Fixed points and attractors of reaction systems

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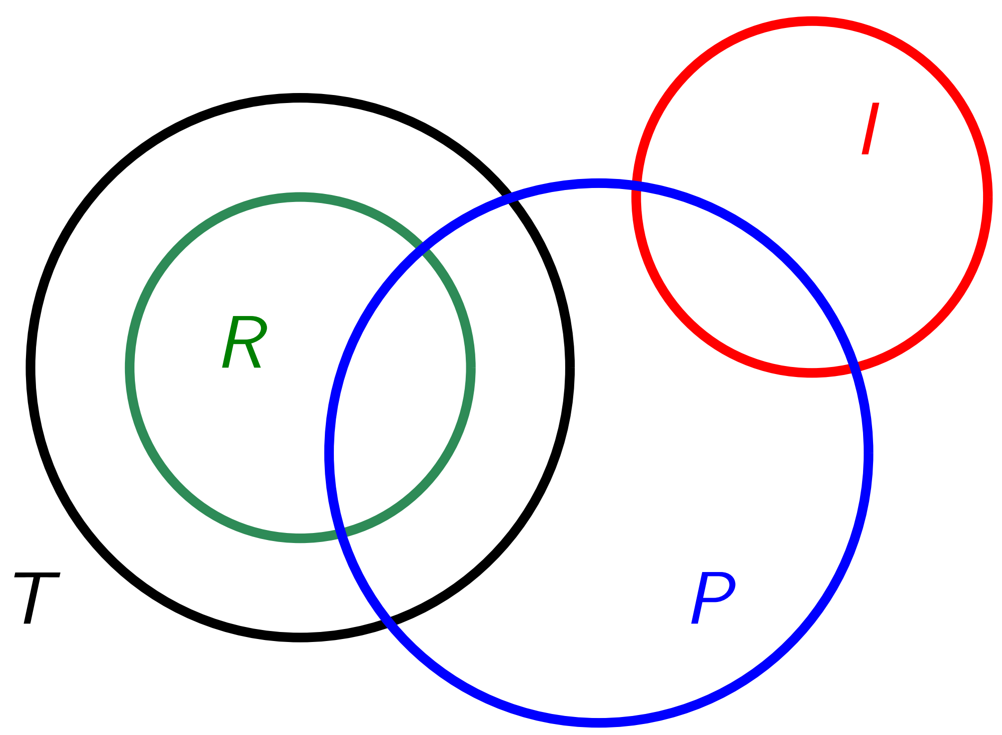
Reactions

$$a = (R, I, P) \qquad \in 2^S \times 2^S \times 2^S$$

$$\text{res}_a(T) = \begin{cases} P & \text{if } R \subseteq T \text{ and } I \cap T = \emptyset \\ \emptyset & \text{otherwise} \end{cases}$$

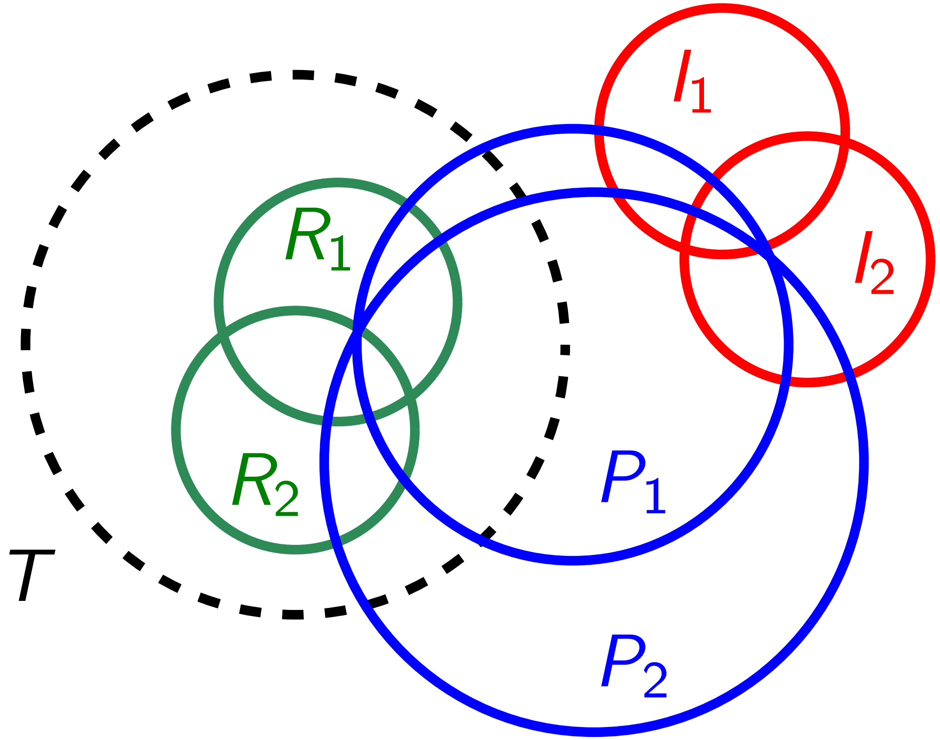


Reaction systems

$$\mathcal{A} = (S, A)$$

$$\text{res}_{\mathcal{A}}(T) = \bigcup\{\text{res}_a(T) : a \in A\}$$

$$\begin{cases} \text{no multiplicities} \\ \text{no competition} \\ \text{no permanence} \end{cases}$$

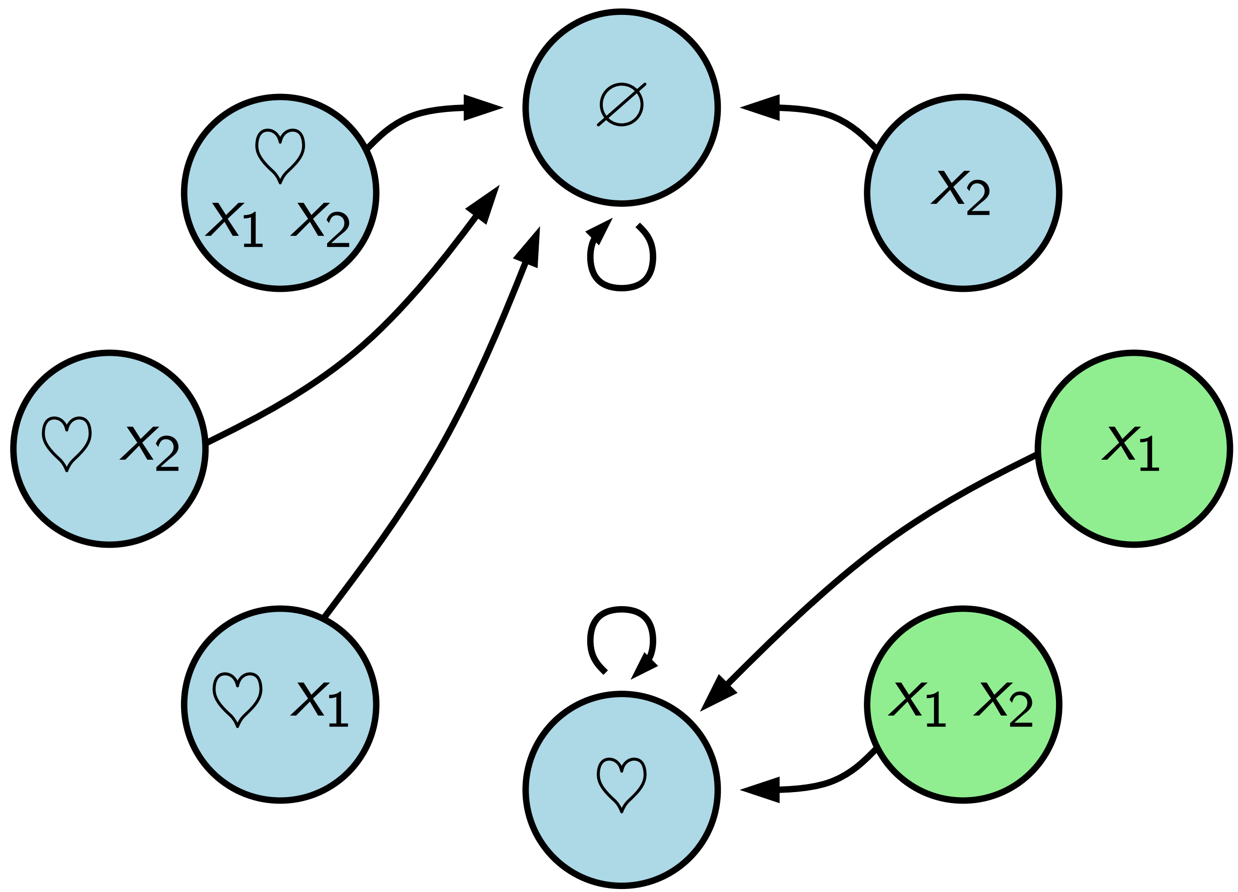


An example: evaluating DNF formulae

$\varphi \equiv (x_1 \wedge \neg x_2) \vee (x_1 \wedge x_2)$

$(\mathrm{pos}(\varphi_j), \mathrm{neg}(\varphi_j) \cup \{\heartsuit\}, \{\heartsuit\})$

