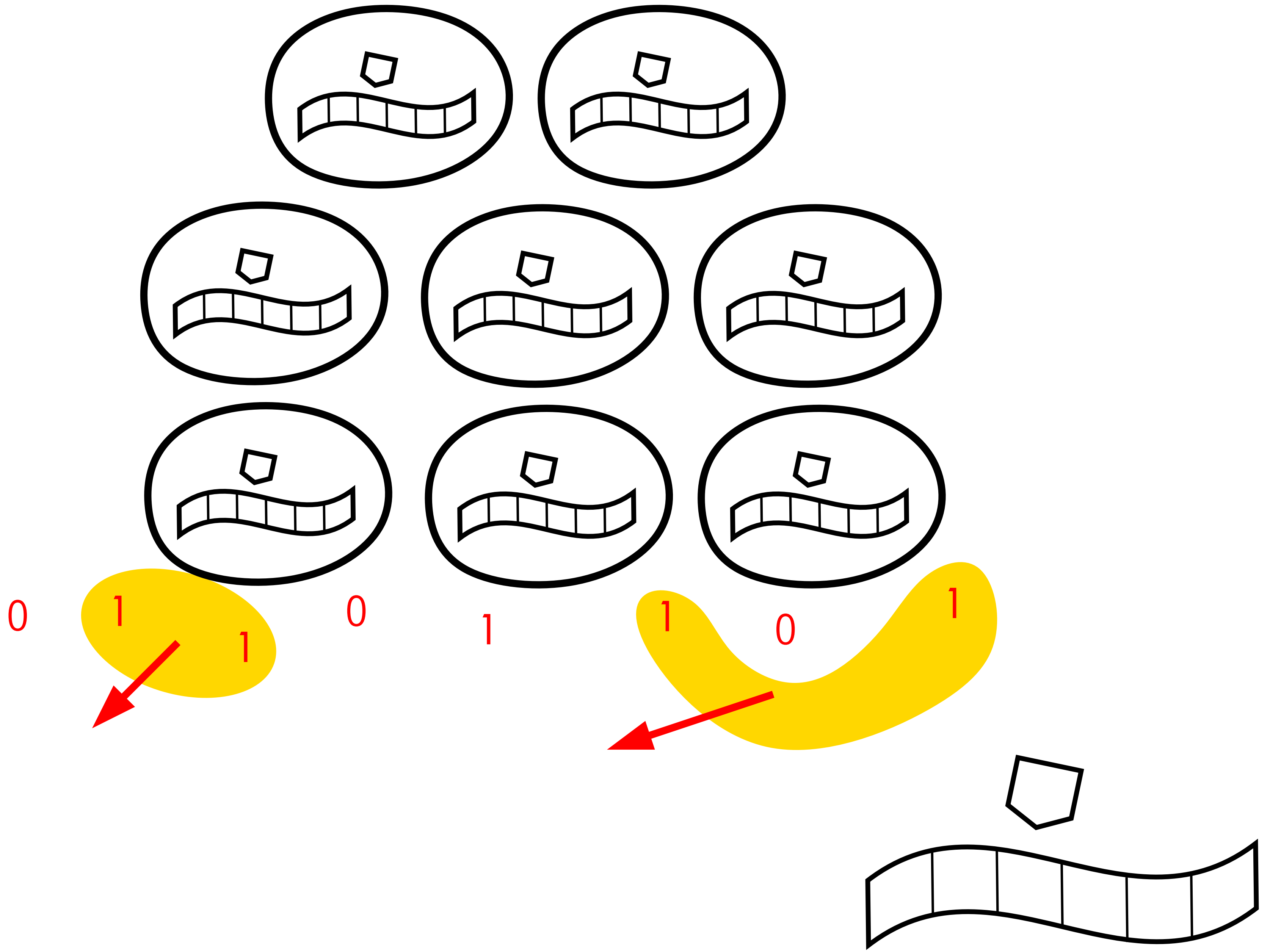

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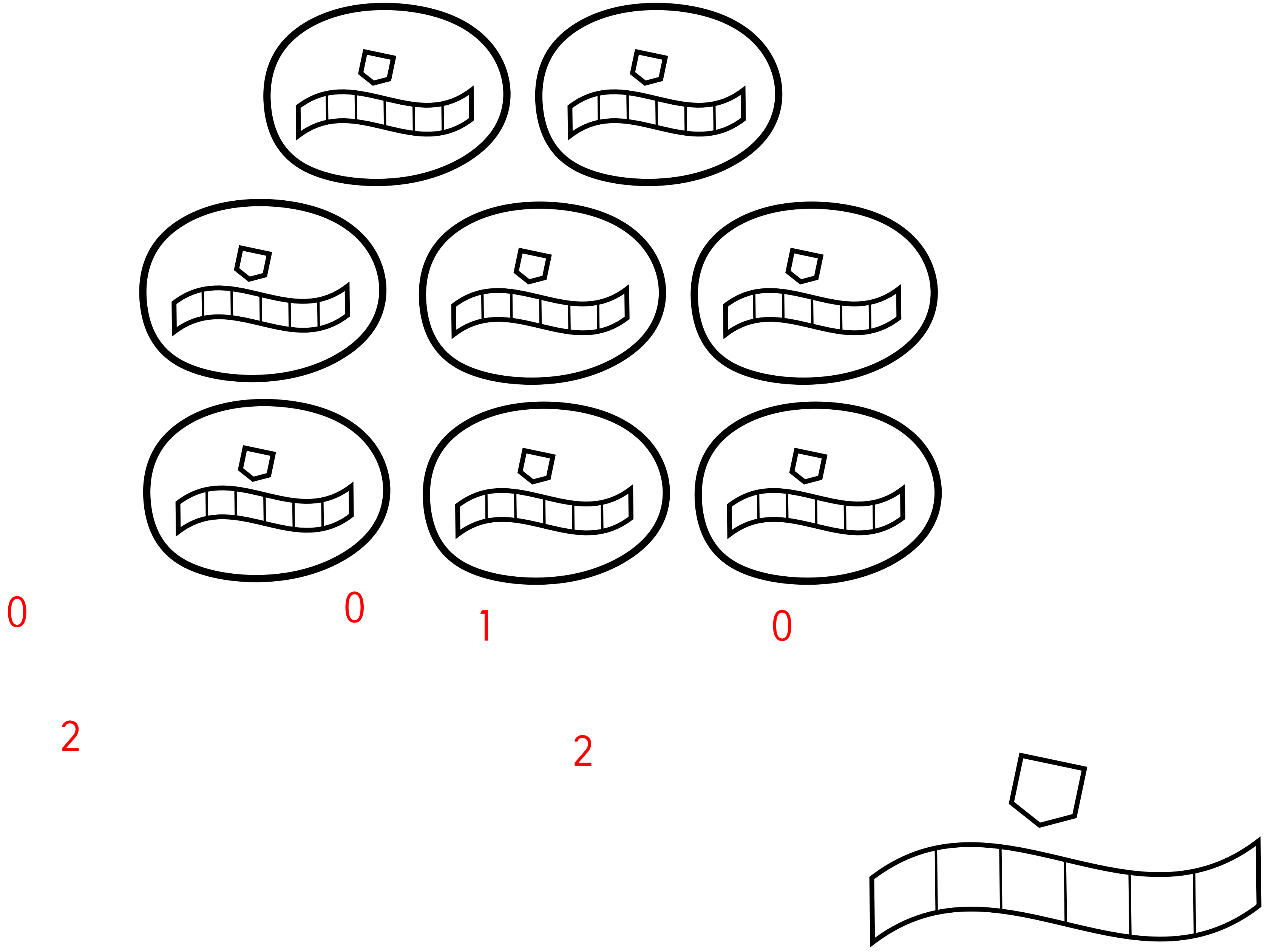

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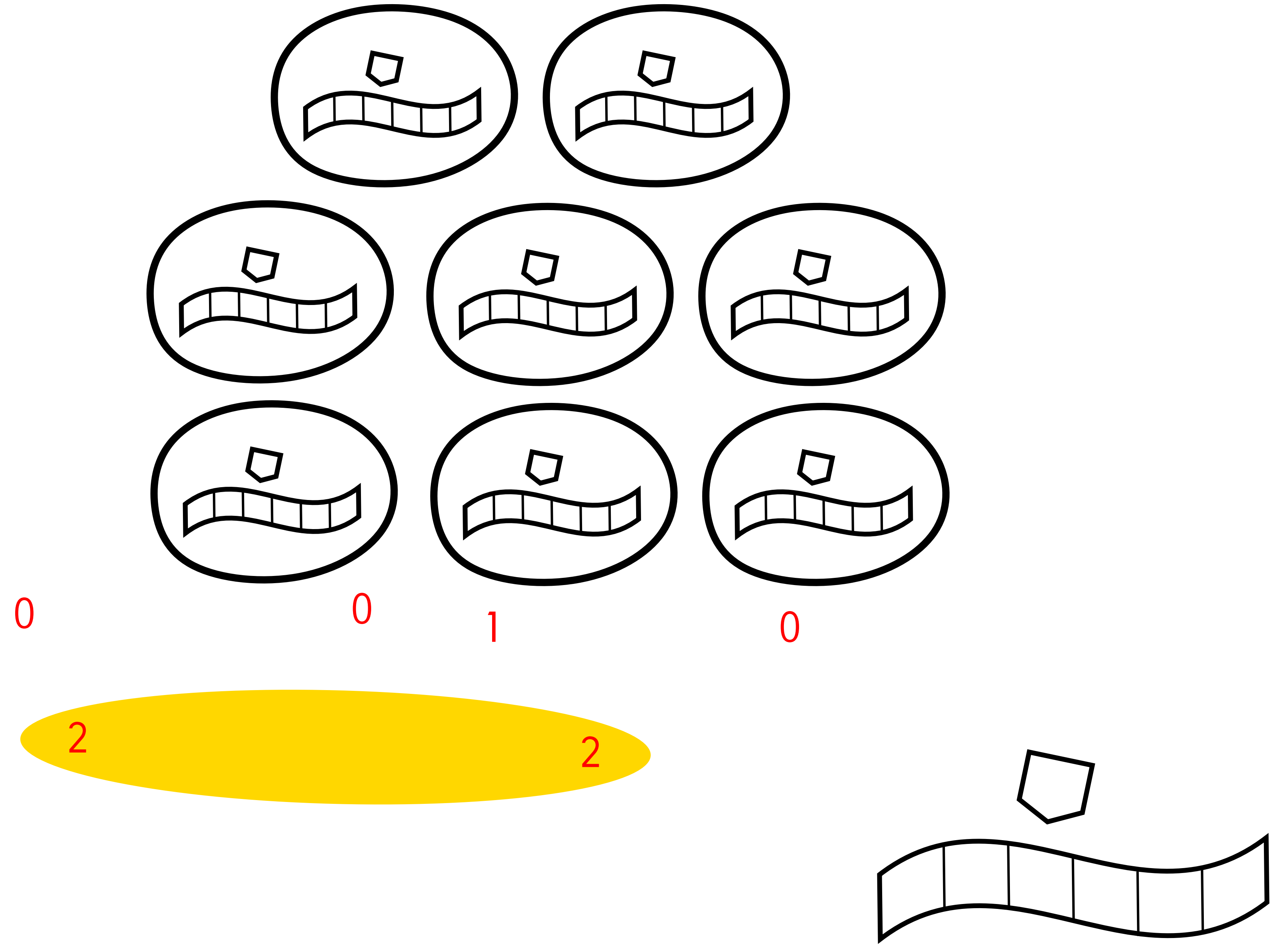

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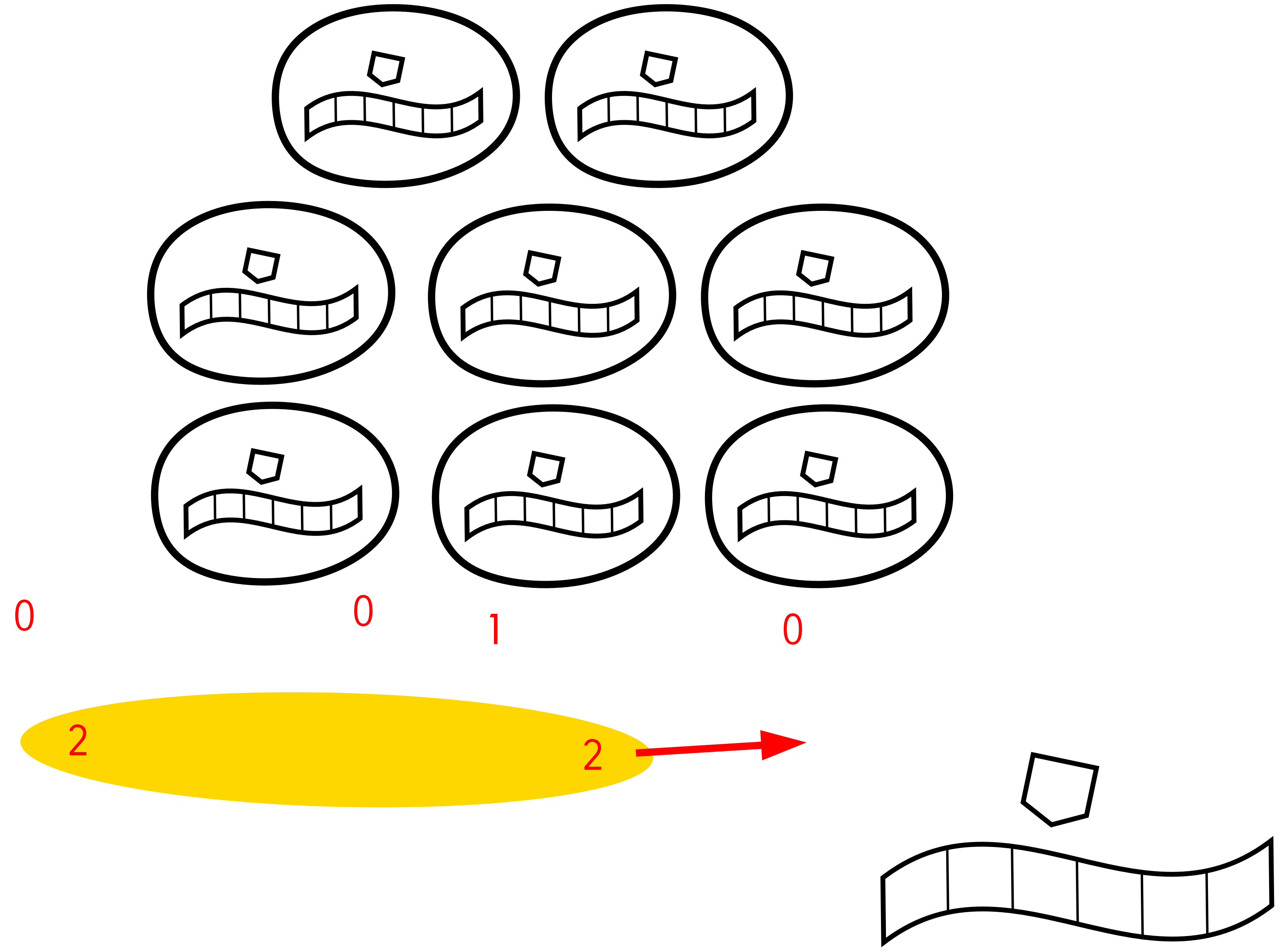

