Natural computing models of unusual complexity

Antonio E. Porreca
Aix-Marseille Université, Lab. Informatique et Systèmes, équipe CANA
https://aeporreca.org

Outline

- The first and second machine classes
- Membrane computing
- Complexity theory of membrane systems
- Communication topologies and their role

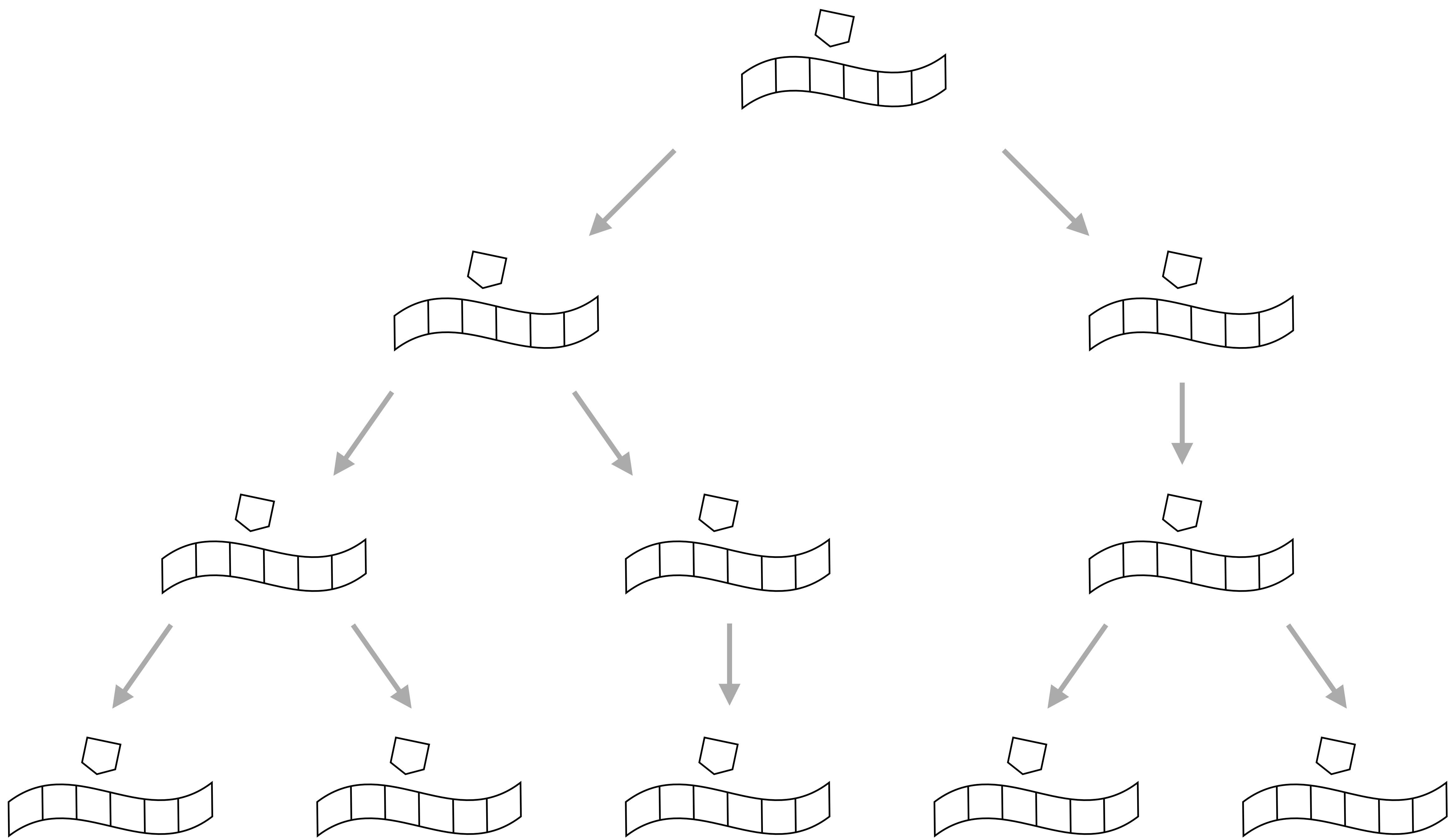
The first machine class and **P**

- The deterministic Turing machine and all models that simulate and are simulated by it efficiently
  - Random access machines with arithmetic operations + and –
  - Cellular automata with finite initial configuration

The second machine class and **PSPACE**

- Computing models that solve in polynomial **time** what a Turing machine solves in polynomial **space**
  - Alternating Turing machines
  - Random access machines with arithmetic operations + − x ÷
  - Parallel processes generated by `fork(2)` running on an unbounded number of processors
  - Cellular automata over hyperbolic grids

Nondeterministic Turing machines: **NP**



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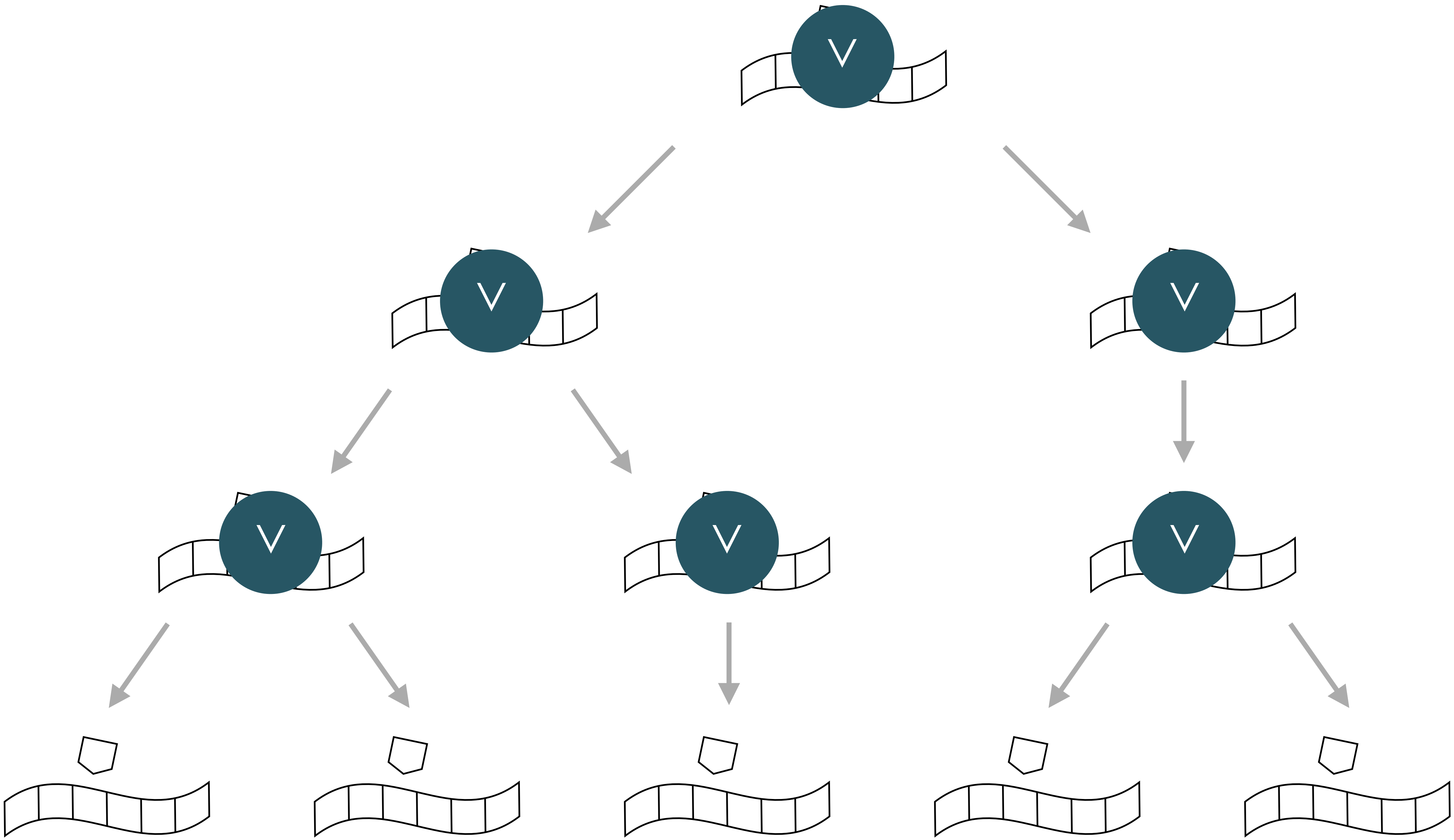
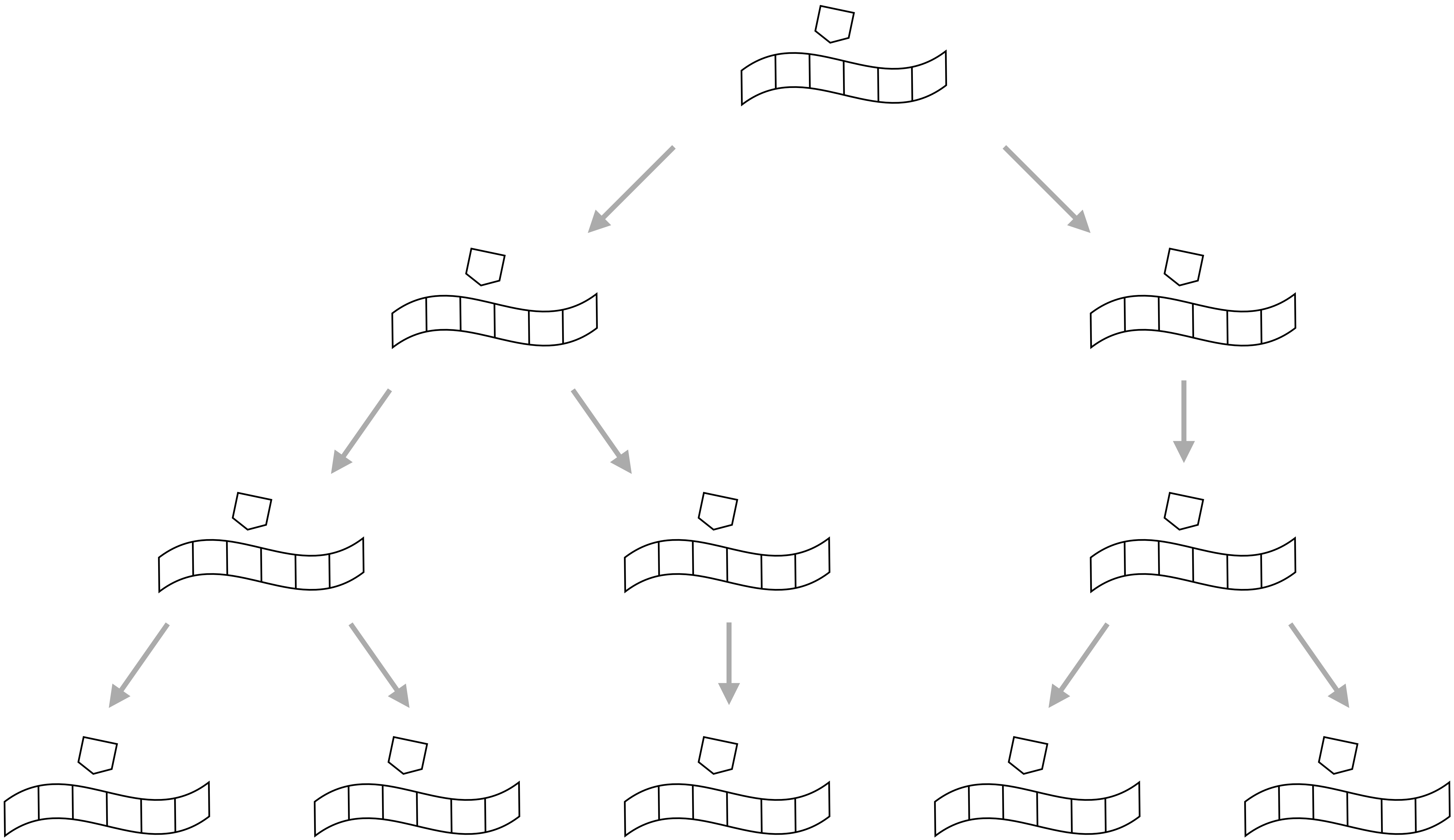


Alternating Turing machines: **PSPACE**



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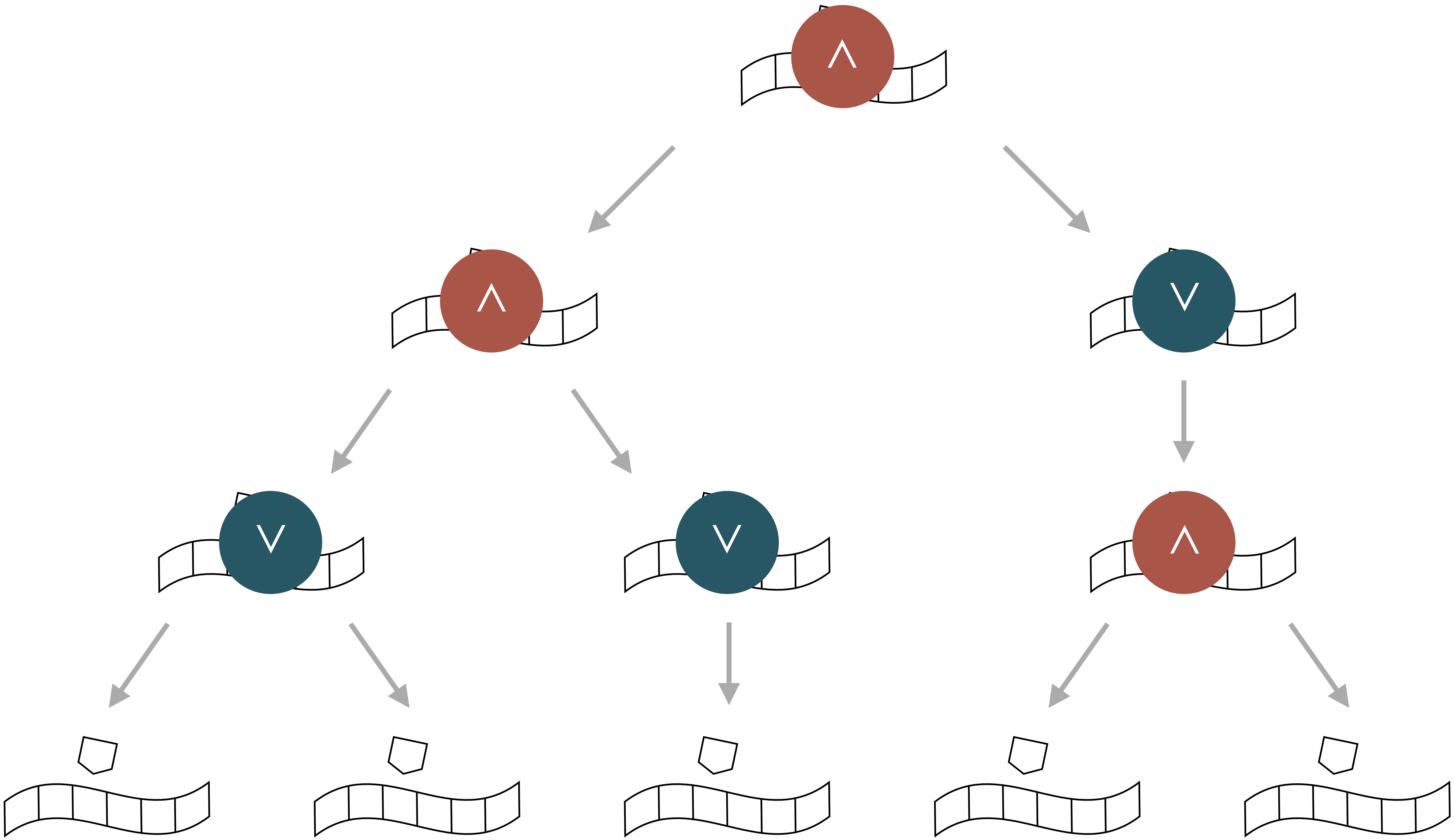
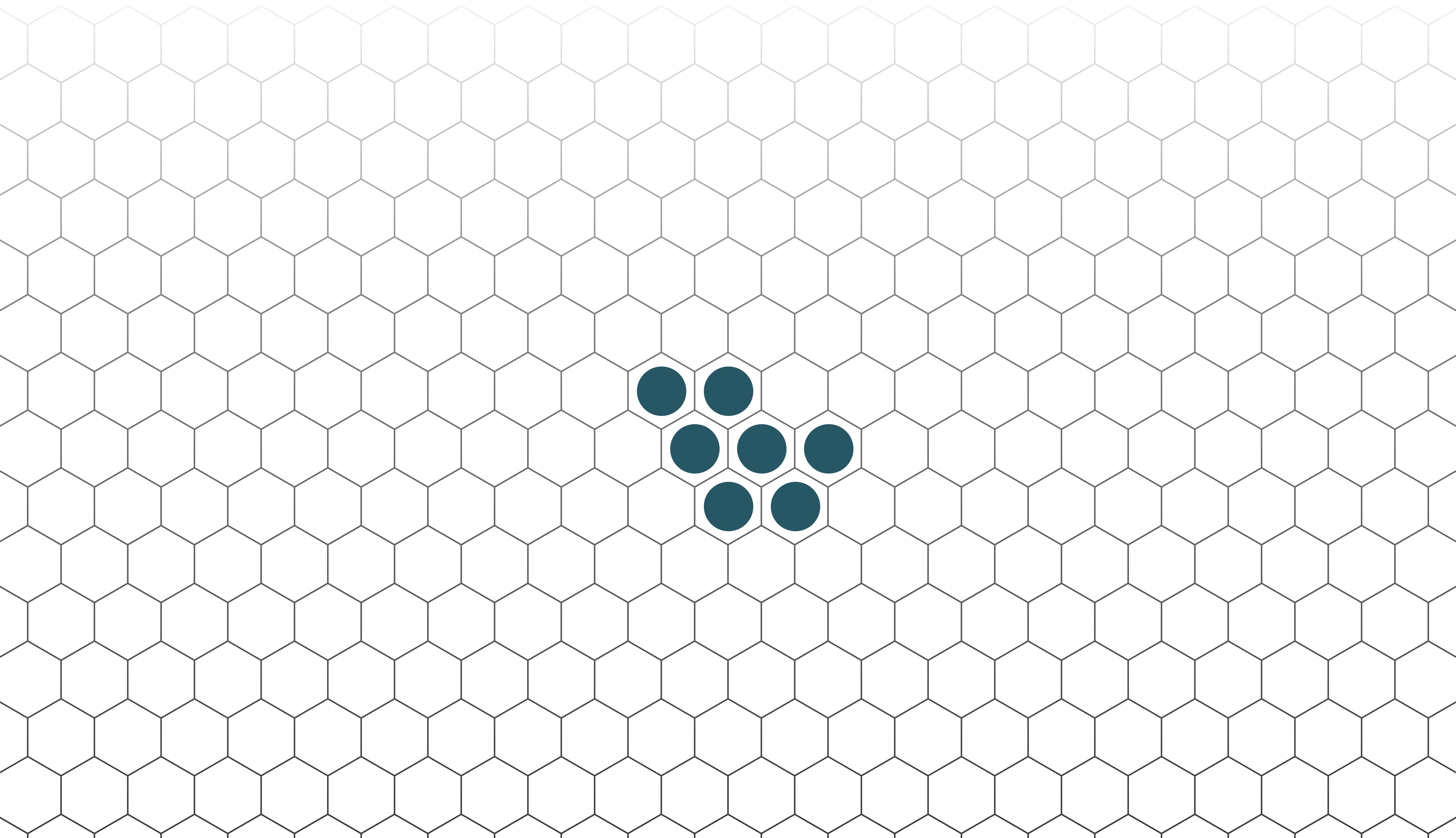


Cellular automata over a Euclidean grid



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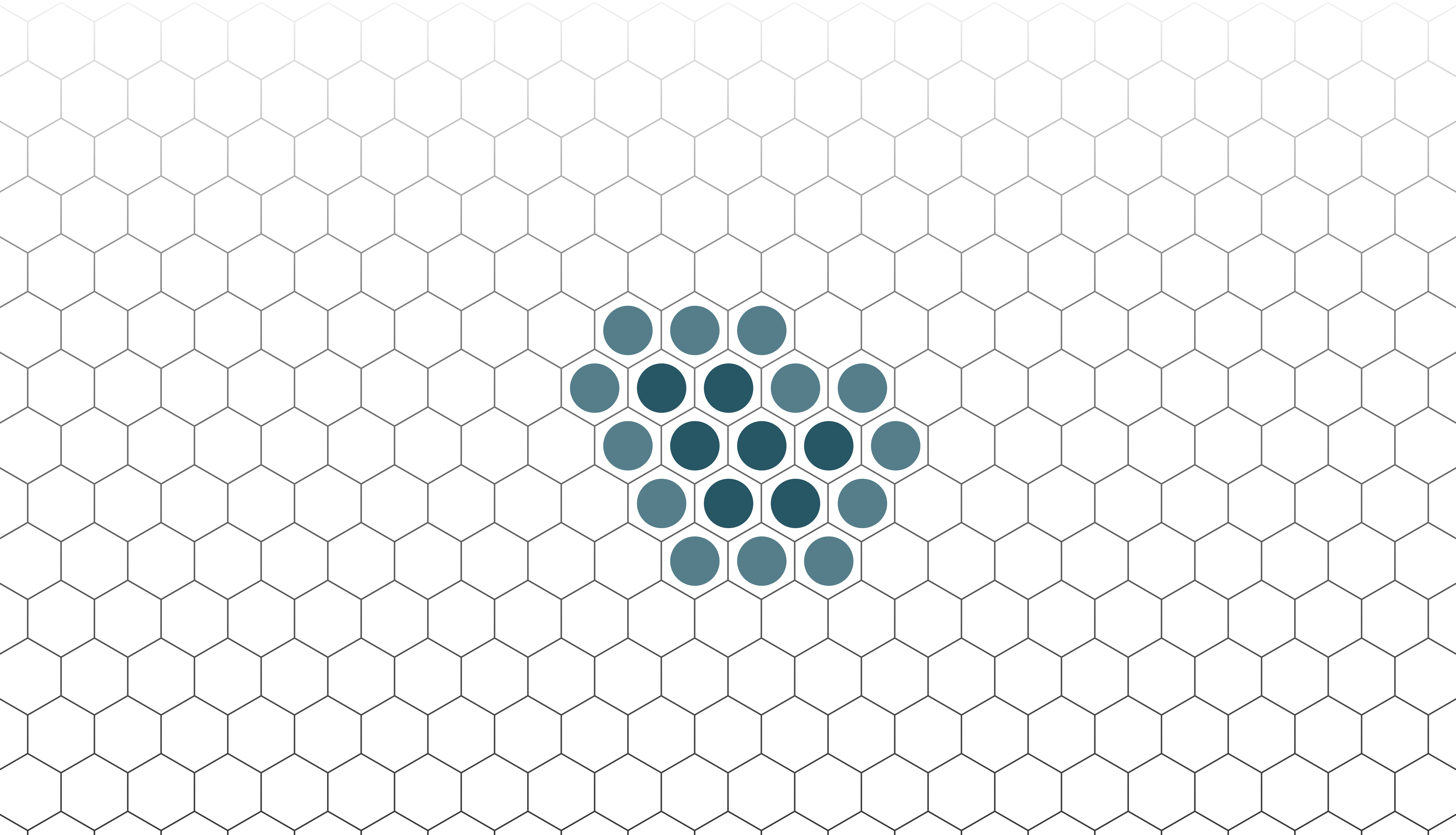

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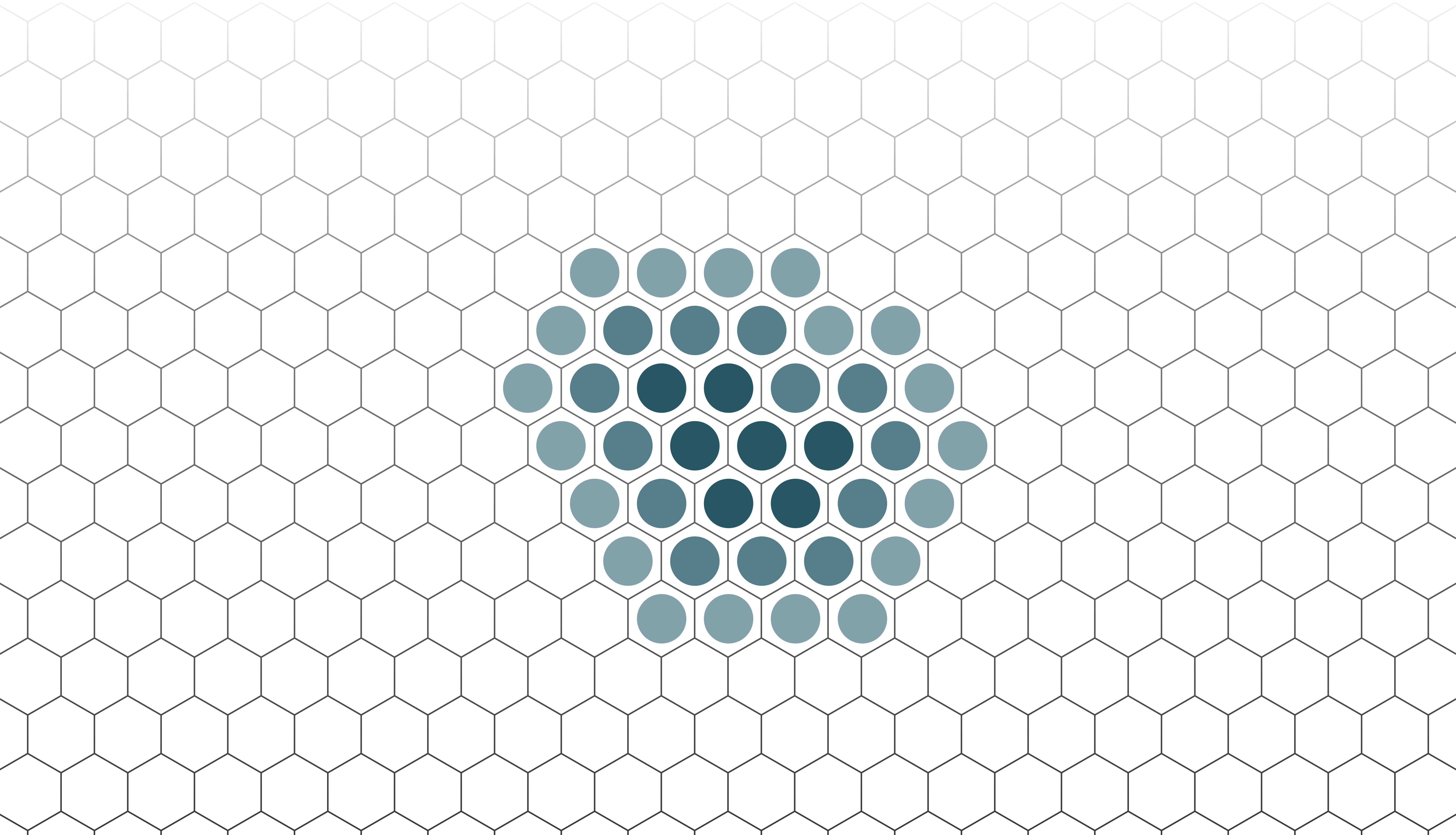