$(\{\heartsuit\}, \{x_i\}, \{\heartsuit\})$



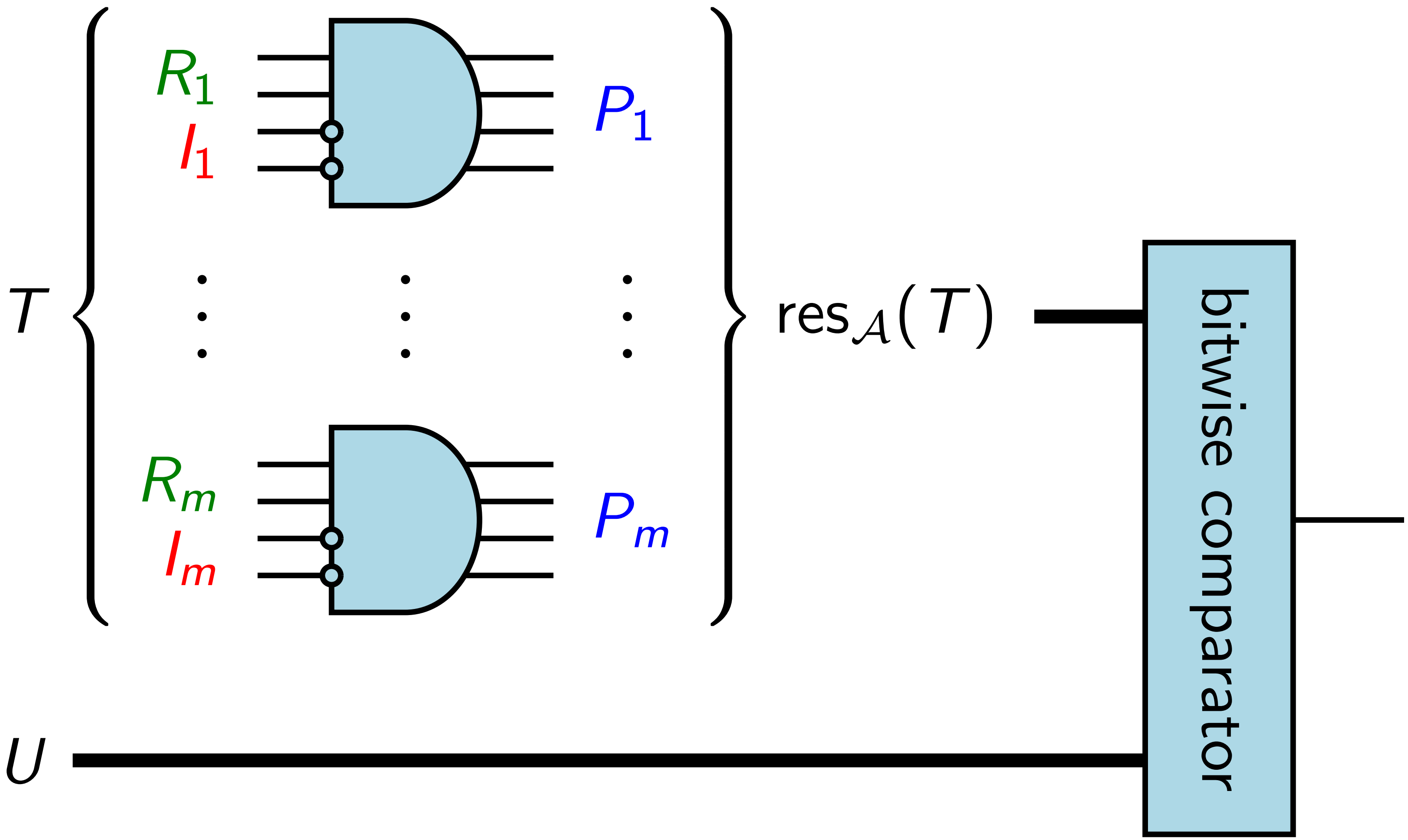
Dynamics of reaction systems

$$\mathcal{A} = (S, A) \quad \Longrightarrow \quad (\mathrm{res}_{\mathcal{A}}^{t}(T))_{t \in \mathbf{N}}$$

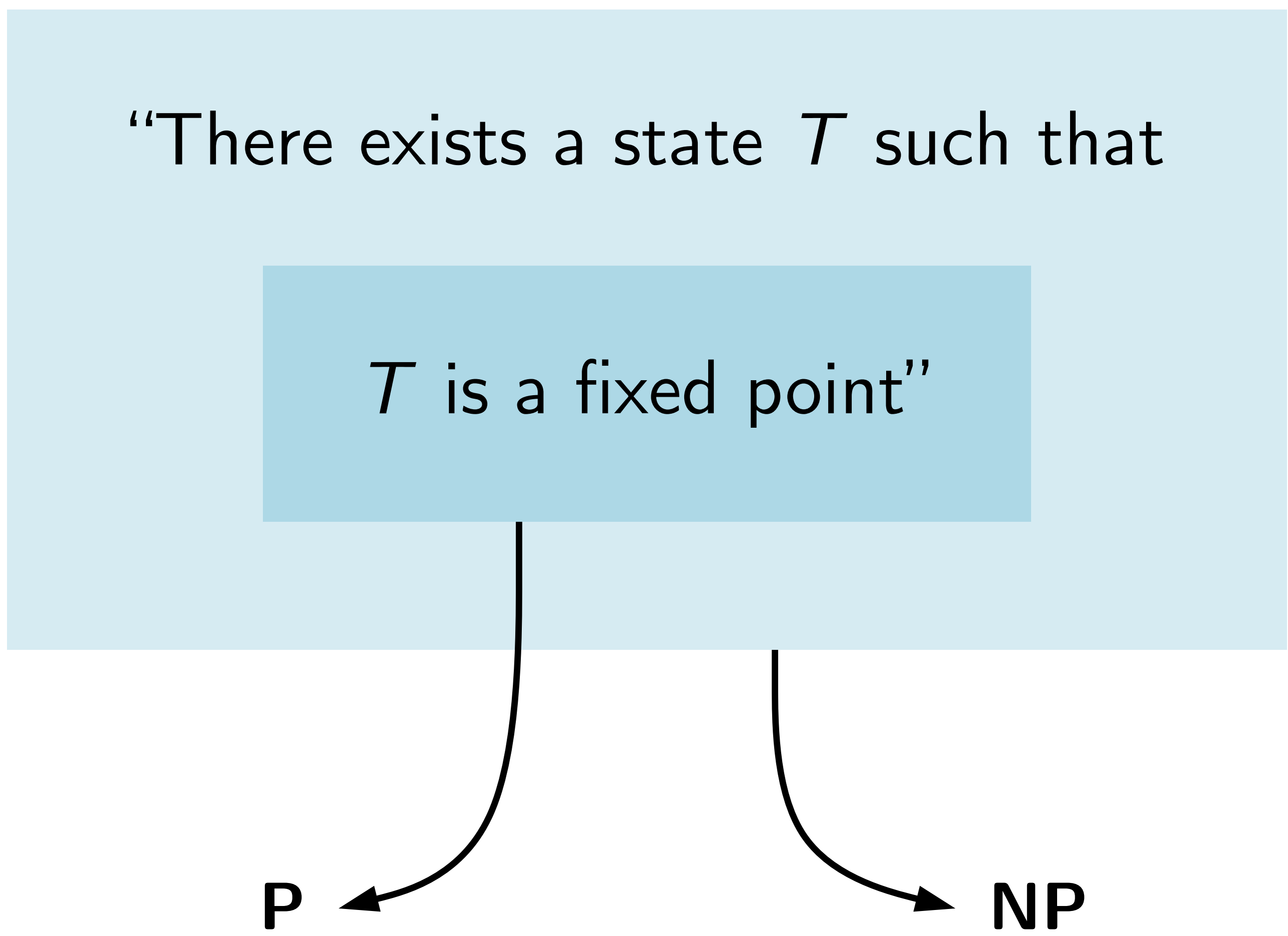
Questions (biologically inspired):

- Is T a fixed point?
- Does $\mathcal{A}$ have fixed points?
- Similarity of $\mathcal{A}$ and $\mathcal{B}$ wrt fixed points
- Does $\mathcal{A}$ have (local) fixed point attractors?
- Similarity of $\mathcal{A}$ and $\mathcal{B}$ wrt fixed point attractors

Computing $\mathrm{res}_{\mathcal{A}}$ and "fixedness" $\in$ **AC**^{0} (**FO**-uniform)