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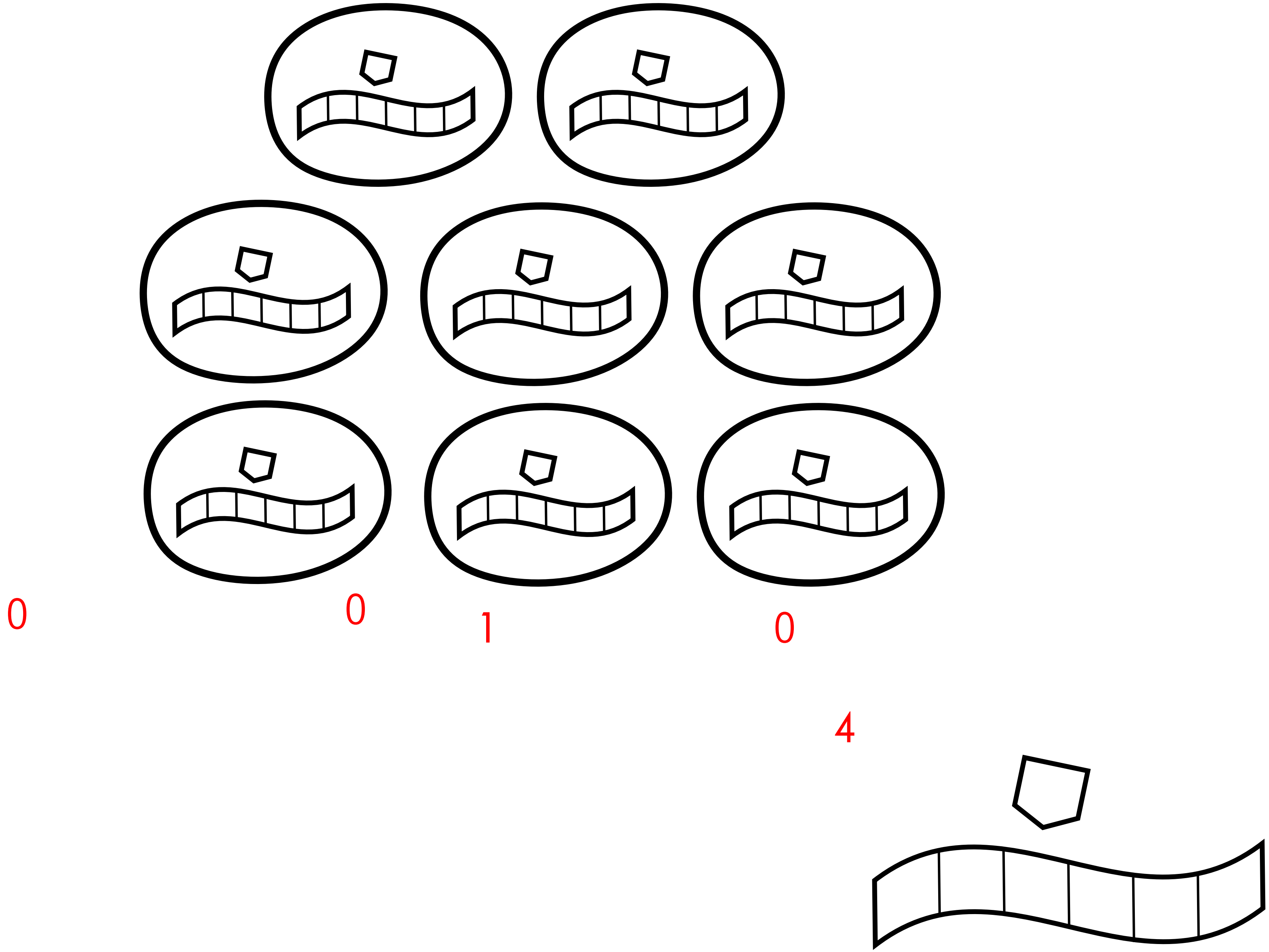

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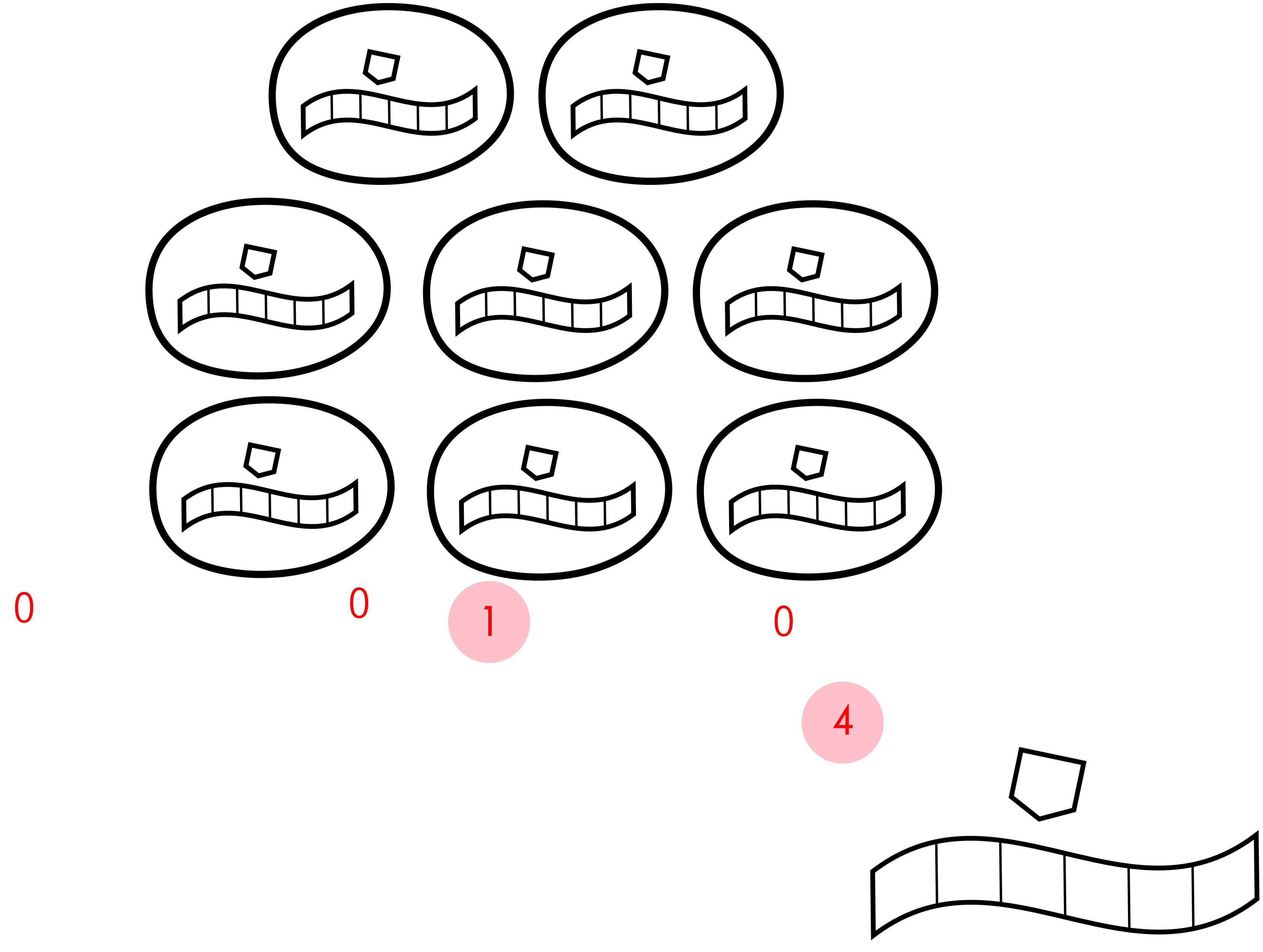

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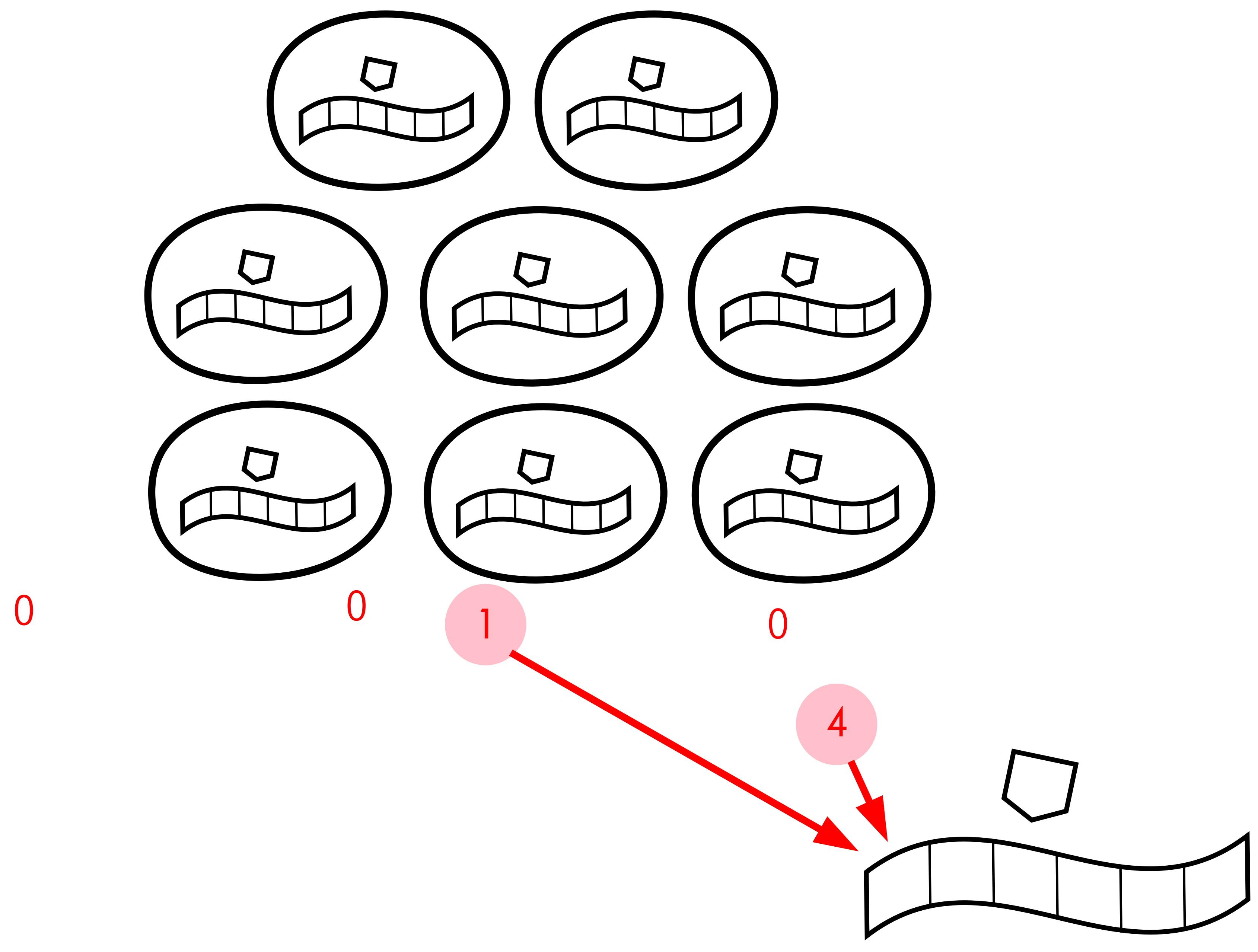
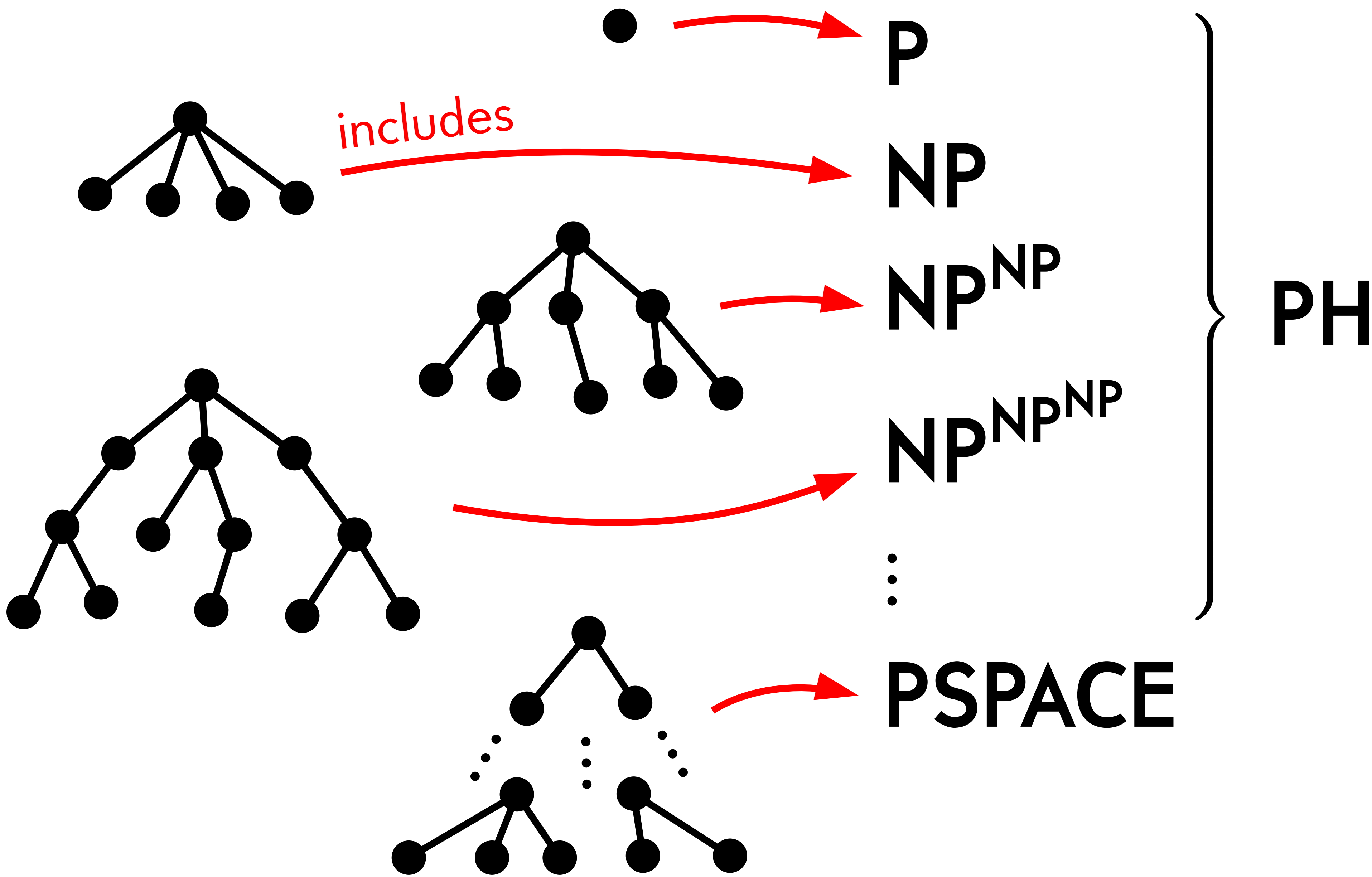