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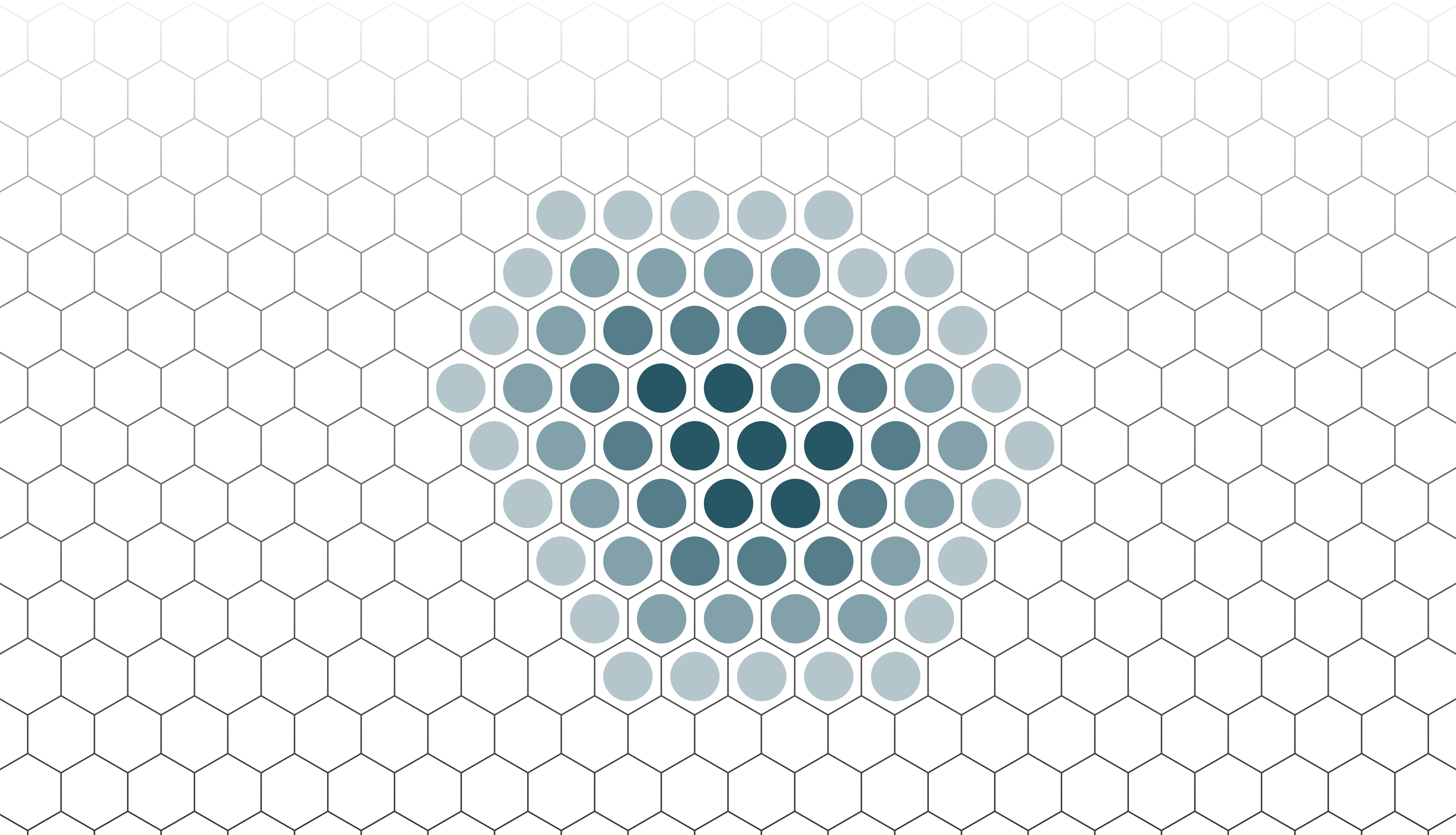
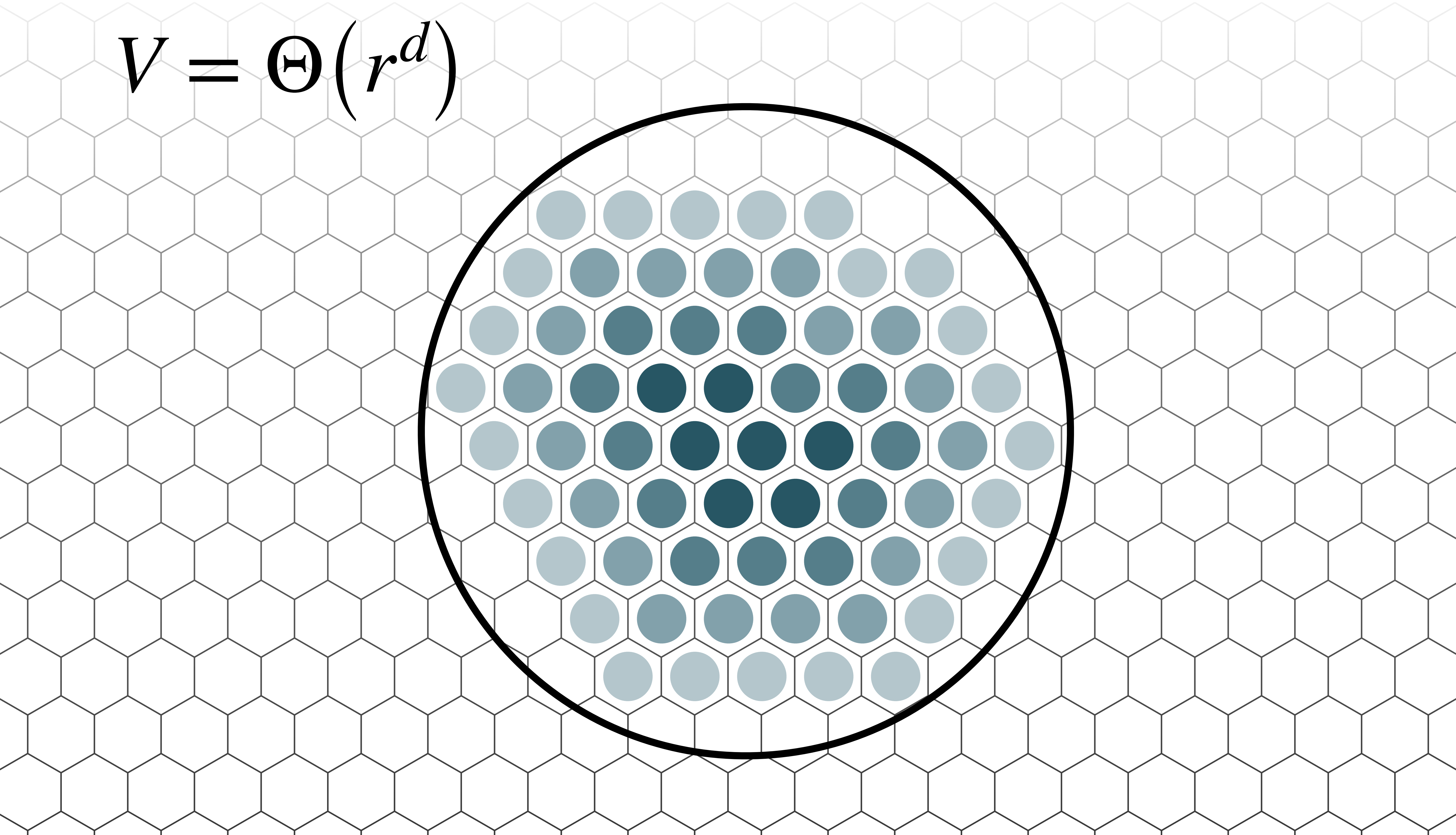
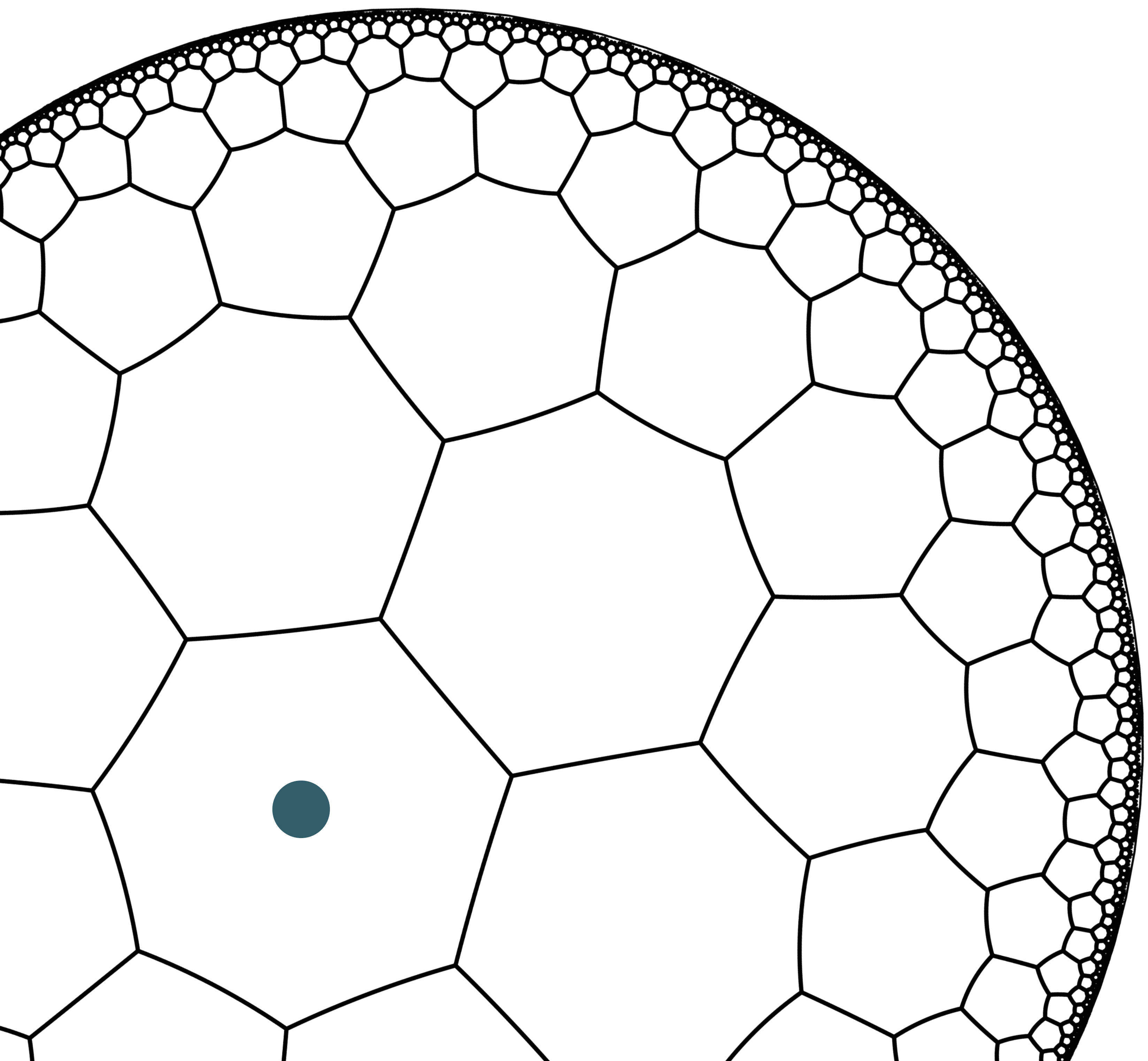


Cellular automata over a Euclidean grid

$$V = \Theta\left(r^d\right)$$

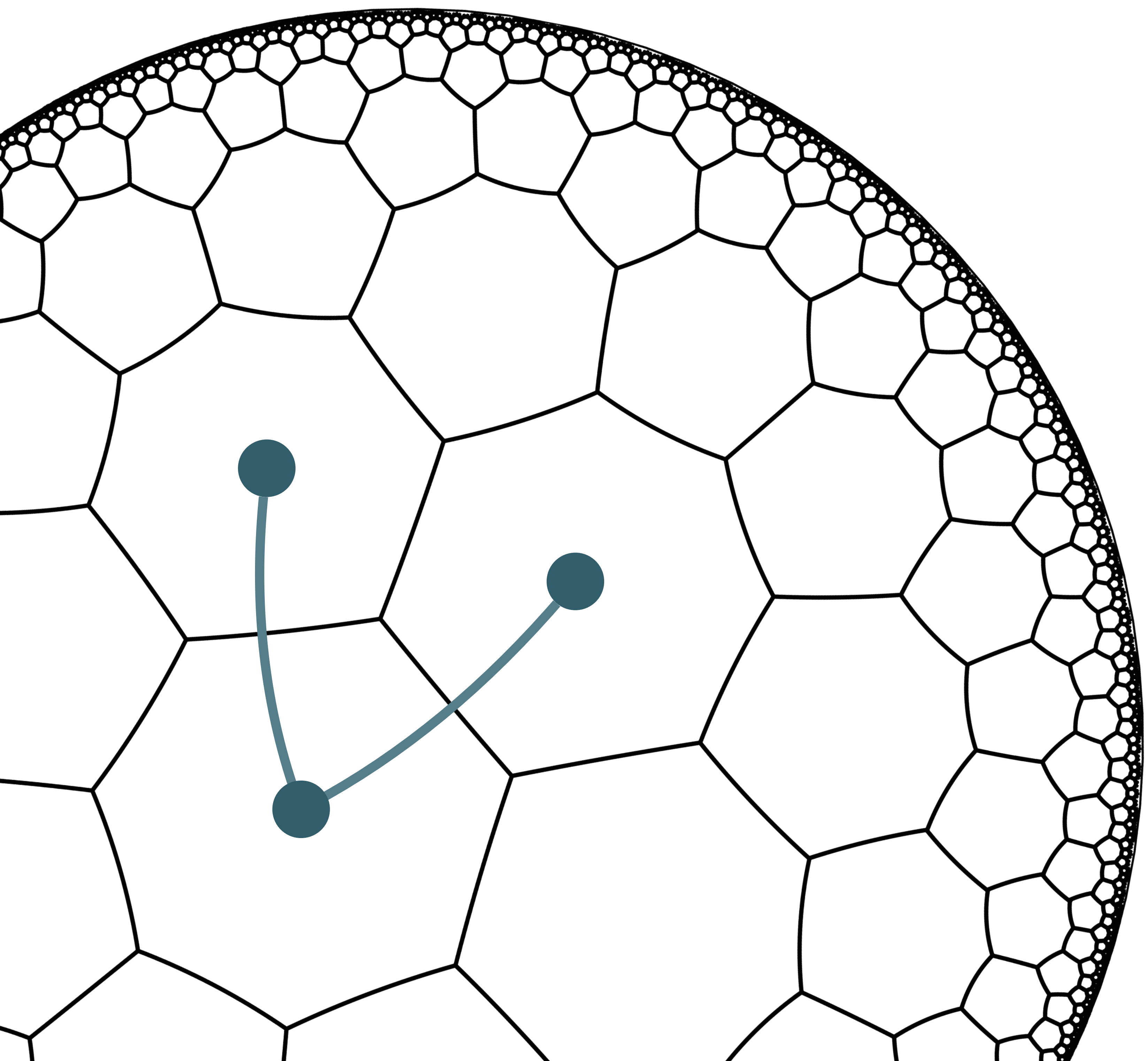


Hyperbolic cellular automata



Pavage du plan hyperbolique par des heptagones, dans le modèle du disque de Poincaré. By Theon, used under CC BY-SA 3.0 https:/en.wikipedia.org/wiki/File:PavageHypPoincare2.svg

Hyperbolic cellular automata



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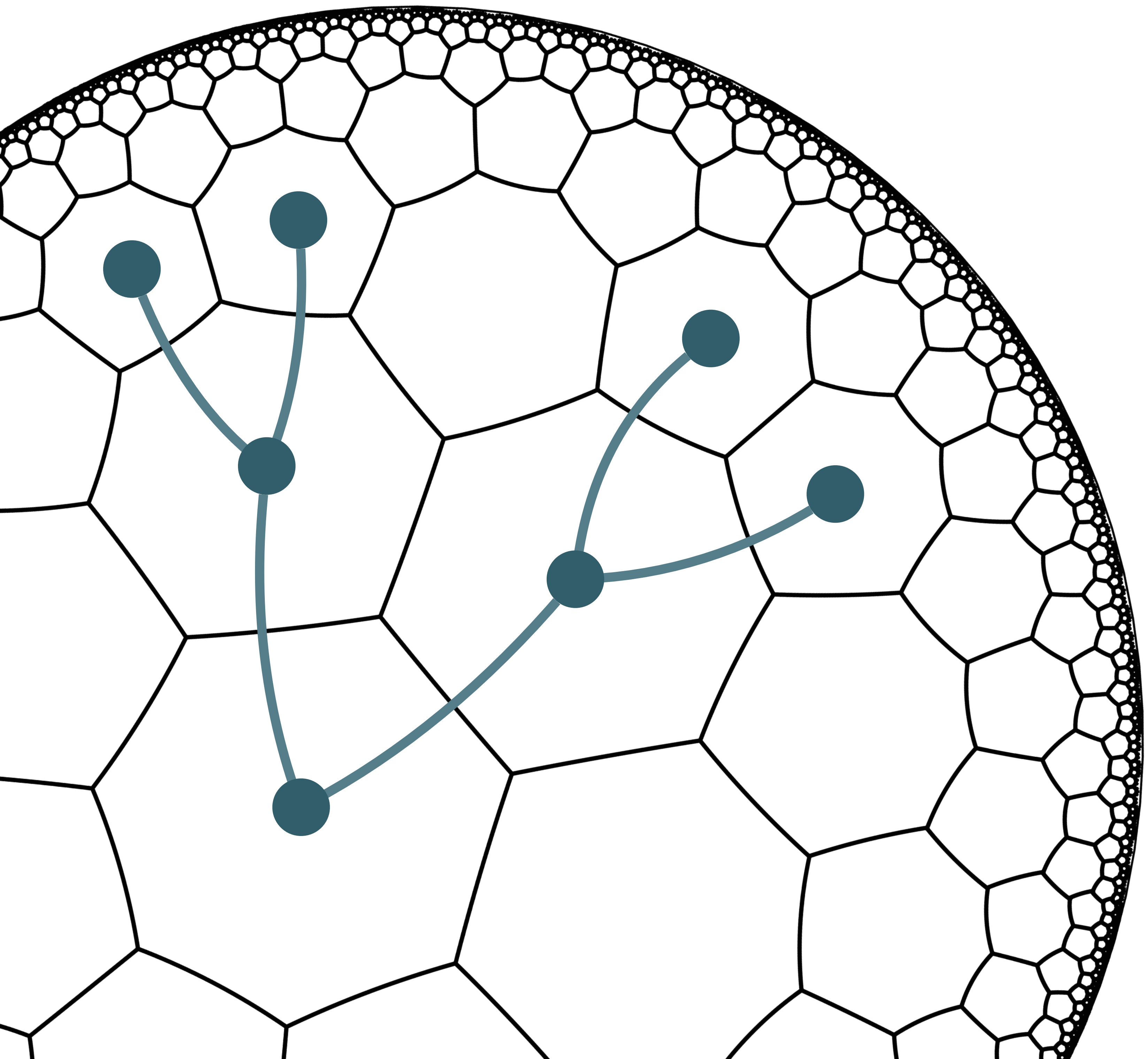

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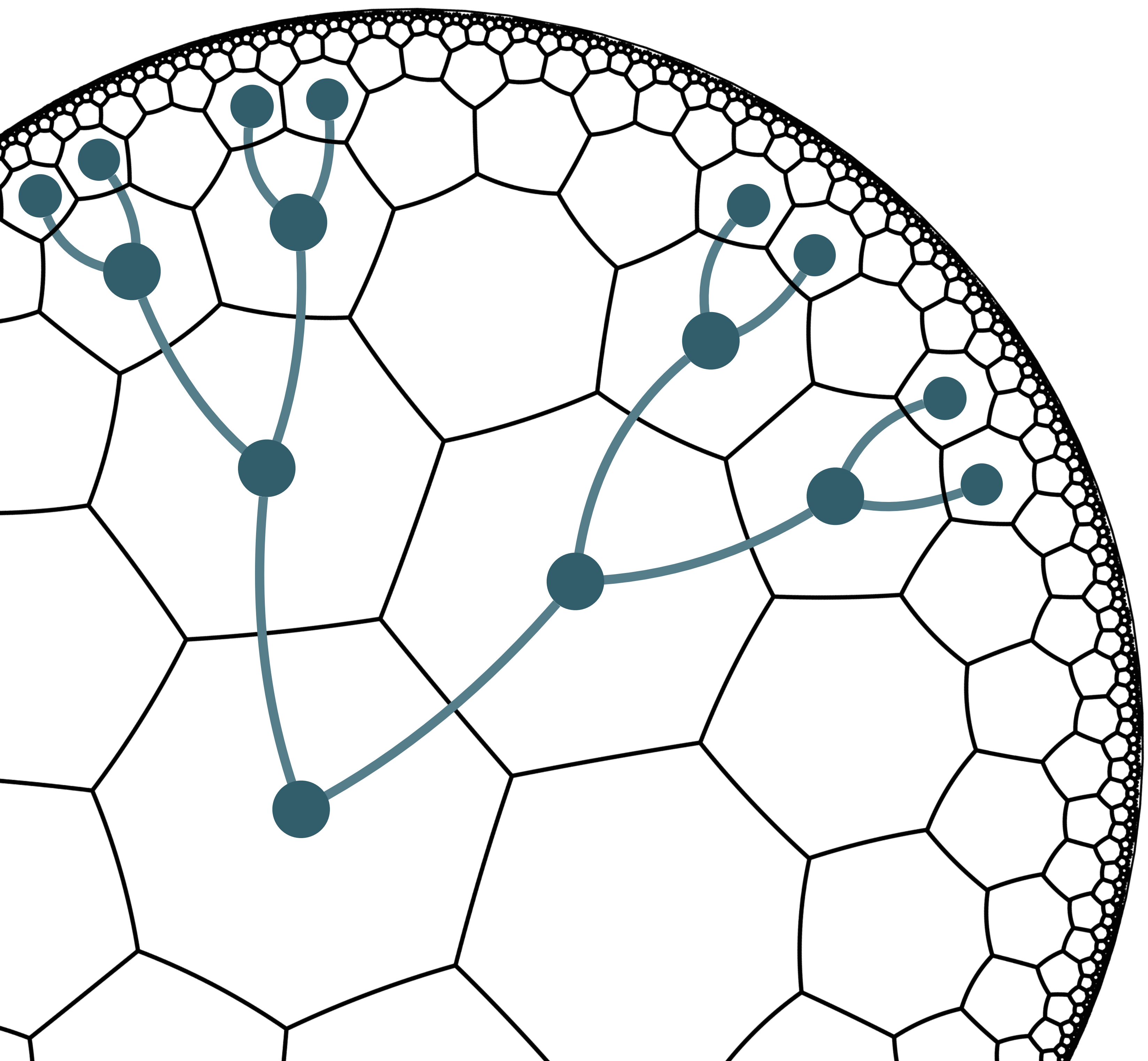

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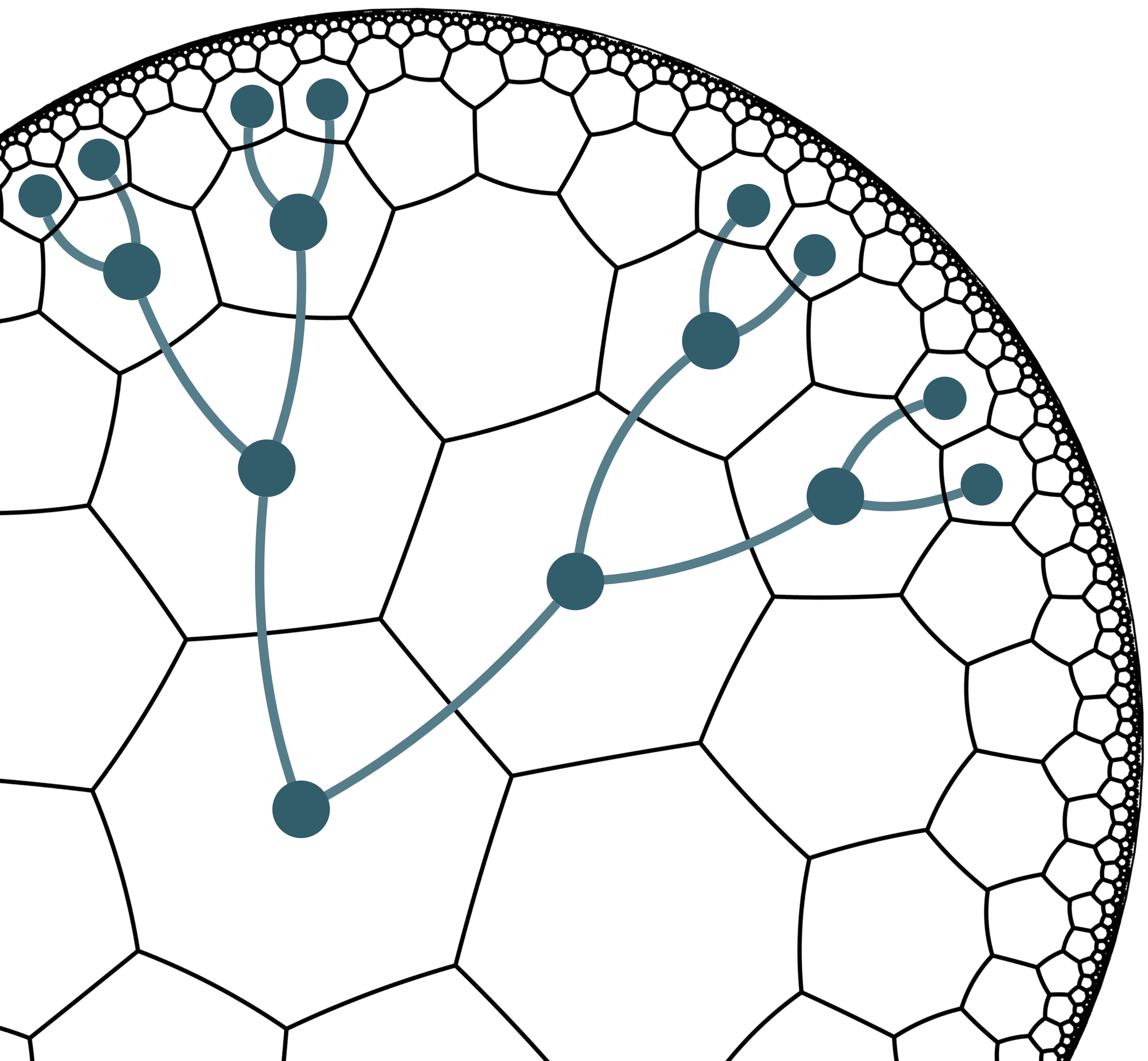