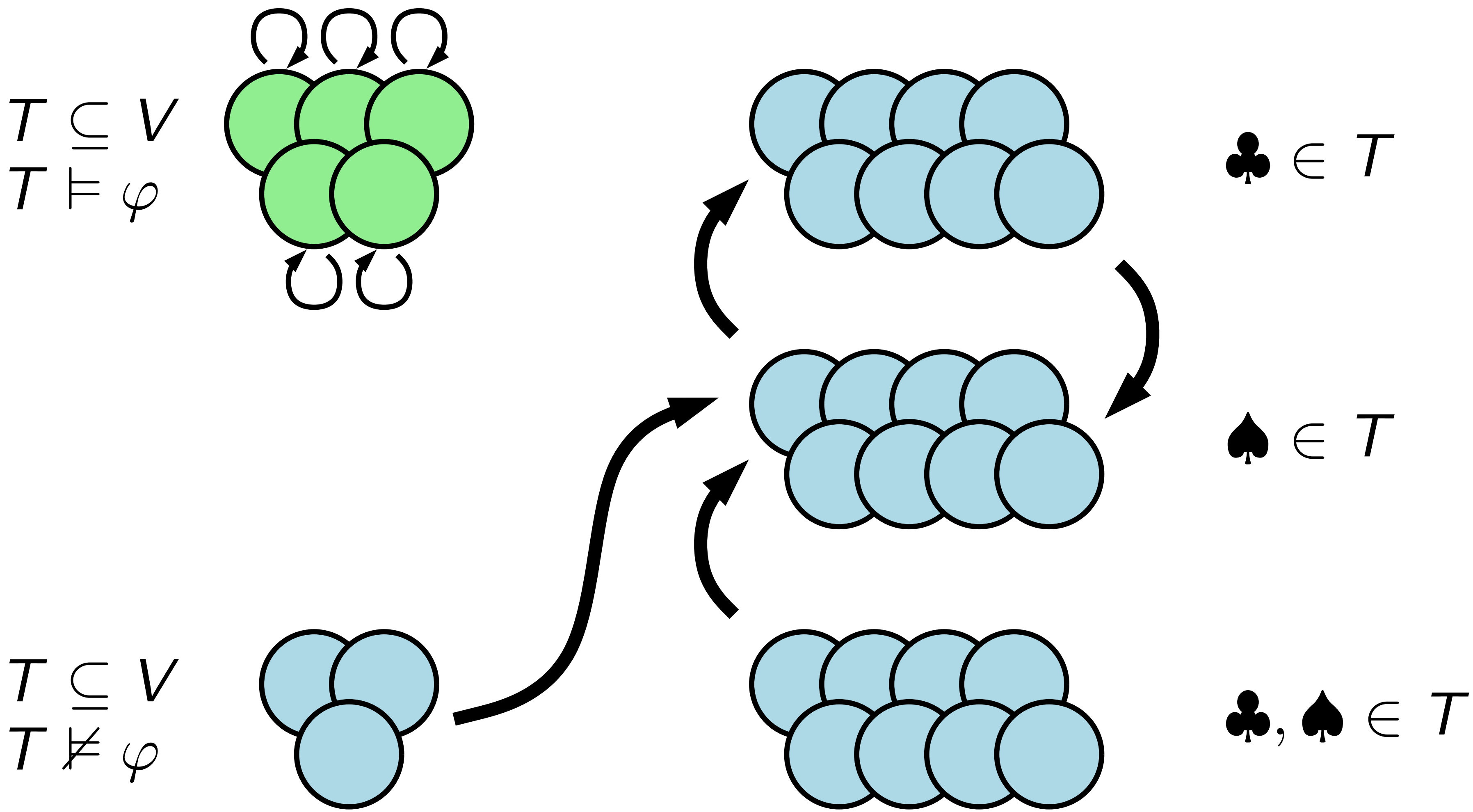


Existence of fixed points $\in$ **NP**

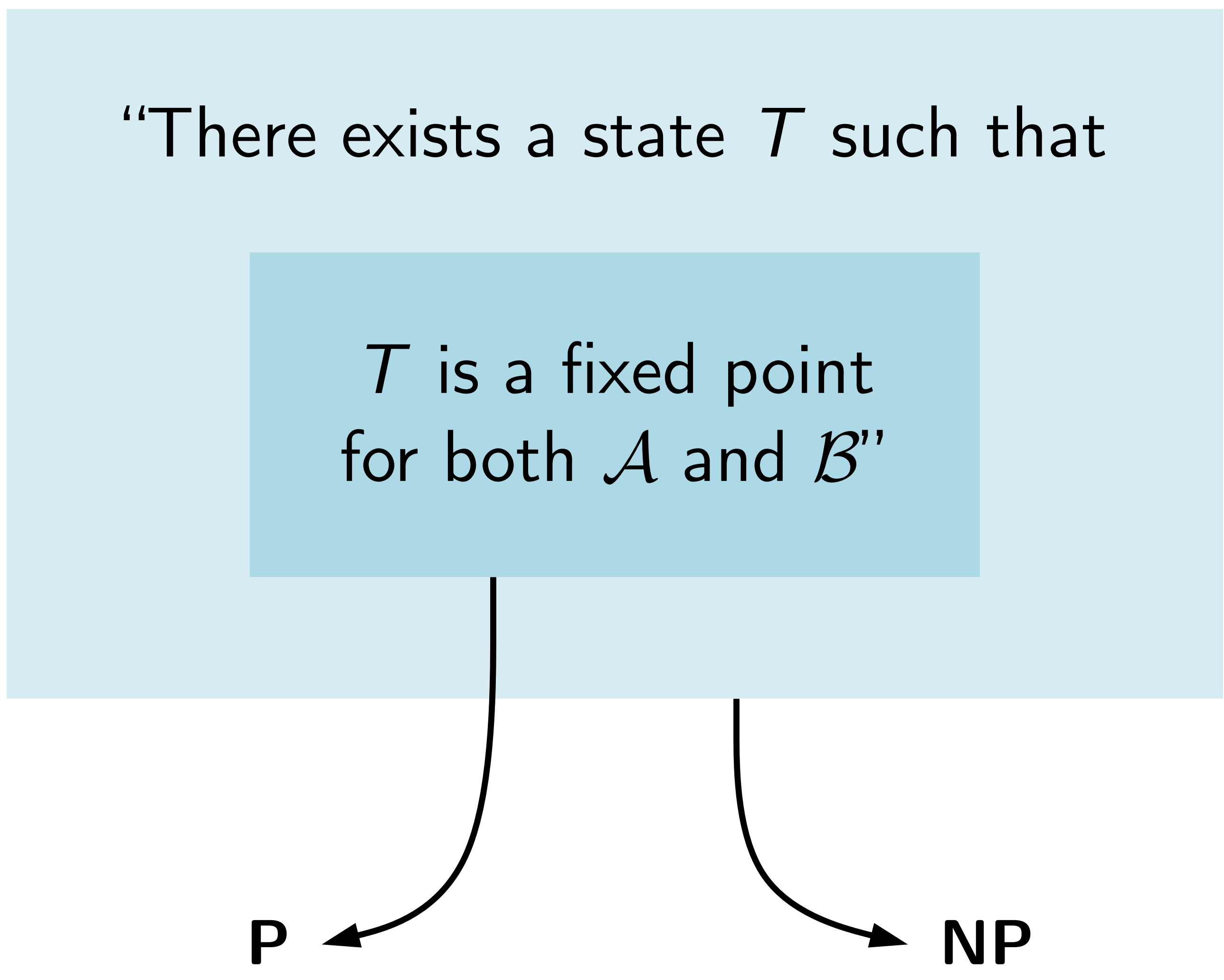


Existence of fixed points is **NP**-hard

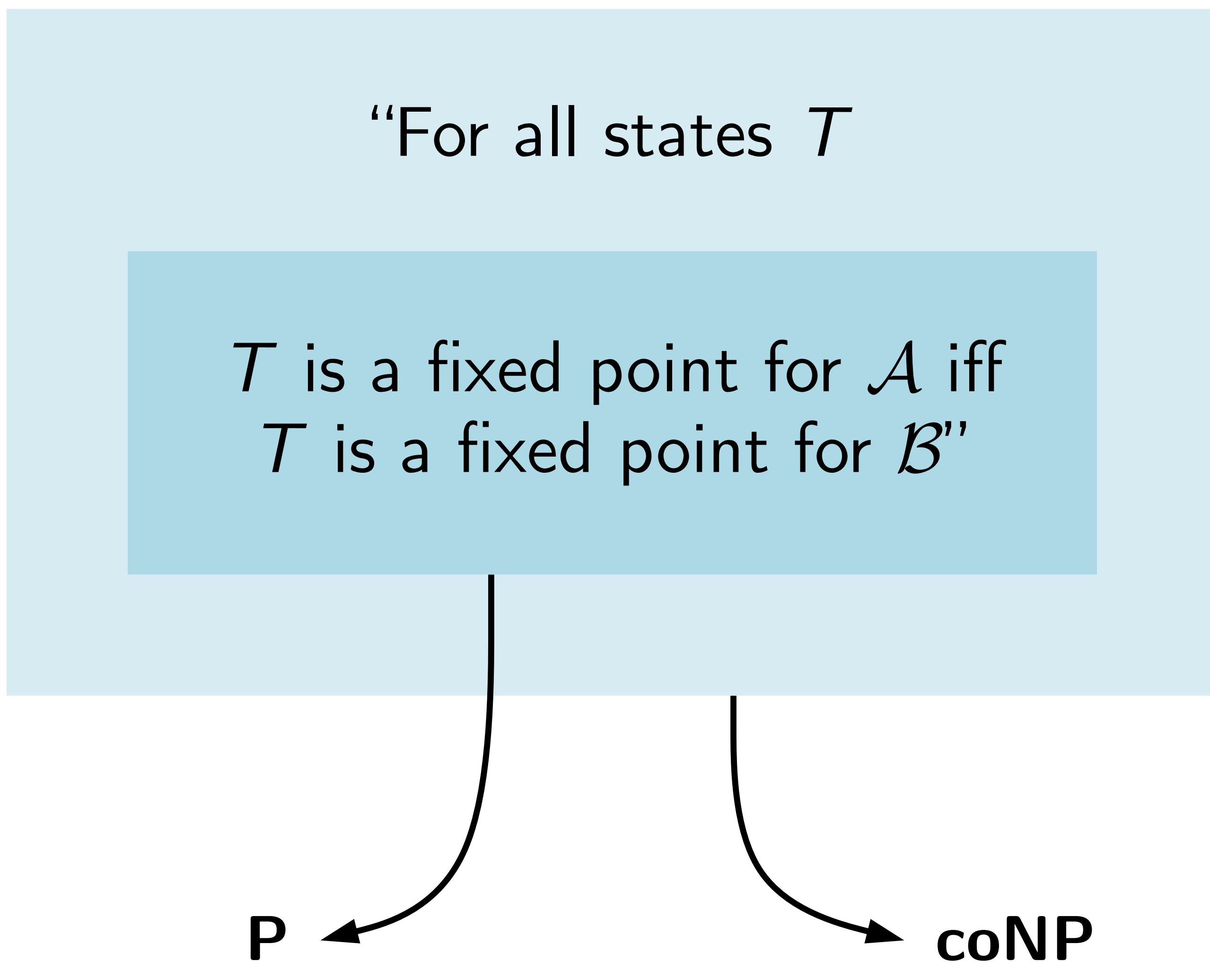
CNF formula $\exists V\ \varphi$



Existence of shared fixed points is **NP**-complete

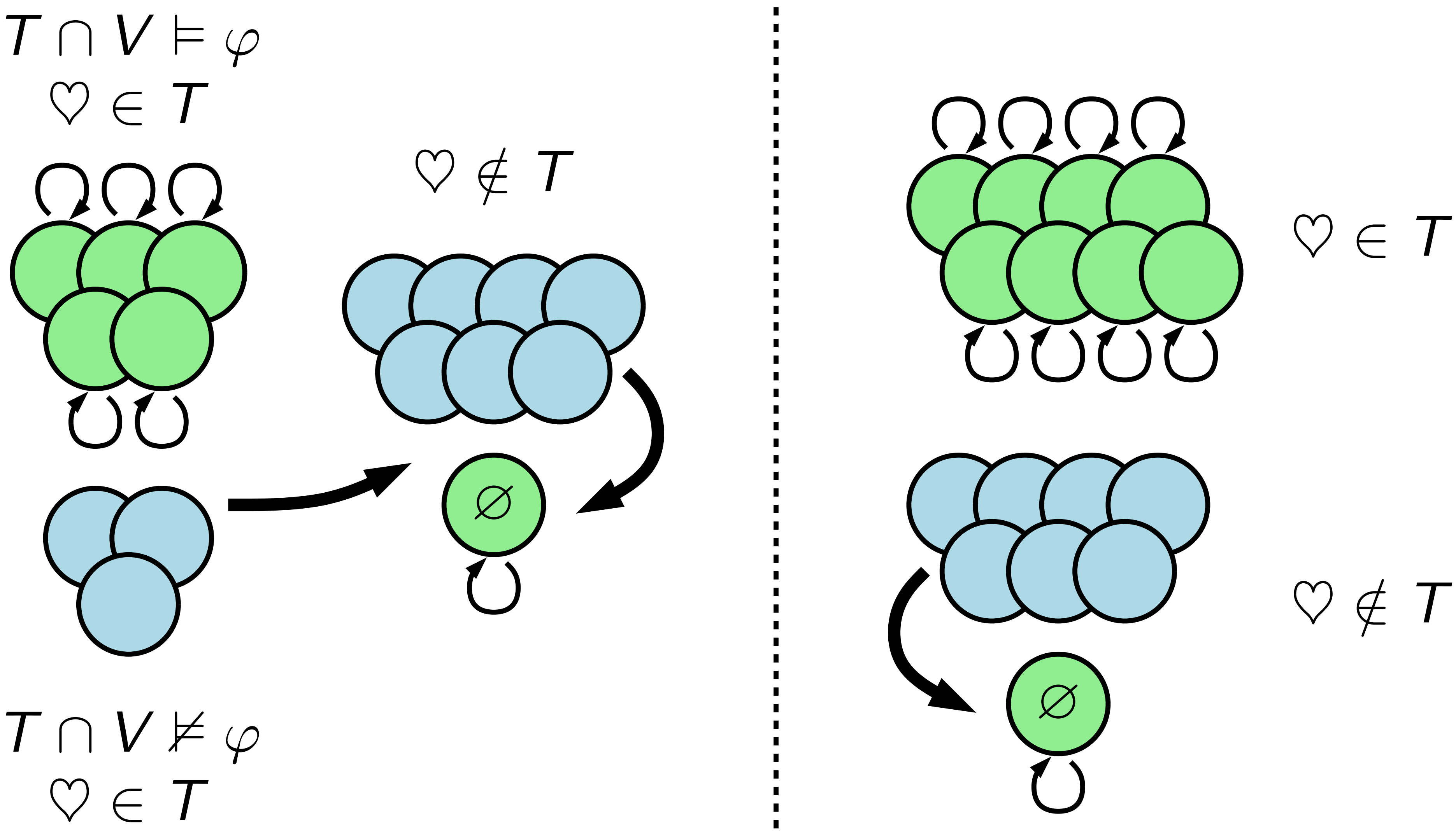