The counting hierarchy



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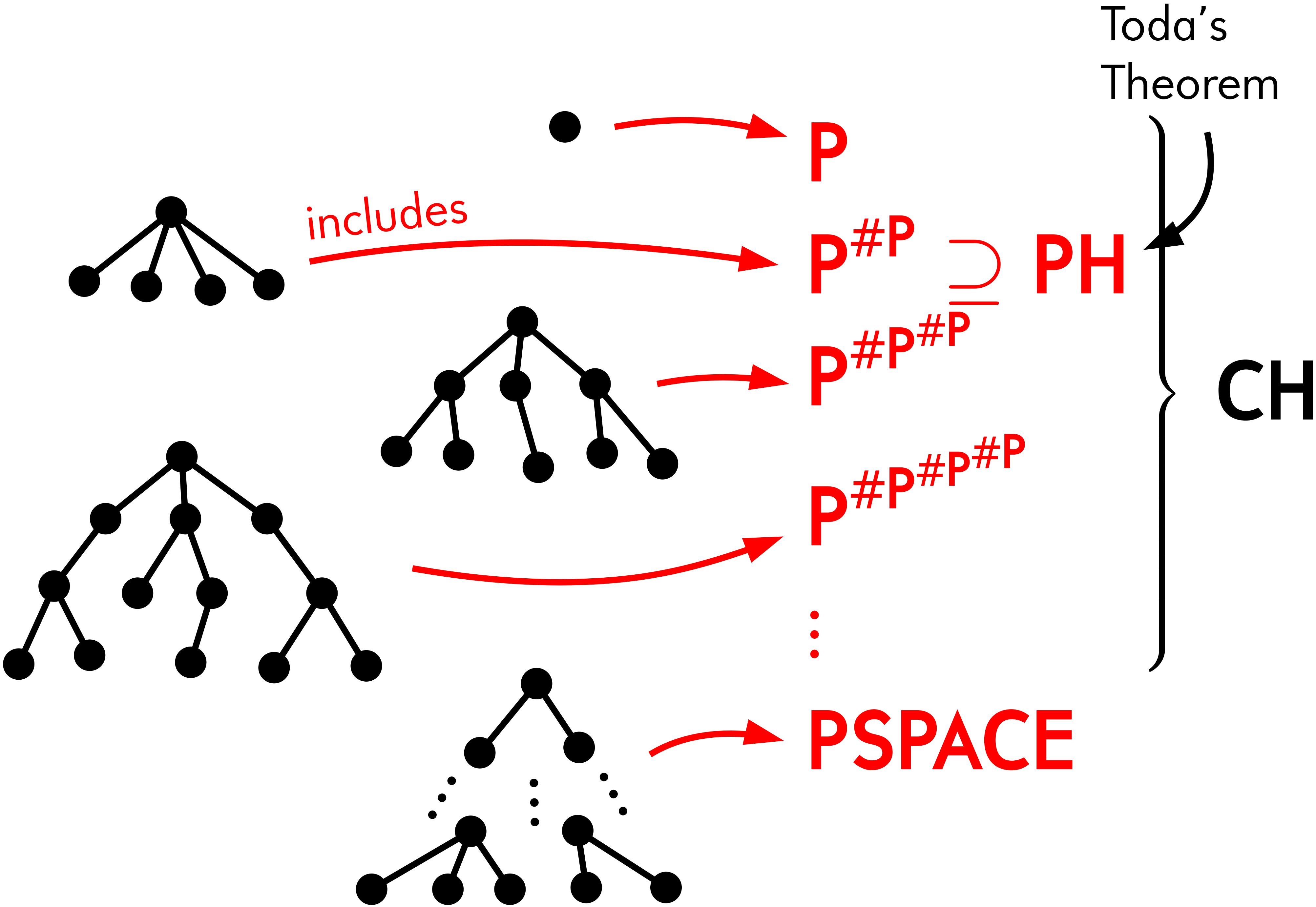
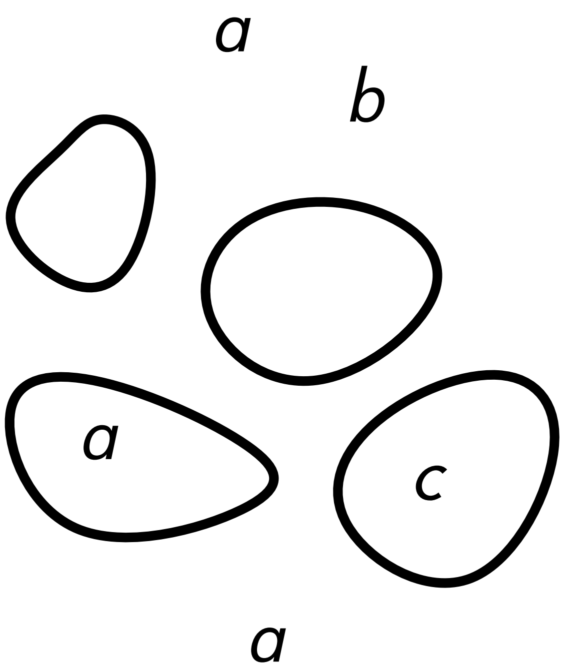


Exact characterisation of $\mathbf{P}^{\#\mathbf{P}}$



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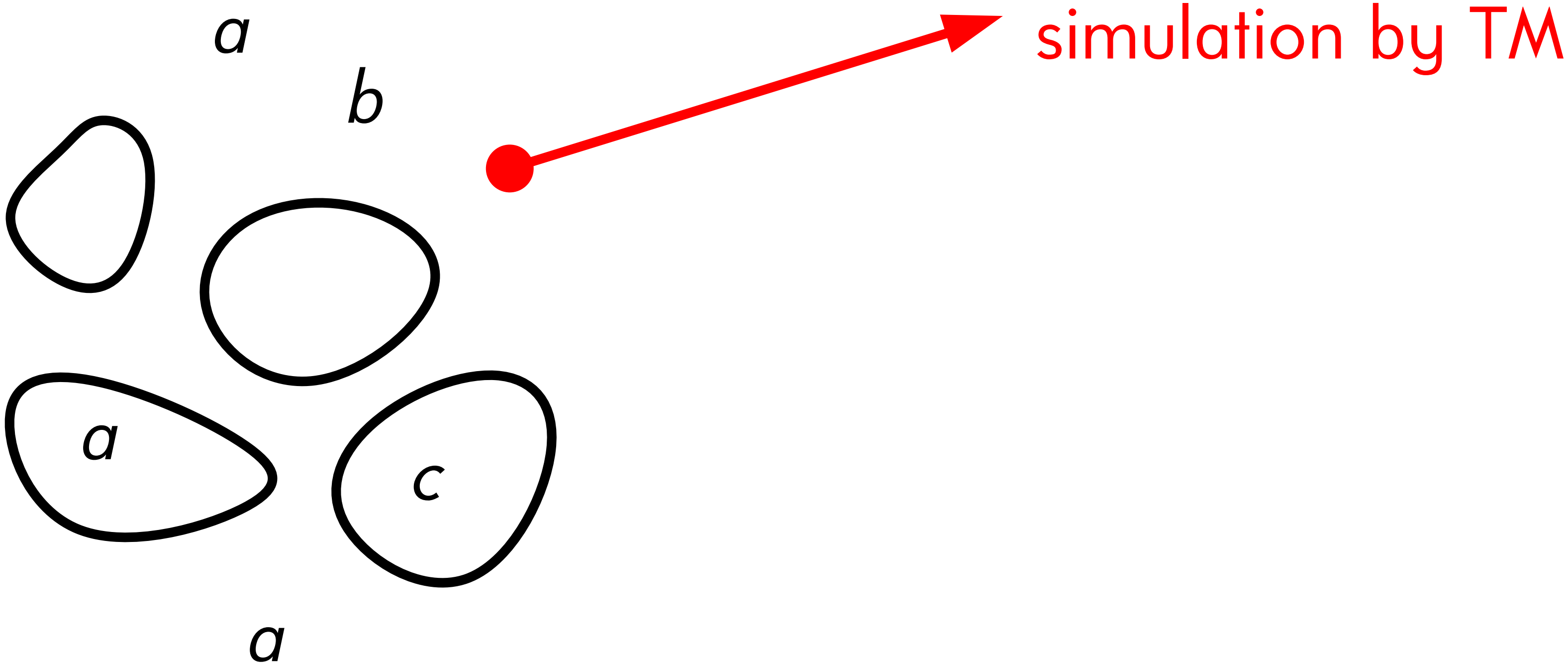

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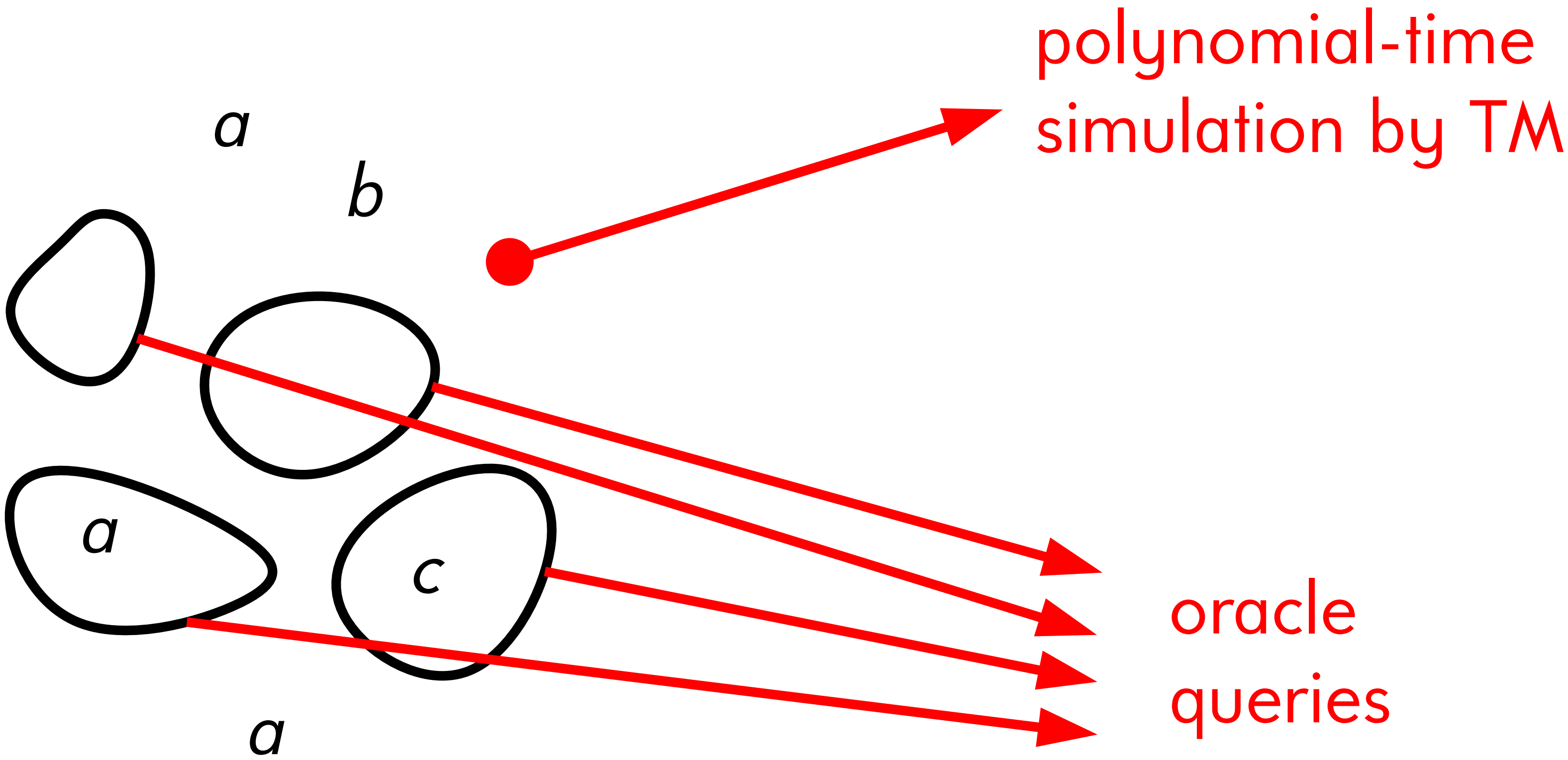

Query simulating dividing membranes

If the dividing membranes receive the sequence of inputs $(m_1, \dots, m_t)$, how many instances of object a are sent out at time $t + 1$?