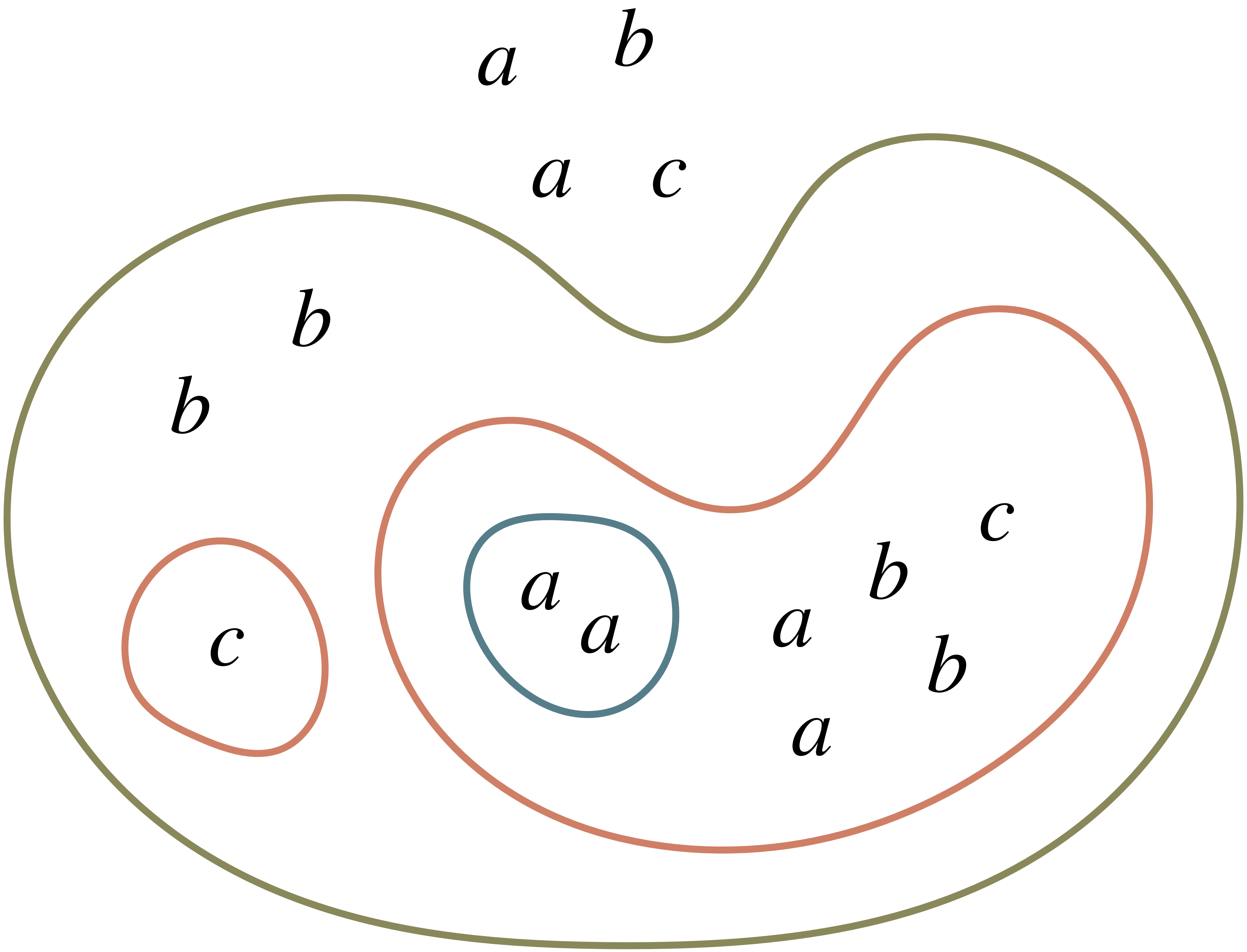
$$V = \Omega\left(2^r\right)$$

Pavage du plan hyperbolique par des heptagones, dans le modèle du disque de Poincaré. By Theon, used under CC BY-SA 3.0 https:/en.wikipedia.org/wiki/File:PavageHypPoincare2.svg

Rule of thumb

- Sequential machines are first class
- Bounded parallel machines are first class
- Unbounded parallel machines are second class

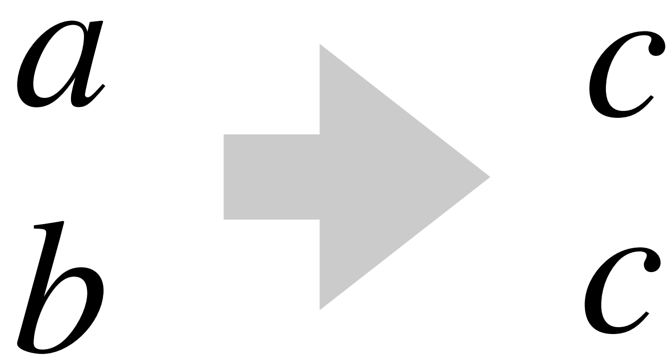
Membrane systems (P systems)

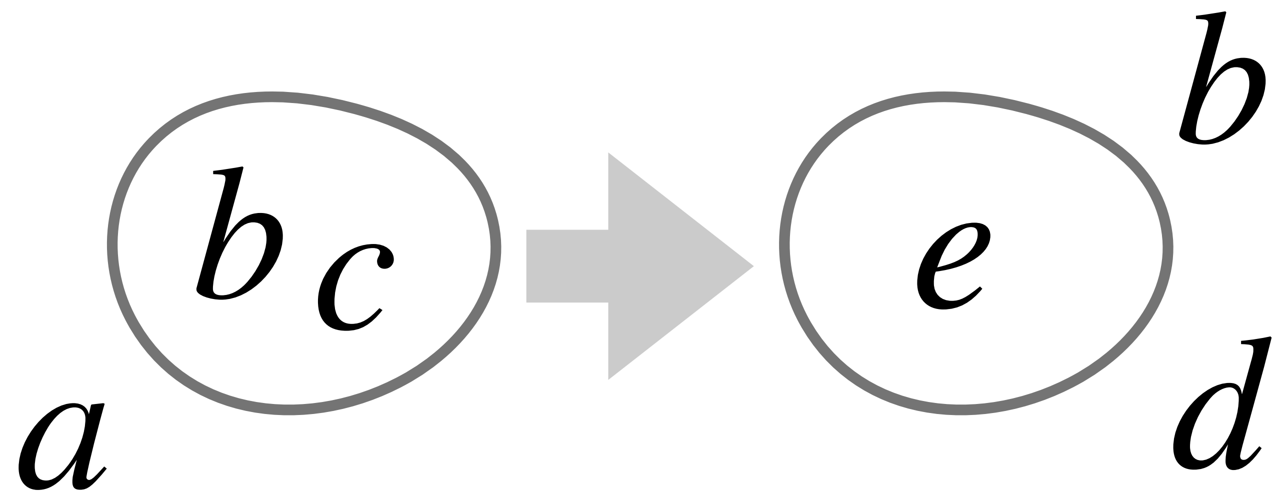


Evolution rules and maximally parallel semantics

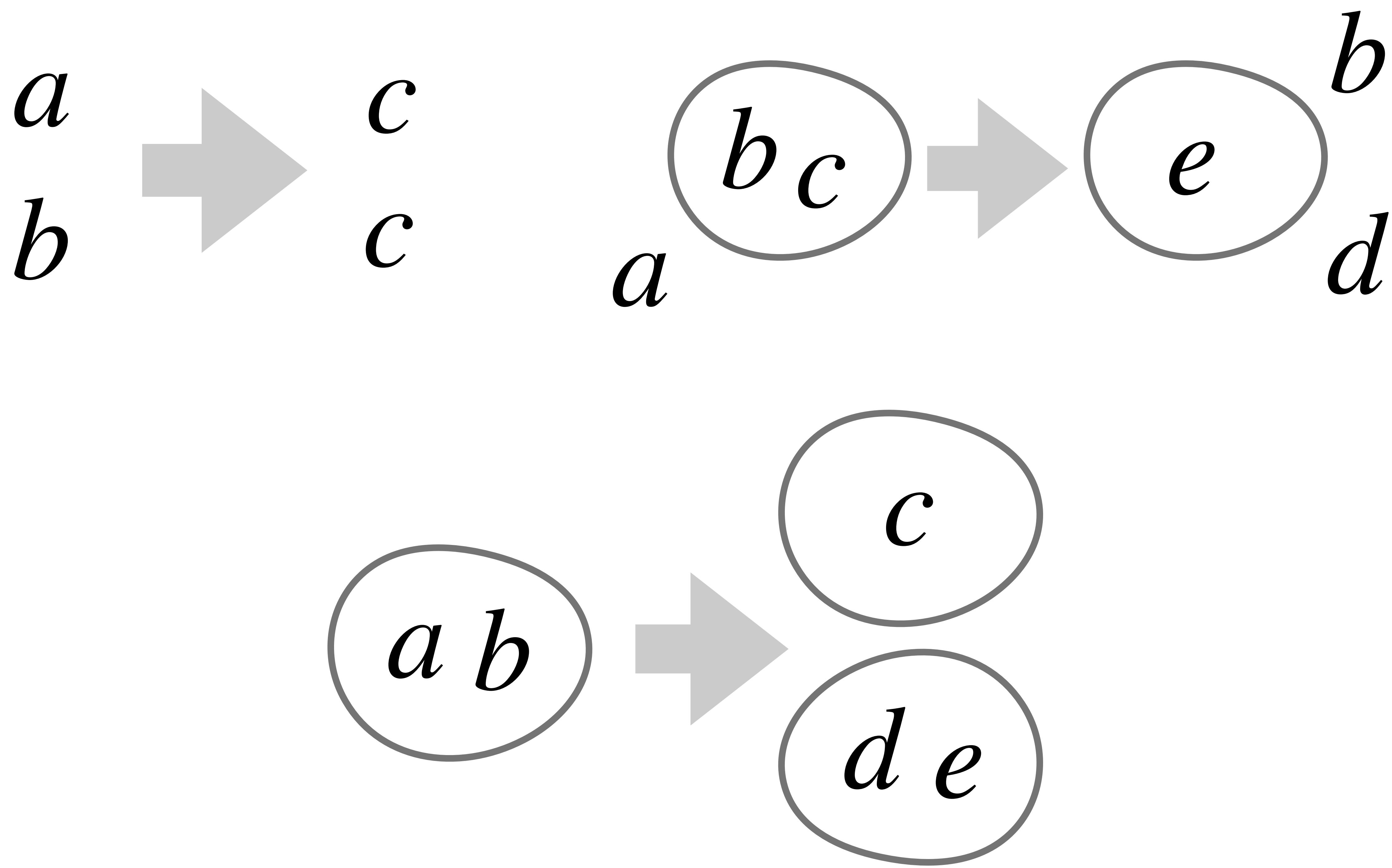
$$\begin{matrix} a \\ b \end{matrix} \;\Rightarrow\; \begin{matrix} c \\ c \end{matrix}$$

Evolution rules and maximally parallel semantics





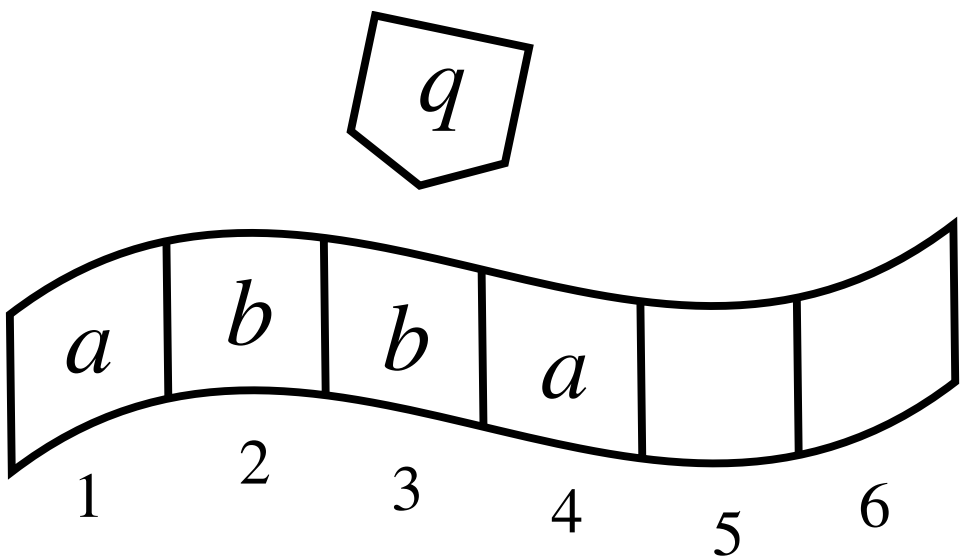
Evolution rules and maximally parallel semantics



Computational universality of membrane systems

- Able to simulate, e.g., counter machines
  when working in maximally parallel way
- Equivalent to Petri nets or vector addition systems
  when working in an asynchronous mode
- Which means that they are not computationally universal
- But see W. Czerwinski, S. Lasota, R. Lazic, J. Leroux, F. Mazowiecki, **The Reachability Problem for Petri Nets is Not Elementary**, https://arxiv.org/abs/1809.07115

Simulating bounded-tape Turing machines



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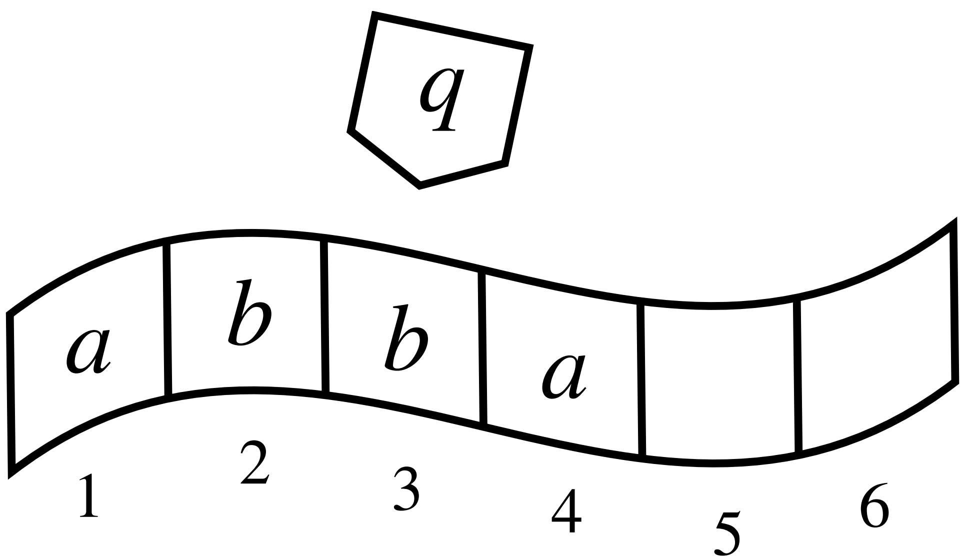


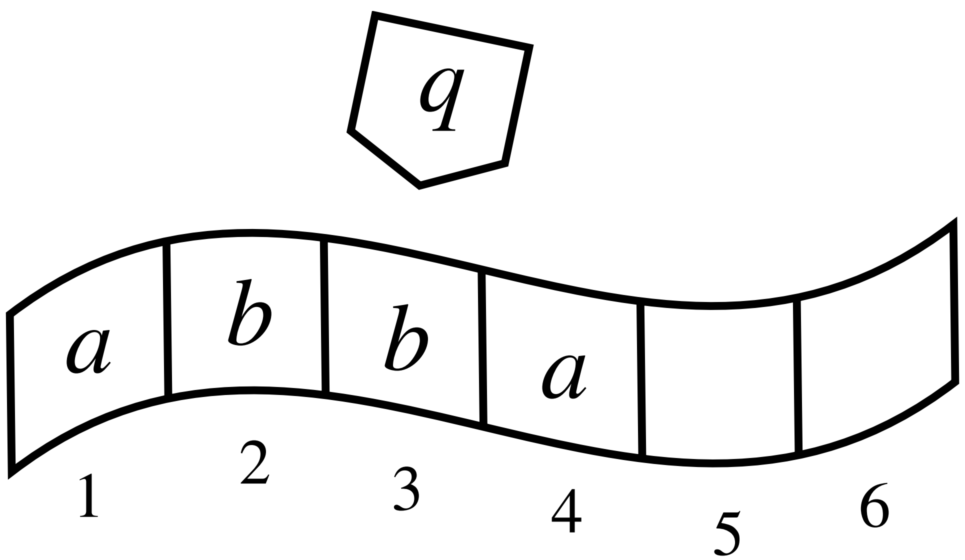
b_2 b_3

a_1

a_4

q_3

Simulating bounded-tape Turing machines



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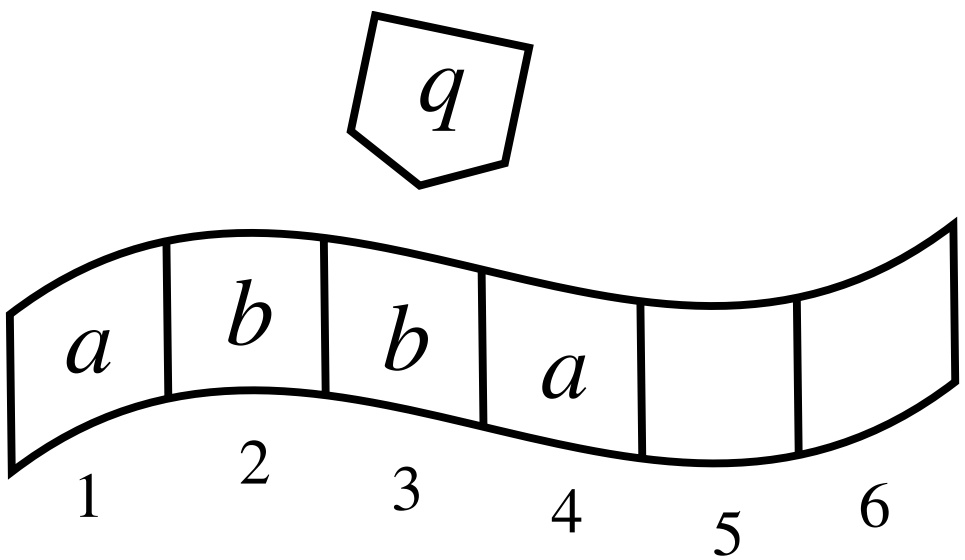
a_1

a_4

q_3

$$\delta(q, b) = (r, a, +1)$$

Simulating bounded-tape Turing machines

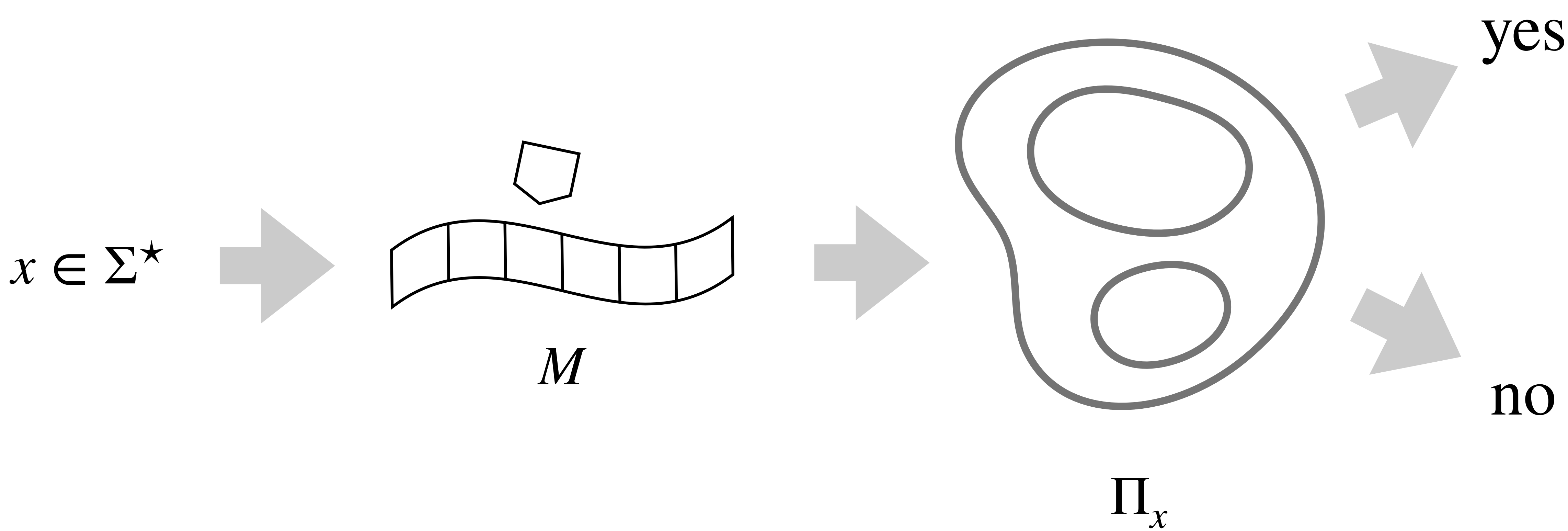


$$b_2 \quad b_3$$
$$a_1 \qquad\qquad a_4$$
$$q_3$$

$$\delta(q, b) = (r, a, +1)$$

$$\begin{matrix} b_i \\ q_i \end{matrix} \Rightarrow \begin{matrix} a_i \\ r_{i+1} \end{matrix}$$

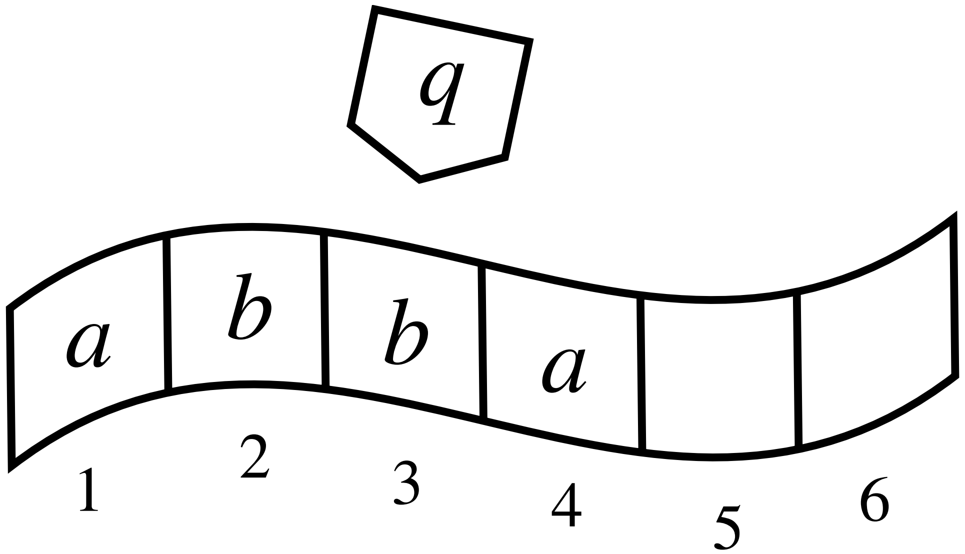
Semi-uniform confluent families of P systems



“Milano Theorem”

Semi-uniform families of confluent membrane systems **without** membrane division characterise **P** in polynomial time.