All fixed points are shared ∈ **coNP**



All fixed points are shared is **coNP**-hard

DNF formula $\forall V \, \varphi$



Is T a fixed point attractor? $\in$ **NP**

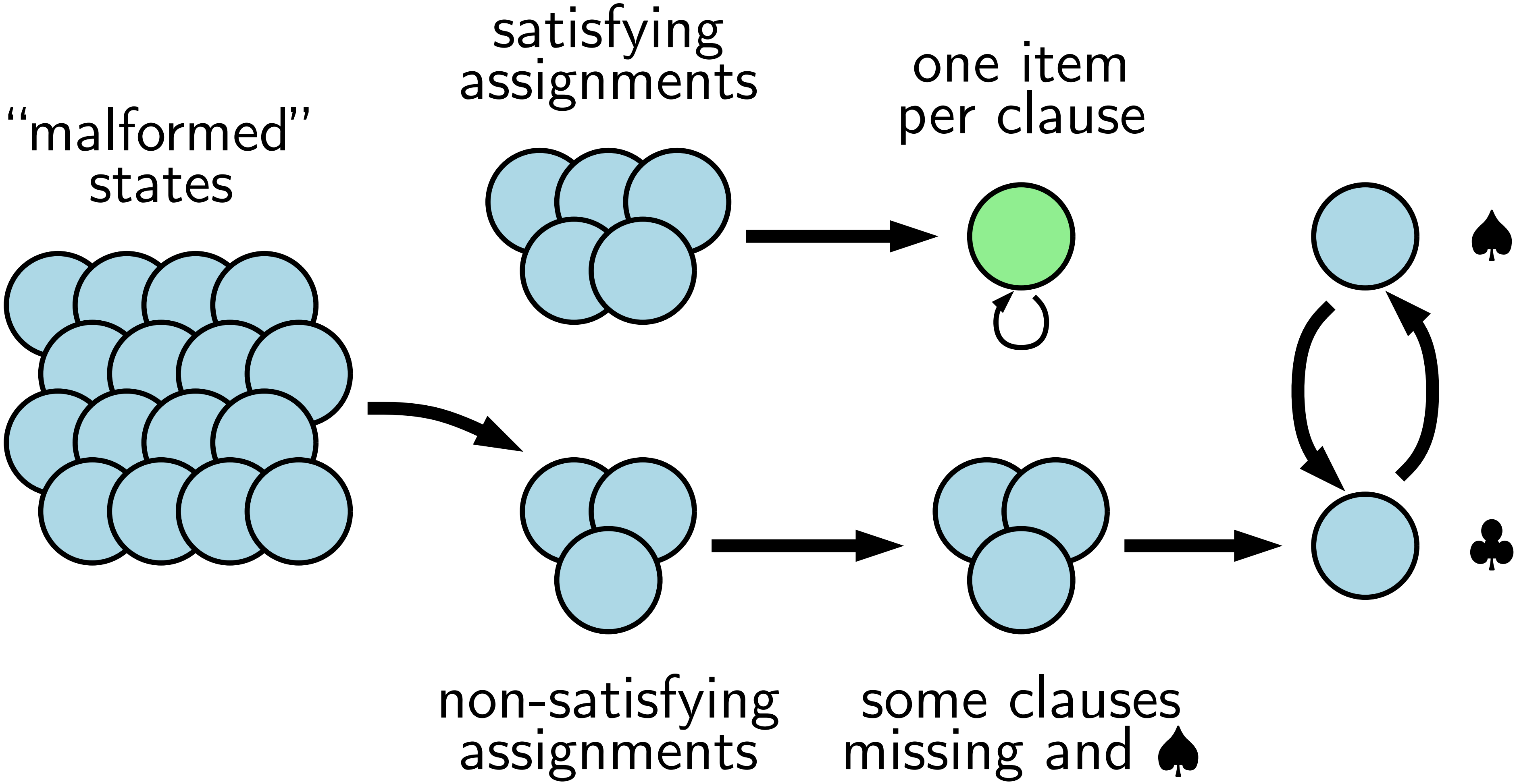
"There exists a state U such that

T is a fixed point and
$U \neq T$ and $\text{res}_{\mathcal{A}}(U) = T$"