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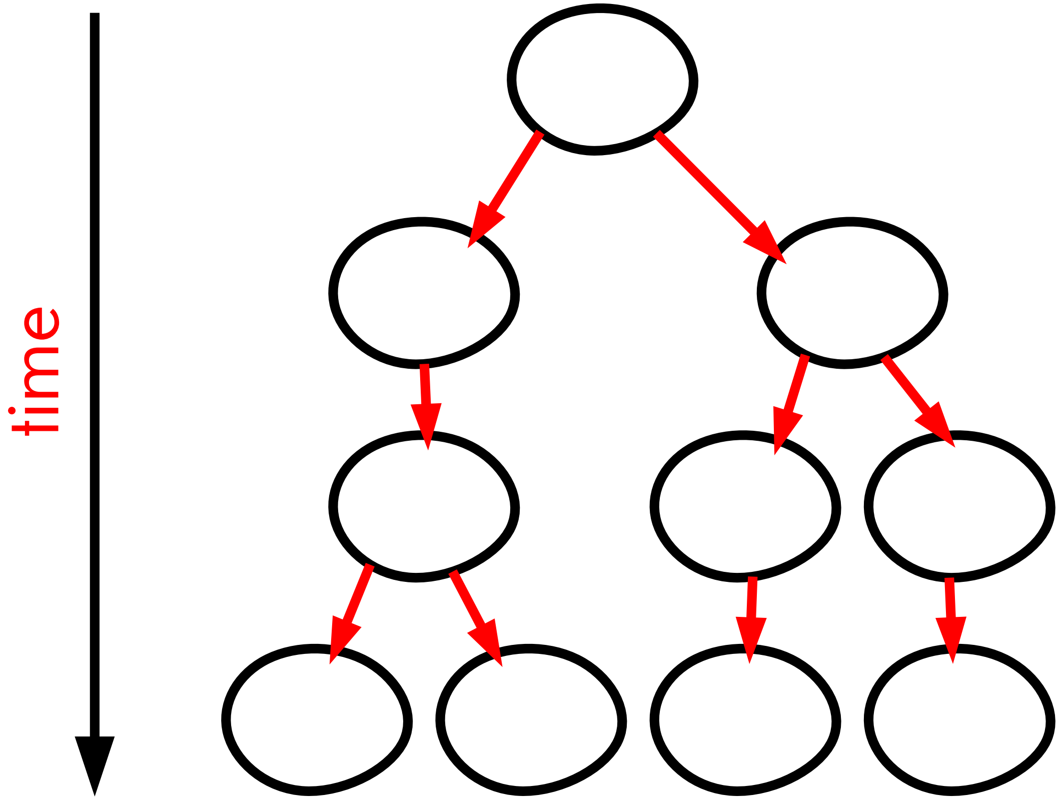

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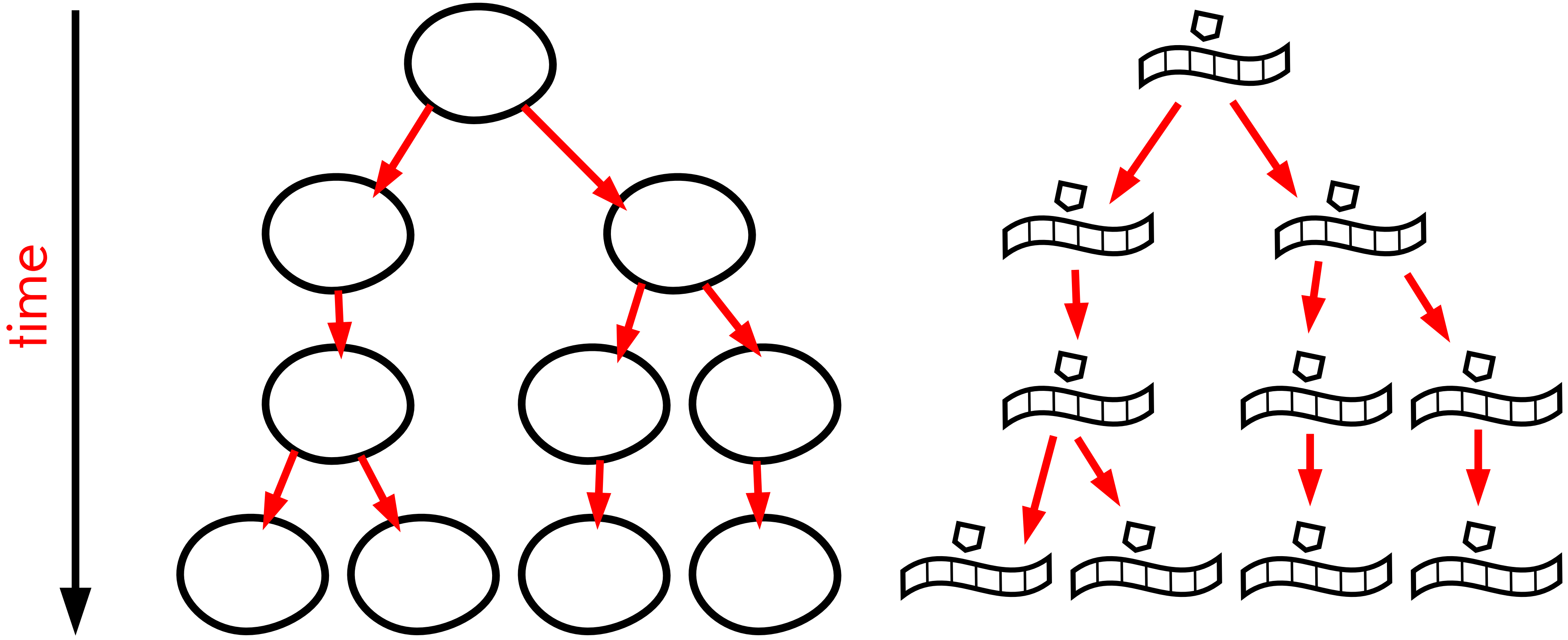

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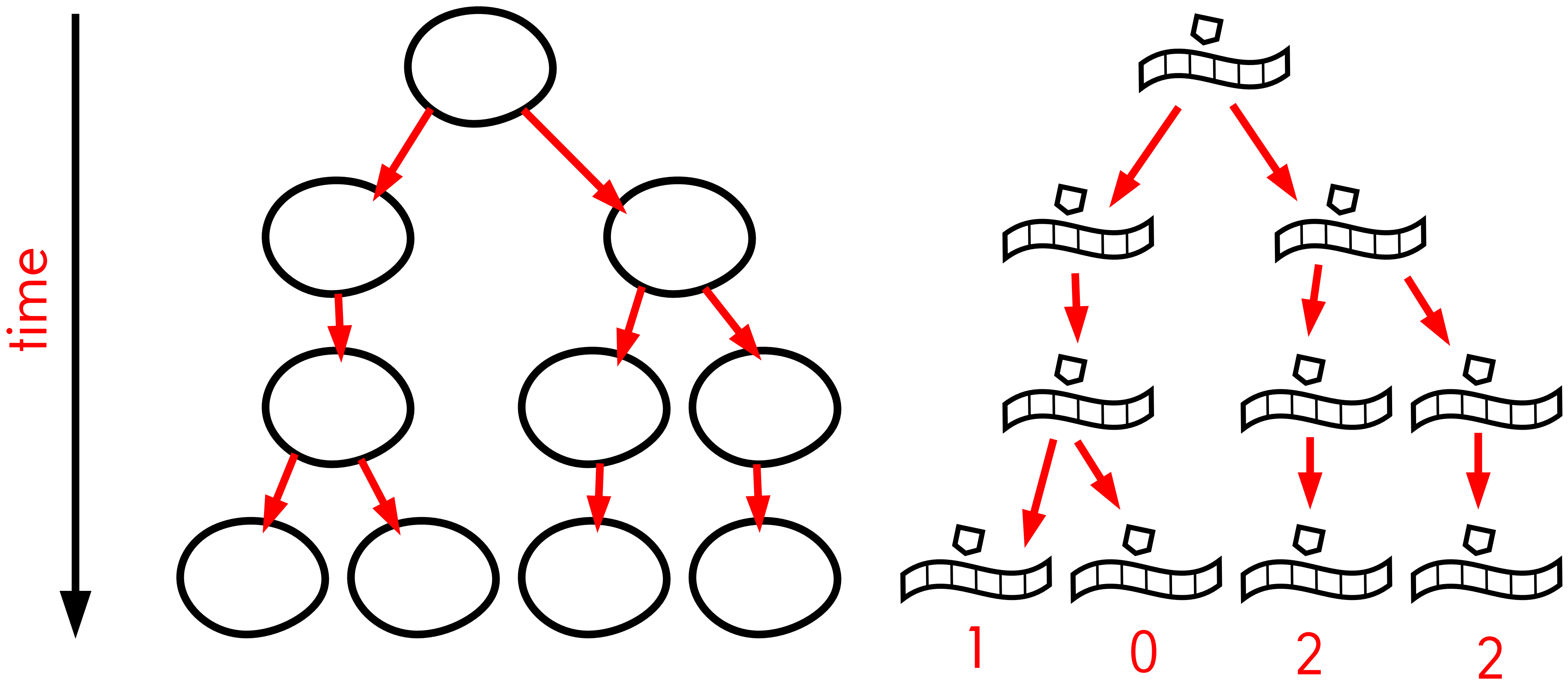

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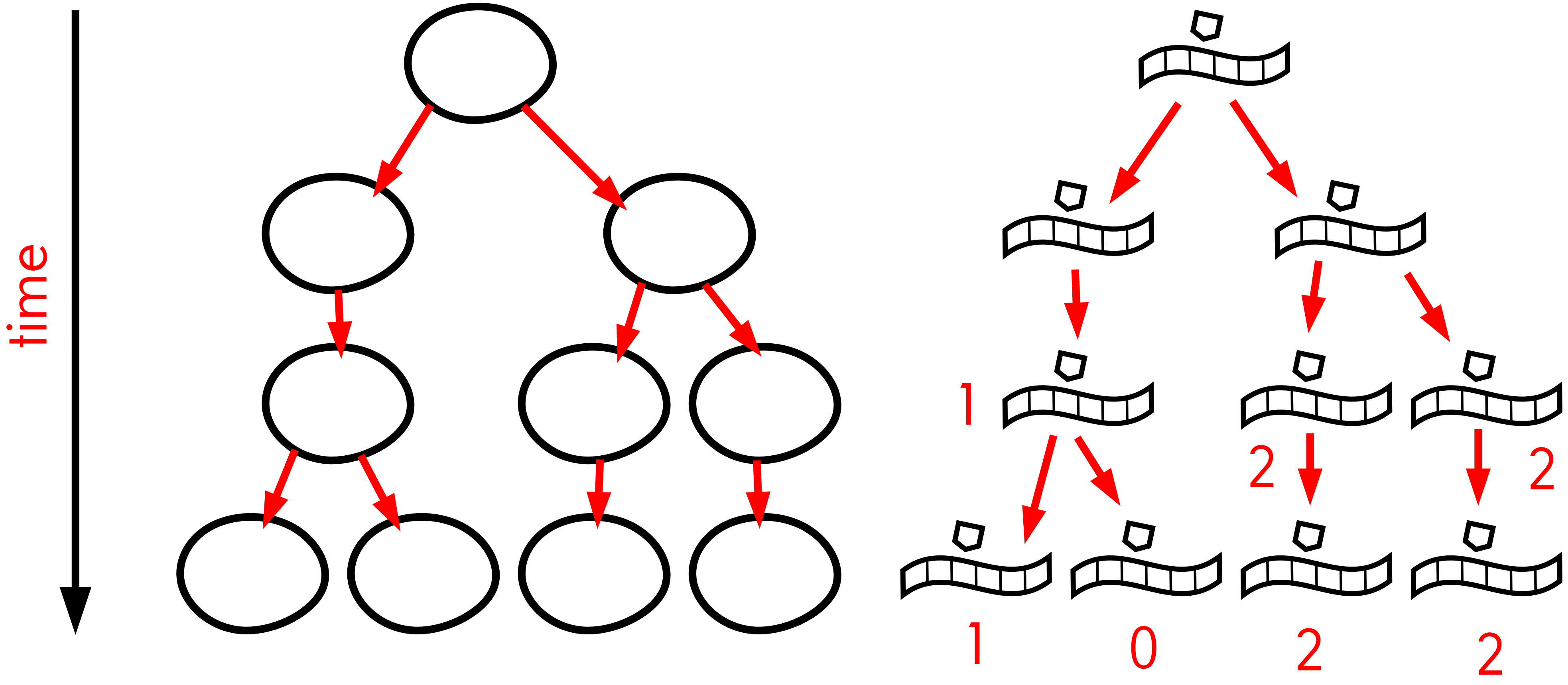

