Simulating elementary active membranes

With an application to the P conjecture

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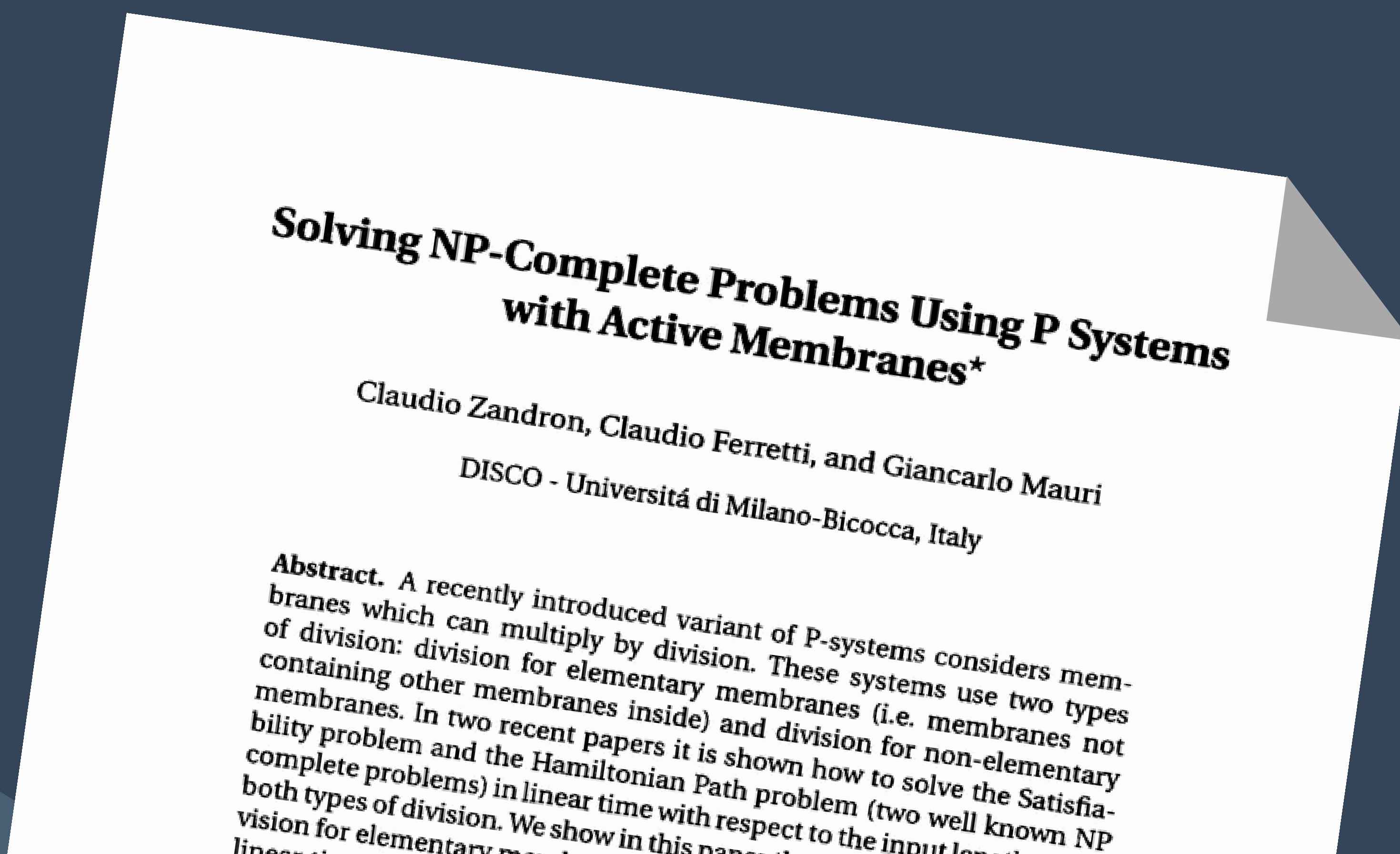
In previous episodes…