P **NP**

Is T a fixed point attractor? is **NP**-hard

CNF formula $\exists V\ \varphi$

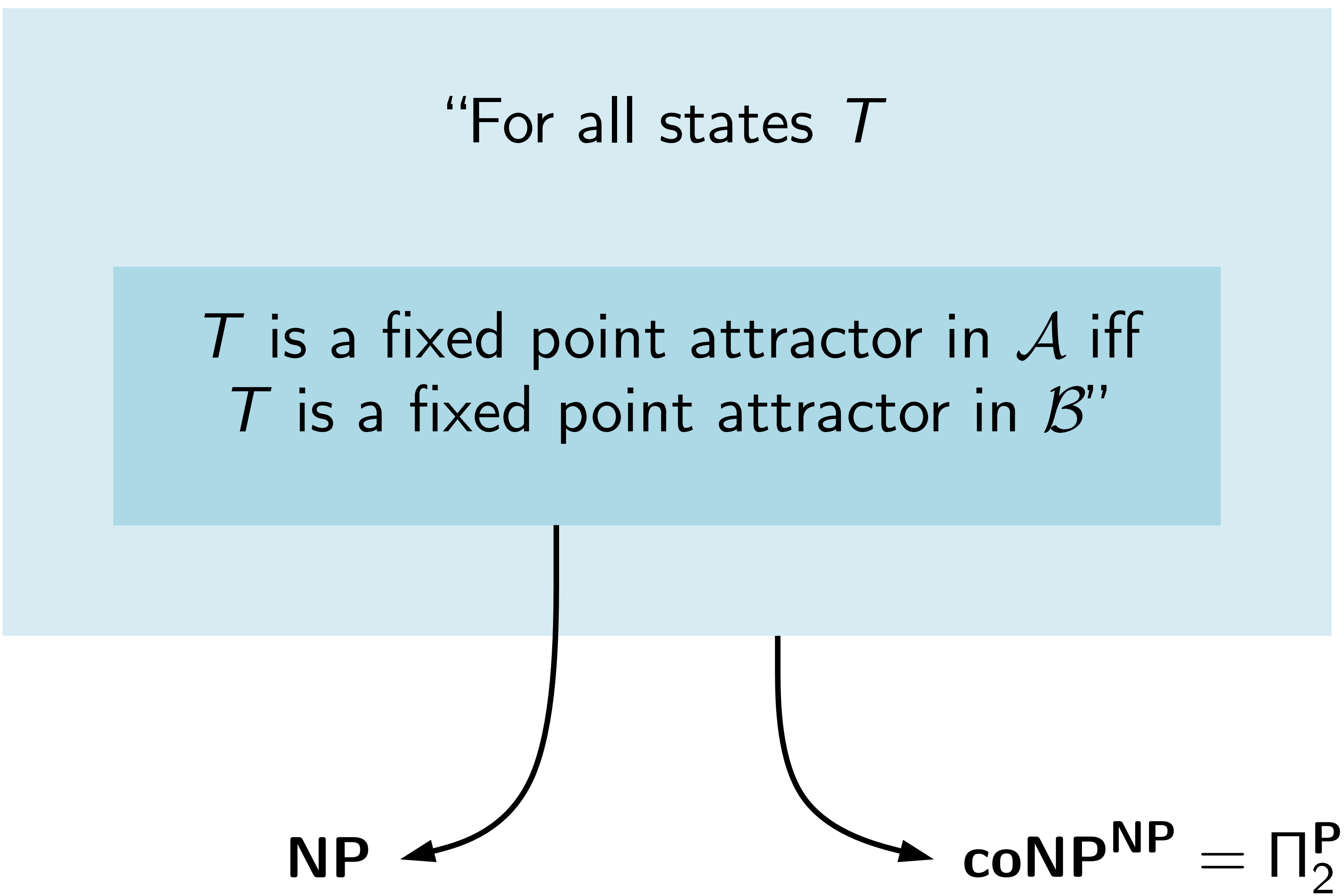


Other problems about fixed point attractors

Has $\mathcal{A}$ got a fixed point attractor? **NP**-complete

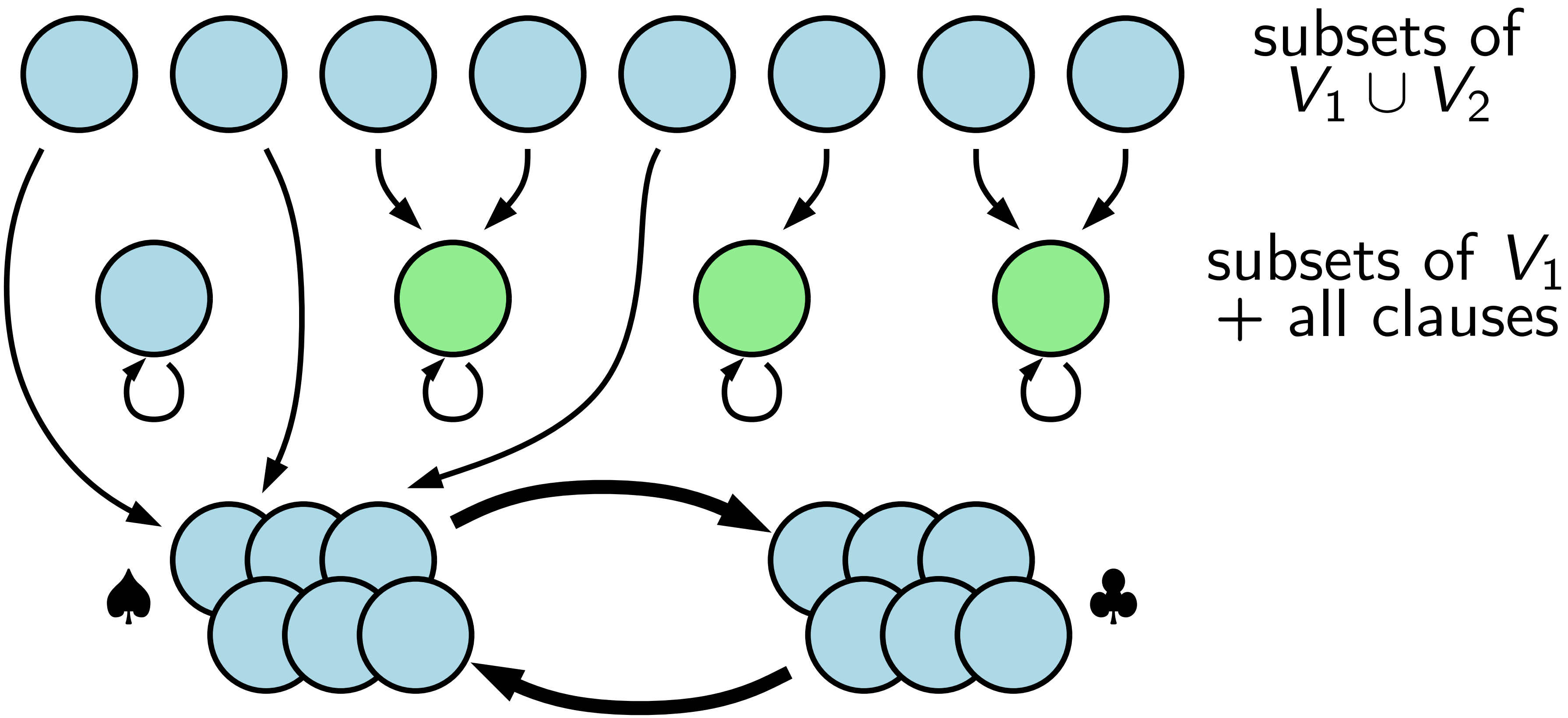
Do $\mathcal{A}$ and $\mathcal{B}$ share a fixed point attractor? **NP**-complete

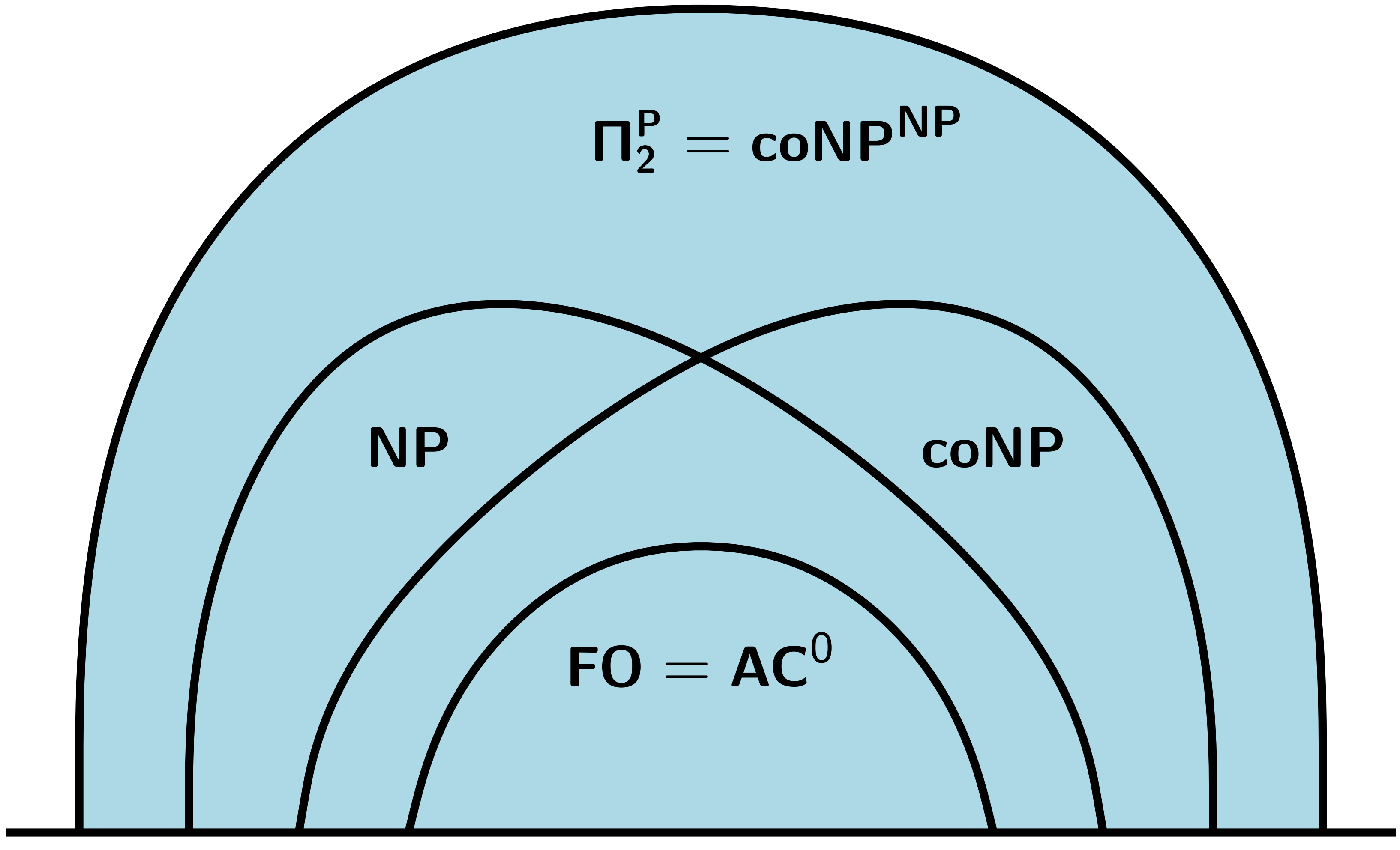
All fixed point attractors are shared $\in \Pi_2^{\mathbf{P}} = \mathbf{coNP}^{\mathbf{NP}}$