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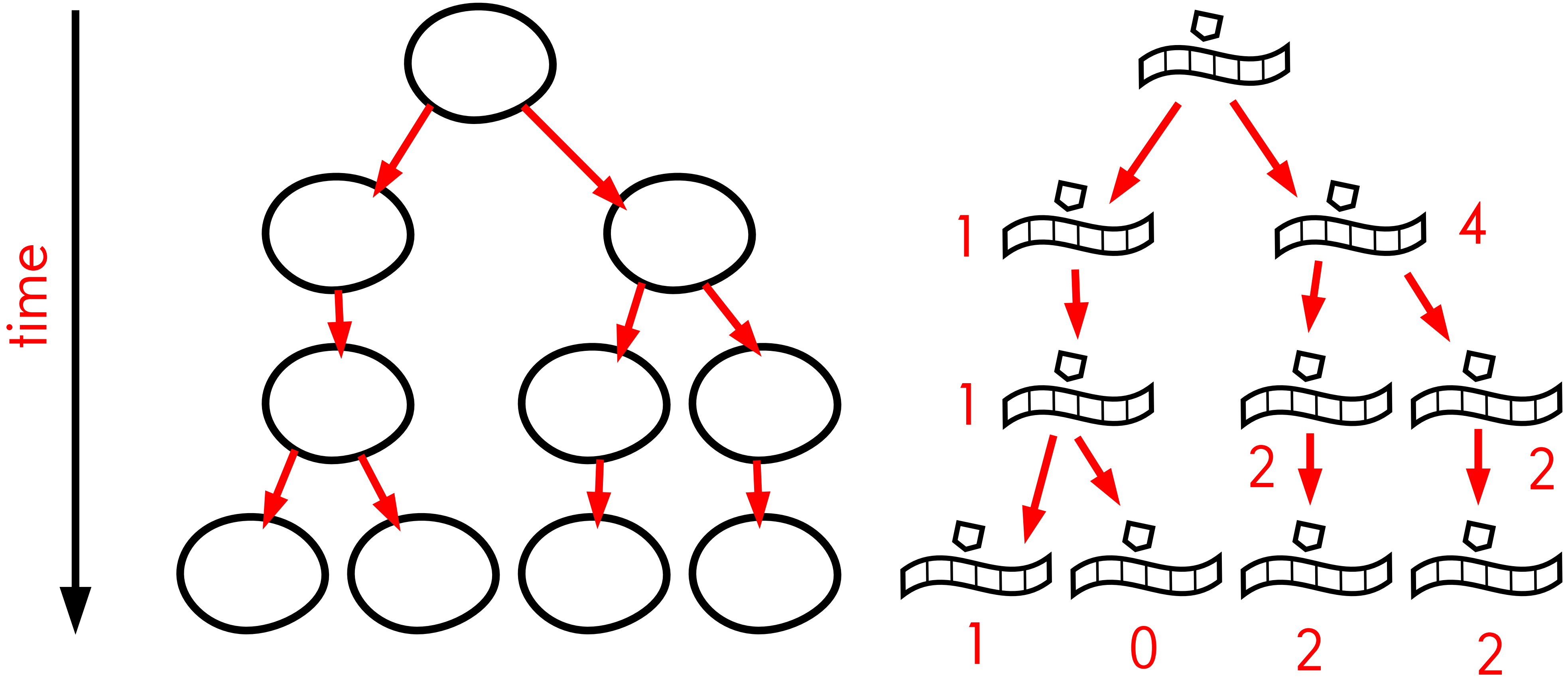

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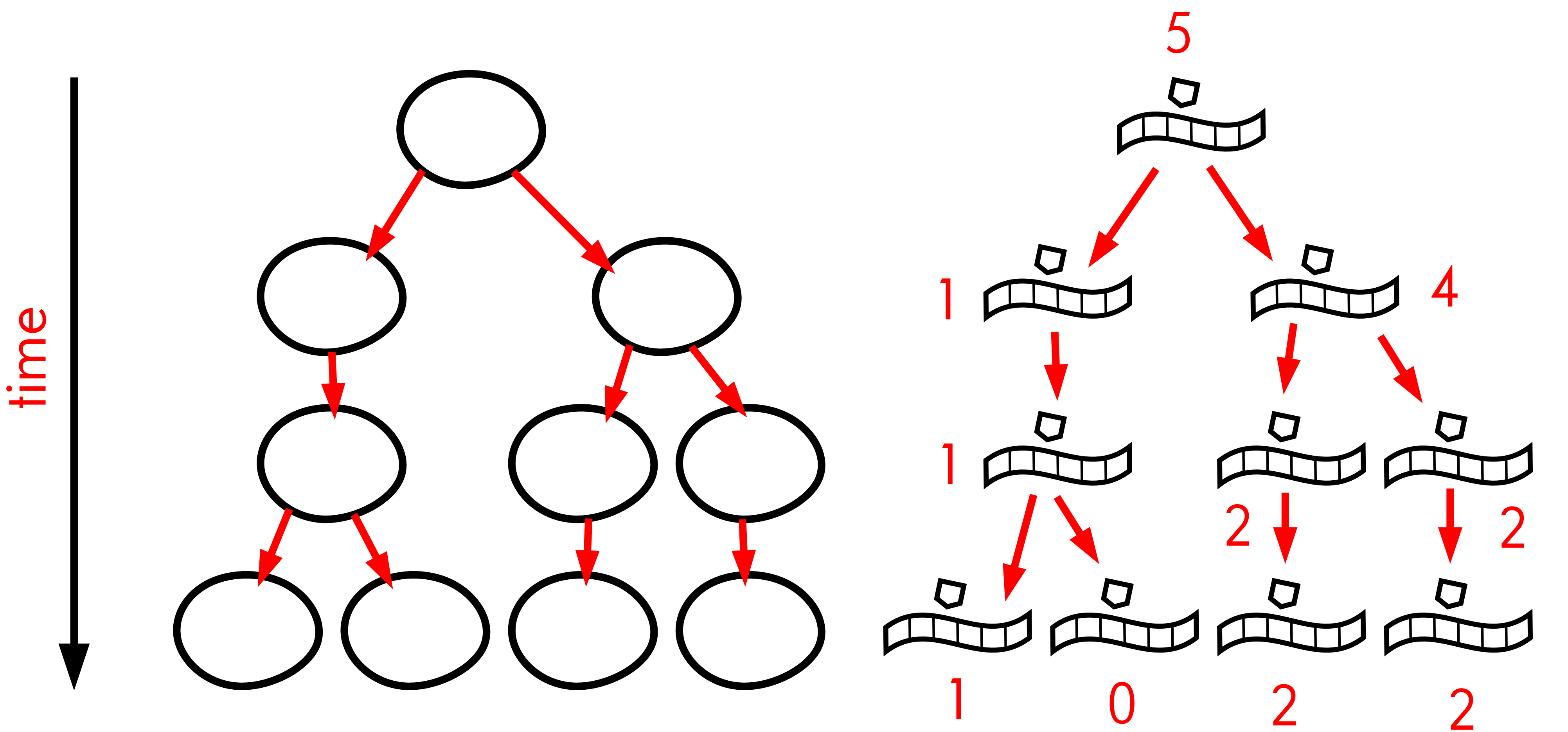
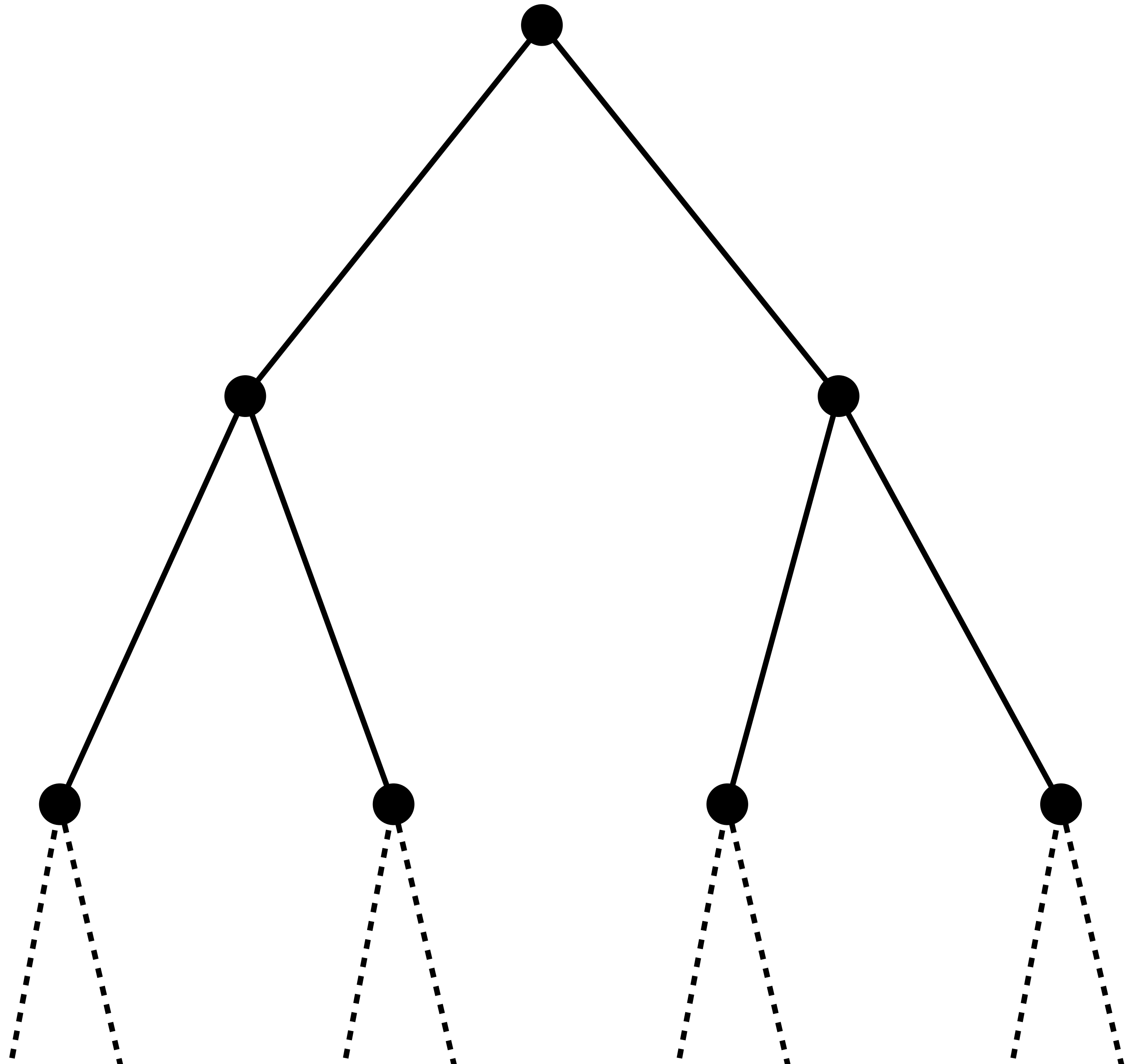


Exact characterisation of $P^{\#P}$

Theorem. Membrane systems where membranes can only divide if they do not contain recursively other membranes characterise $P^{\#P}$ in polynomial time

Solving **PSPACE** efficiently in the “binary tree space”



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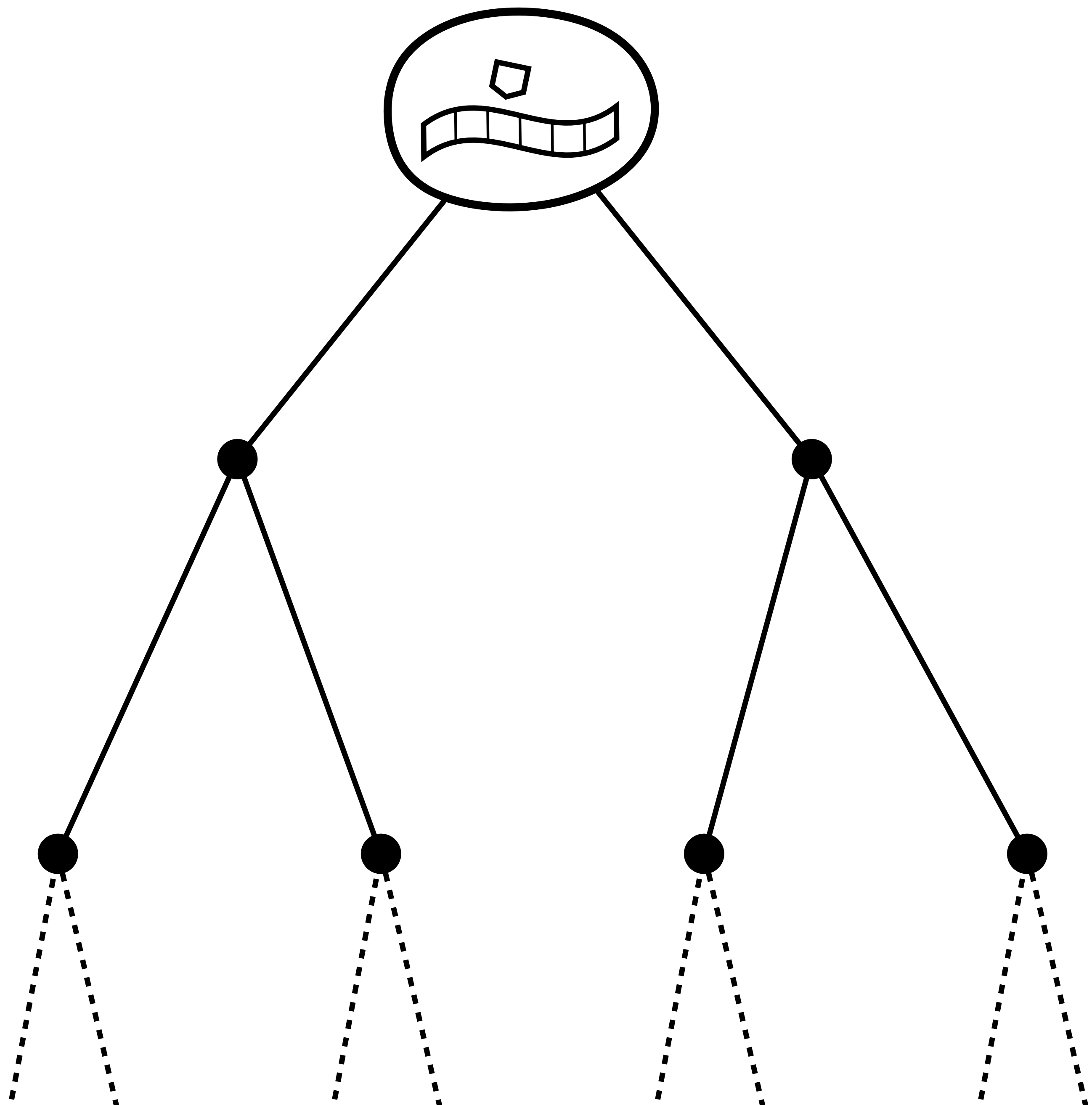