$$\mathbf{NP} \subseteq \mathbf{PMC}^{\star}_{AM(-d,-n)}$$



Solving NP-Complete Problems Using P Systems with Active Membranes*

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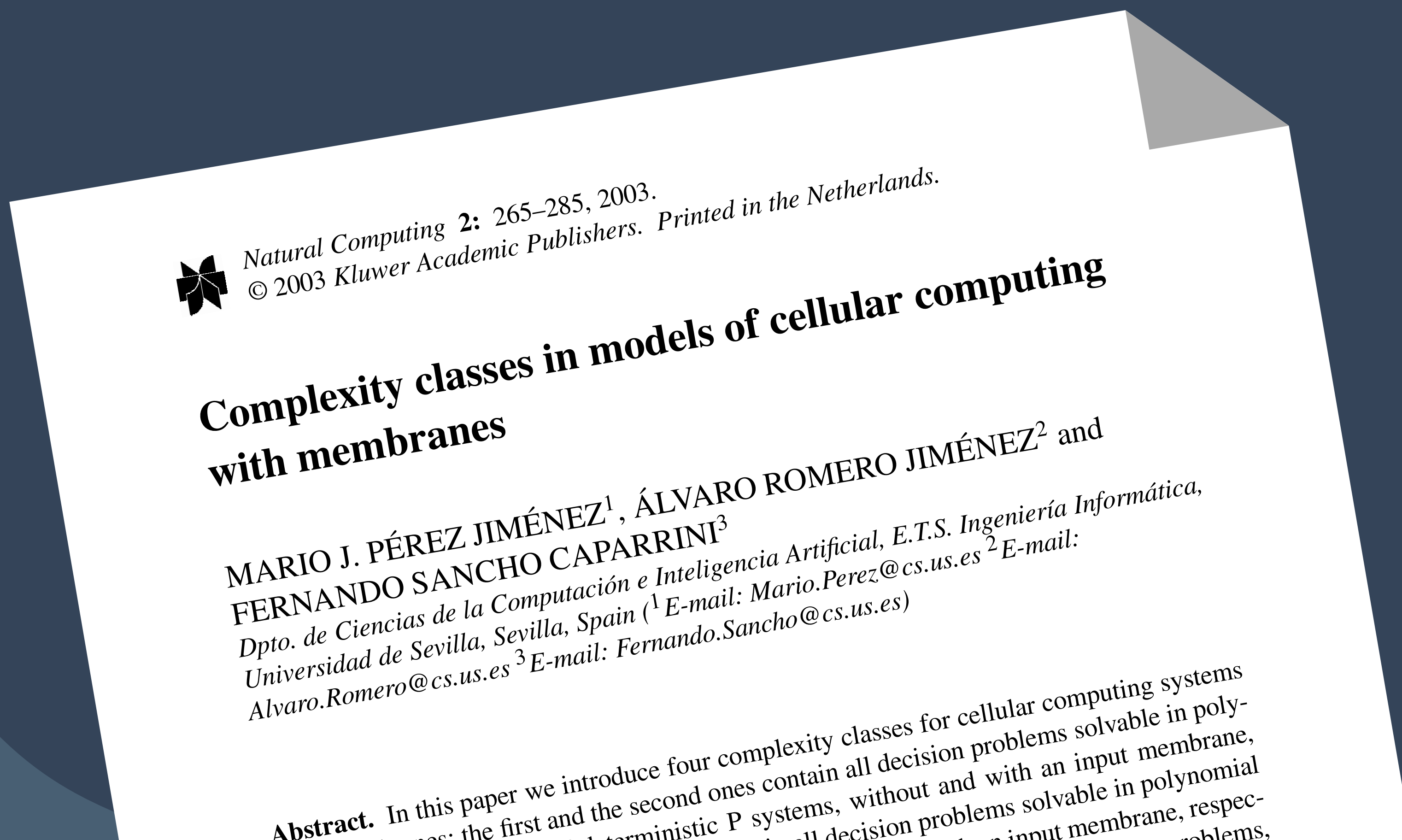
Abstract. A recently introduced variant of P-systems considers membranes which can multiply by division. These systems use two types of division: division for elementary membranes (i.e. membranes not containing other membranes inside) and division for non-elementary membranes. In two recent papers it is shown how to solve the Satisfiability problem and the Hamiltonian Path problem (two well known NP complete problems) in linear time with respect to the input length using both types of division. We show in this paper that ...

$$\mathbf{NP} \cup \mathbf{coNP} \subseteq \mathbf{PMC}_{\mathcal{AM}(-d,-n)}$$



Natural Computing **2:** 265–285, 2003.