Membrane systems solving **NP** problems



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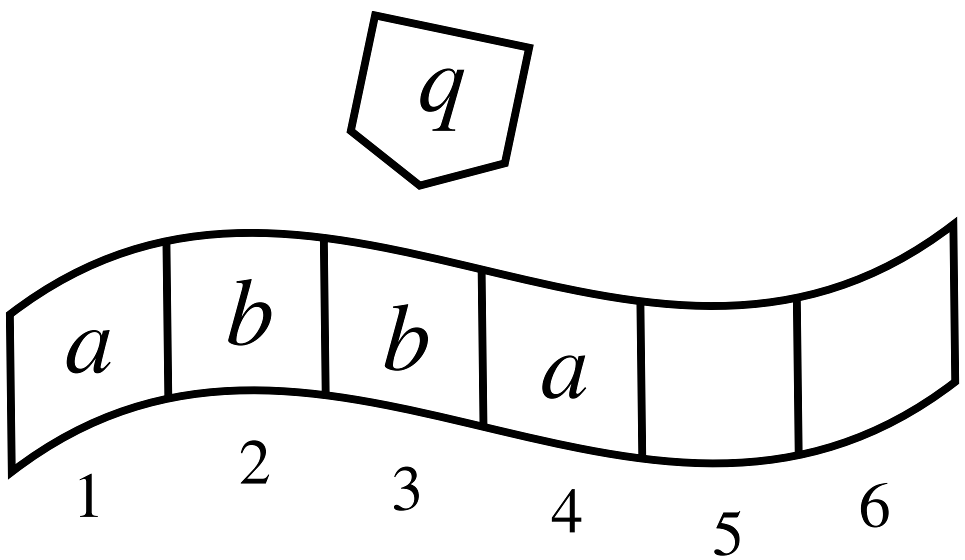

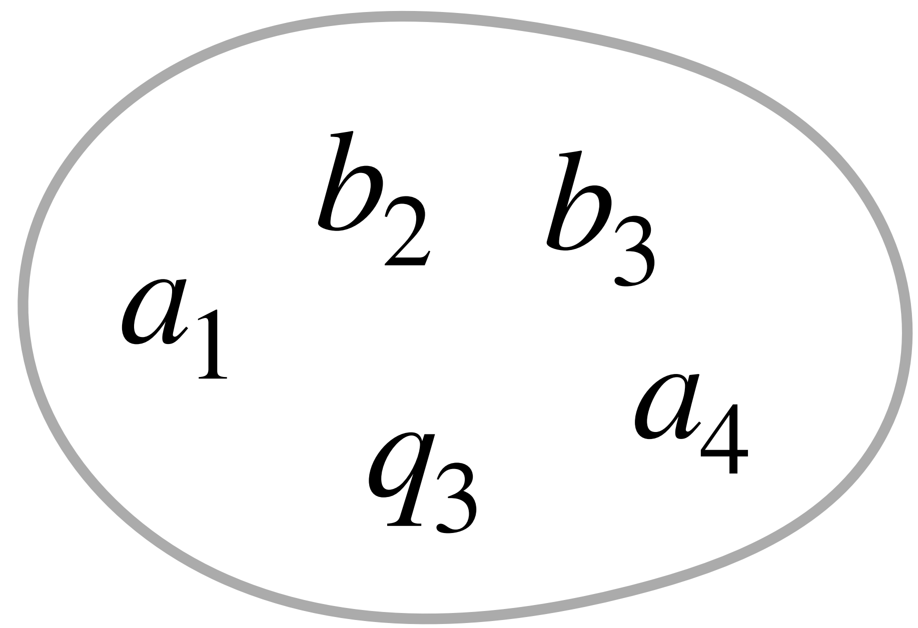

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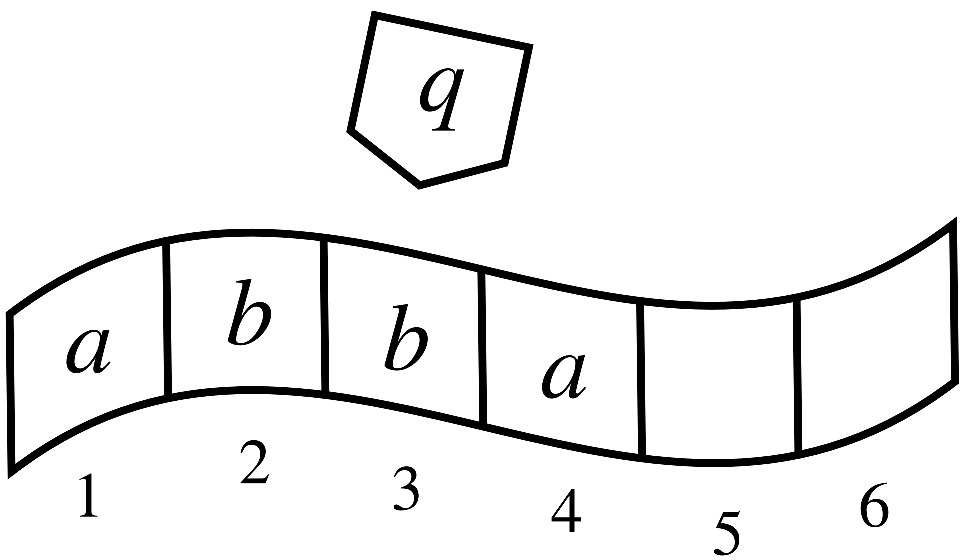
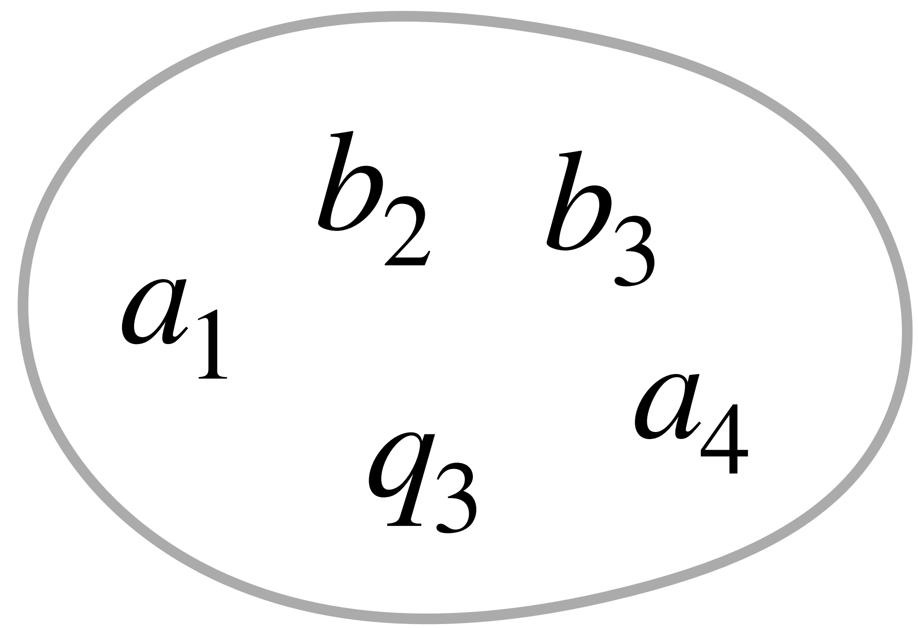
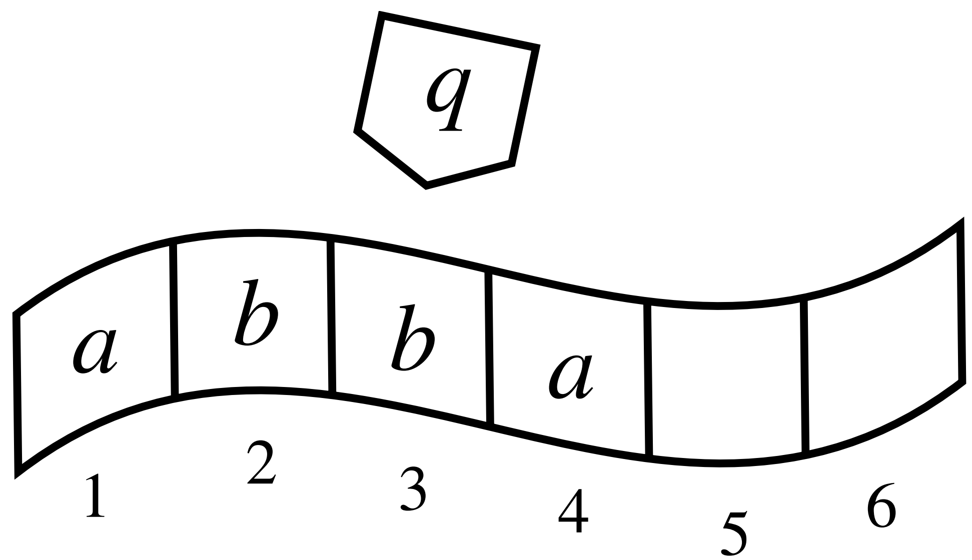
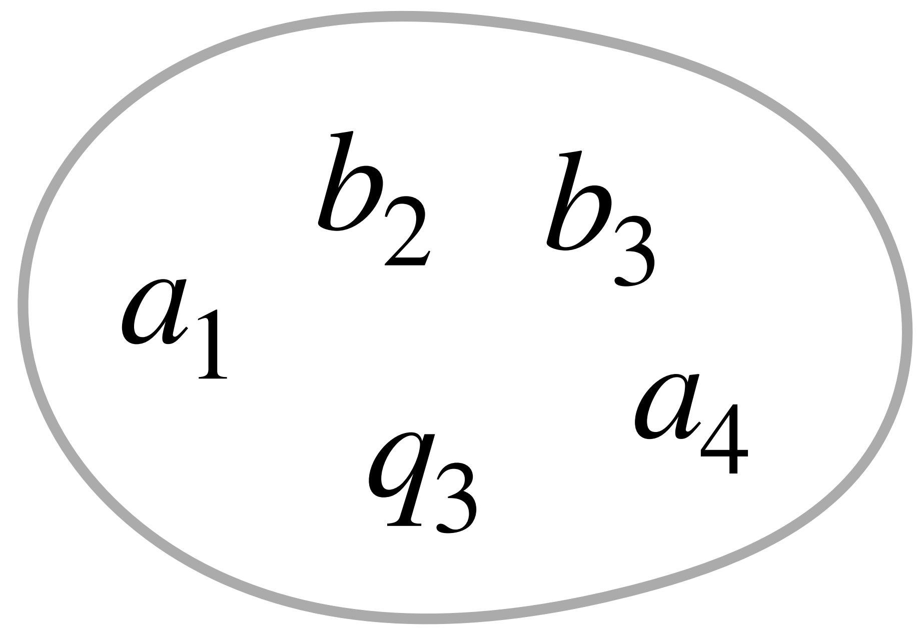




$$\delta(q, b) = \begin{cases} (r, a, +1) \\ (s, b, -1) \end{cases}$$

Membrane systems solving **NP** problems





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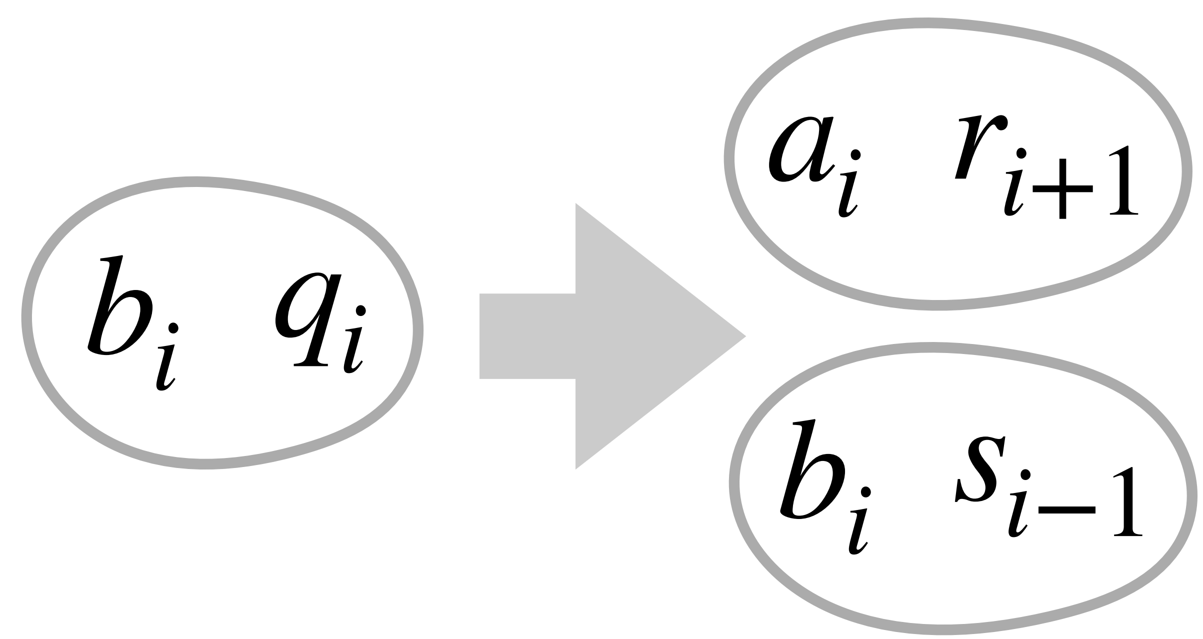
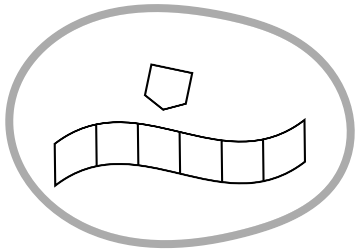


Simulating nondeterminism with parallelism



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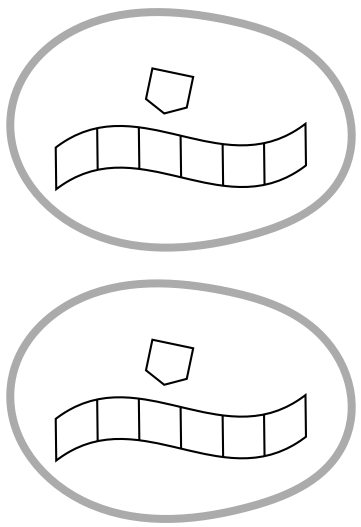

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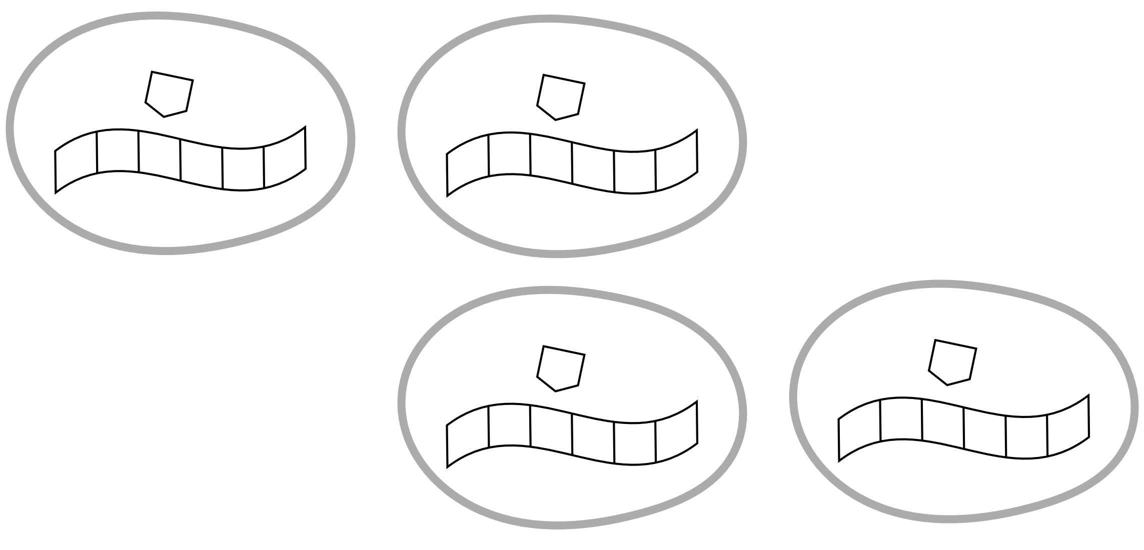

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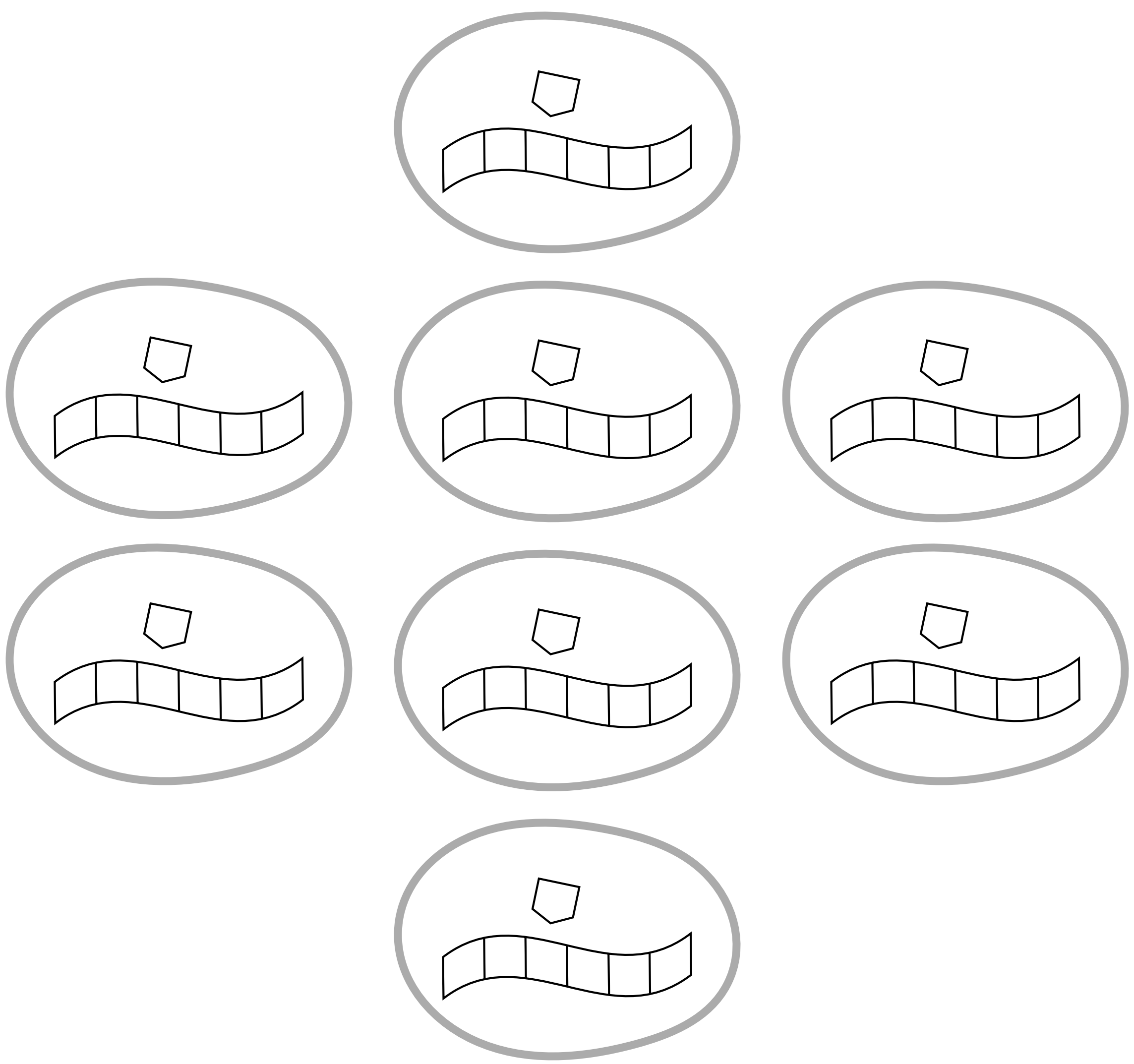

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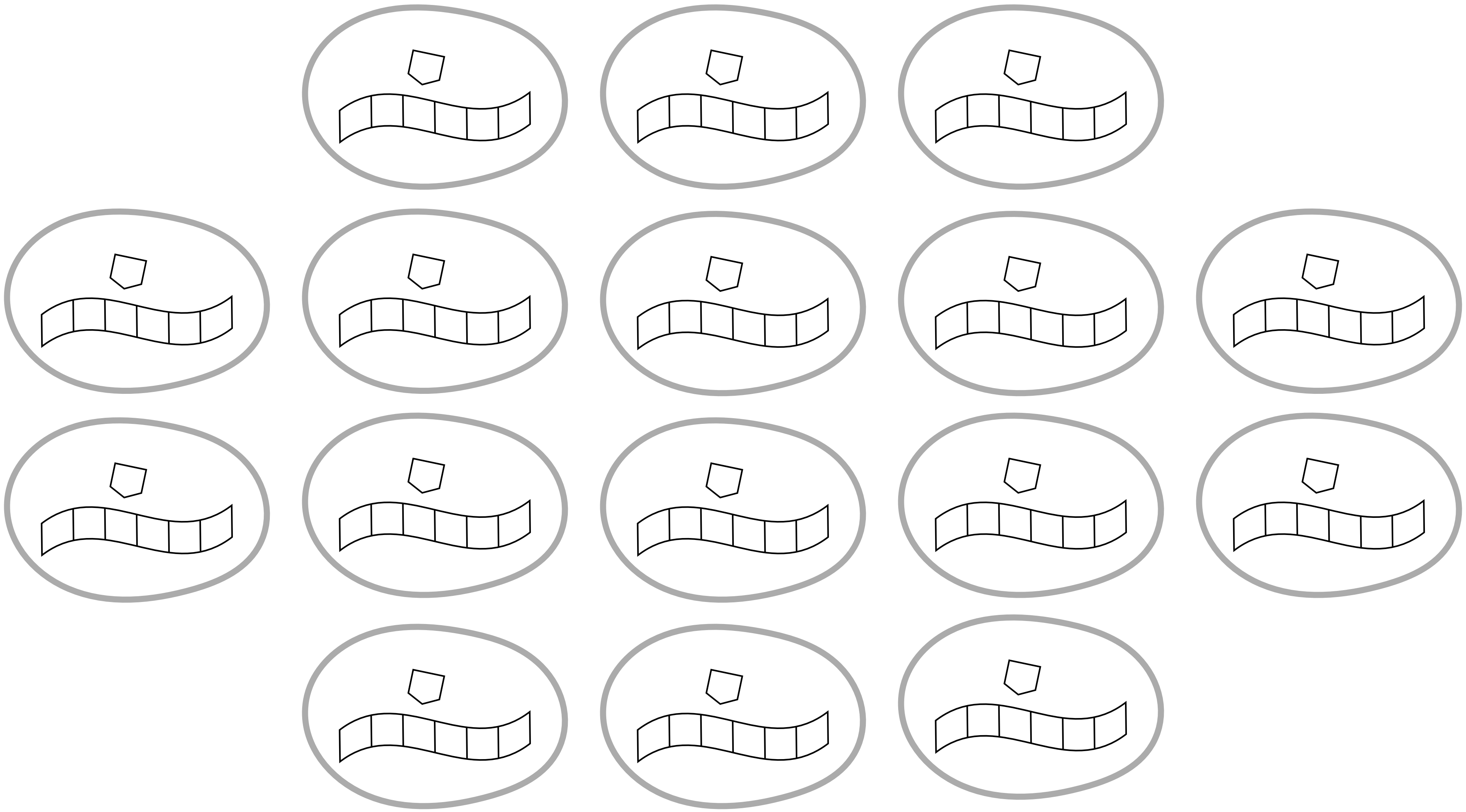

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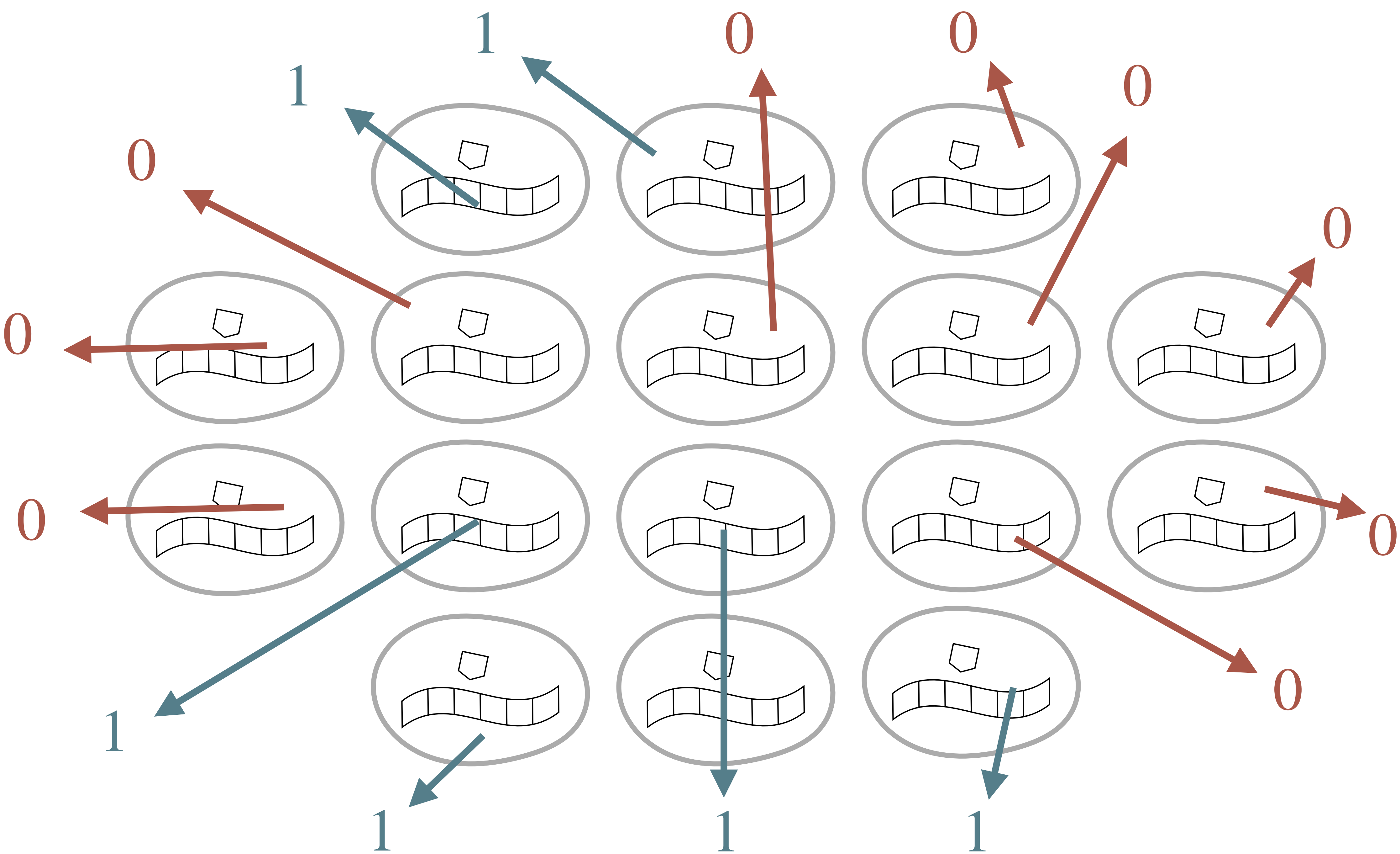
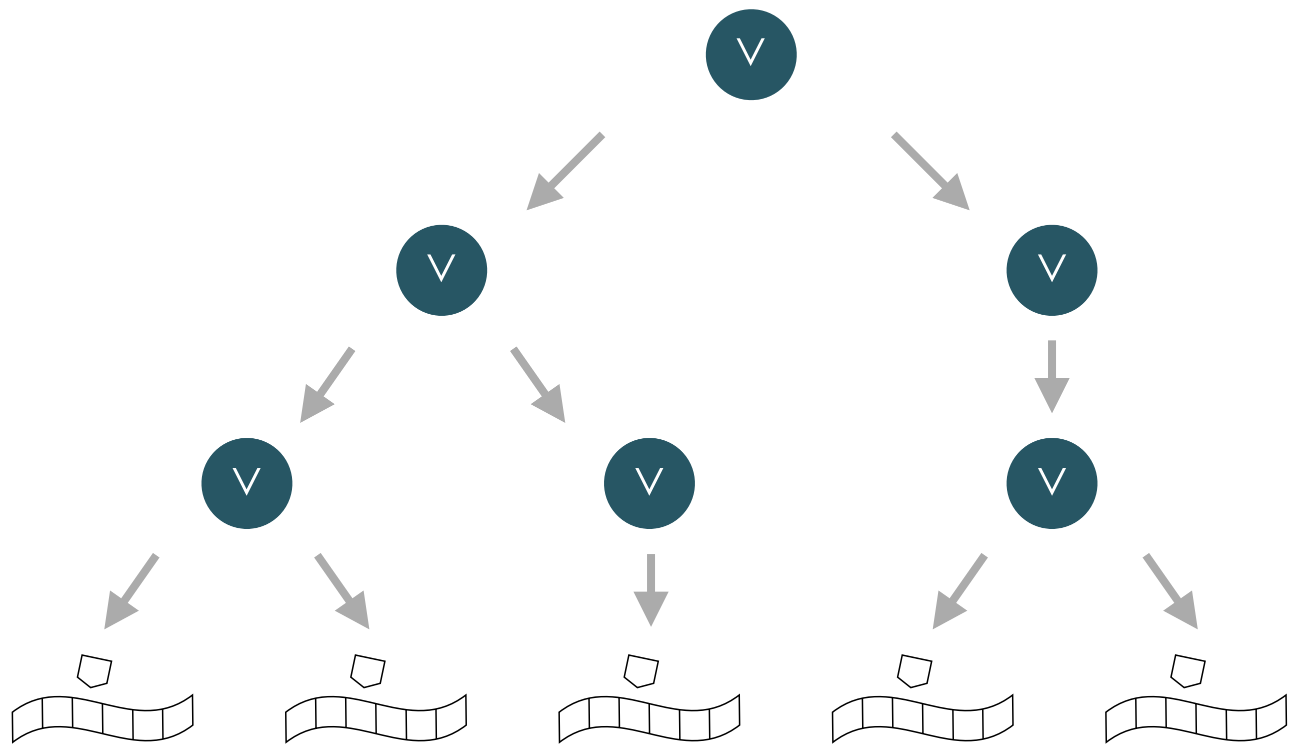