All fixed point attractors are shared is $\Pi_2^{\mathbf{P}}$-complete

CNF formula $\forall V_1 \exists V_2\, \varphi$

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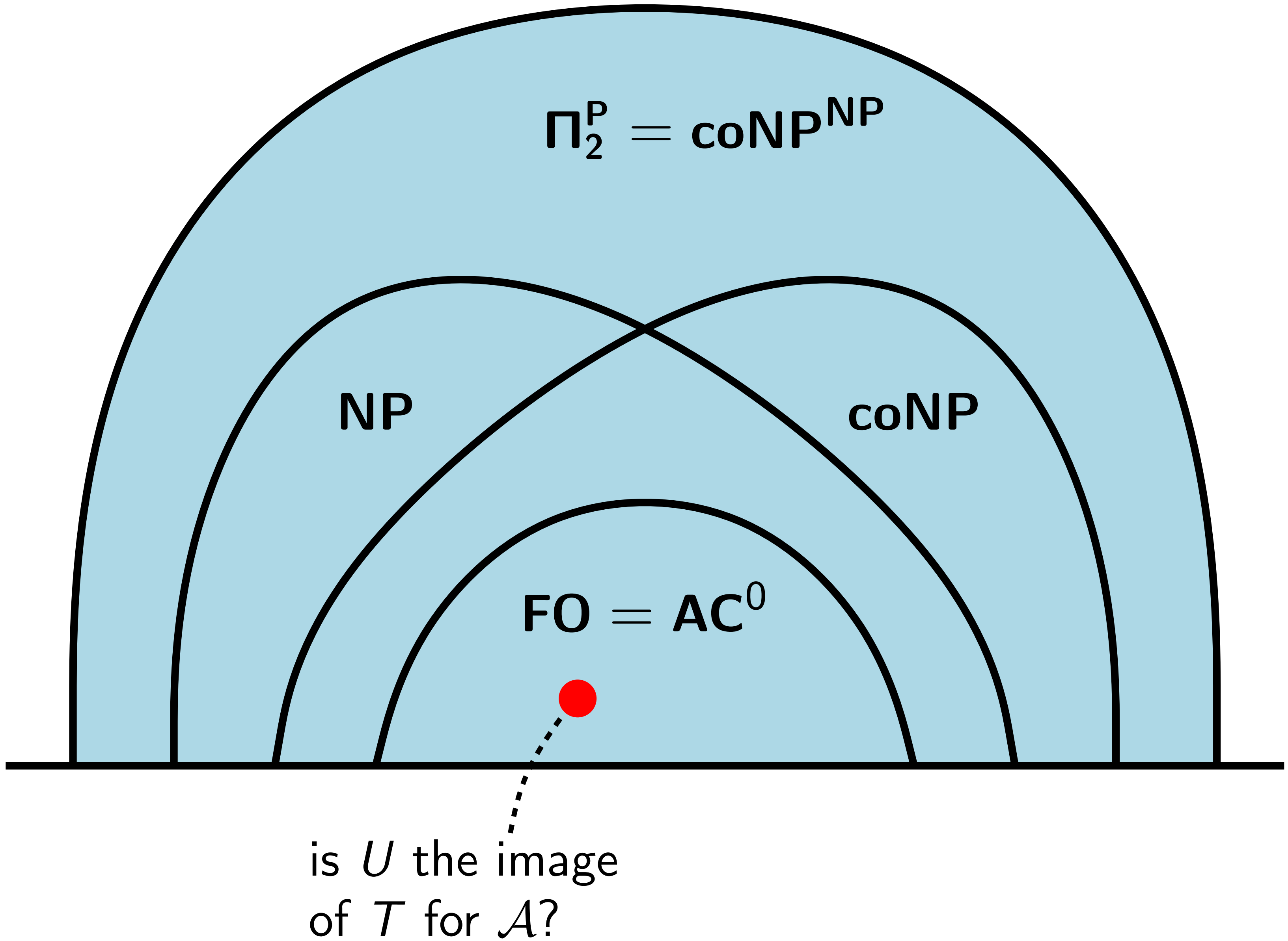
NP
coNP
$FO = AC^0$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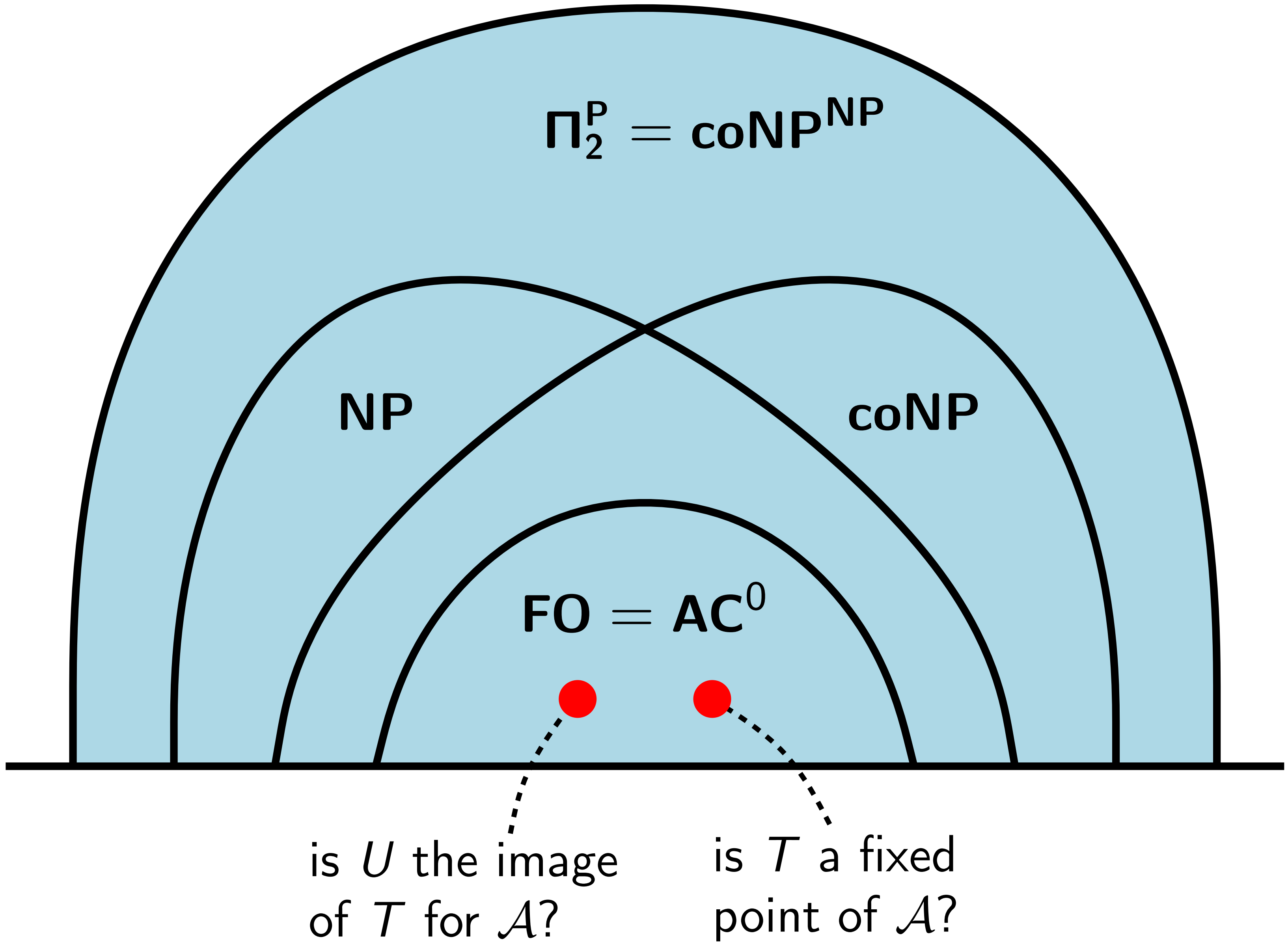

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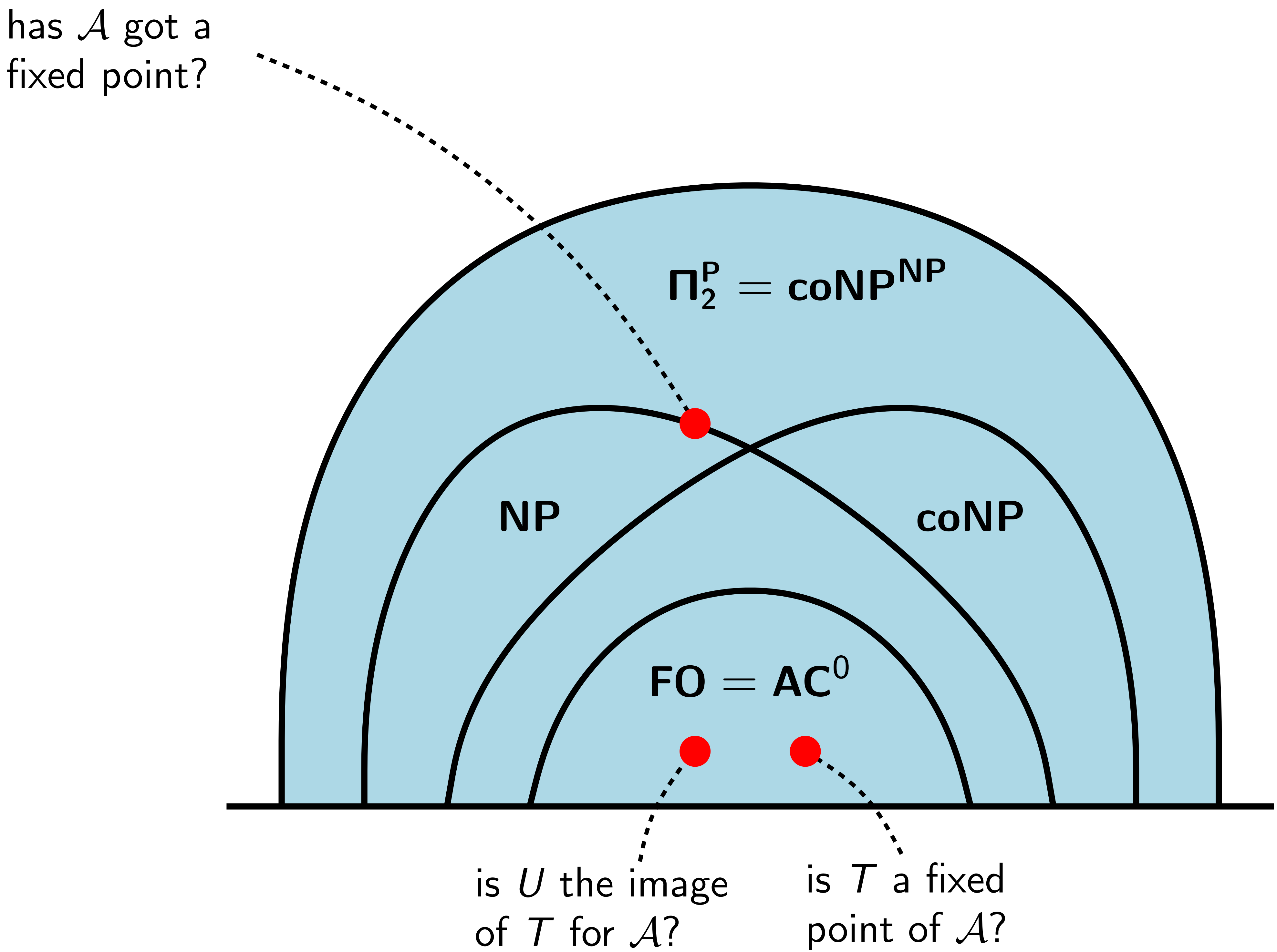
is T a fixed point of $\mathcal{A}$?