Complexity classes in models of cellular computing with membranes

MARIO J. PÉREZ JIMÉNEZ[1], ÁLVARO ROMERO JIMÉNEZ[2] and FERNANDO SANCHO CAPARRINI[3]
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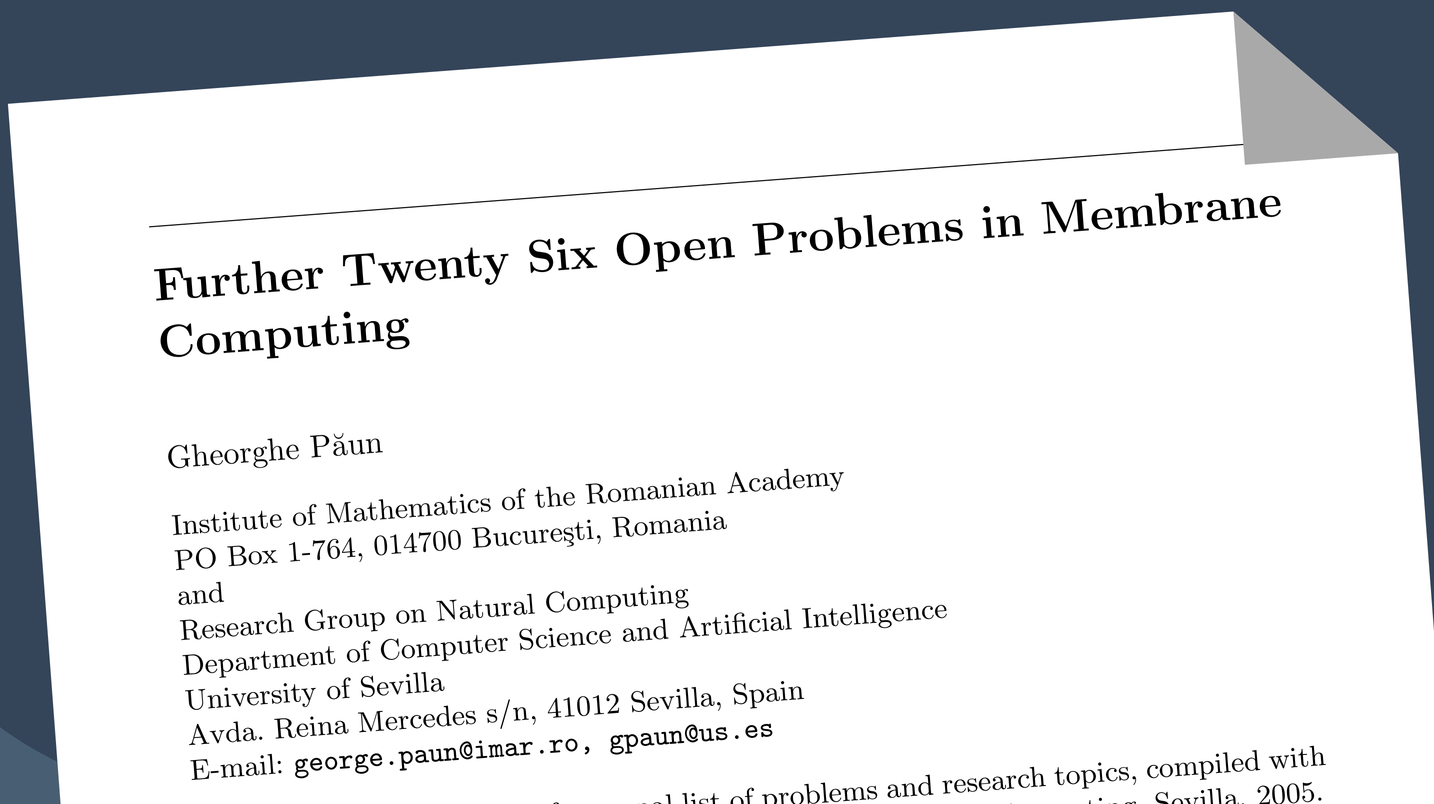
Abstract. In this paper we introduce four complexity classes for cellular computing systems ... the first and the second ones contain all decision problems solvable in poly- ... P systems, without and with an input membrane, ... all decision problems solvable in polynomial ... input membrane, respec- ... problems,

“It is an intriguing question here whether also **PSPACE** $\subseteq$ [**PMC**$_{AM(-n)}$] holds (the conjecture was formulated – P. Sosík, M. J. Pérez-Jiménez – that it is not true [...]).”