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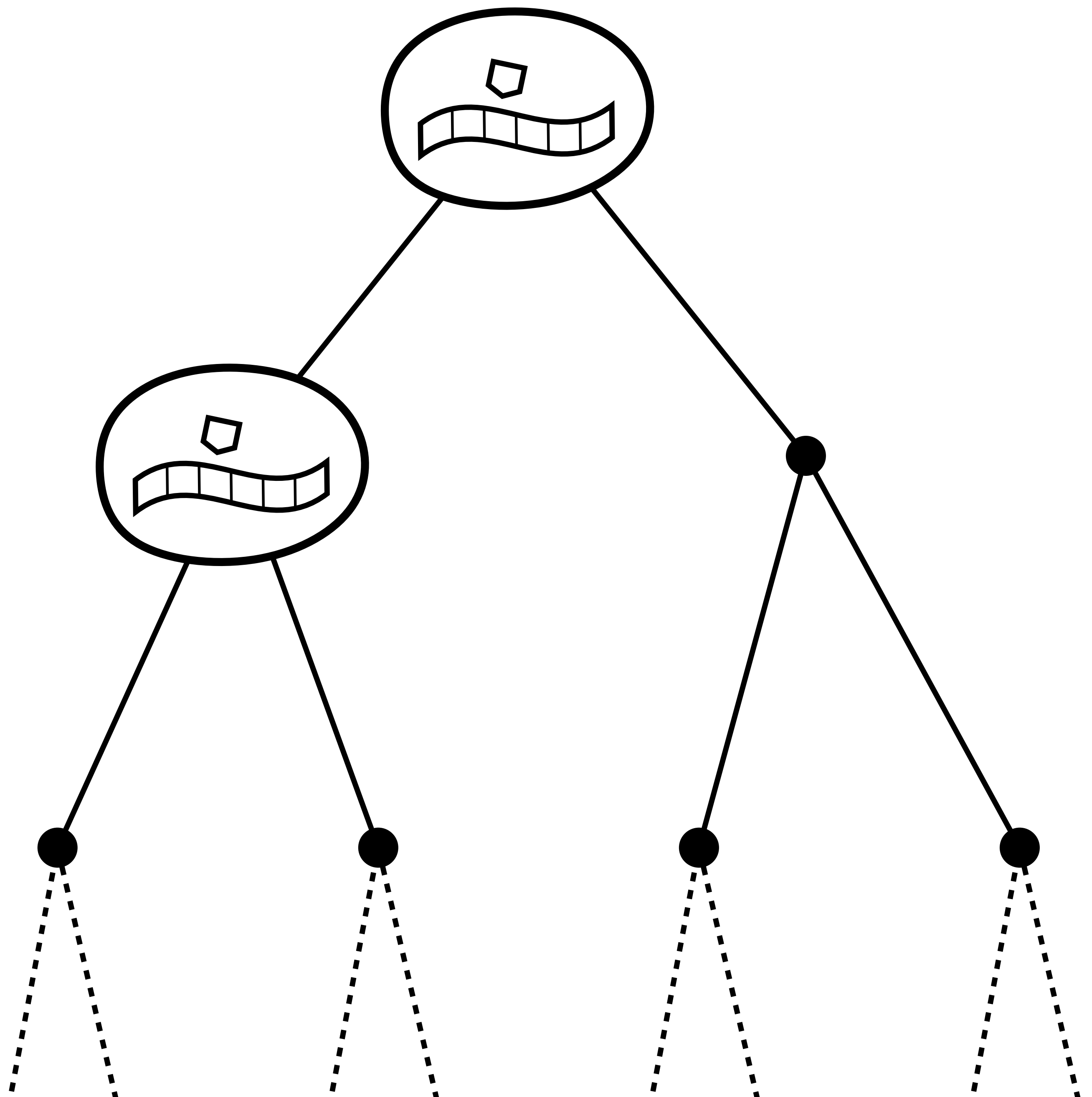

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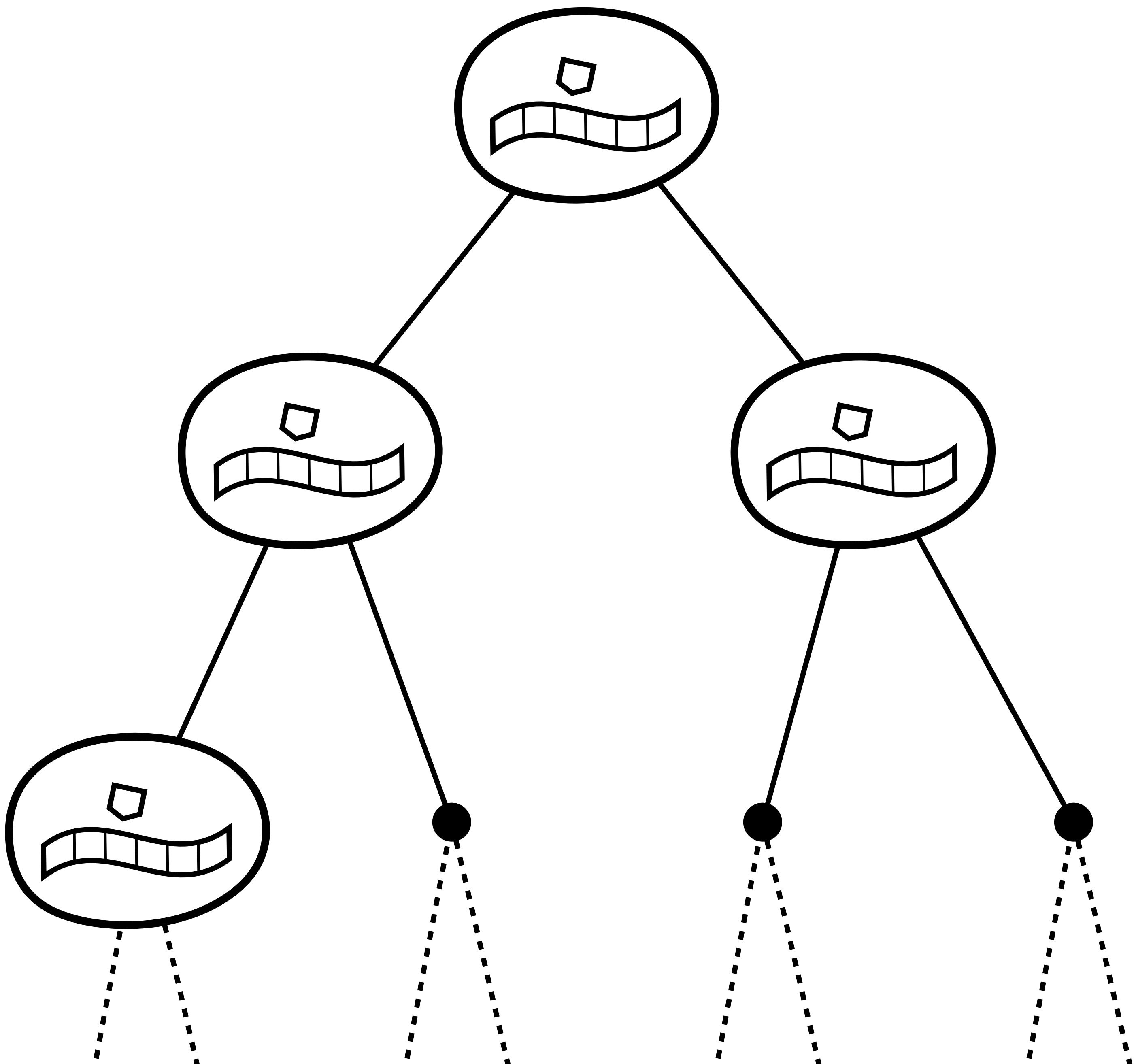

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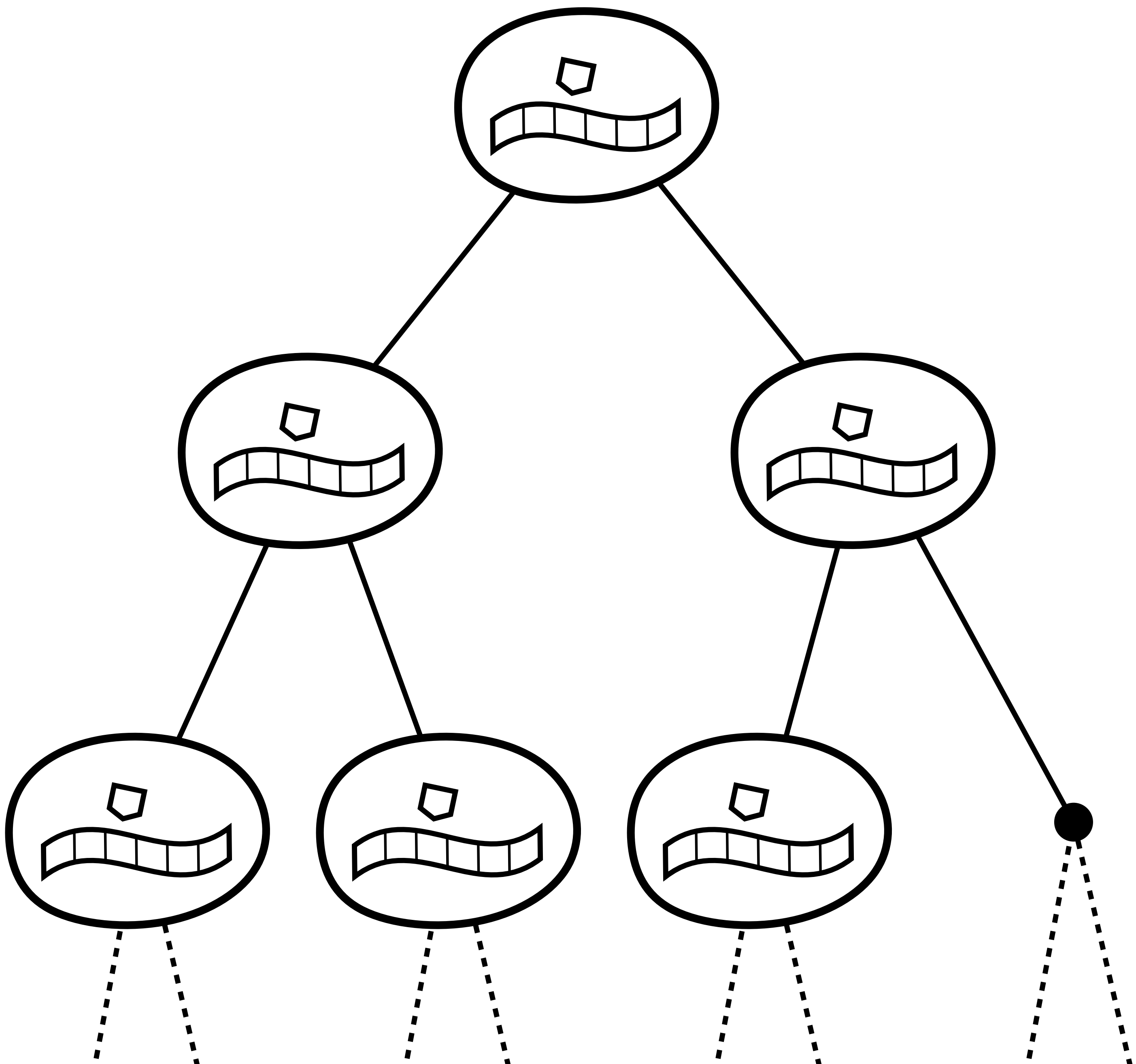