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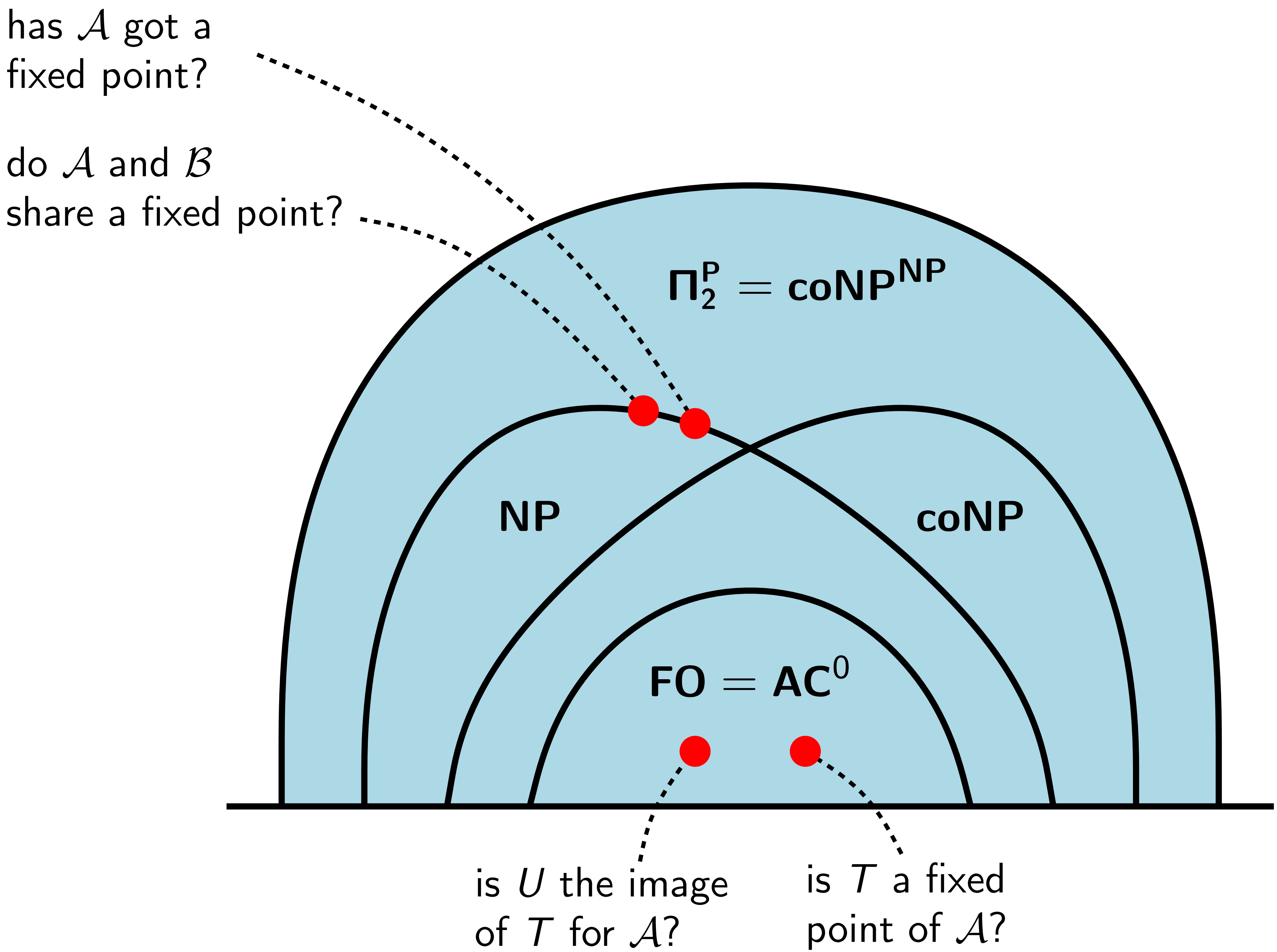
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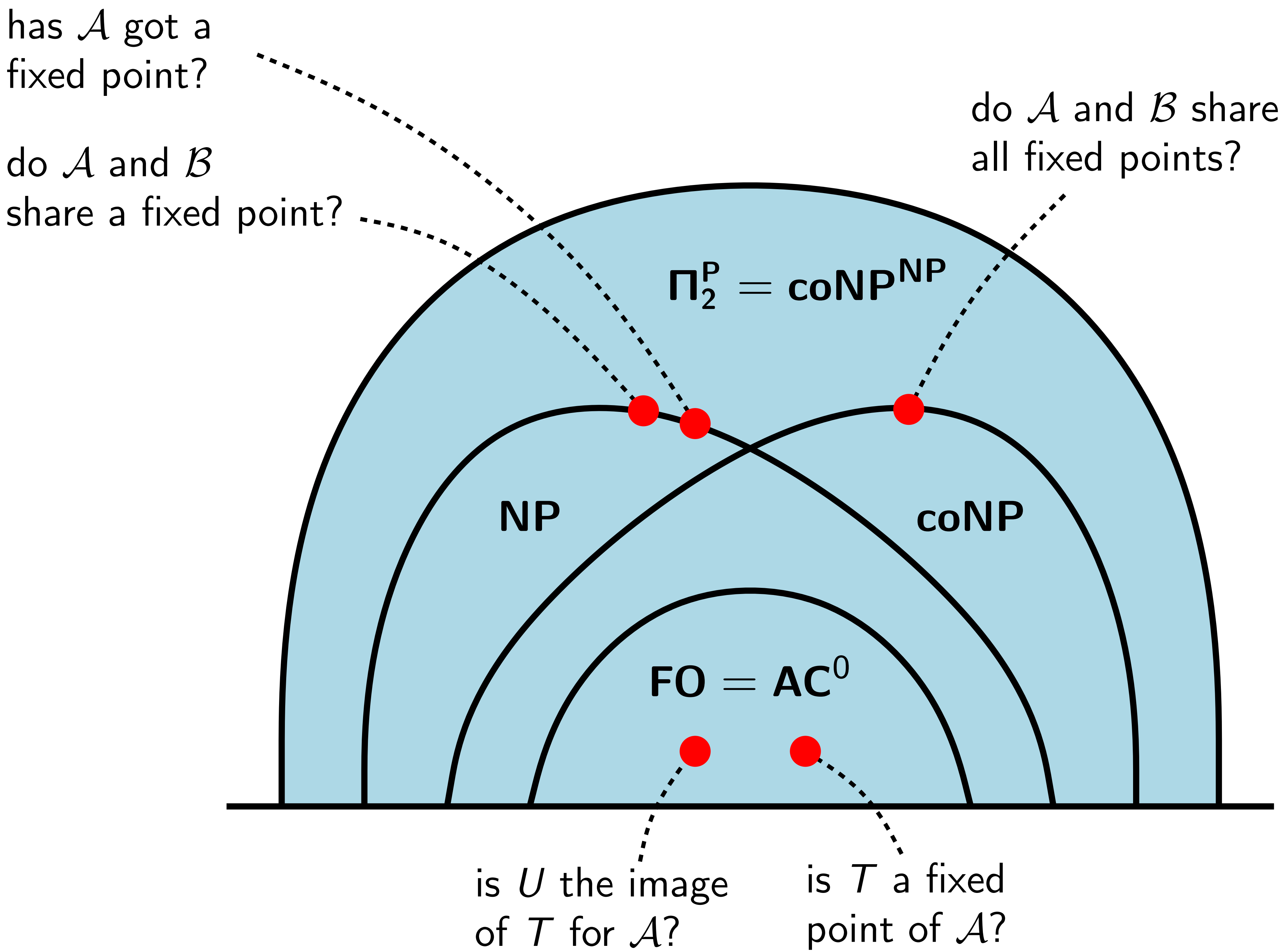
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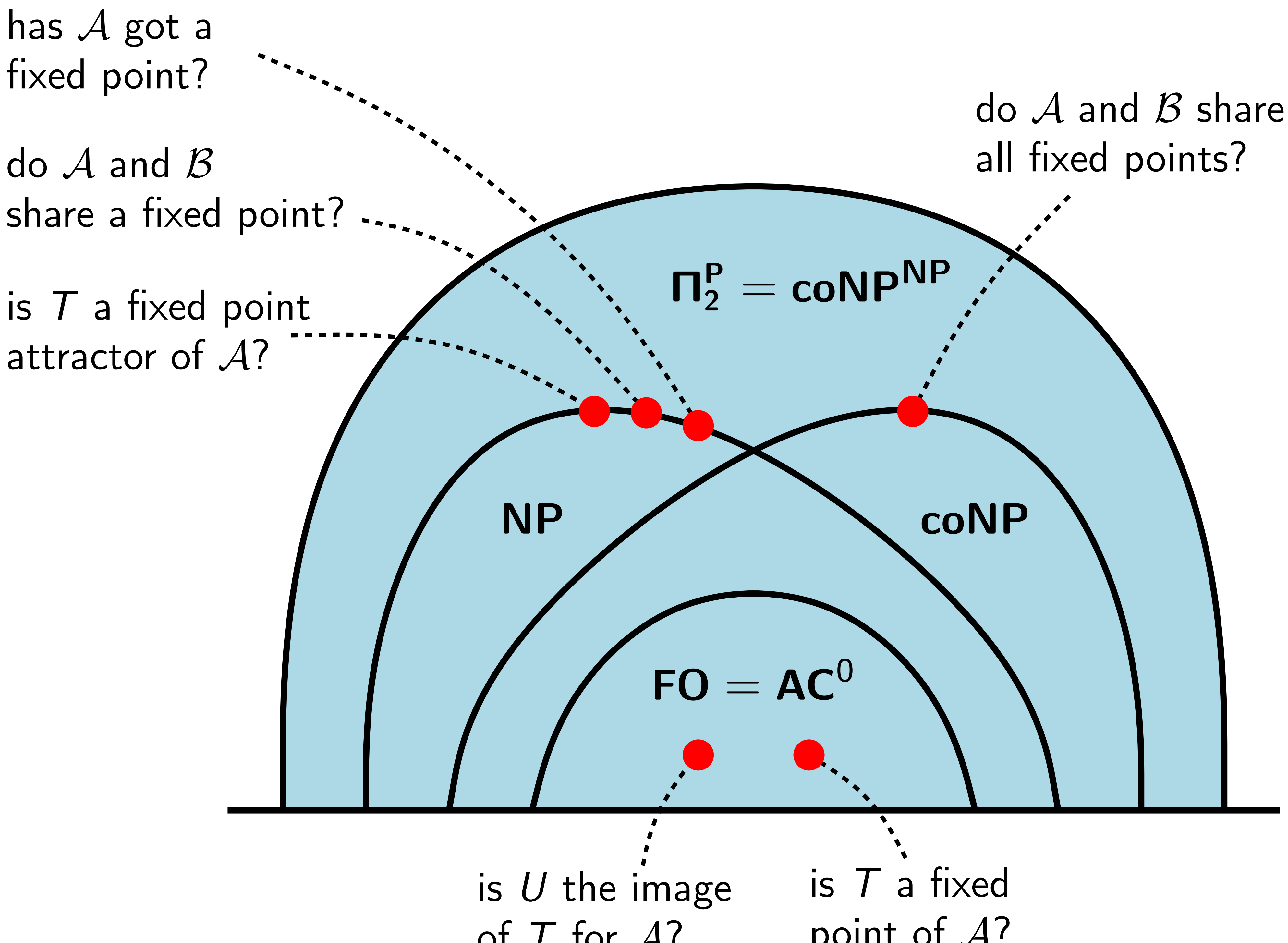
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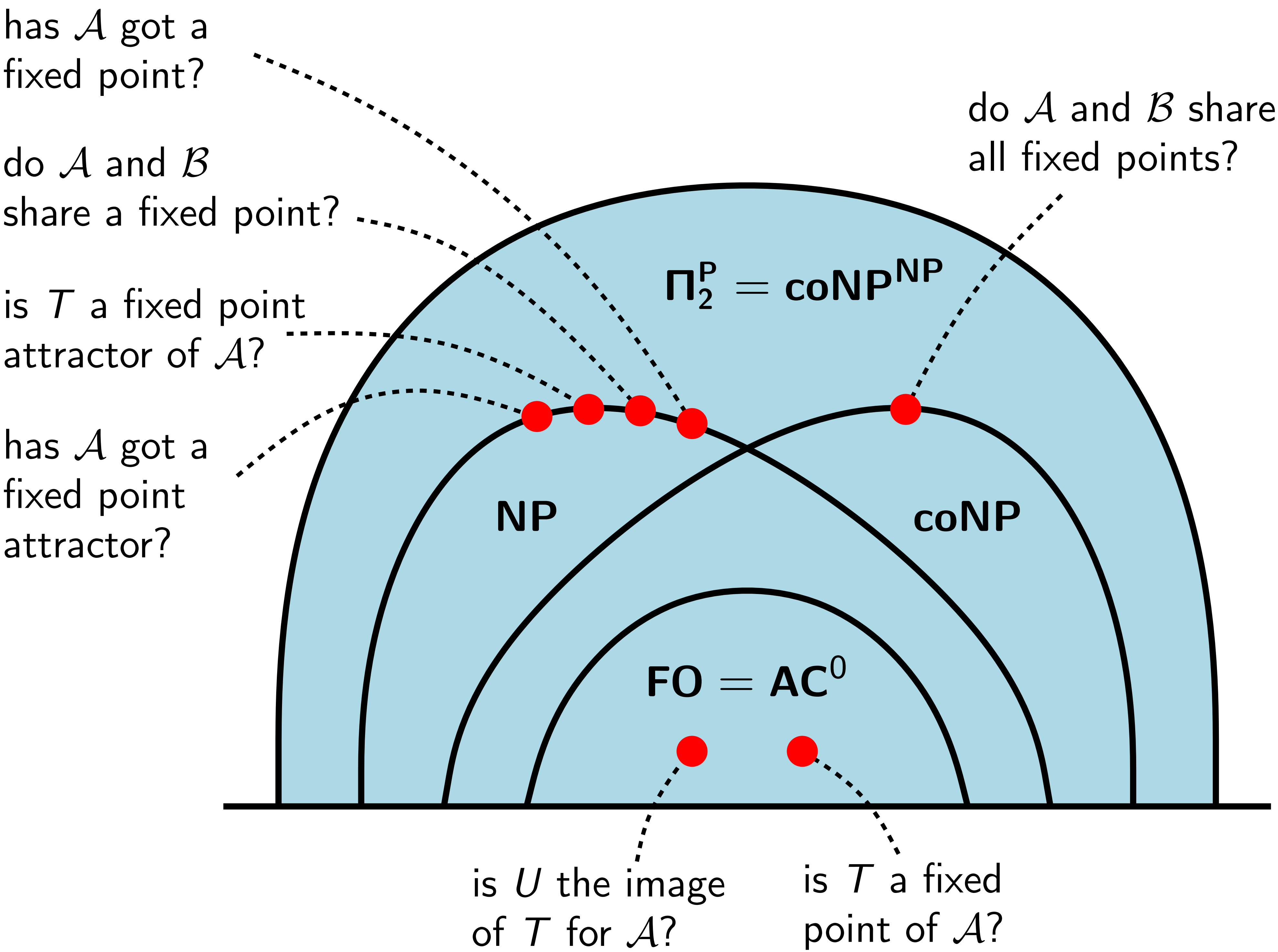
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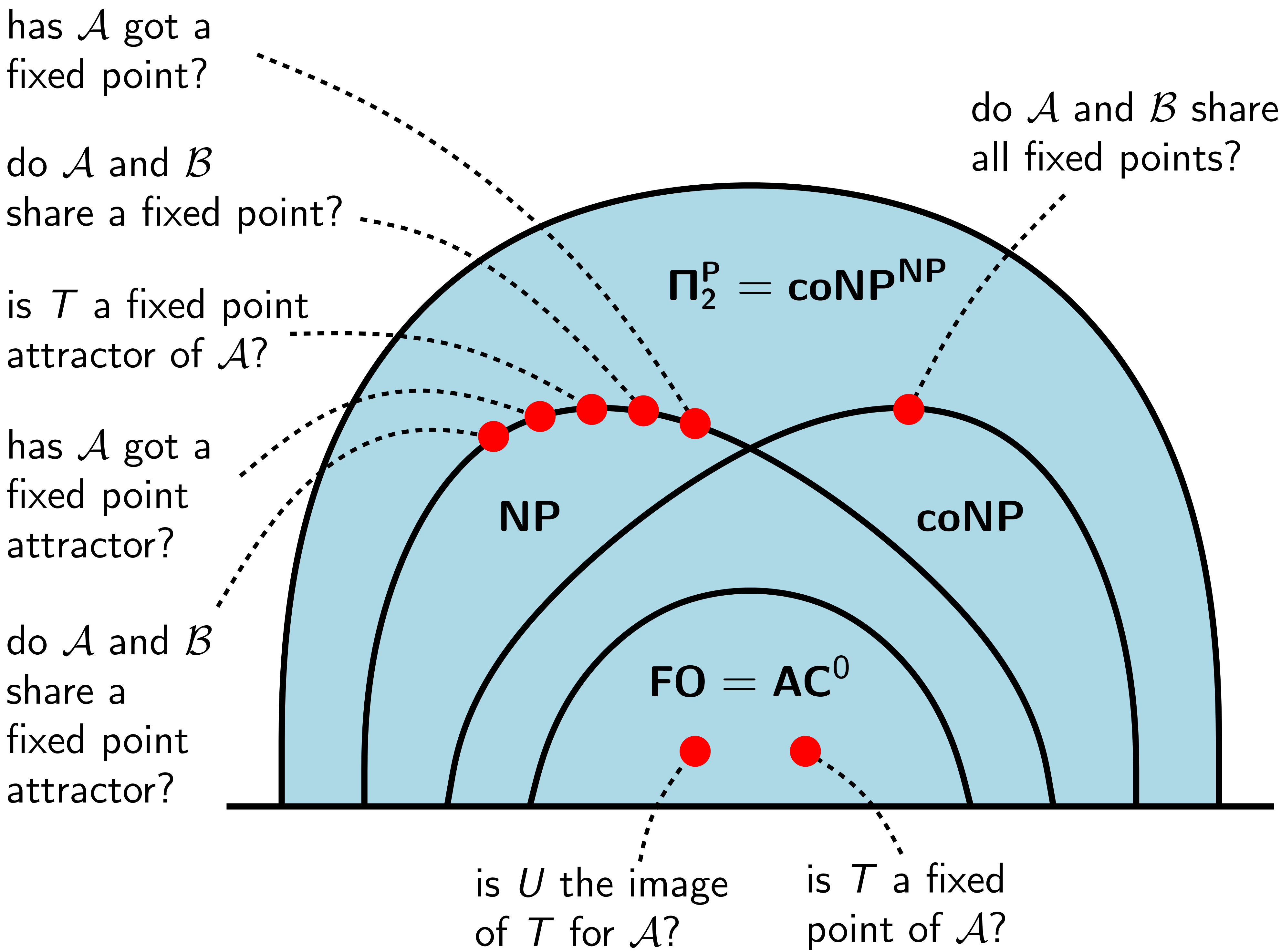