Further Twenty Six Open Problems in Membrane Computing

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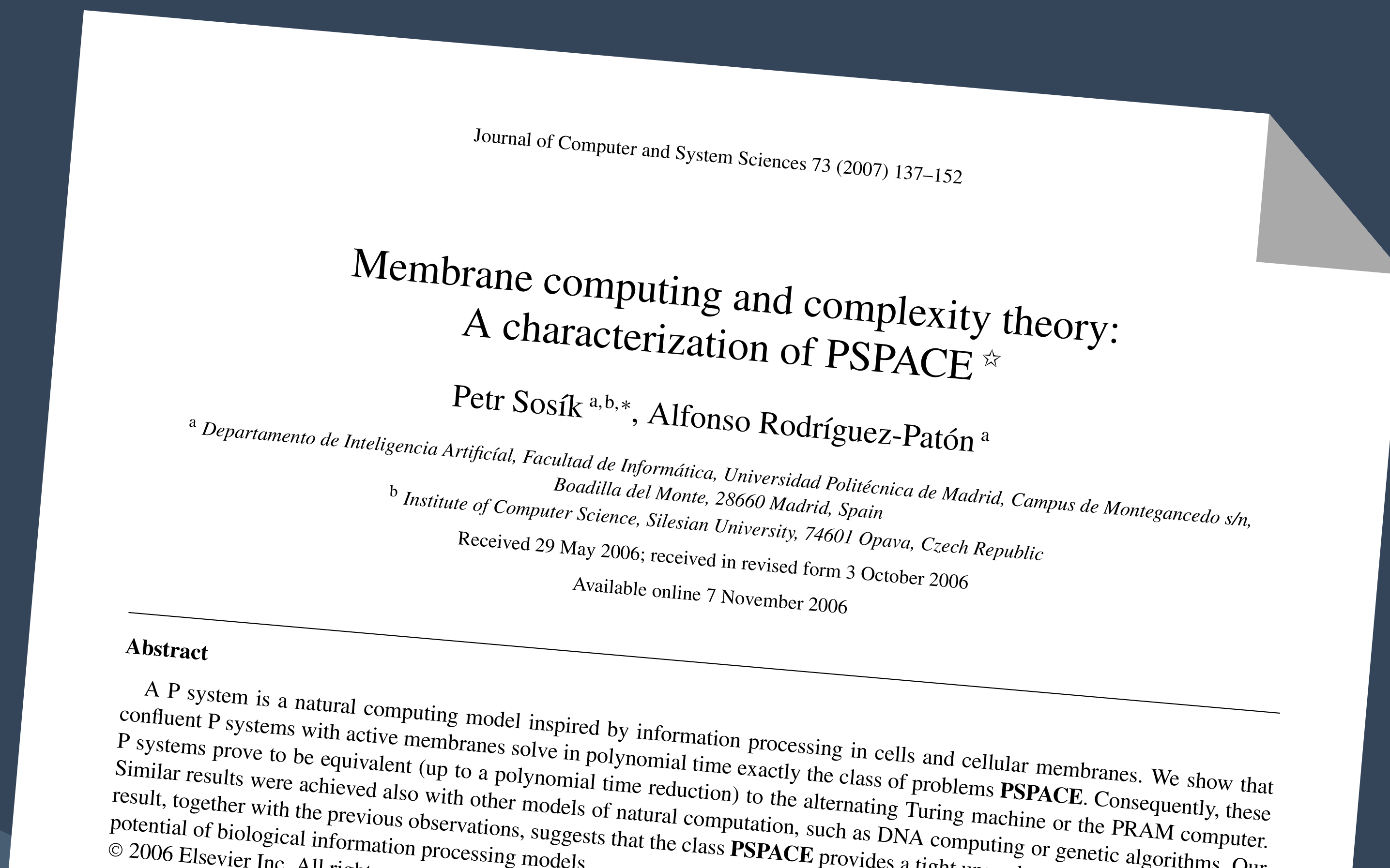
$$\mathrm{PMC}^{\star}_{\mathrm{AM}(-n)} \subseteq \mathrm{PSPACE}$$


Journal of Computer and System Sciences 73 (2007) 137–152

Membrane computing and complexity theory: A characterization of PSPACE ☆

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Abstract

A P system is a natural computing model inspired by information processing in cells and cellular membranes. We show that confluent P systems with active membranes solve in polynomial time exactly the class of problems **PSPACE**. Consequently, these P systems prove to be equivalent (up to a polynomial time reduction) to the alternating Turing machine or the PRAM computer. Similar results were achieved also with other models of natural computation, such as DNA computing or genetic algorithms. Our result, together with the previous observations, suggests that the class **PSPACE** provides a tight
potential of biological information processing models.

