Flattening and simulation of asynchronous divisionless P systems with active membranes

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Summary

- We want to characterise the effect of asynchronicity on the computational power of P systems with active membranes
- Here we show that P systems with active membranes without division can be simulated by one-region transition P systems with cooperative rules...
- ...which can be simulated by Petri nets (non-universal)

Divisionless P systems with active membranes

$$\Pi = (\Gamma, \Lambda, \mu, w_{h_1}, \ldots, w_{h_d}, R)$$

Rules

- Evolution $[a \to w]_h^\alpha$
- Send-in $a\,[\;]_h^\alpha \to [b]_h^\beta$
- Send-out $[a]_h^\alpha \to [\;]_h^\beta\, b$
- Dissolution $[a]_h^\alpha \to b$

Asynchronous parallel mode (any multiset of rules is applicable)

Asynchronicity and sequentiality

Proposition

Let Π be a P system with active membranes using object evolution, communication, and dissolution rules. Then, the asynchronous and the sequential updating policies of Π are equivalent in the following sense: for each asynchronous (resp., sequential) computation step $\mathcal{C} \to \mathcal{D}$ *we have a series of sequential (resp., asynchronous) steps* $\mathcal{C} = \mathcal{C}_0 \to \cdots \to \mathcal{C}_n = \mathcal{D}$ *for some* $n \in \mathbb{N}$*.*

Proof.

First apply all evolution rules, then all communication rules, then all division rules sequentially □

One-region transition P systems

$$\Pi = (\Gamma, w, R)$$

- Rules $v \to w$
- Sequential parallelism policy

Flattened encoding of P systems with active membranes

The *flattened encoding* of $\mathcal{C}$ is the multiset $E(\mathcal{C})$ over $(\Gamma \cup \{-, 0, +\}) \times \Lambda$ defined as follows:

- If there are n copies of the object a contained in a membrane h in $\mathcal{C}$, then $E(\mathcal{C})$ contains n copies of the element (a, h)
- If a membrane h has charge α, then (α, h) is in $E(\mathcal{C})$

From active membranes to transition P systems I

Proposition

Let $\Pi = (\Gamma, \Lambda, \mu, w_{h_1}, \ldots, w_{h_d}, R)$ *be a P system with active membranes working in the sequential mode and using object evolution, communication, and dissolution rules, with initial configuration* $\mathcal{C}_0$*. Then, there exists a single-membrane transition P system* $\Pi' = \big((\Gamma \cup \{-, 0, +\} \cup \{\bullet\}) \times \Lambda, v, R'\big)$*, for some initial multiset* v*, working in the sequential mode, such that:*

From active membranes to transition P systems II

(i) *If* $\vec{\mathcal{C}} = (\mathcal{C}_0, \mathcal{C}_1, \ldots, \mathcal{C}_m)$ *is a halting computation of* Π*, then there exists a halting computation* $\vec{\mathcal{D}} = (E(\mathcal{C}_0), \mathcal{D}_1, \ldots, \mathcal{D}_n)$ *of* Π' *such that* $\mathcal{D}_n$ *is the union of* $E(\mathcal{C}_m)$ *and the set of all the elements in the form* $(\bullet, h)$ *where* h *is a membrane that has been dissolved in* $\vec{\mathcal{C}}$*.*

(ii) *If* $\vec{\mathcal{D}} = (E(\mathcal{C}_0), \mathcal{D}_1, \ldots, \mathcal{D}_n)$ *is a halting computation of* Π'*, then there exists a halting computation* $\vec{\mathcal{C}} = (\mathcal{C}_0, \mathcal{C}_1, \ldots, \mathcal{C}_m)$ *of* Π *such that* $\mathcal{D}_n$ *can be written as the union of the set of elements in the form* $(\bullet, h)$*, where* h *is a membrane that was dissolved in* $\vec{\mathcal{C}}$*, and* $E(\mathcal{C}_m)$*.*

(iii) Π *admits a non-halting computation* $(\mathcal{C}_0, \mathcal{C}_1, \ldots)$ *if and only if* Π' *admits a non-halting computation* $(E(\mathcal{C}_0), \mathcal{D}_1, \ldots)$*.*