Flattening nondeterministic computation trees



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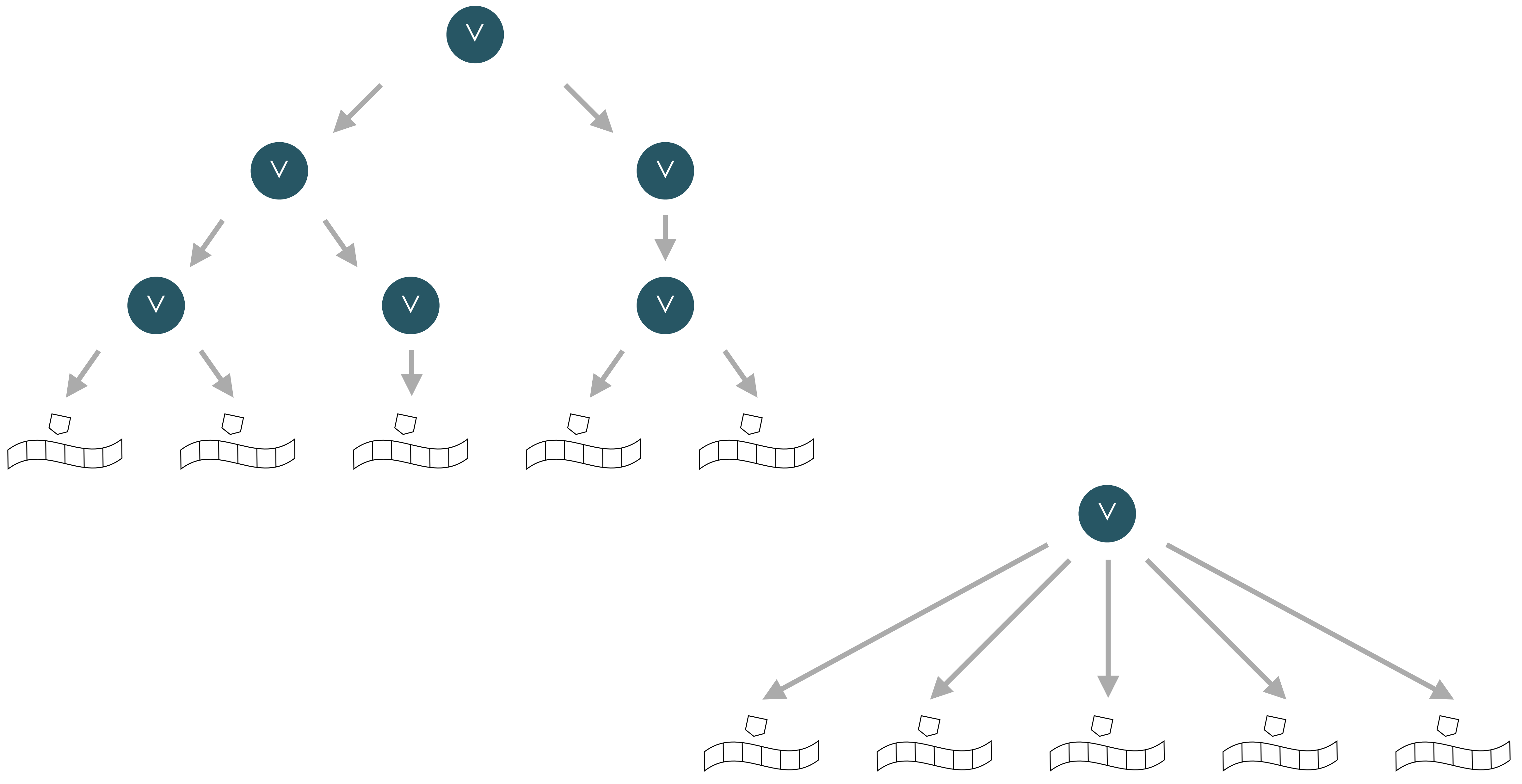
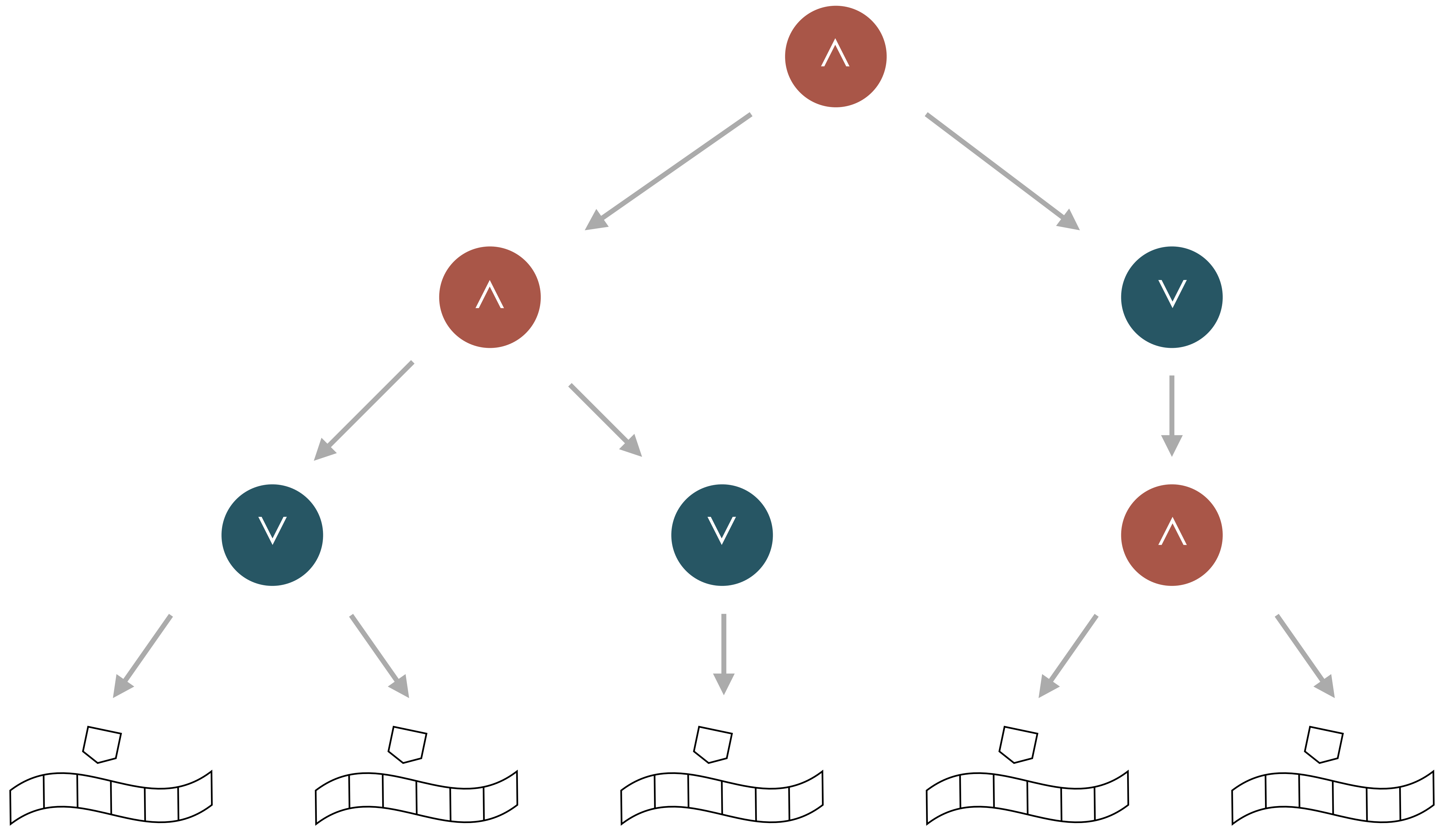
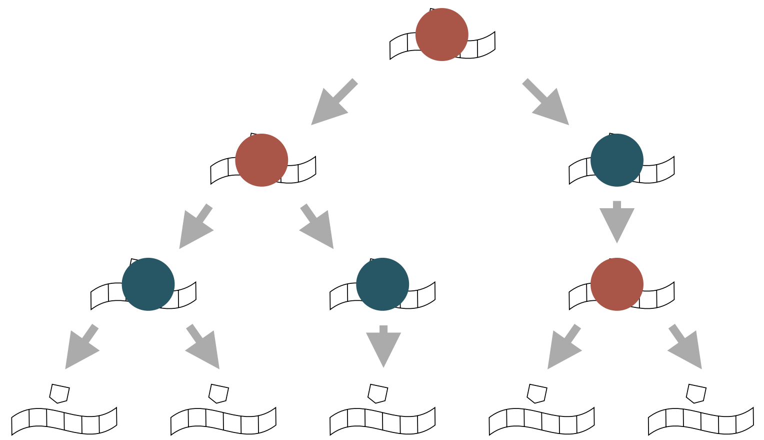


Cannot flatten alternating computation trees!



Membrane systems are second class



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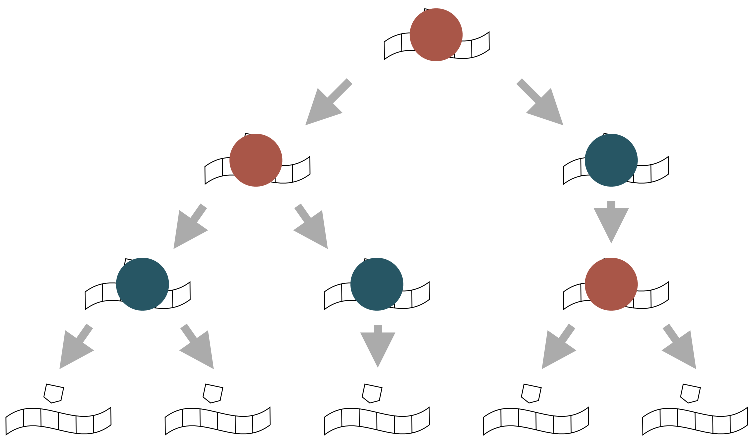

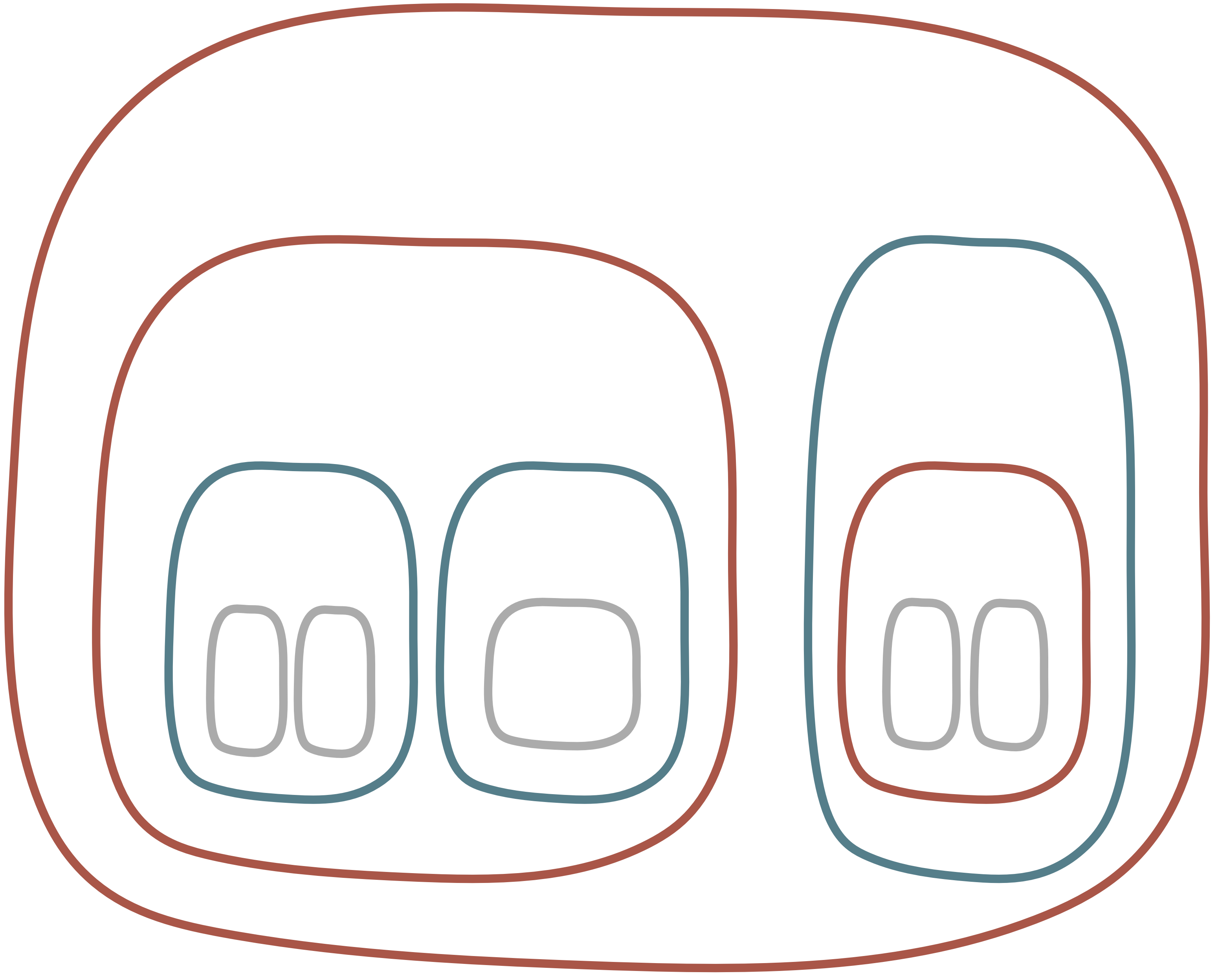

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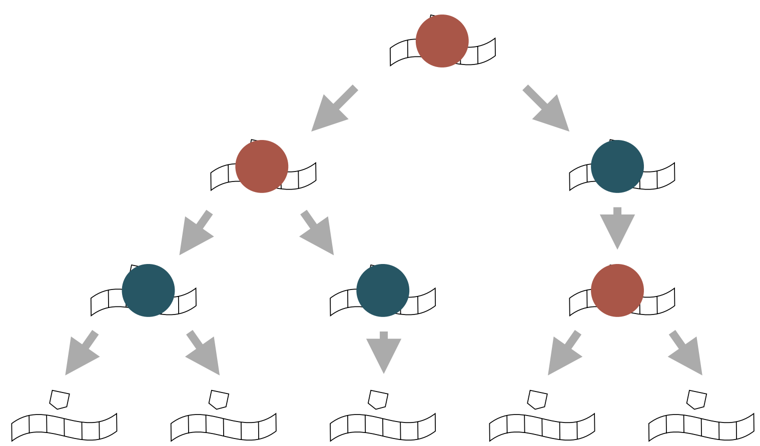
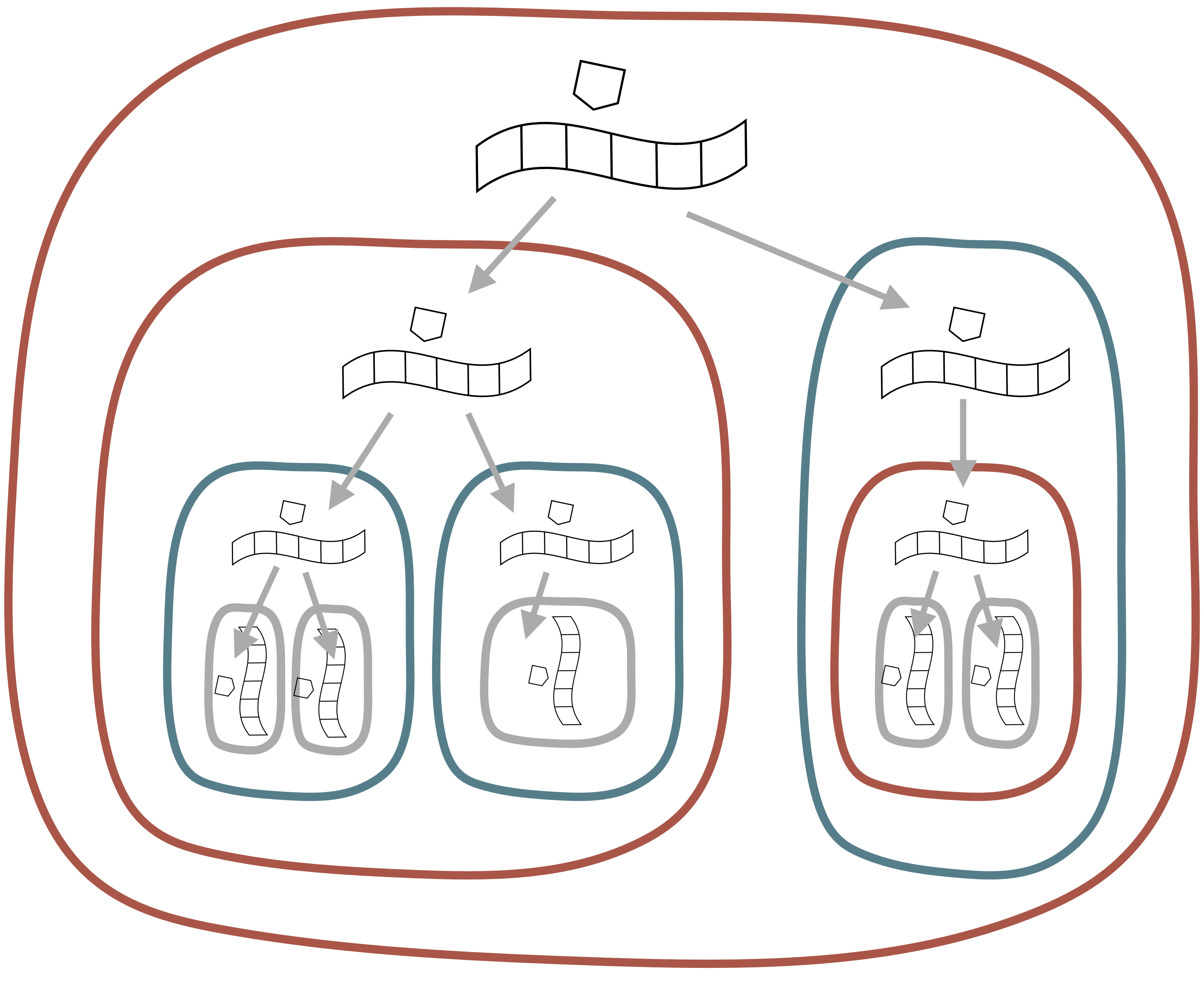
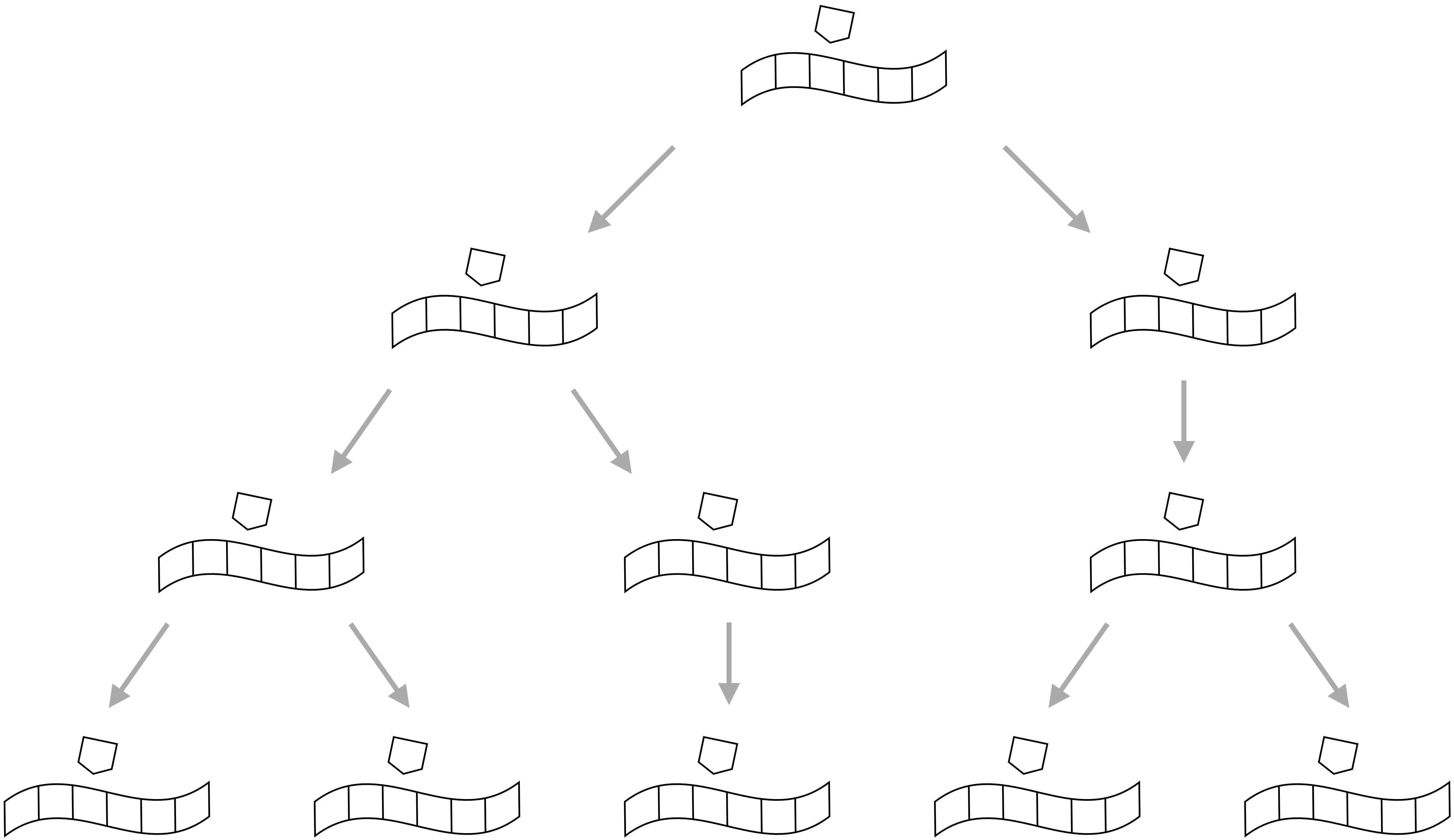




Counting Turing machines: **#P**



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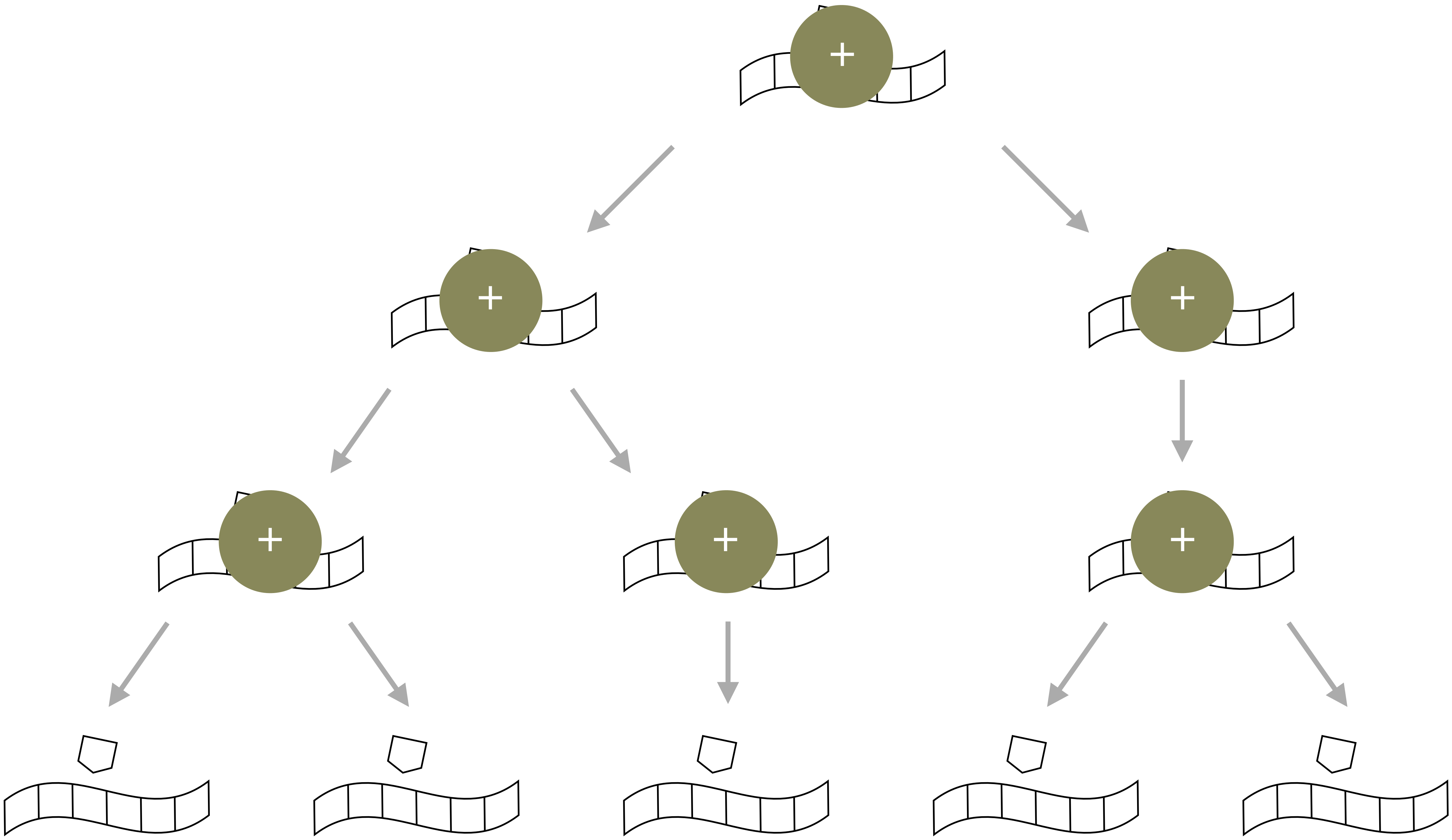
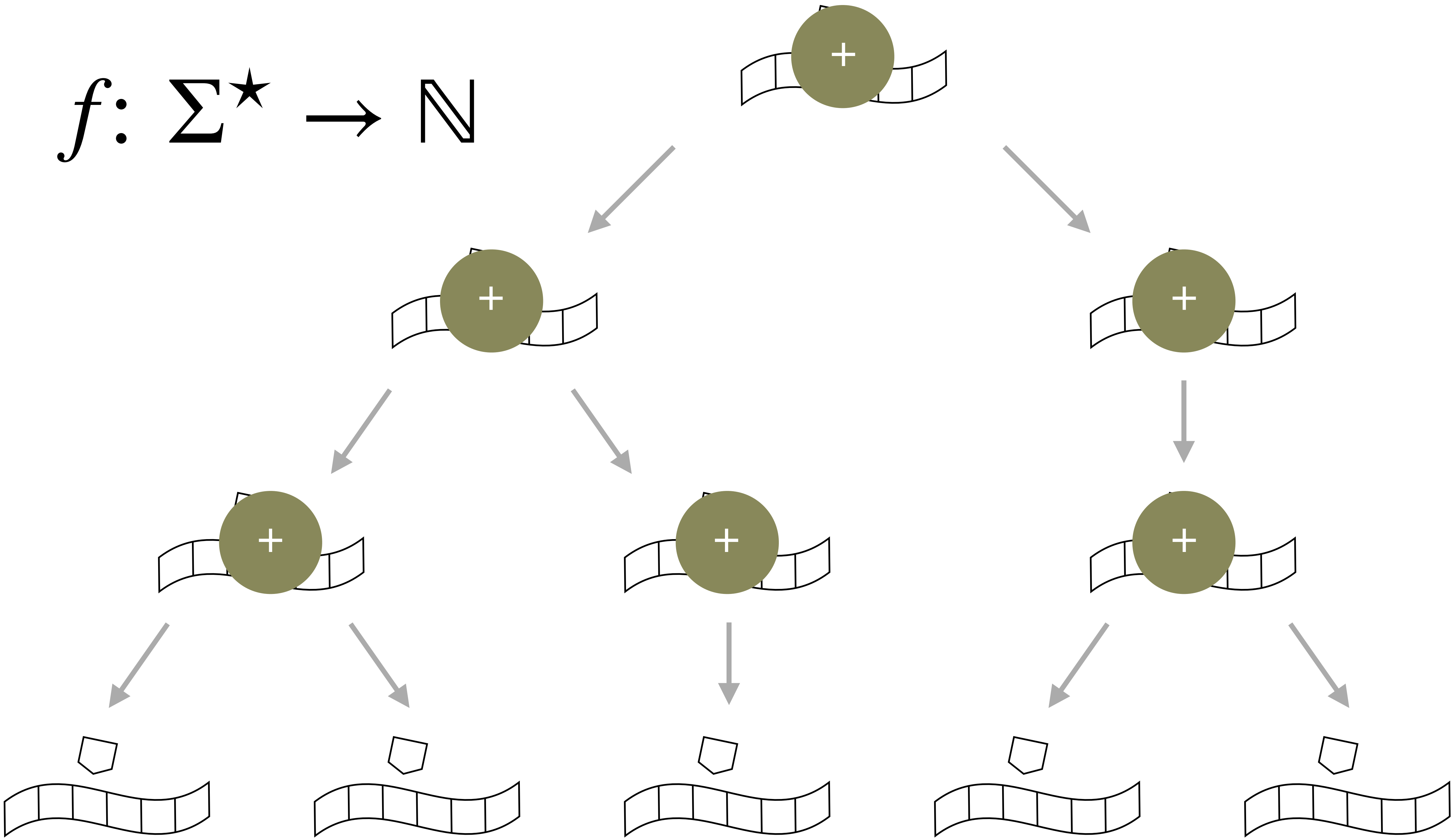
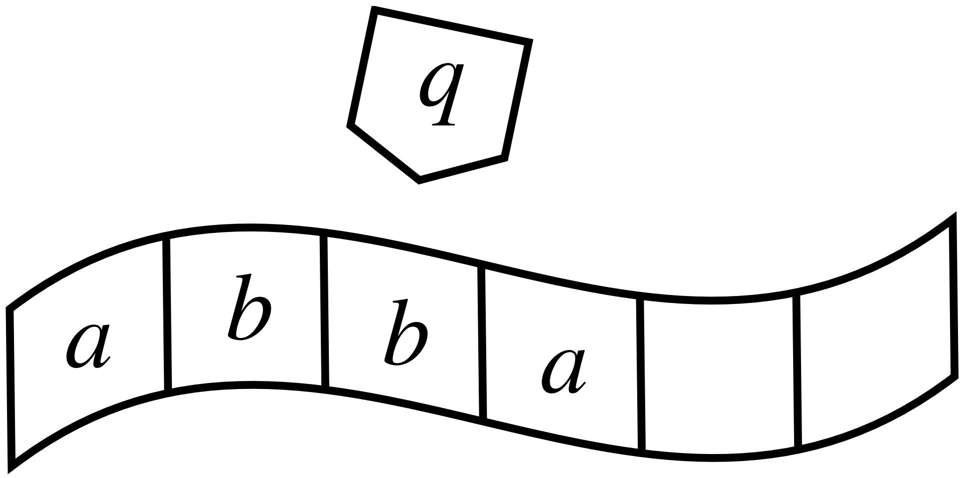