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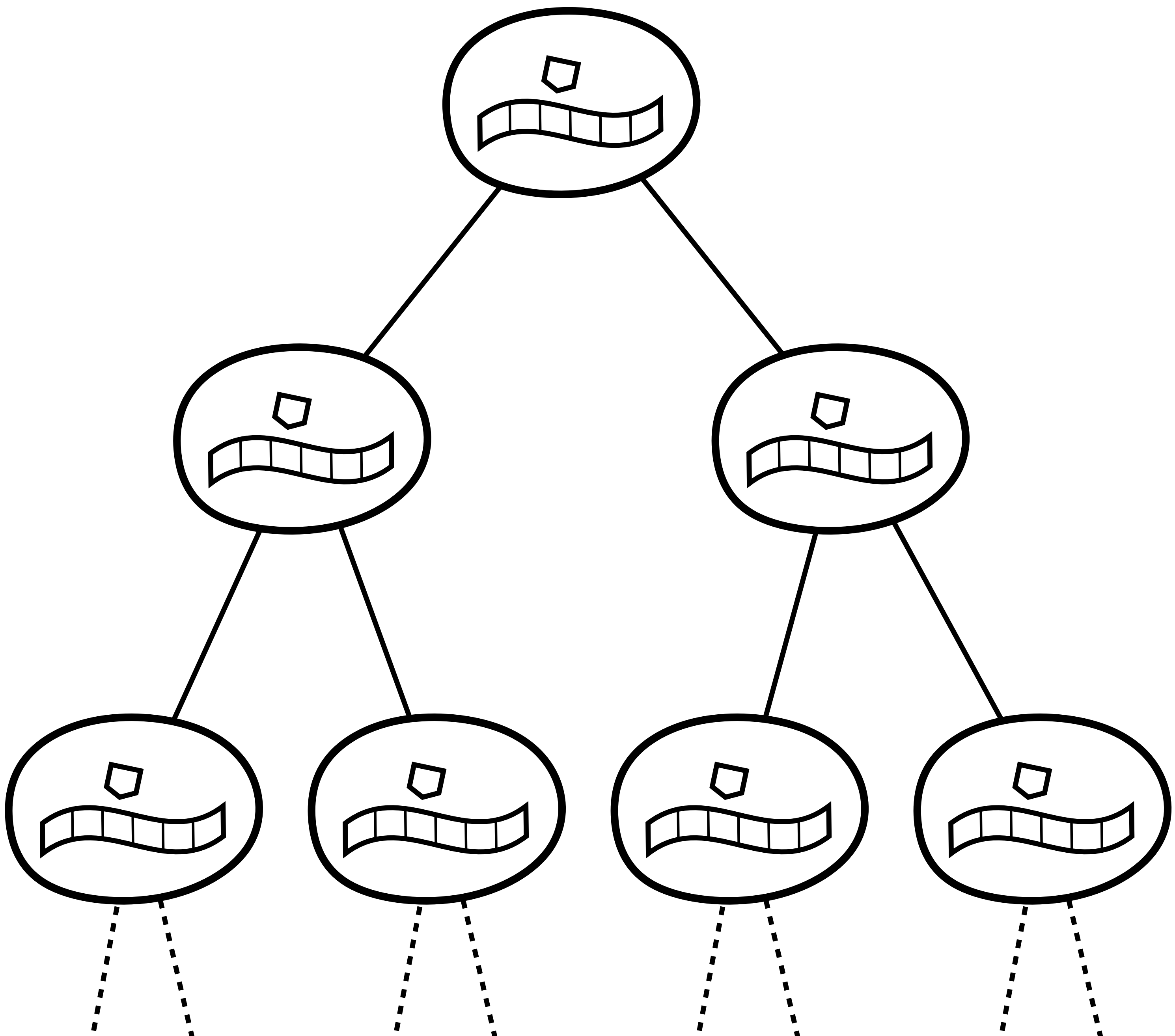
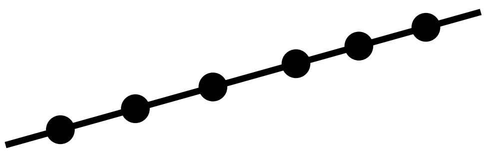
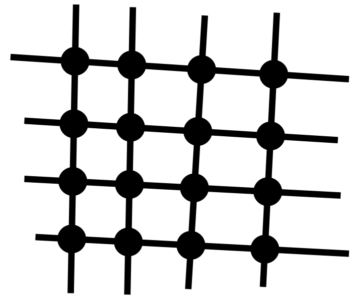
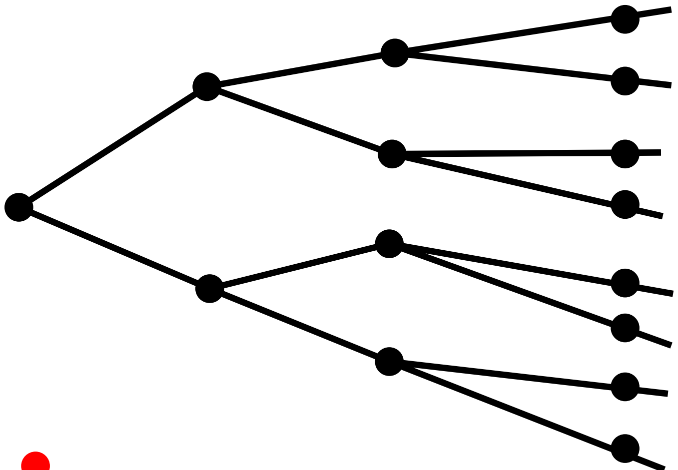