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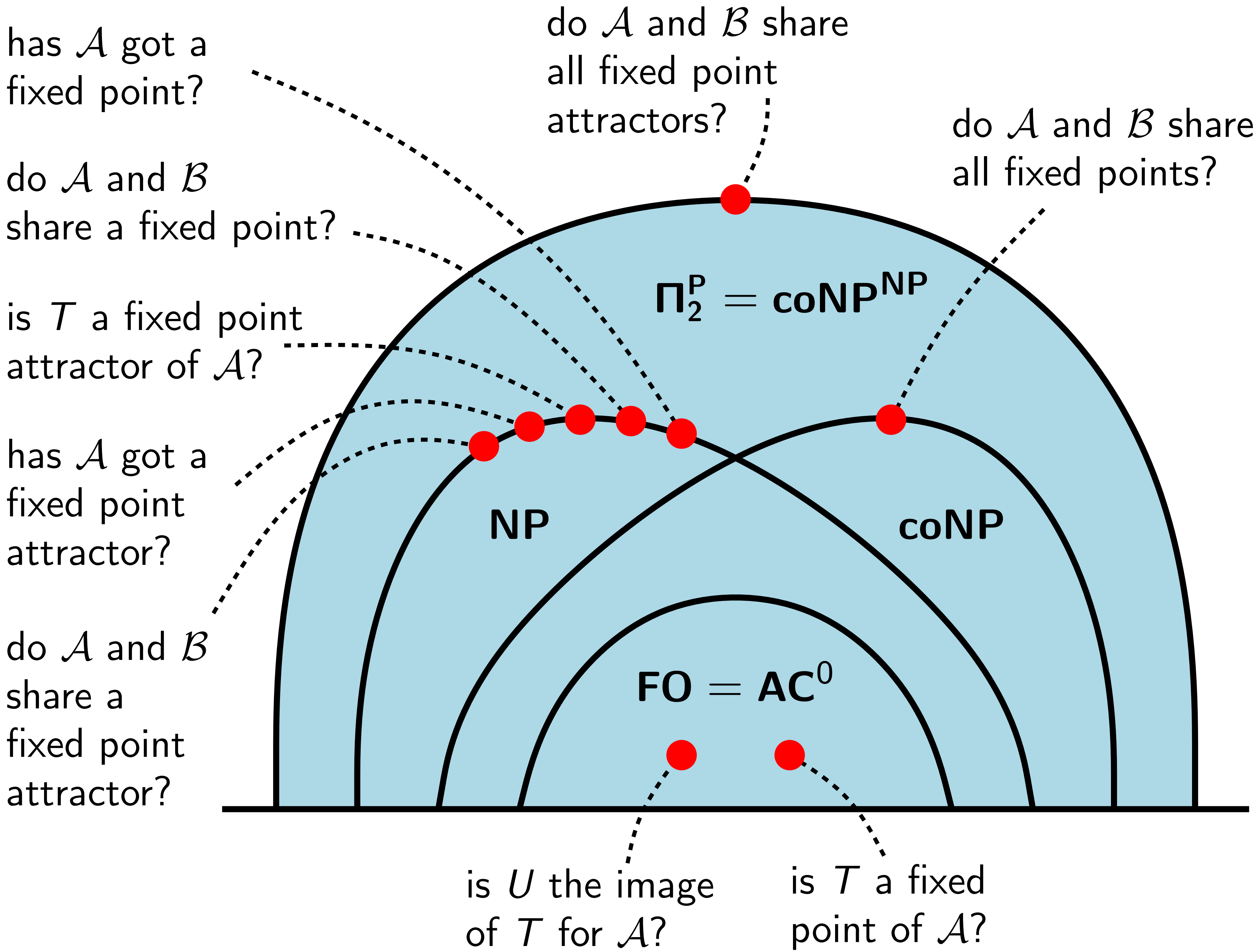

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Open problems

- Complexity of reachability
- Complexity of finding cycles
- Complexity of finding global attractors

SPOILER: everything becomes **PSPACE**-complete!

See our DCFS 2014 paper

(preprint at `http://aeporreca.org`)

- Dynamics of RS with context ("nondeterministic")
- Finding minimal RS with given $\text{res}_{\mathcal{A}}$

Köszönöm a figyelmet!

Thanks for your attention!

Any questions?