Counting Turing machines: **#P**

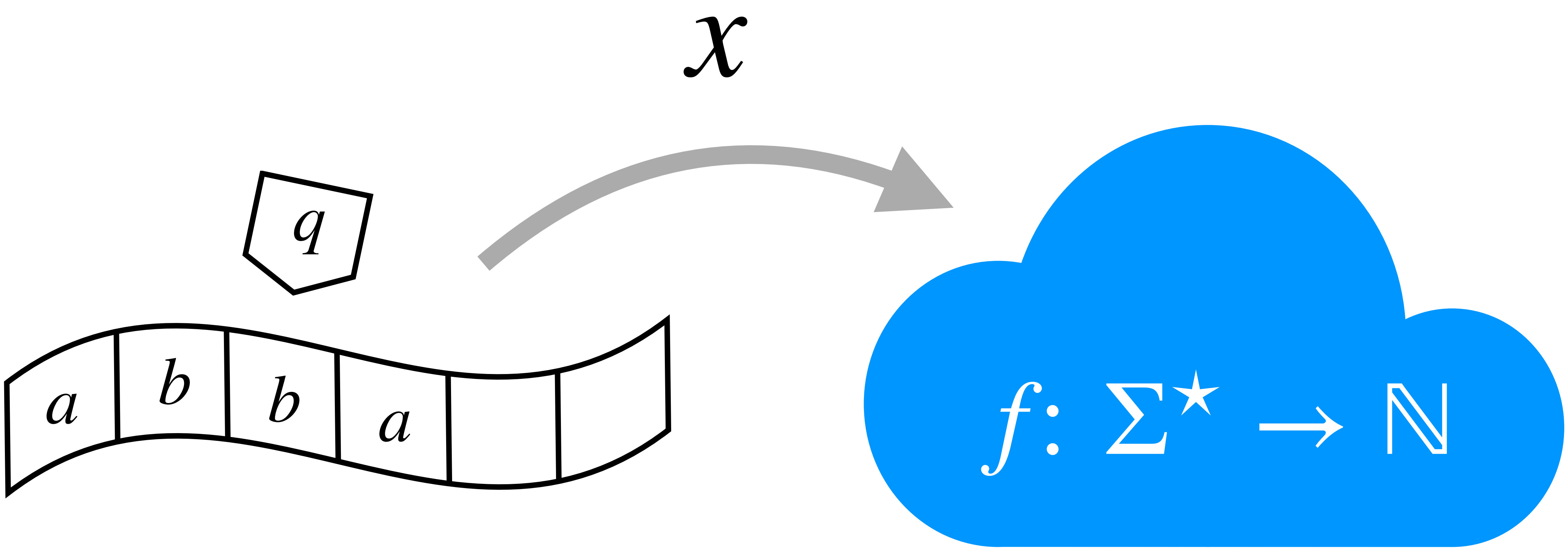
$$f\colon \Sigma^\star \to \mathbb{N}$$



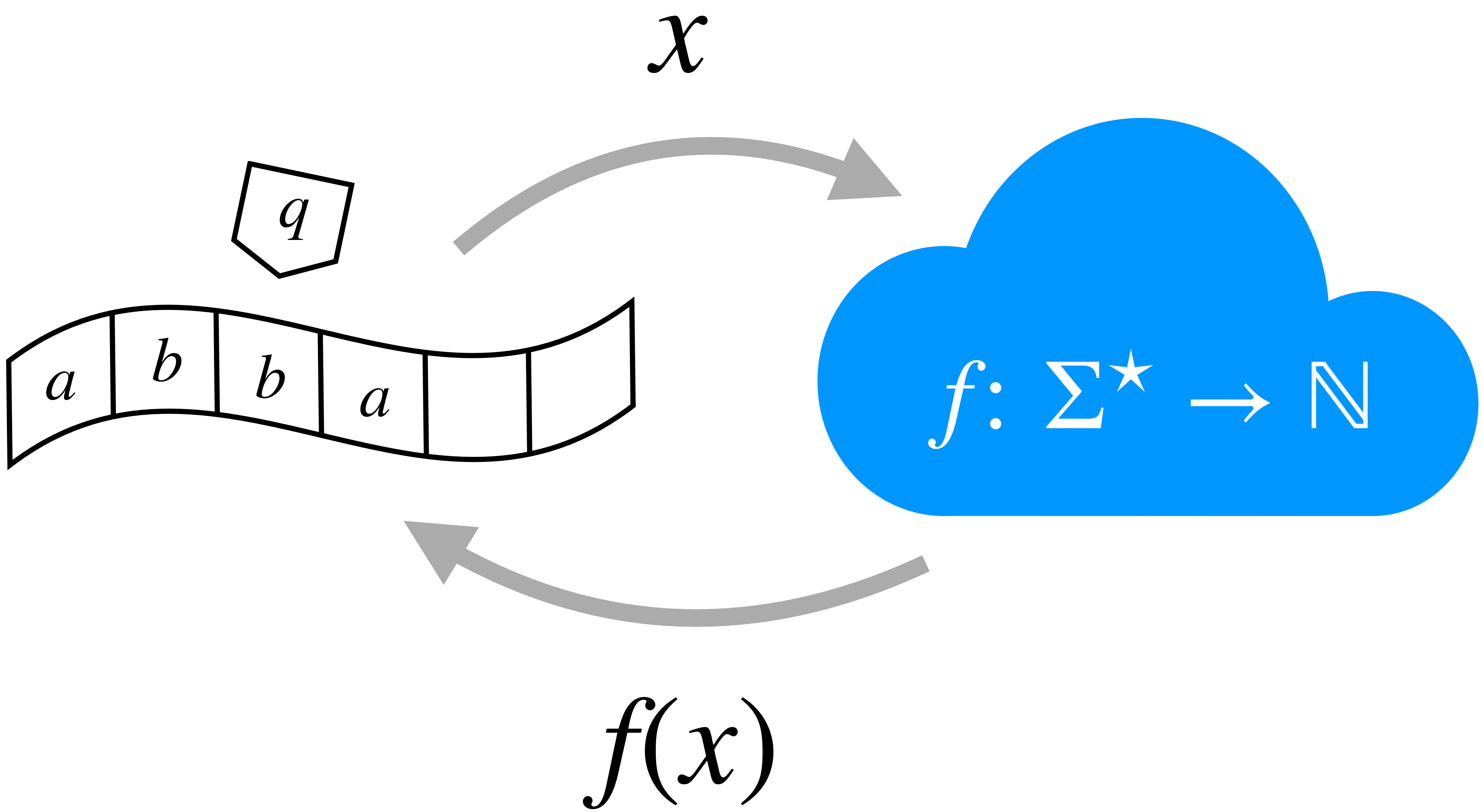
Oracle Turing machines and the class $\mathbf{P}^{\#\mathbf{P}}$



Oracle Turing machines and the class **P#P**



Oracle Turing machines and the class **P**$^{\#\mathbf{P}}$



The class $\mathbf{P}^{\#\mathbf{P}}$ in context

$$\mathbf{P}^{\#\mathbf{P}}$$

The class $\mathbf{P}^{\#\mathbf{P}}$ in context

$$\mathbf{P}^{\#\mathbf{P}}$$

$$\subseteq$$

$$\mathbf{PSPACE}$$

The class $\mathbf{P}^{\mathbf{\#P}}$ in context

$$\overbrace{\mathbf{P} \subseteq \mathbf{NP} \subseteq \mathbf{NP}^{\mathbf{NP}} \subseteq \mathbf{NP}^{\mathbf{NP}^{\mathbf{NP}}} \subseteq \cdots}^{\mathbf{PH}}$$

$$\rotatebox{90}{$\subseteq$}$$

$$\mathbf{P}^{\mathbf{\#P}}$$

$$\rotatebox{90}{$\subseteq$}$$

$$\mathbf{PSPACE}$$

The class $\mathbf{P}^{\#\mathbf{P}}$ in context

$$\overbrace{\mathbf{P} \subseteq \mathbf{NP} \subseteq \mathbf{NP}^{\mathbf{NP}} \subseteq \mathbf{NP}^{\mathbf{NP}^{\mathbf{NP}}} \subseteq \cdots}^{\mathbf{PH}}$$

$$\rotatebox{90}{$\subseteq$}$$

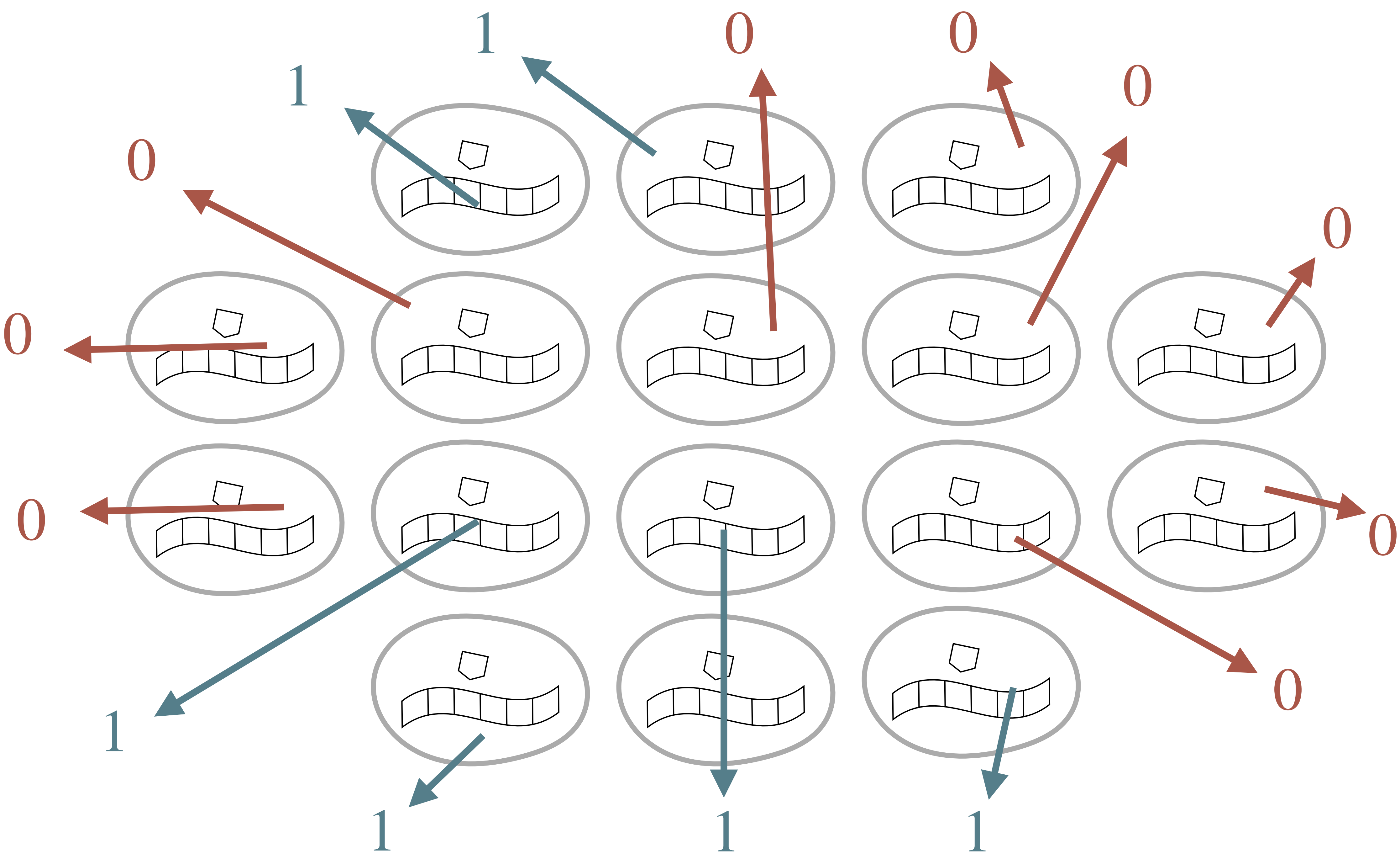
Toda's theorem

$$\mathbf{P}^{\#\mathbf{P}}$$

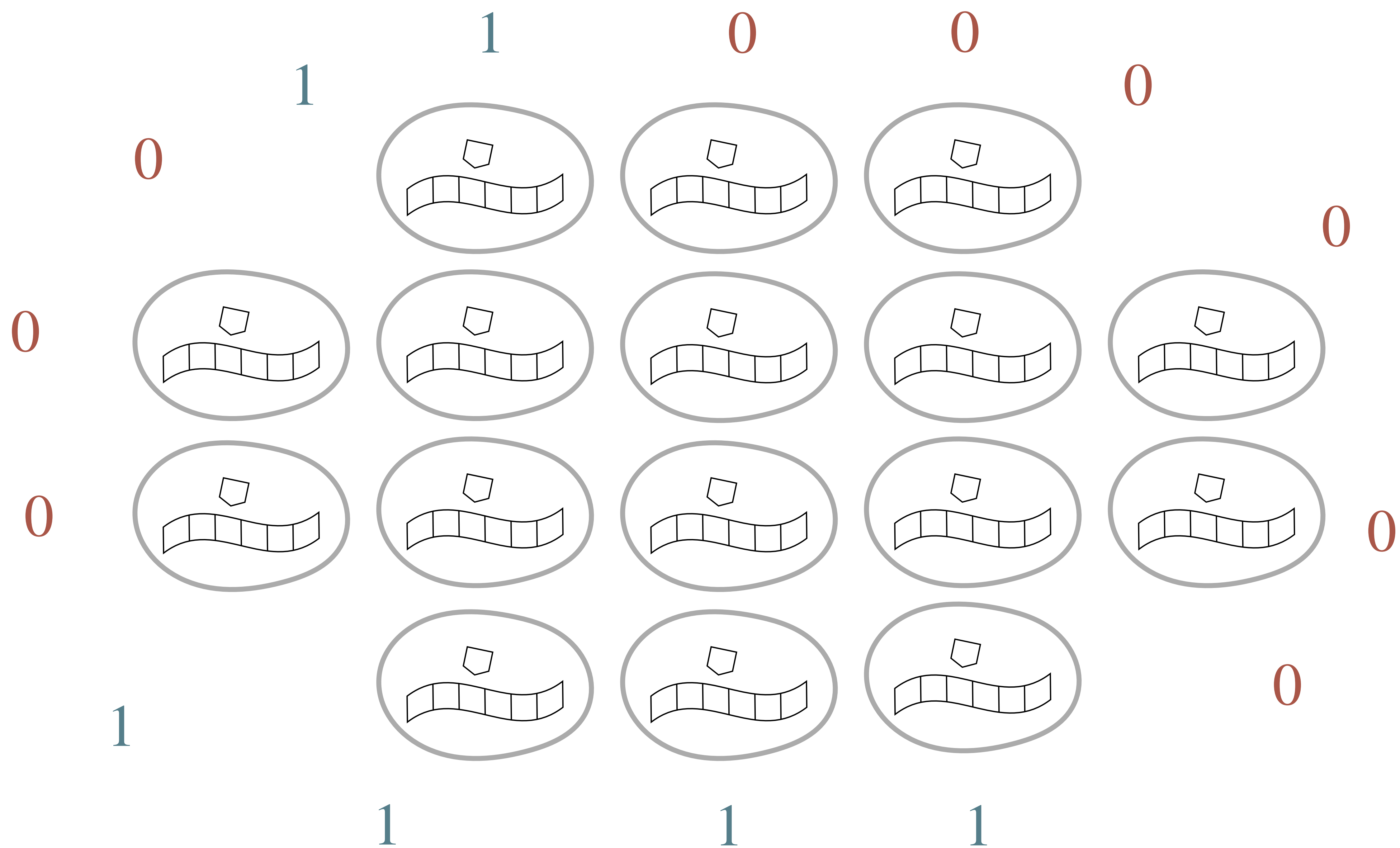
$$\rotatebox{90}{$\subseteq$}$$

$$\mathbf{PSPACE}$$

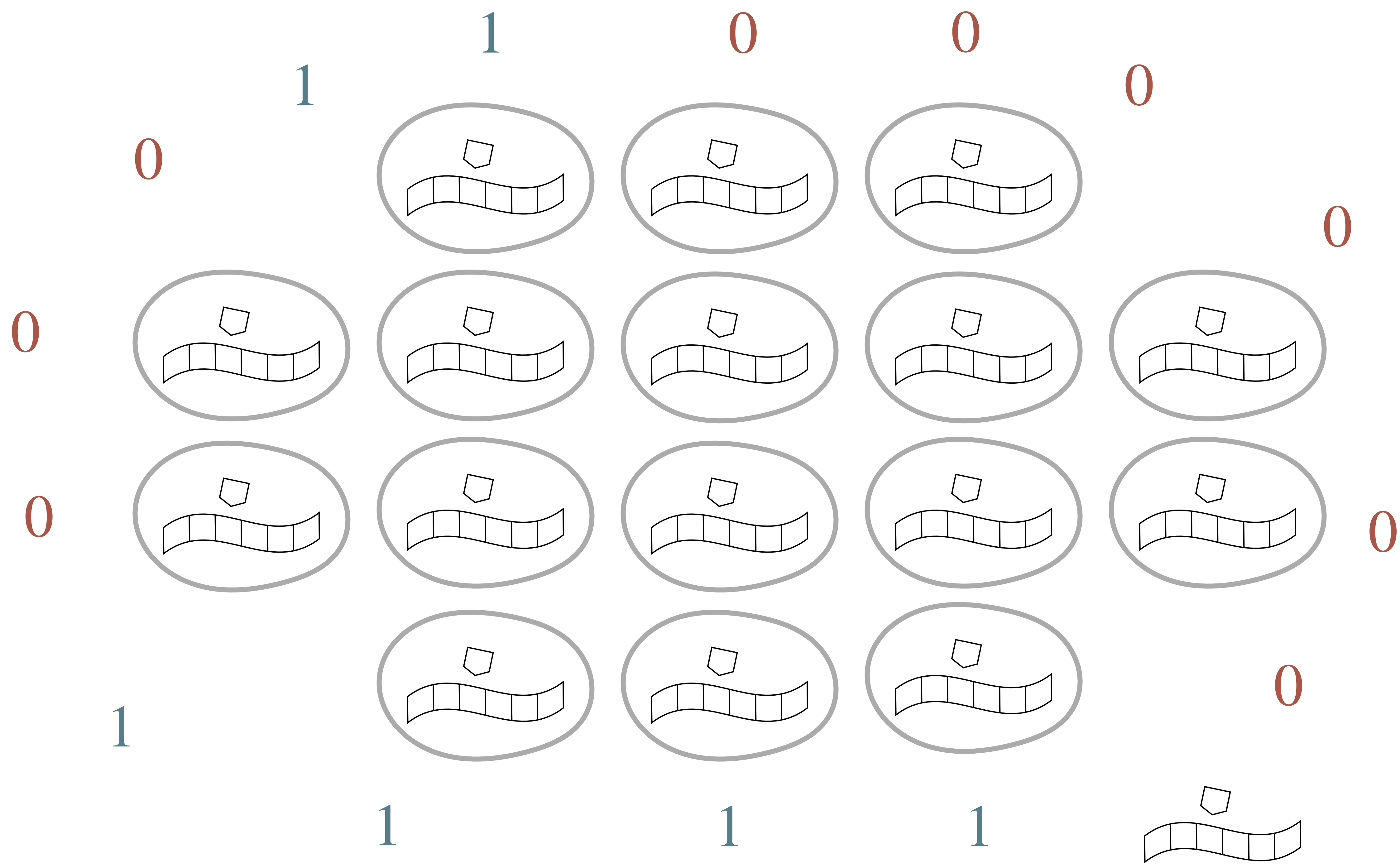
Shallow membrane systems solve **P$^{\#P}$**



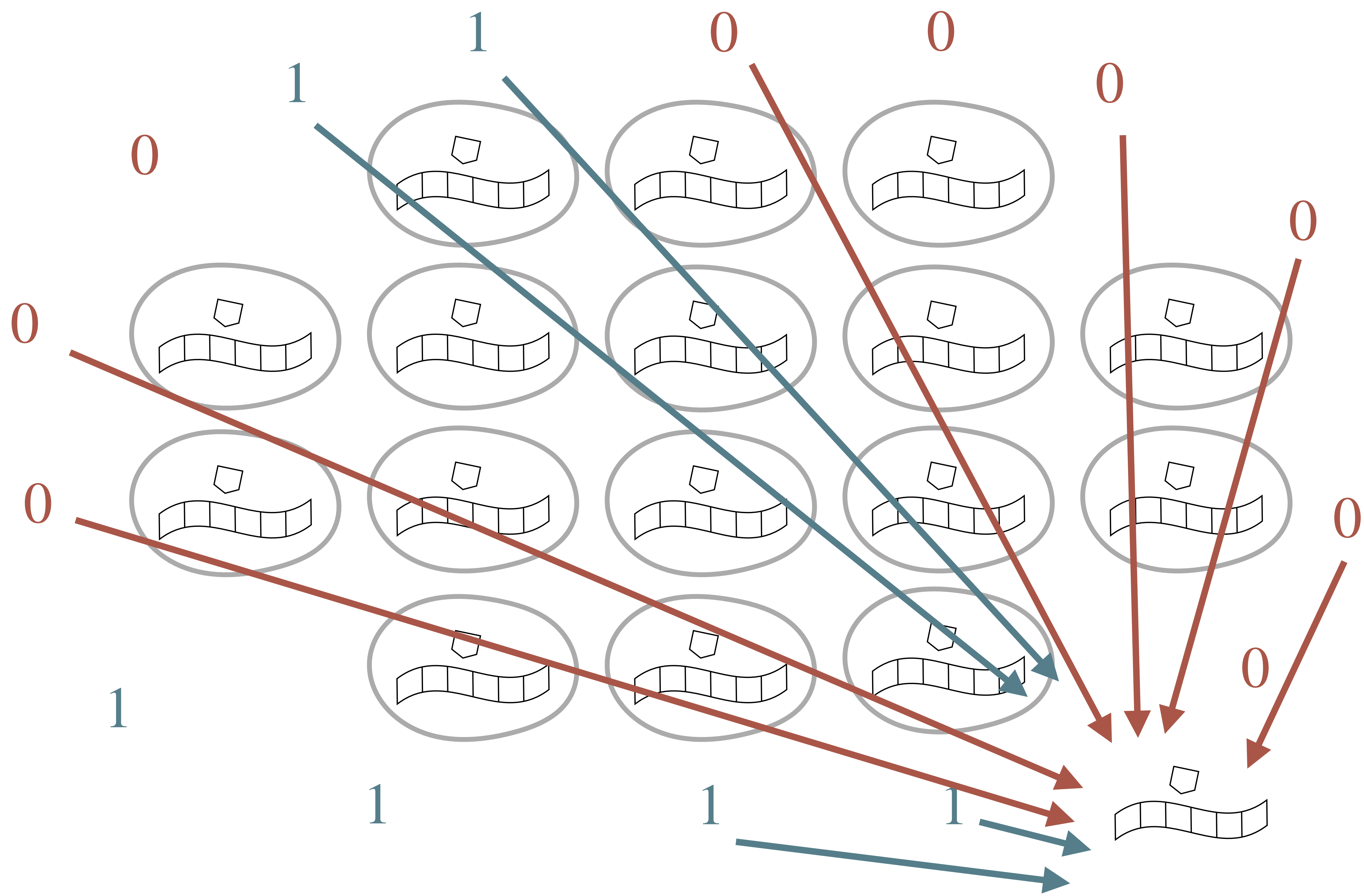
Shallow membrane systems solve $\mathbf{P}^{\mathbf{\#P}}$