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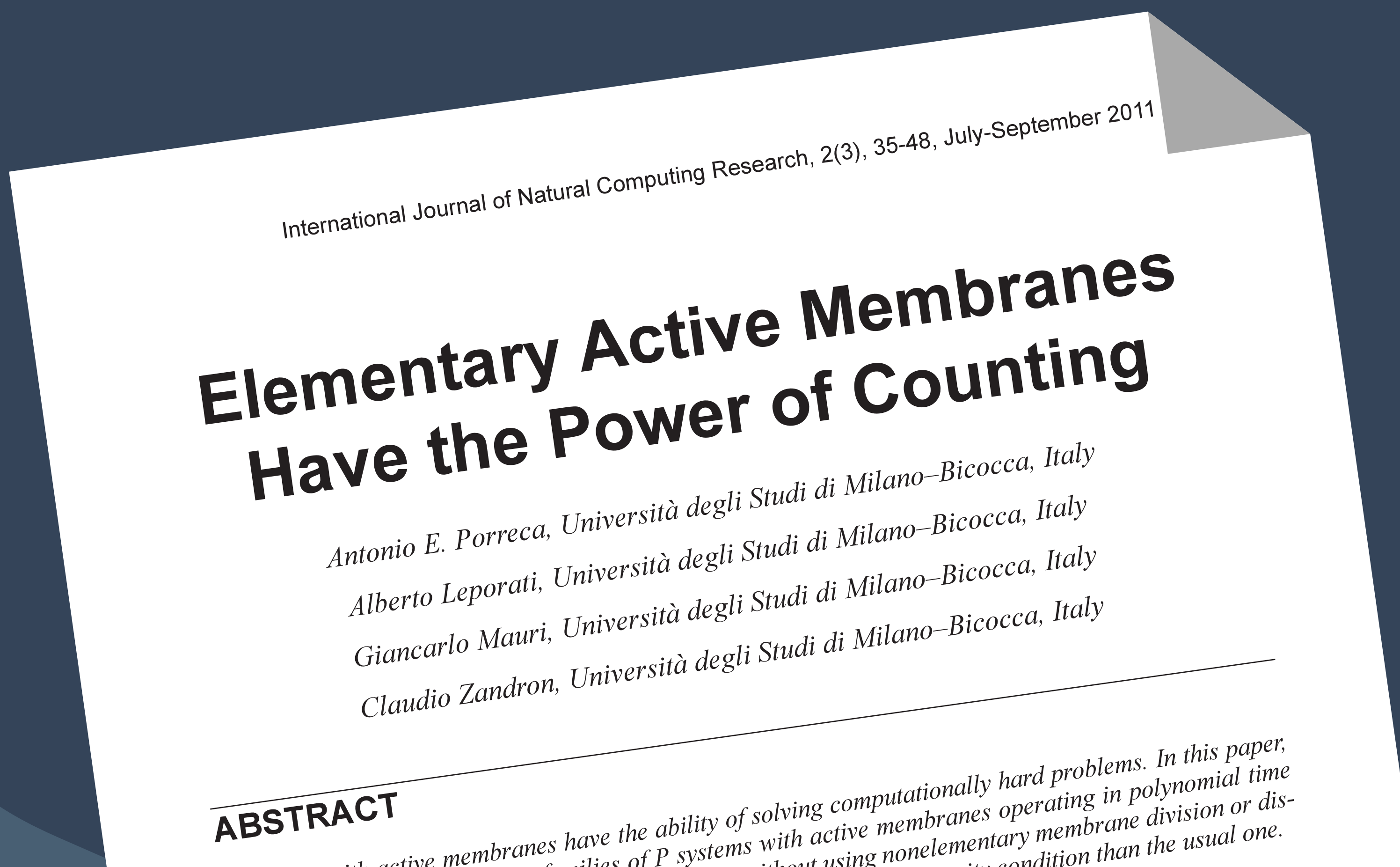
$$\mathbf{PP} \subseteq \mathbf{PMC}_{\mathcal{AM}(-d,-n)}$$



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Elementary Active Membranes Have the Power of Counting

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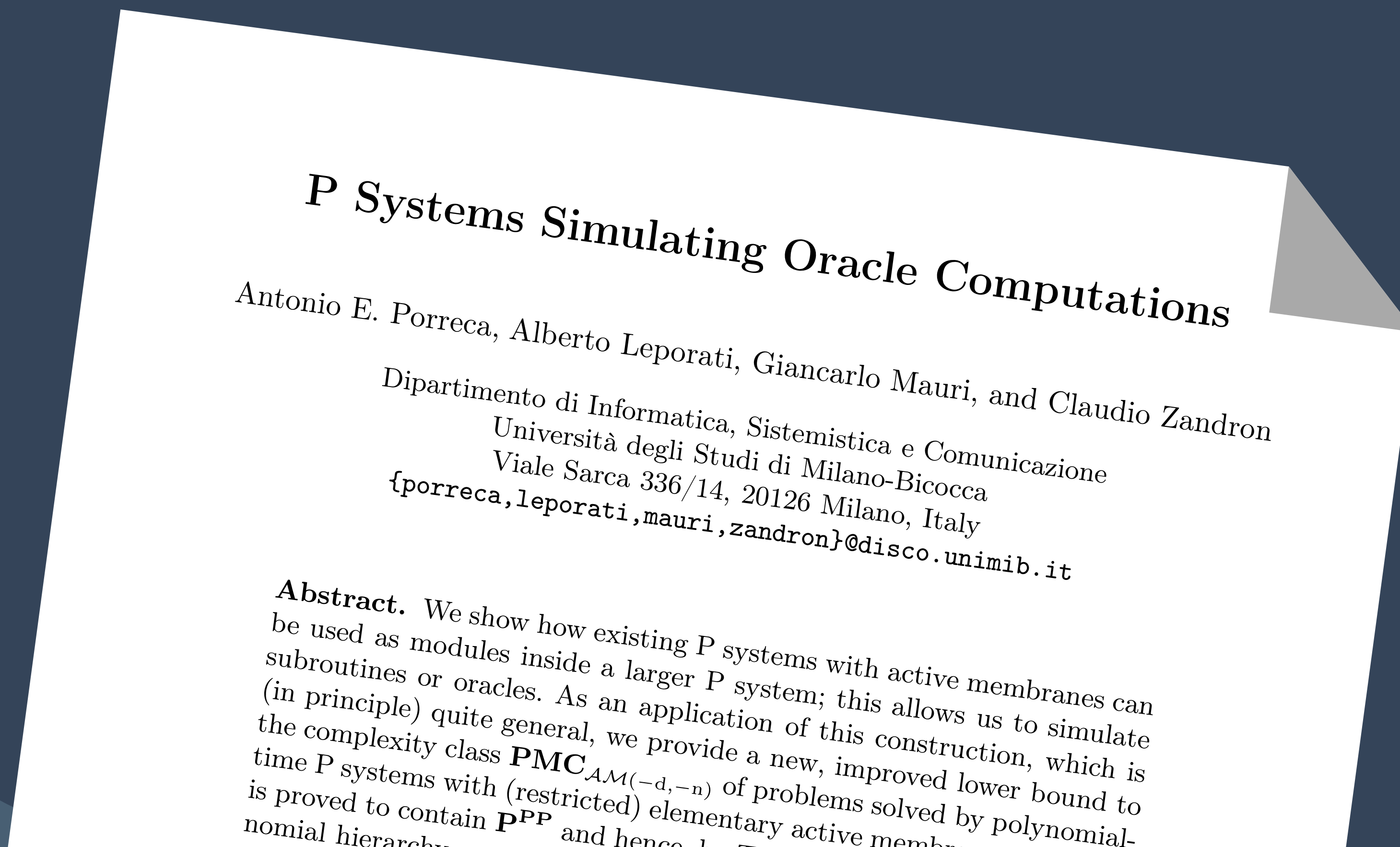
ABSTRACT

