Do complexity classes for P systems have complete problems?

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Classes with known complete problems

- In several cases, the answer is known and affirmative...
- ...simply because several complexity classes for P systems are simply **P** or **PSPACE**
- **P** and **PSPACE** have well-known complete problems
- Complexity classes for P systems inherit them

The importance of complete problems

- Suppose L complete for $\mathbf{C}$ and $\mathbf{C}$ closed under reductions
- Then solving L implies solving every problem in $\mathbf{C}$
  - Useful to prove upper bounds on $\mathbf{C}$
- Most important classes for TMs have complete problems
- But we conjecture that some do not (for instance $\mathbf{PH}$)

Bounded acceptance problem for Turing machines

- Let L be the set of triples $(M, x, 1^t)$ such that
  - M is (the description of a) DTM
  - x is a string
  - 1^t is the unary notation for $t \in \mathbf{N}$
  - M accepts x in t steps

Theorem

L is **P***-complete w.r.t. logspace reductions* □

Let's change the notion of acceptance for P systems

Just for the duration of this talk!

- A P system accepts iff it sends out *yes*
- A P system rejects iff it sends out *no*
- Only one between *yes*, *no* is sent out
- All computations agree (confluence)
- But the P system is not required to halt after having produced the result

Bounded acceptance for P systems is complete

- Let $\mathcal{AM}(-\mathrm{ne})$ denote P systems with elementary active membranes
  - Three polarizations
  - All rules except nonelementary division
- Let BAP be the set of pairs $(\Pi, 1^t)$ where
  - $\Pi \in \mathcal{AM}(-\mathrm{ne})$
  - 1^t is the unary notation for $t \in \mathbf{N}$
  - The "first computation" of Π accepts in t steps

Theorem

BAP *is* $\mathbf{PMC}^{\star}_{\mathcal{AM}(-\mathrm{ne})}$*-complete w.r.t. polytime reductions, assuming the alternative definition of acceptance*

Lemma 1. BAP is $\mathbf{PMC}^{\star}_{\mathcal{AM}(-\mathrm{ne})}$-hard

Proof.

- Let $L \in \mathbf{PMC}^{\star}_{\mathcal{AM}(-\mathrm{ne})}$
- Let M be a DTM constructing $\mathbf{\Pi}$ in polytime
- $\mathbf{\Pi} = \{\Pi_x \mid x \in \Sigma^\star\} \subseteq \mathcal{AM}(-\mathrm{ne})$ is a family deciding L in polytime for some polynomial p
- Let $R(x) = \left(M(x), 1^{p(|x|)}\right) = \left(\Pi_x, 1^{p(|x|)}\right)$
- R is polytime computable
- The following assertions are equivalent
  - $x \in L$
  - The first computation of Π_x accepts in $p(|x|)$ steps
  - $R(x) \in \mathrm{BAP}$
- Thus R is a polytime reduction $L \leq \mathrm{BAP}$ □

Lemma 2. BAP $\in \mathbf{PMC}^{\star}_{\mathcal{AM}(-\mathrm{ne})}$

Proof.

- Let M be the DTM defined by $M(\Pi, 1^t) = \Pi'$
- Π' is Π modified as follows
  - If Π accepts in t steps, then Π' accepts
  - Otherwise Π' rejects
- Idea of the construction (details omitted for brevity)
  - Enclose Π with a few extra membranes, which stop any output from Π after t steps
  - In that case, object *no* is sent out
  - Only $O(1)$ extra time required
- M runs in polytime $\Rightarrow$ semi-uniform construction □

Generalisation to other variants of P systems

- The construction of Lemma 2 uses only evolution and send-out communication rules
  - Thus we can find complete problems for several other classes
- We can prove Lemma 2 using dissolution rules instead of polarizations
  - A connection to the P conjecture?
- The argument also works in the uniform case

Fundamental open question

- Recall that we changed the notion of acceptance
- Lemma 1 (BAP is $\mathbf{PMC}^{\star}_{\mathcal{AM}(-\mathrm{ne})}$-hard) does not depend on that
- The proof of Lemma 2 (BAP $\in \mathbf{PMC}^{\star}_{\mathcal{AM}(-\mathrm{ne})}$) unfortunately does
- Can we prove it in the usual setting?
- Recall that $\mathbf{NP} \cup \mathbf{coNP} \subseteq \mathbf{PMC}^{\star}_{\mathcal{AM}(-\mathrm{ne})} \subseteq \mathbf{PSPACE}$
  - Also $\mathbf{NP} \cup \mathbf{coNP} \subseteq \mathbf{PH} \subseteq \mathbf{PSPACE}$
  - If $\mathbf{PMC}^{\star}_{\mathcal{AM}(-\mathrm{ne})}$ has complete problems then $\mathbf{PMC}^{\star}_{\mathcal{AM}(-\mathrm{ne})} \neq \mathbf{PH}$ (probably)

True or false?

Problem

For each alternative recogniser P system Π, giving output in polynomial time, we can construct a regular recogniser P system Π' such that

- *The construction only takes polynomial time*
- *They give the same result*
- *Π' always halts*
- *The slowdown is at most polynomial*

The last point seems to be the difficult one!

Any questions?

¡Gracias!

Thank you!