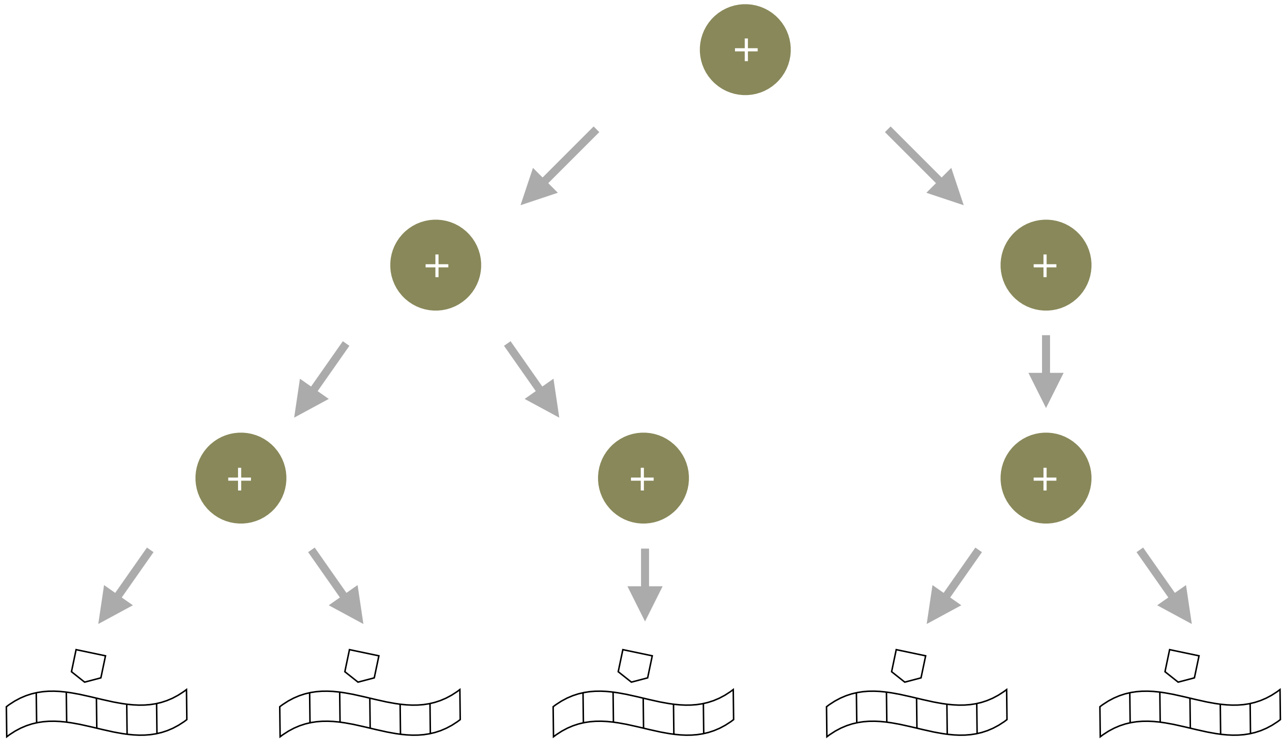
Shallow membrane systems solve $\mathbf{P}^{\#\mathbf{P}}$



Shallow membrane systems solve $\mathbf{P}^{\#\mathbf{P}}$



Flattening counting computation trees



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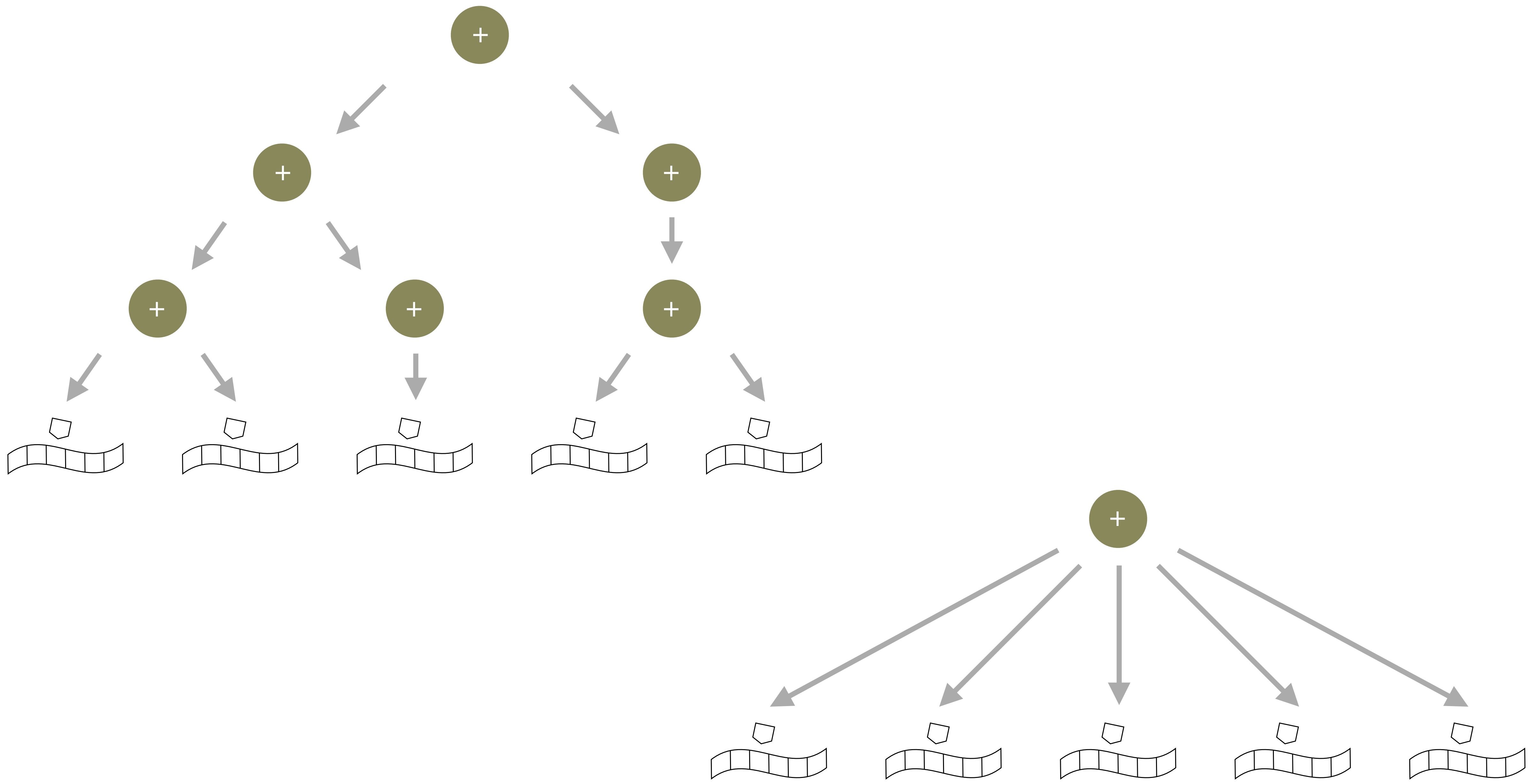


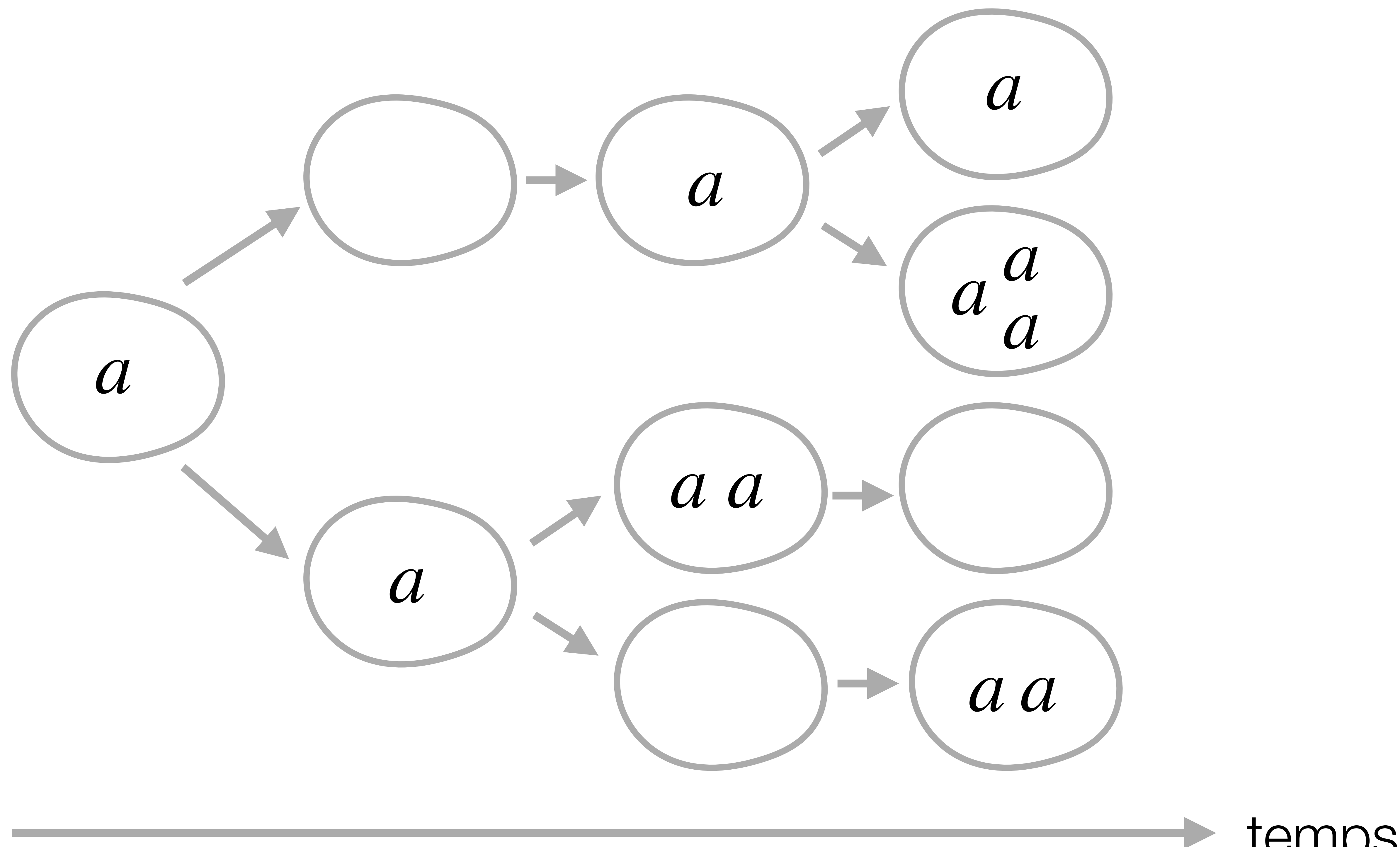
The results up to now

- No membranes (equivalently, no division) = **P**
- Shallow (depth 1) membrane division ⊇ $\mathbf{P}^{\#\mathbf{P}}$
- Deep membrane division = **PSPACE**

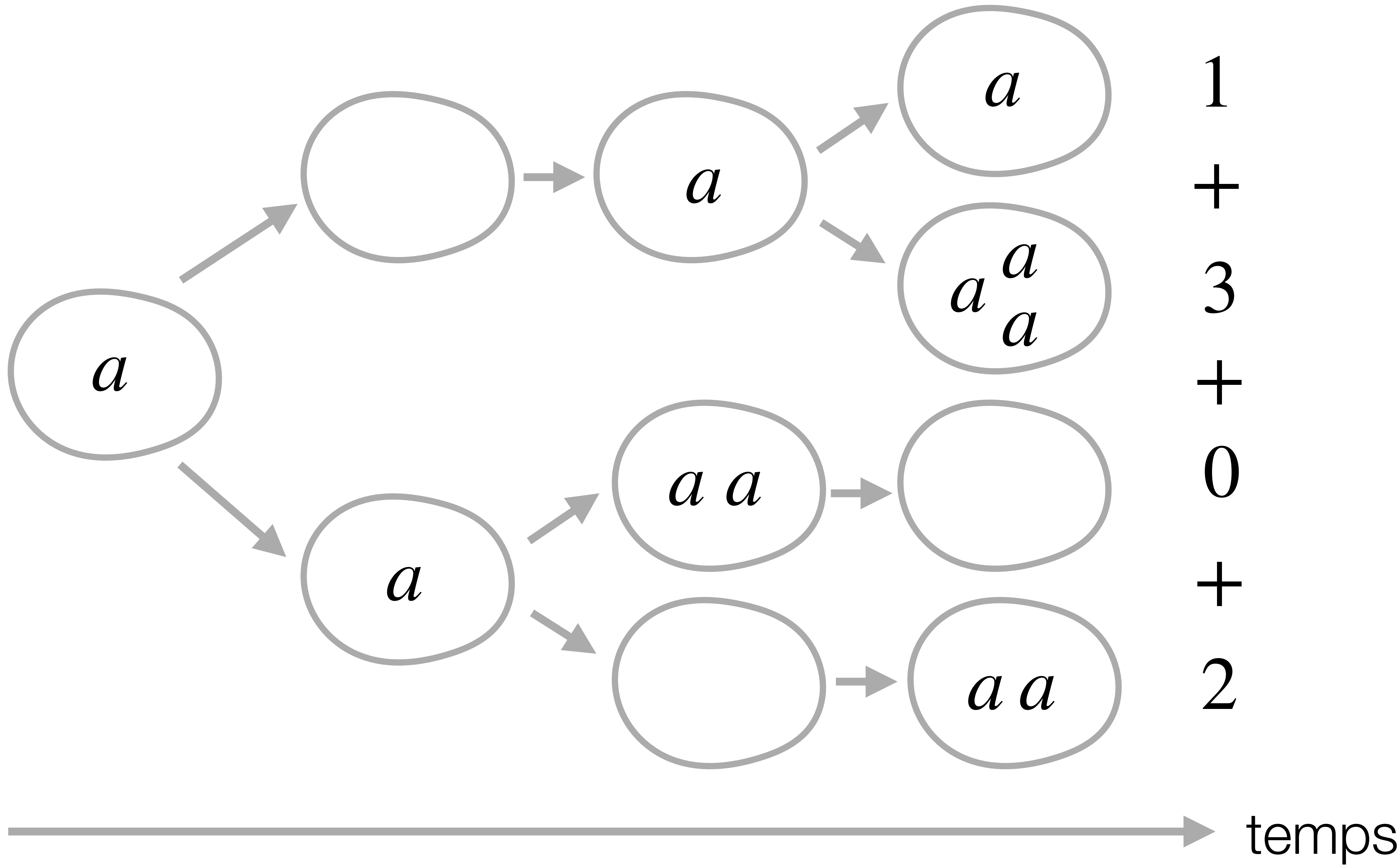
Proving that shallow membrane division = $\mathbf{P}^{\#\mathbf{P}}$

Given the initial configuration
of an elementary membrane,
how many copies of object *a* are sent out
by descendant membranes at time *t*?

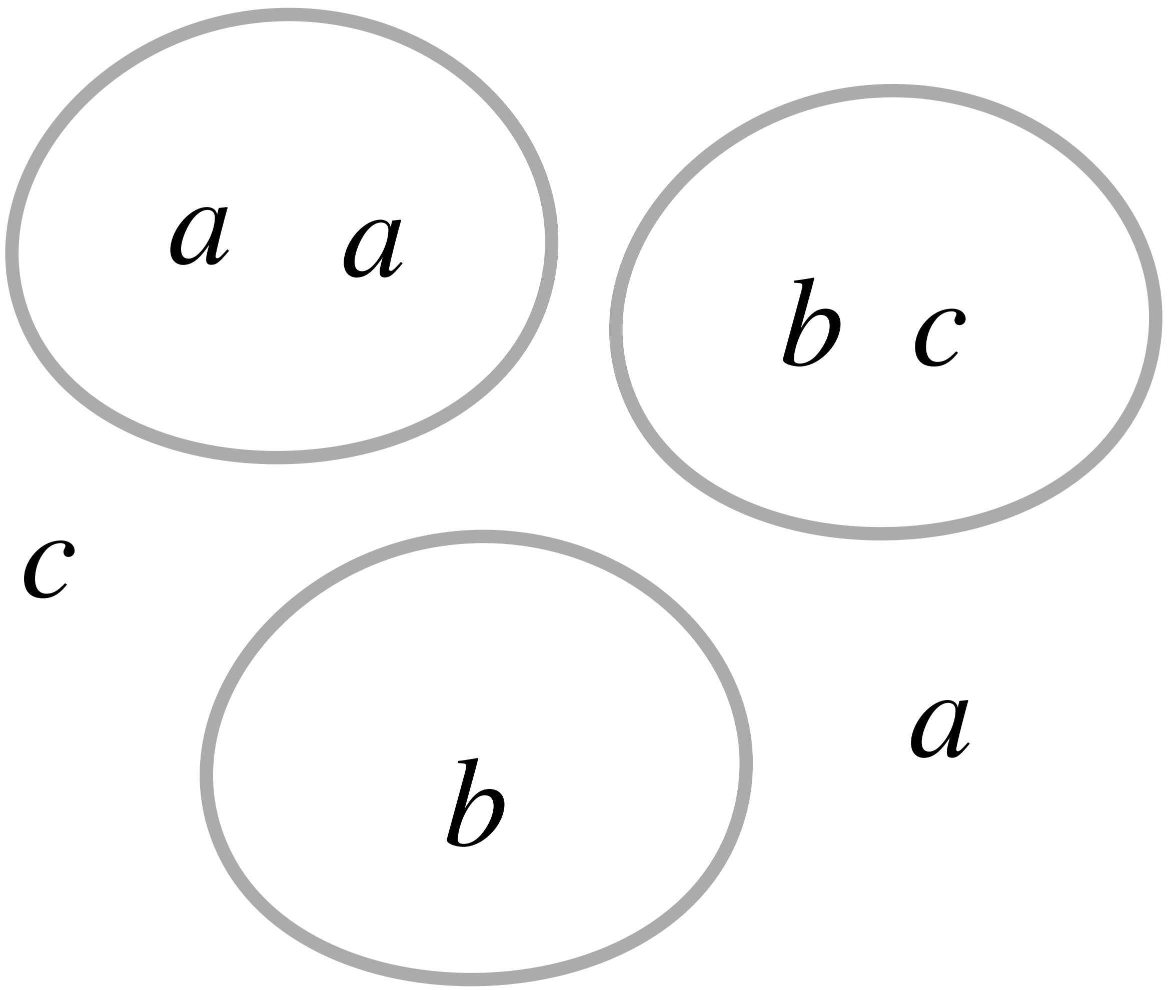
Query in **#P** in the monodirectional case



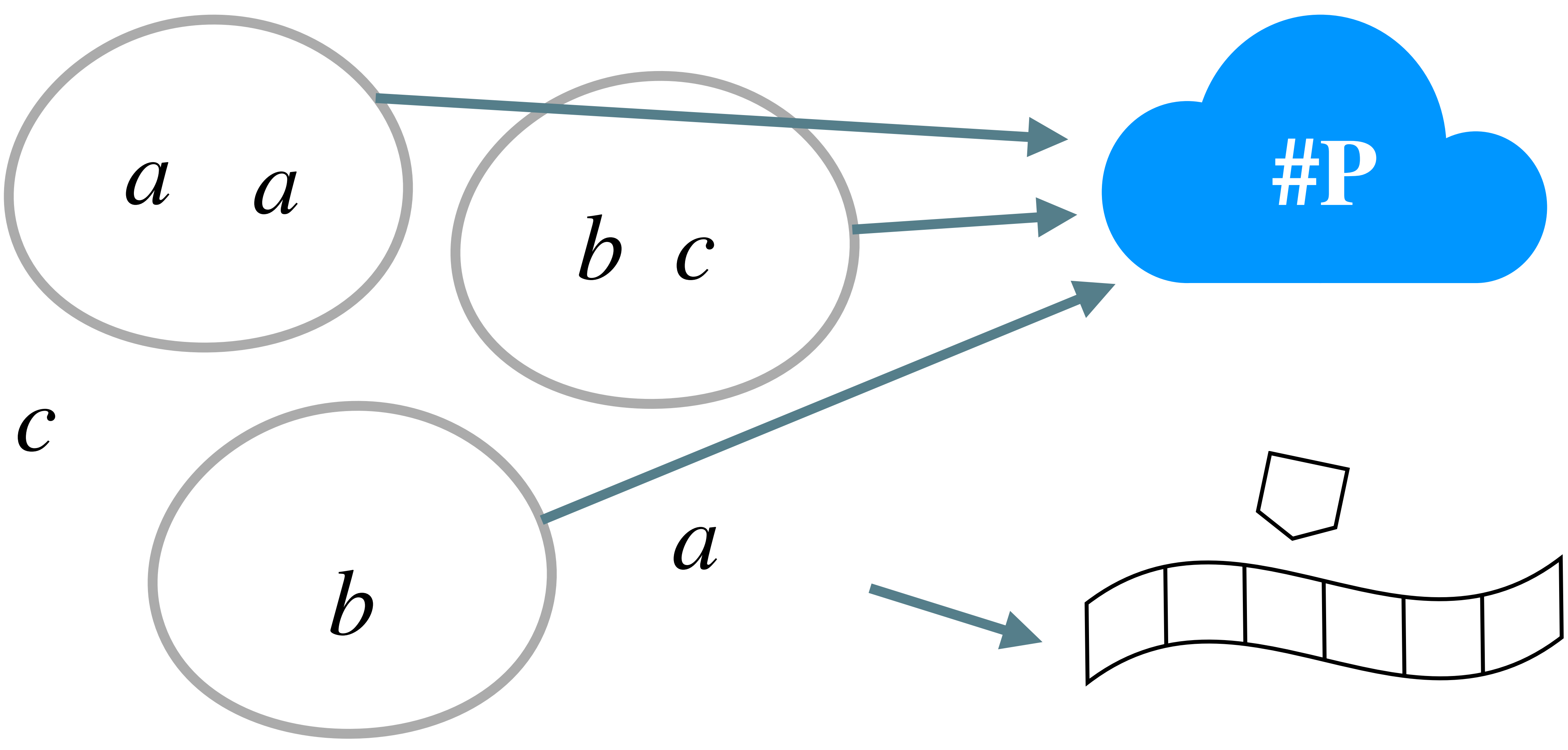
Query in **#P** in the monodirectional case



“Easy” upper bound in the monodirectional case



“Easy” upper bound in the monodirectional case



Simulation algorithm (monodirectional case)

- For each time step *t* until the simulated system halts:
  - Simulate one step of the external environment, updating the stored configuration
  - **Ask the oracle** how many instances of each symbol are sent out at time *t* by each elementary membrane in the initial configuration (not stored)
  - Add the answer to each query to the external environment
- Accept or reject depending on the result of the simulation

“Send-in” communication breaks the algorithm

Given the initial configuration of an elementary membrane and a table describing what communication rules (with multiplicity!) were applied to its descendants until time $t - 1$, how many copies of object a are sent out at time t?

Monodirectional characterisation of $\mathbf{P}^{\mathbf{NP}}$

- Consider membrane systems where deep membrane division is allowed
- But limit inter-membrane communication to monodirectional (outwards)
- Then we obtain a precise characterisation of $\mathbf{P}^{\mathbf{NP}}$ in polynomial time

Finally some unusual complexity classes!