...active membranes have the ability of solving computationally hard problems. In this paper, ...families of P systems with active membranes operating in polynomial time ...without using nonelementary membrane division or dis- ...condition than the usual one.

$\mathbf{P}^{\#\mathbf{P}} \subseteq \mathbf{PMC}_{\mathcal{AM}(-d,-n)}$



P Systems Simulating Oracle Computations

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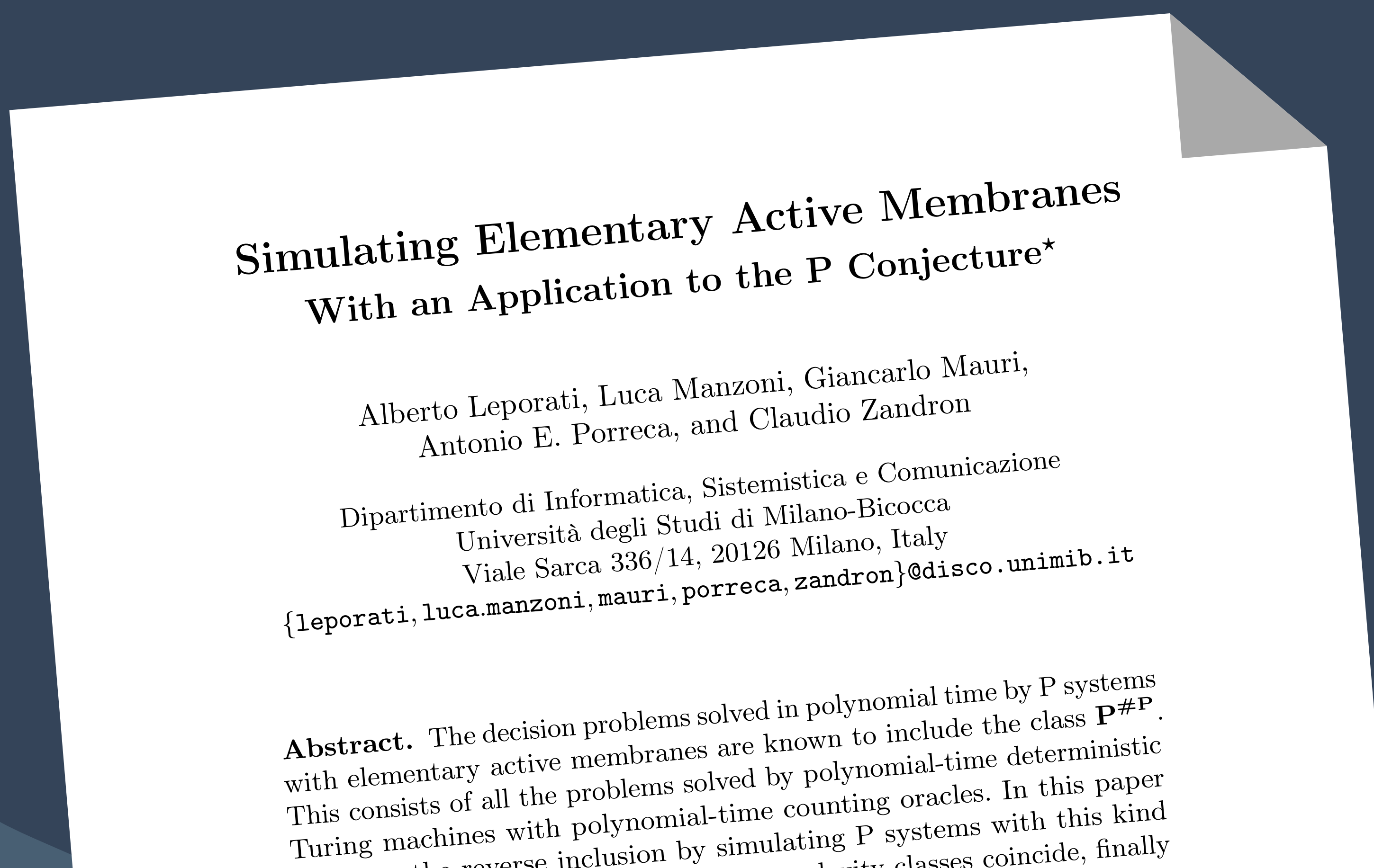
Abstract. We show how existing P systems with active membranes can be used as modules inside a larger P system; this allows us to simulate subroutines or oracles. As an application of this construction, which is (in principle) quite general, we provide a new, improved lower bound to the complexity class $\mathbf{PMC}_{\mathcal{AM}(-d,-n)}$ of problems solved by polynomial-time P systems with (restricted) elementary active membranes: this class is proved to contain $\mathbf{P}^{\mathbf{PP}}$ and hence