From active membranes to transition P systems III

- *For each dissolution rule* $[a]_{h_1}^{\alpha} \to b$*:*

$$(a, h_1)(\alpha, h_1) \to (b, h_1)(\bullet, h_1)$$
$$(a, h_1)(\bullet, h_1) \to (a, h_2)(\bullet, h_1)$$

where h_2 *is the parent of* h_1

- *For each evolution rule* $[a \to w]_{h}^{\alpha}$*:*

$$(a, h)(\alpha, h) \to (w_1, h) \ldots (w_n, h)(\alpha, h)$$

- *For each send-out communication rule* $[a]_{h_1}^{\alpha} \to [\ ]_{h_1}^{\beta} b$*:*

$$(a, h_1)(\alpha, h_1) \to (b, h_2)(\beta, h_1)$$

From active membranes to transition P systems IV

- *For each send-in rule* $a\,[\;]_{h_1}^{\alpha} \to [b]_{h_1}^{\beta}$, *for each sequence* $(h_n, h_{n-1}, \ldots, h_2, h_1)$ *of nested membranes surrounding* h_1

$$(\bullet, h_{n-1}) \cdots (\bullet, h_2)(\alpha, h_1)(a, h_n)$$
$$\downarrow$$
$$(\bullet, h_{n-1}) \cdots (\bullet, h_2)(\beta, h_1)(b, h_1)$$

From active membranes to transition P systems V

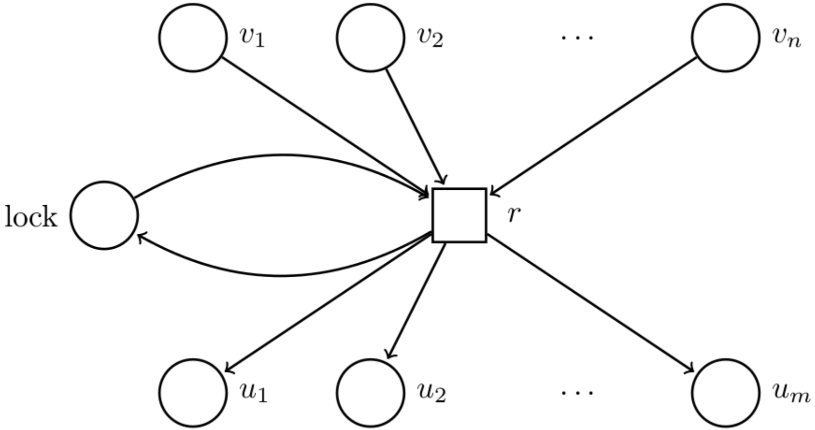
(i) *If* $\vec{\mathcal{C}} = (\mathcal{C}_0, \mathcal{C}_1, \ldots, \mathcal{C}_m)$ *is a halting computation of* Π*, then there exists a halting computation* $\vec{\mathcal{D}} = (E(\mathcal{C}_0), \mathcal{D}_1, \ldots, \mathcal{D}_n)$ *of* Π' *such that* $\mathcal{D}_n$ *is the union of* $E(\mathcal{C}_m)$ *and the set of all the elements in the form* $(\bullet, h)$ *where* h *is a membrane that has been dissolved in* $\vec{\mathcal{C}}$*.*

(ii) *If* $\vec{\mathcal{D}} = (E(\mathcal{C}_0), \mathcal{D}_1, \ldots, \mathcal{D}_n)$ *is a halting computation of* Π'*, then there exists a halting computation* $\vec{\mathcal{C}} = (\mathcal{C}_0, \mathcal{C}_1, \ldots, \mathcal{C}_m)$ *of* Π *such that* $\mathcal{D}_n$ *can be written as the union of the set of elements in the form* $(\bullet, h)$*, where* h *is a membrane that was dissolved in* $\vec{\mathcal{C}}$*, and* $E(\mathcal{C}_m)$*.*

(iii) Π *admits a non-halting computation* $(\mathcal{C}_0, \mathcal{C}_1, \ldots)$ *if and only if* Π' *admits a non-halting computation* $(E(\mathcal{C}_0), \mathcal{D}_1, \ldots)$*.*

Simulation with Petri nets

For each cooperative rule $v_1 \cdots v_n \to u_1 \cdots u_m$



Main result

Theorem

For every asynchronous P system with active membranes Π using evolution, communication, and dissolution rules, there exists a Petri net N such that

(i) *every halting configuration of Π corresponds to a halting configuration of N and vice versa*

(ii) *every non-halting computation of Π corresponds to a non-halting computation of N and vice versa* □

This holds for P systems computing functions, generators and recognisers

Conclusions

- Asynchronous divisionless active membranes can be flattened and simulated by Petri nets
- Are they equivalent? (Does not follow immediately from previous results, halting condition is relevant)
- What about division?

Thanks for your attention!

Vă mulțumim pentru atenție!

Спасибо за внимание!