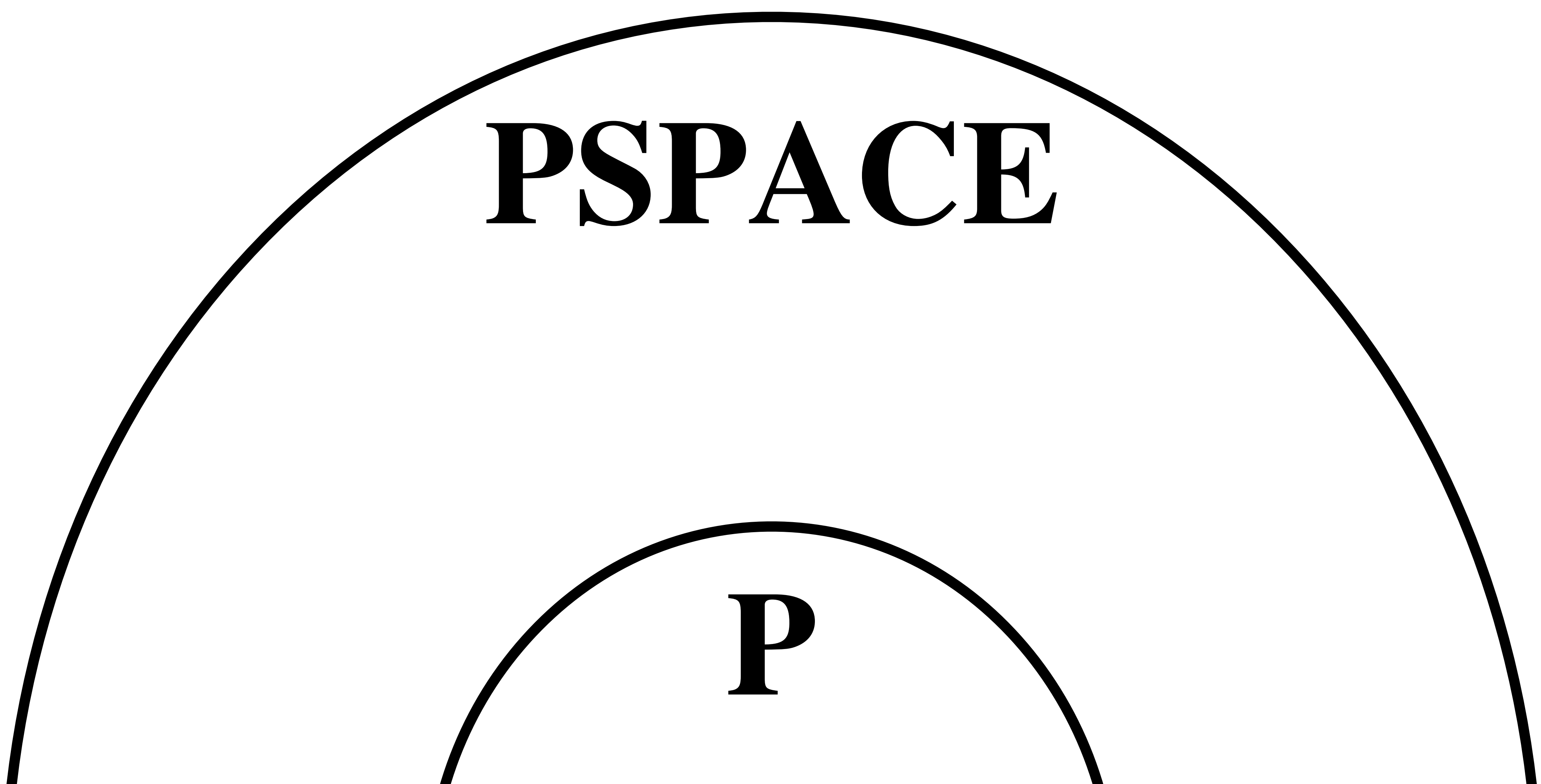
$$\mathbf{P}^{\mathbf{NP}}$$

$$\mathbf{P}^{\#\mathbf{P}}$$

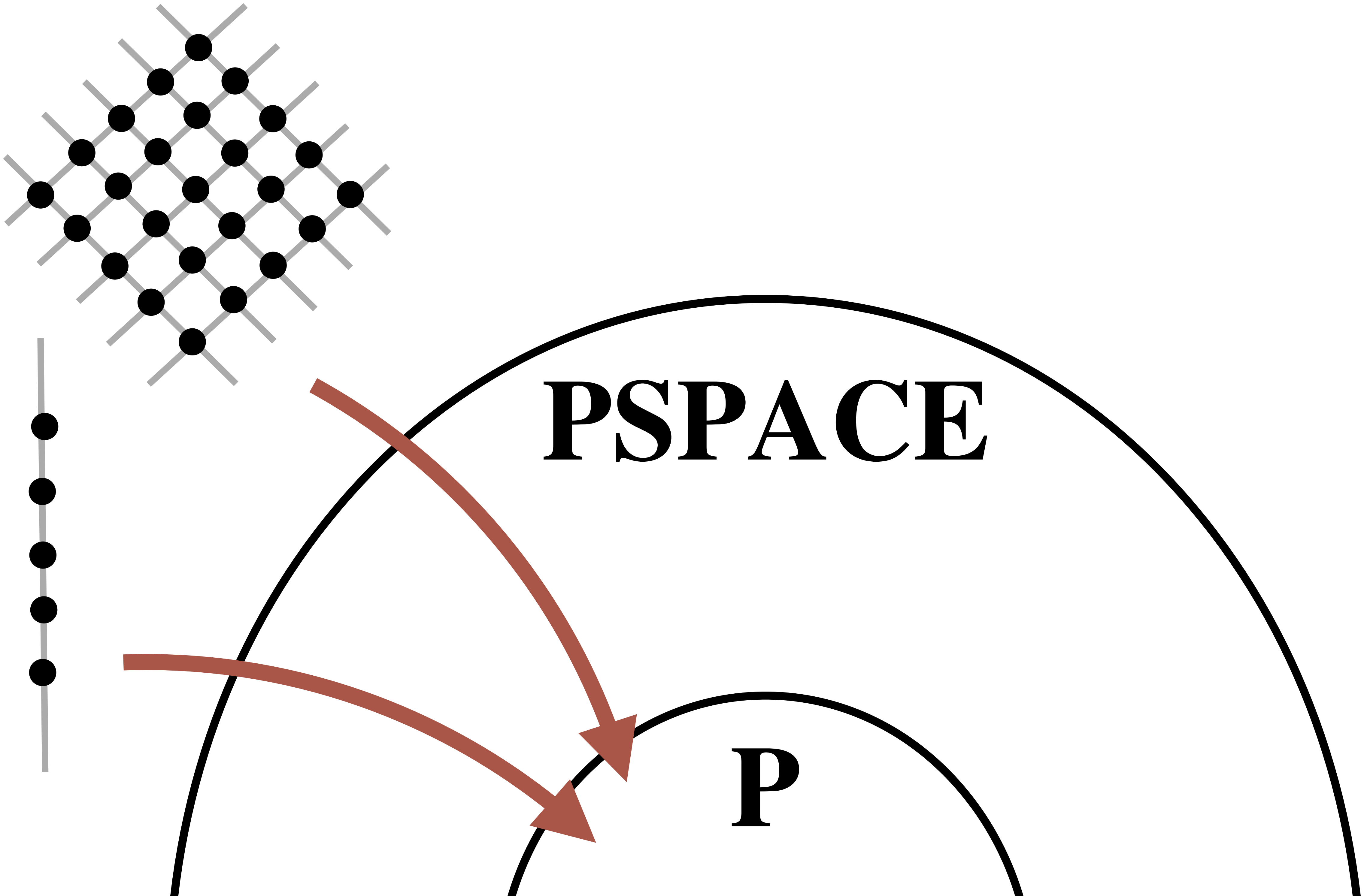
Hierarchy of membrane systems wrt division depth

- No membranes (equivalently, no division) = **P**
- Shallow (depth 1) membrane division = $\mathbf{P}^{\#\mathbf{P}}$
- Deep membrane division = **PSPACE**

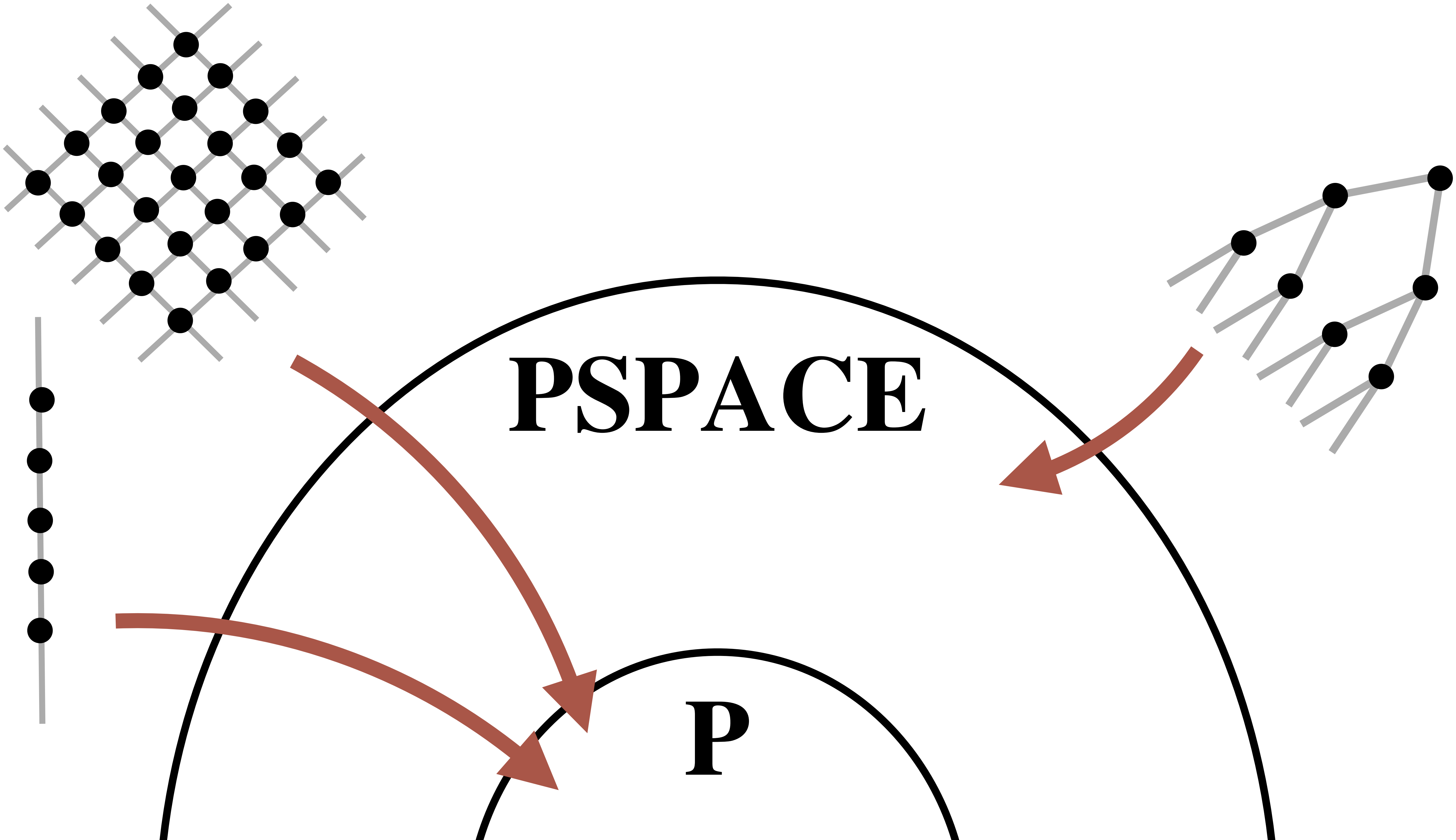
Communication topology and complexity



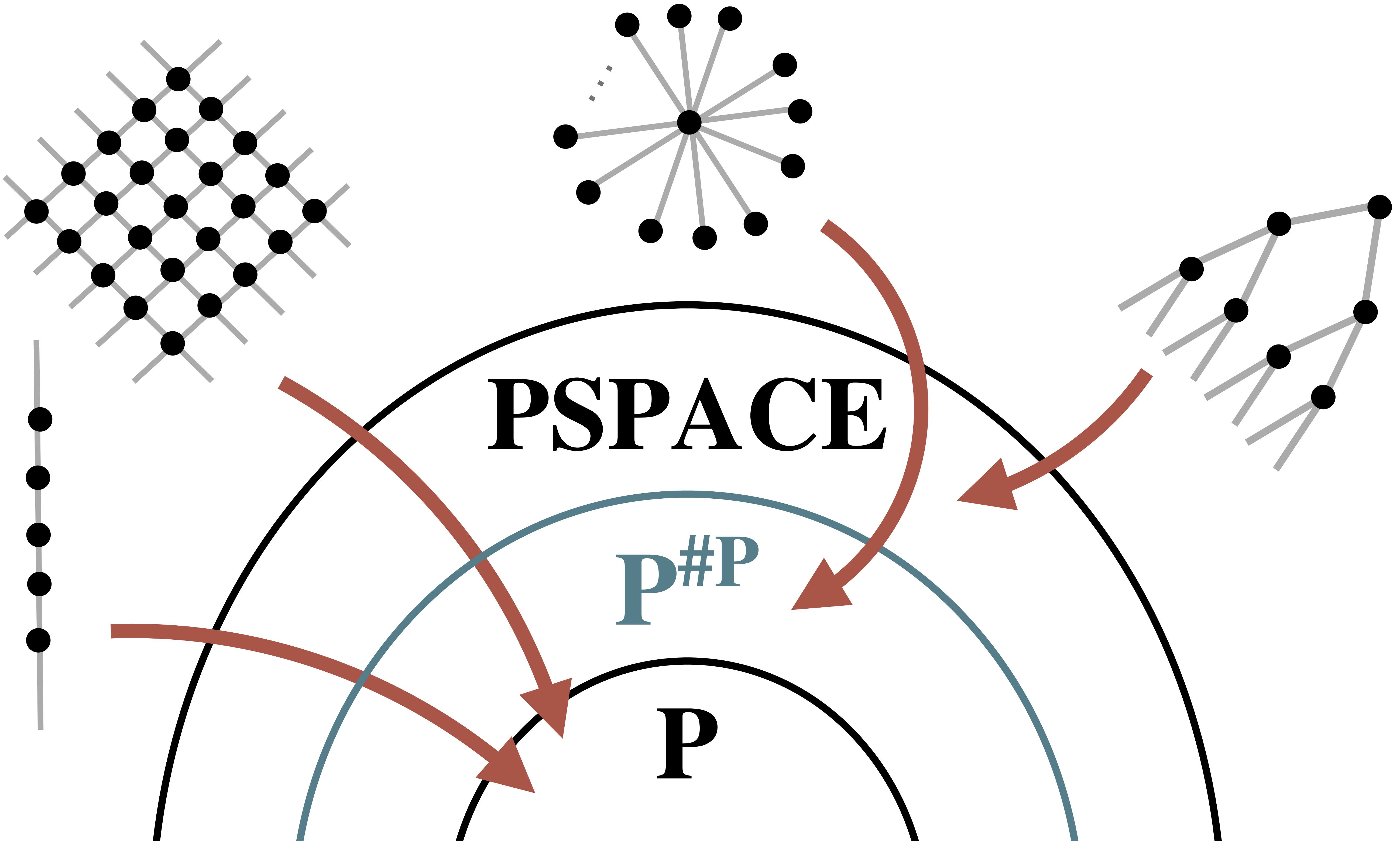
Communication topology and complexity



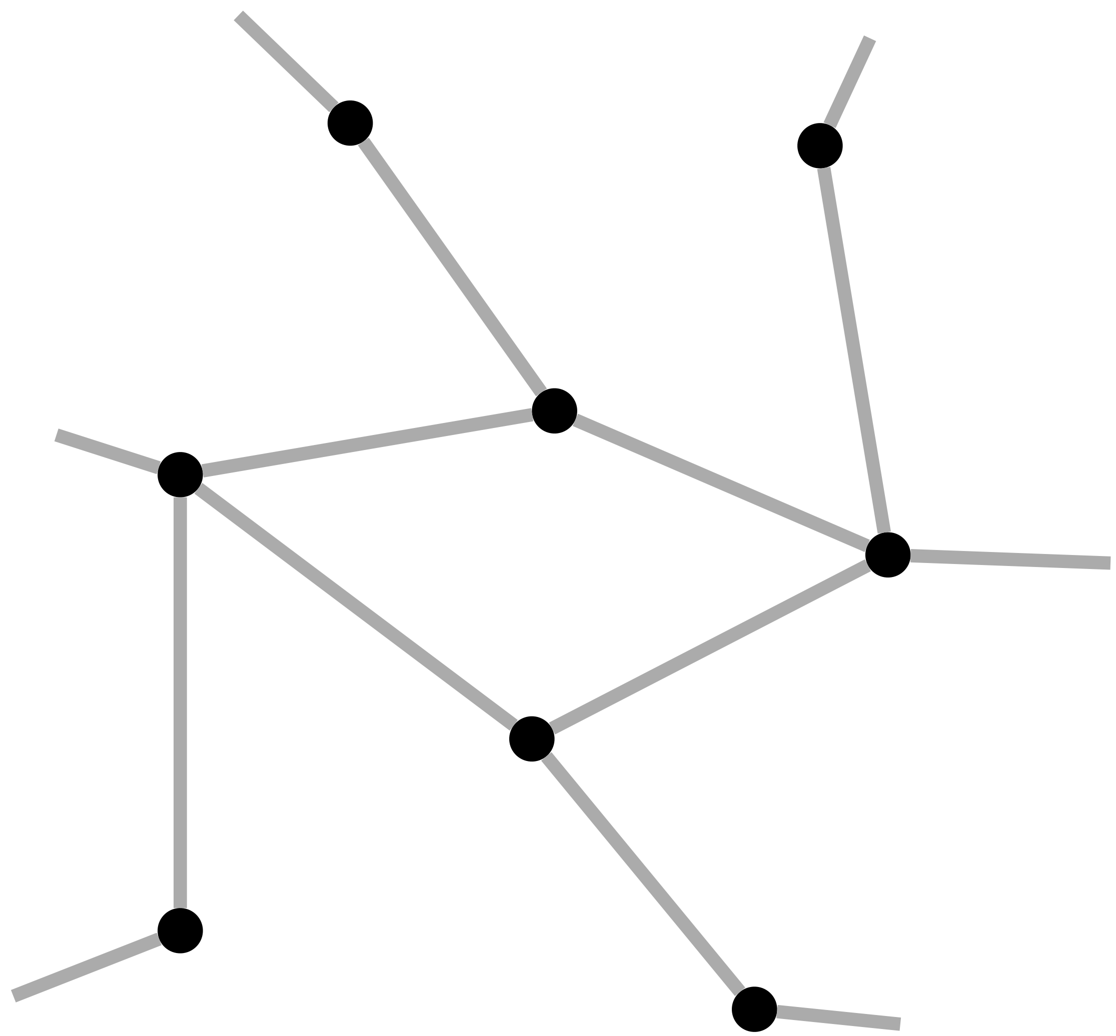
Communication topology and complexity



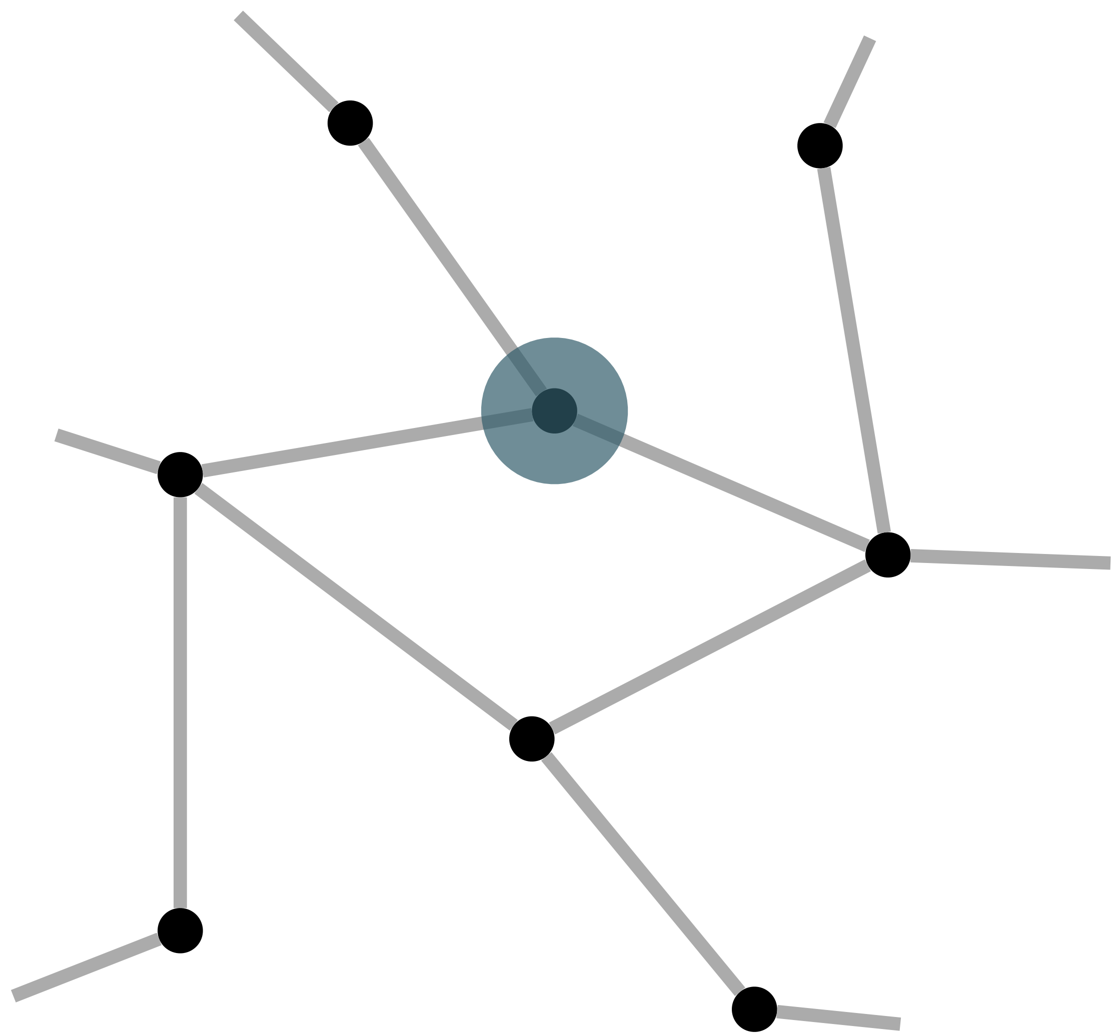
Communication topology and complexity



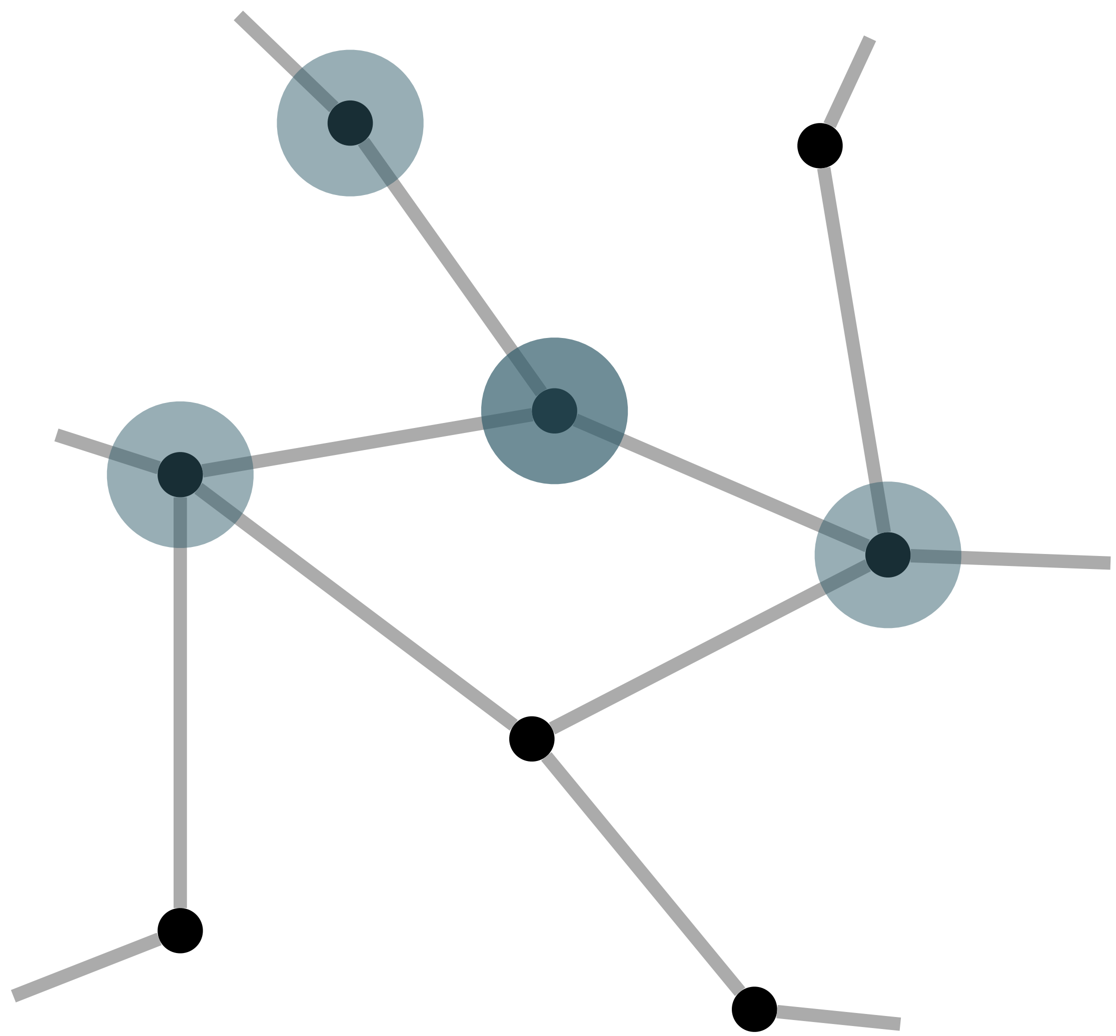
Automata networks over arbitrary infinite graphs as a reference model of computation



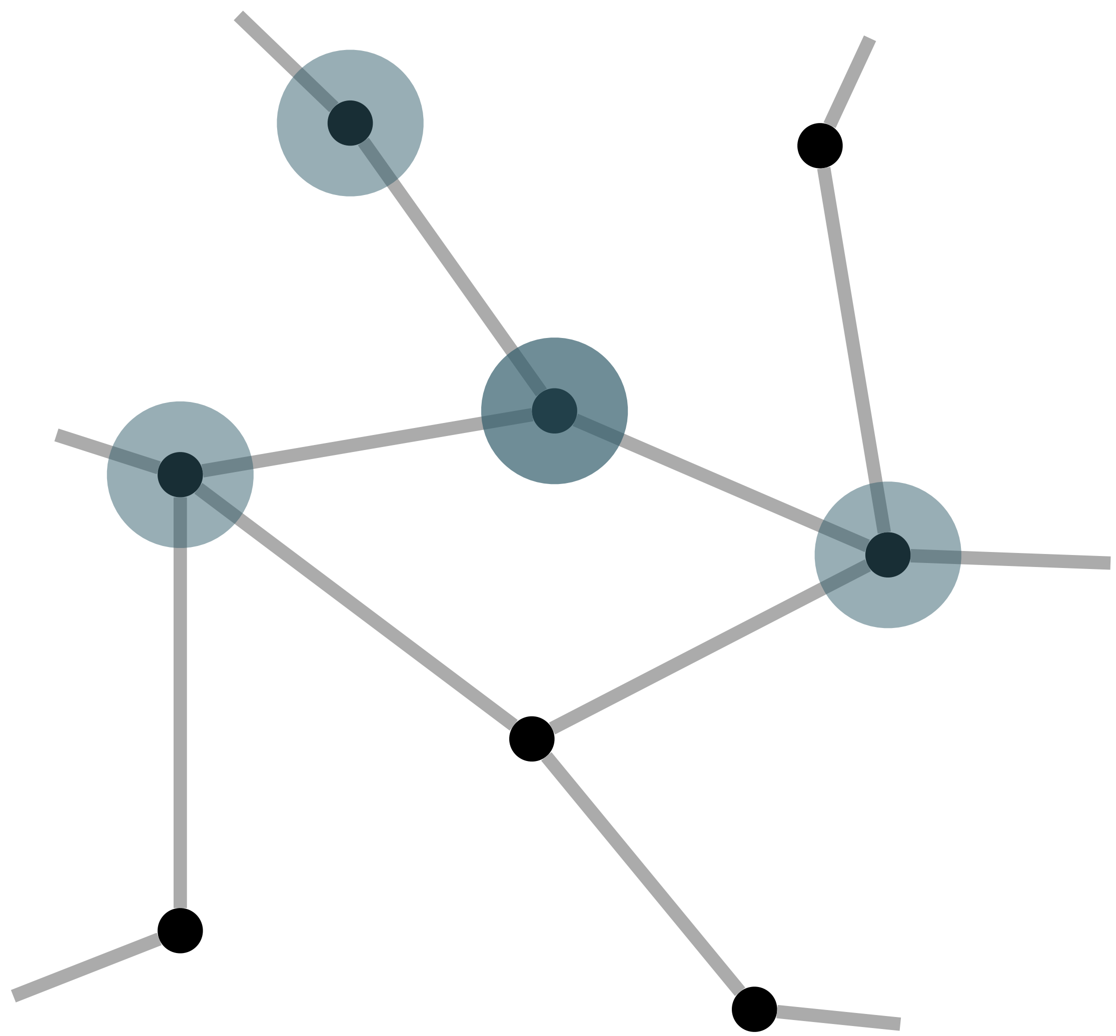
Automata networks over arbitrary infinite graphs as a reference model of computation



Automata networks over arbitrary infinite graphs as a reference model of computation



Automata networks over arbitrary infinite graphs as a reference model of computation



$$f\colon \mathrm{Multisets}(Q) \to Q$$

Generalised complexity classes over a graph G

$\mathbf{PSPACE}(G)$

$\mathbf{P}(G)$

$\mathbf{EXPTIME}(G)$

$\mathbf{LOGTIME}(G)$

$\mathbf{NP}(G)$

Expected results

$$\mathbf{P}(\text{Euclidean grid}) = \mathbf{P}$$

$$\mathbf{P}(\text{infinite binary tree}) = \mathbf{PSPACE}$$

$$\mathbf{P}(\text{variant of infinite star}) = \mathbf{P}^{\#\mathbf{P}}$$

Open problems

- Find explanations to the membrane computing results
- Find graphs characterising the standard complexity classes
- Find complexity classes corresponding to “natural” families of graphs (i.e., with interesting graph-theoretic properties)
- Find how the geometry of the space influence the efficiency of the algorithms

Thanks for your attention!
Merci de votre attention !

Any questions?