Communication topologies and complexity classes

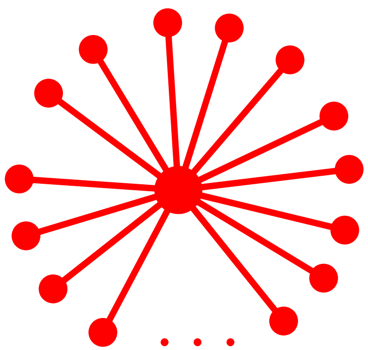




P

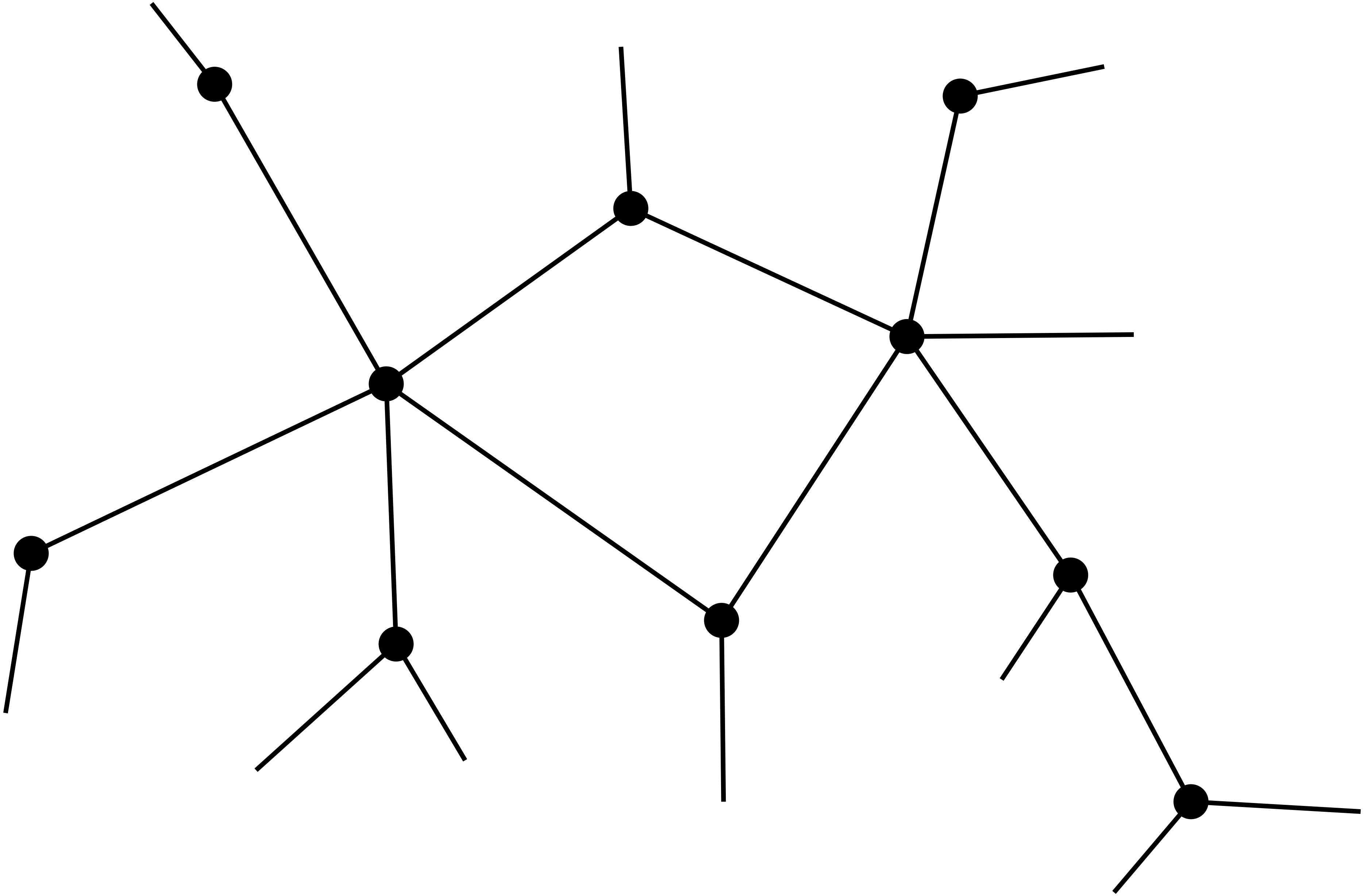


PSPACE

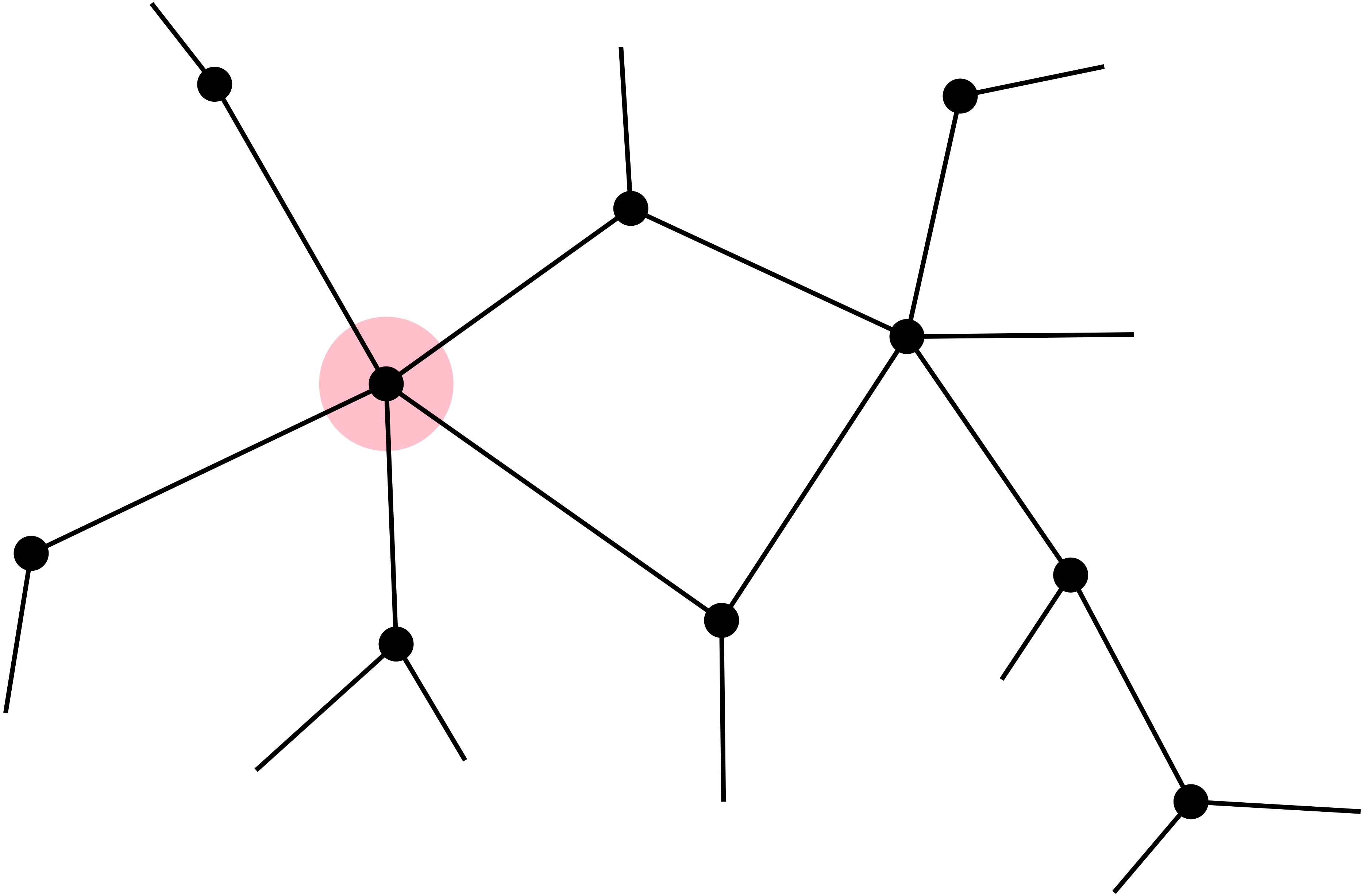


$P^{\#P}$

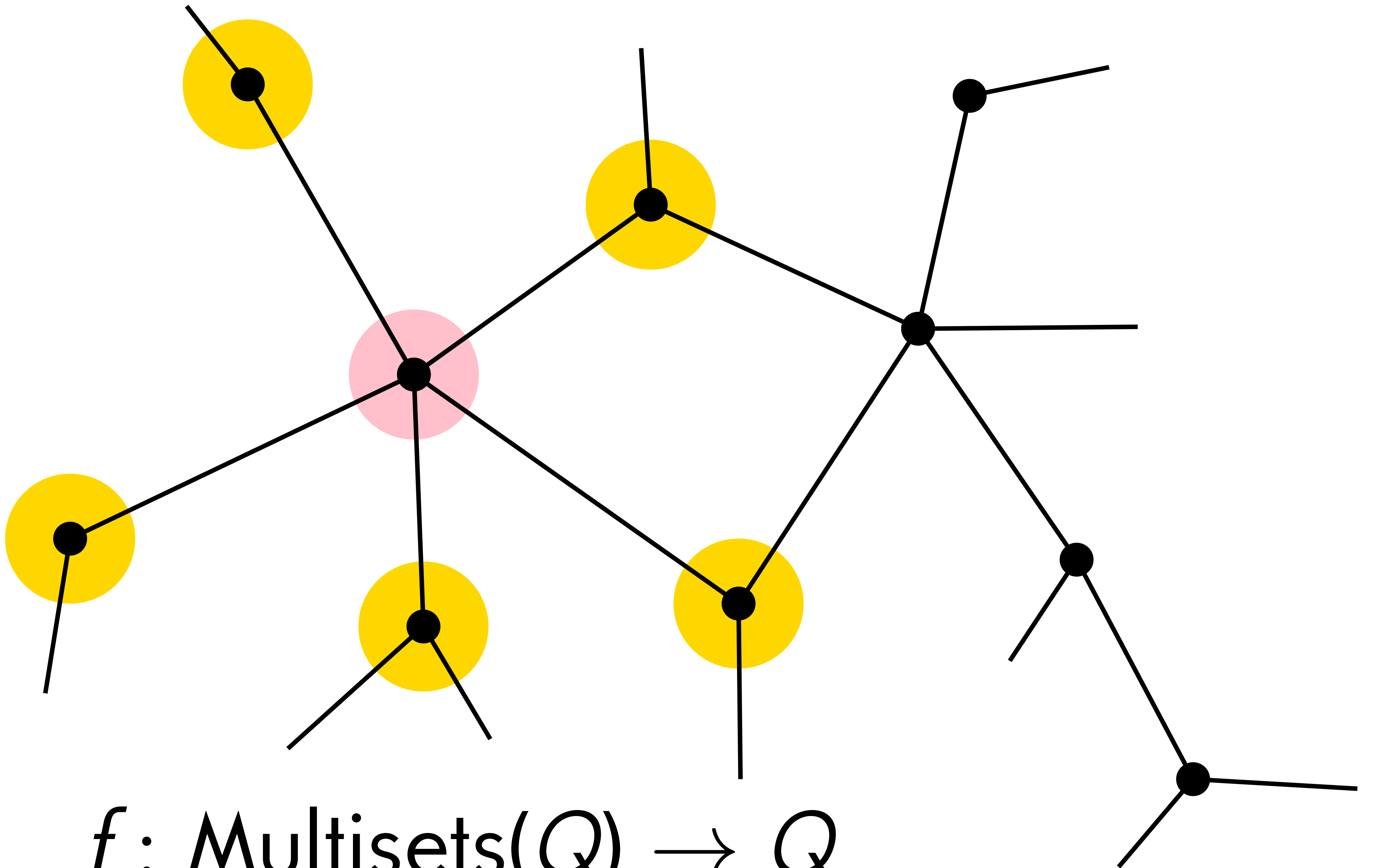
Automata networks over infinite graphs



Automata networks over infinite graphs

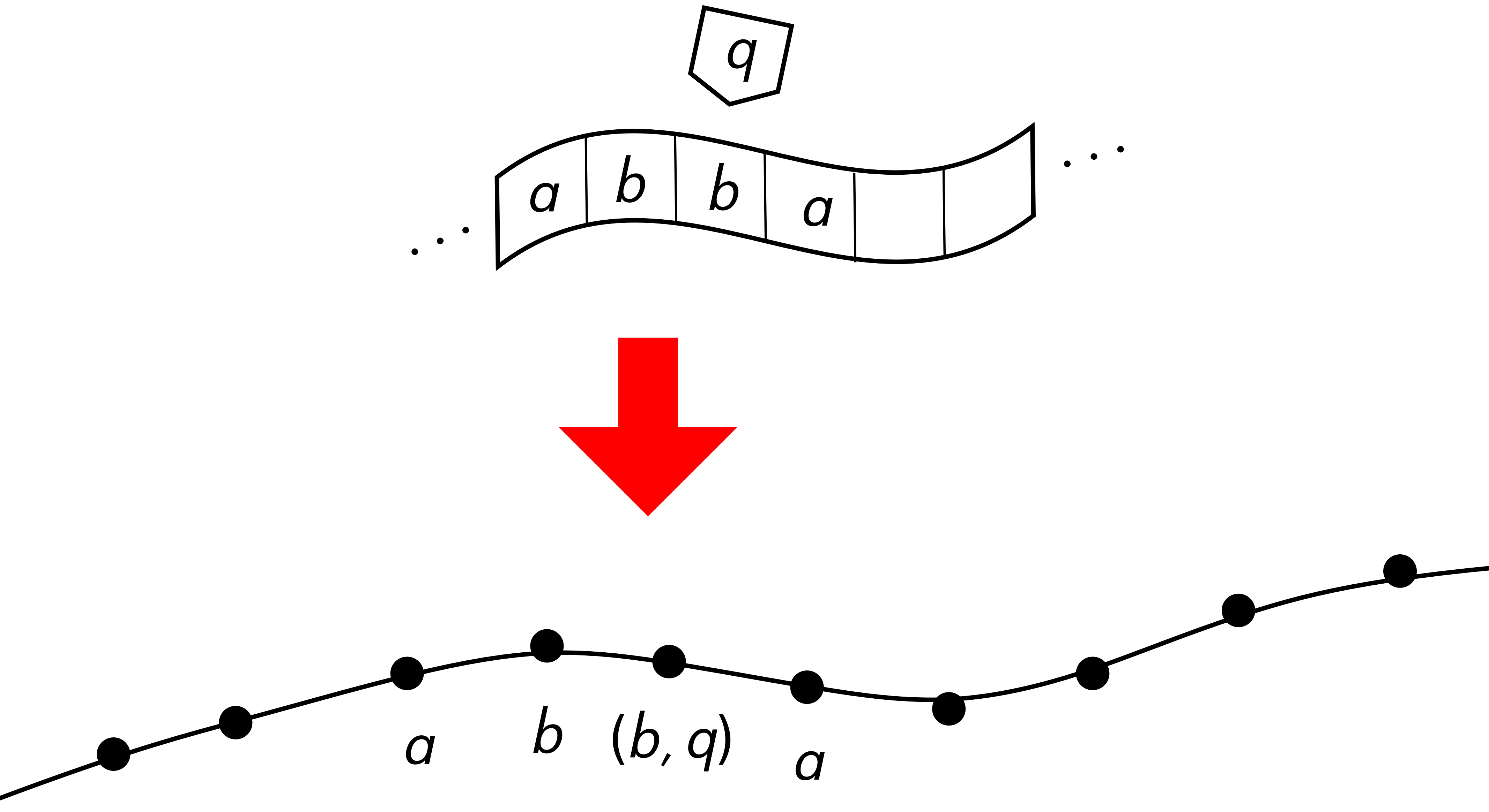


Automata networks over infinite graphs

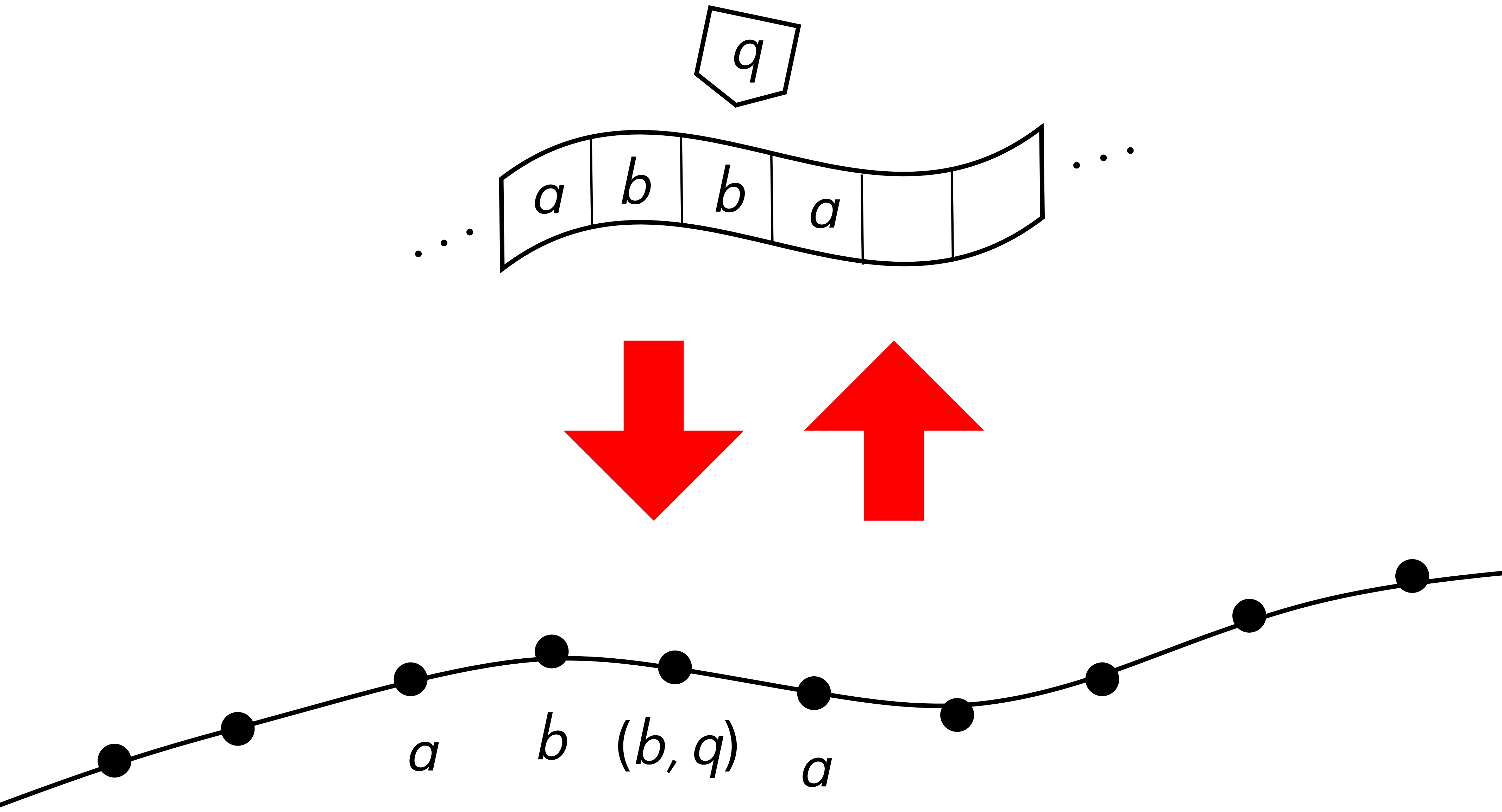


$$f : \mathrm{Multisets}(Q) \to Q$$

1D cellular automata as generalised Turing machines



1D cellular automata as generalised Turing machines



Things to do

- Choose restrictions on the class
  of local transition functions $f\colon \text{Multisets}(Q) \to Q$
  e.g., threshold functions
- Choose a way to encode the input in the initial configuration

Defining new complexity classes

Definition. Given an infinite graph G, let

- **P**(G) be the problems solved by automata networks over G in polynomial time
- **PSPACE**(G) = polynomial space over G
- **EXPTIME**(G) = exponential time over G
- etc.

but also

- **LOGTIME**(G) = logarithmic time over G
- etc.

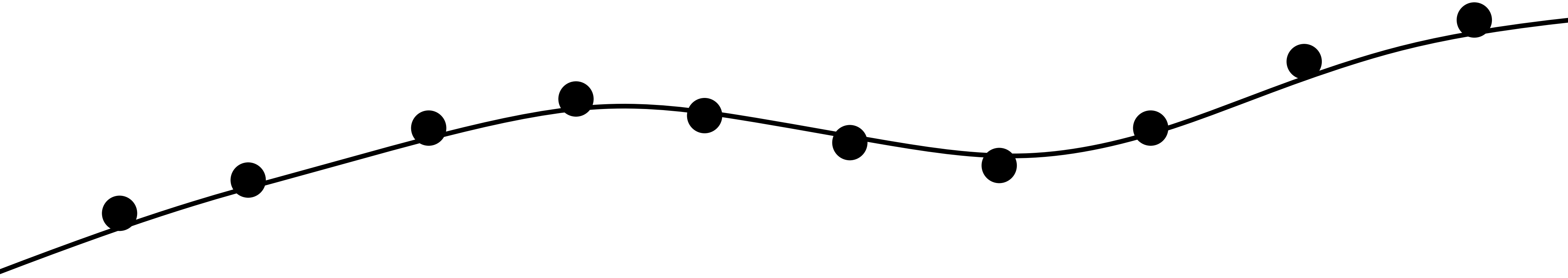
because automata networks are parallel

Preliminary results

- **P**(linear graph) = **P**
- **PSPACE**(linear graph) = **PSPACE**

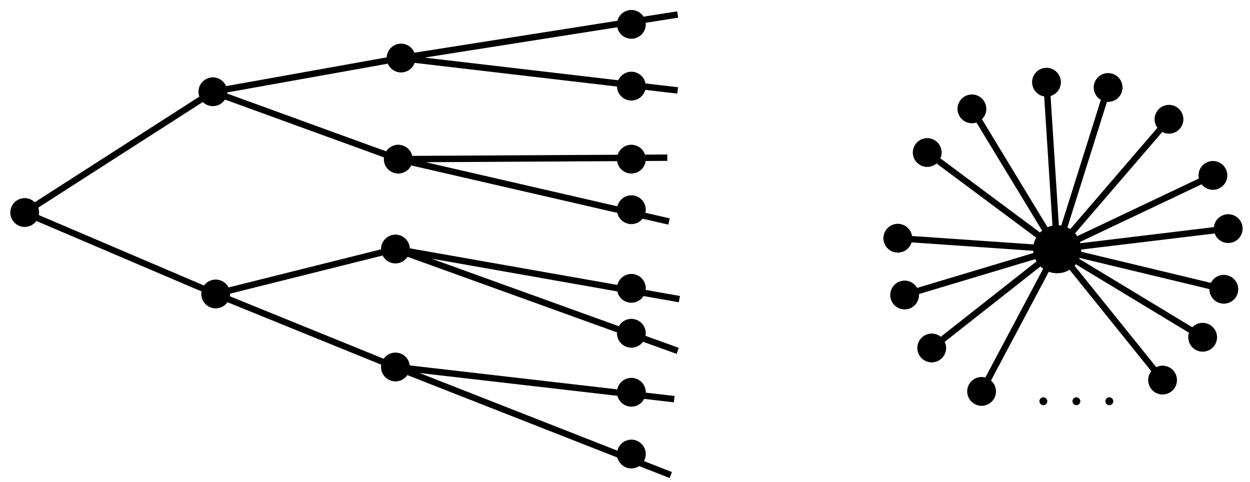
because of the equivalence with Turing machines

- **P**(efficiently computable graph in $\mathbf{R}^d$) = **P**



Expected results

- **P**(infinite binary tree) = **PSPACE**
- **P**(infinite star or variant thereof) = $\mathbf{P}^{\#\mathbf{P}}$



- **P**(non-computable graph) includes undecidable problems

Hopeful developments of the theory

- Find graphs (or classes of graphs) characterising all standard complexity classes
- Find new intermediate complexity classes using "natural" graphs

Hopeful developments of the theory

- Find graphs (or classes of graphs) characterising all standard complexity classes
- Find new intermediate complexity classes using "natural" graphs

- Find algorithms working on all graphs or on certain classes of graphs
- Discover how graph-theoretic or geometric properties of the graphs can speed up or slow down algorithms e.g., algorithms running in time $\Theta(n^{f(d)})$ in $\mathbf{R}^d$

Applications

- Same applications as automata networks (e.g., biology)
- Theory (and practice) of distributed algorithms
- Low-level hardware design
- Machine learning (variants of deep learning? non-Euclidean learning?)

and, of course

- Computability and complexity theory

Thanks for your attention!
Merci de votre attention!

Any questions?