And now, the conclusion. . .

$$\mathsf{PMC}_{\mathsf{AM}(-n)} = \mathsf{P}^{\#\mathsf{P}}$$



Simulating Elementary Active Membranes
With an Application to the P Conjecture*

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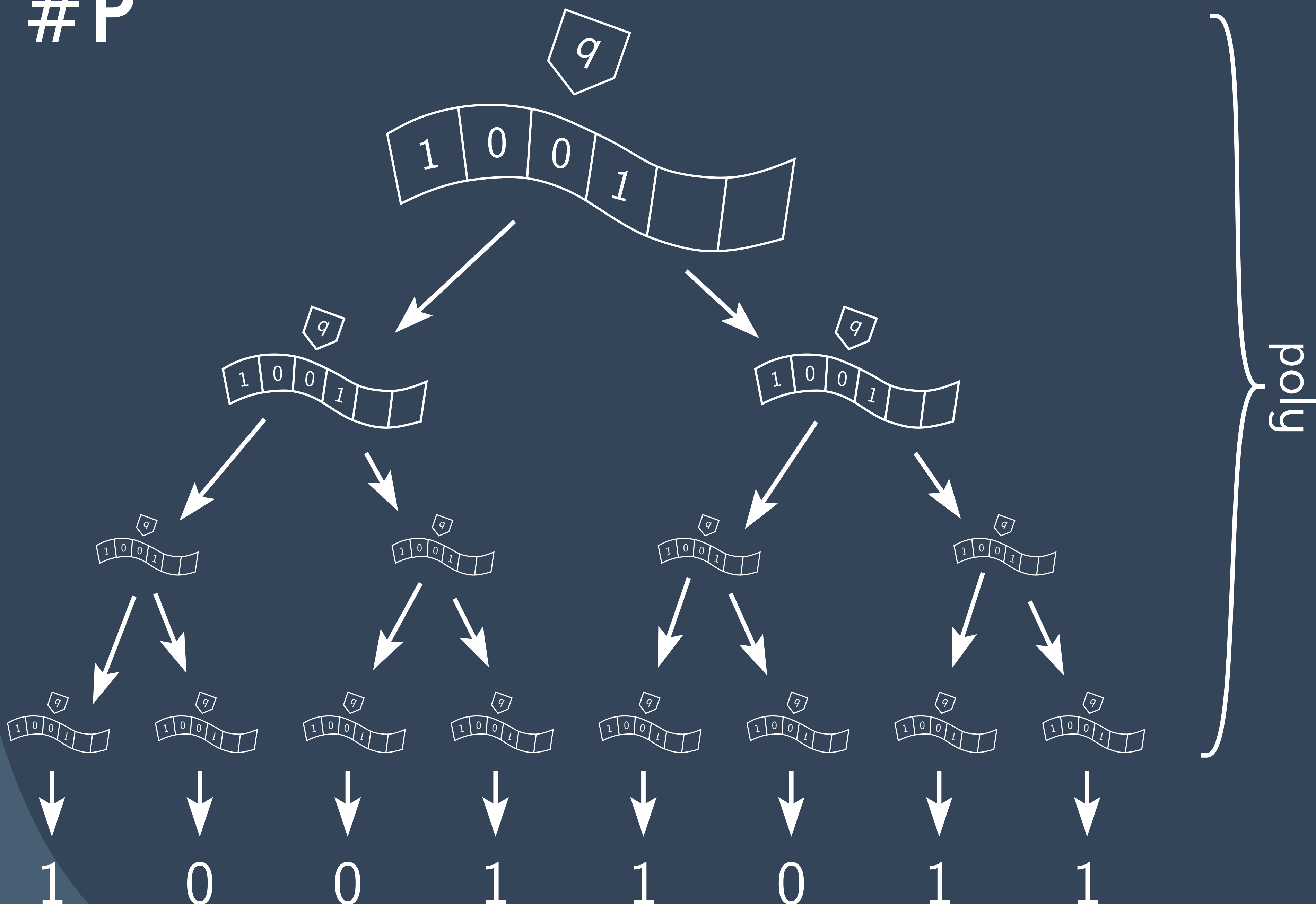
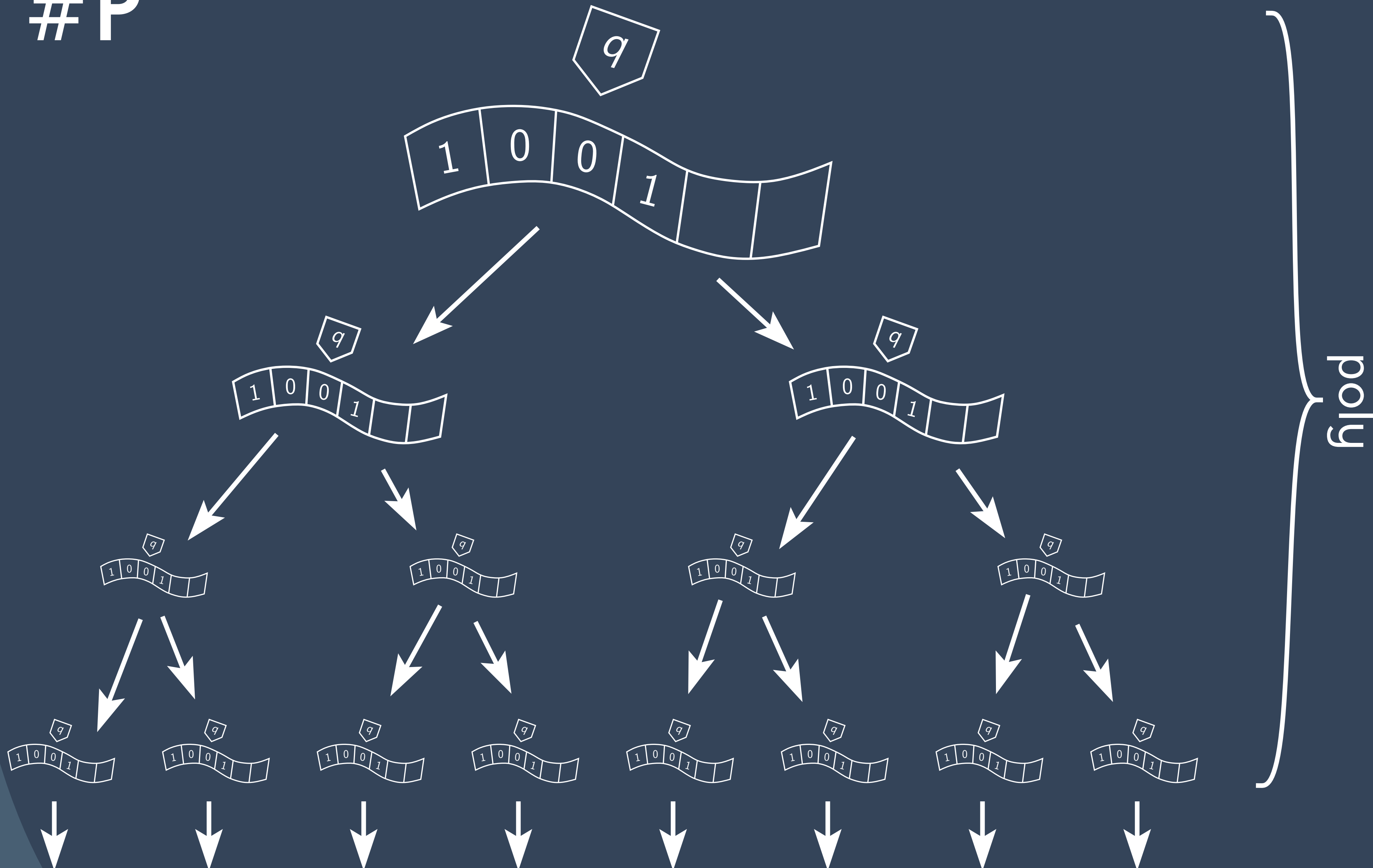
Abstract. The decision problems solved in polynomial time by P systems with elementary active membranes are known to include the class $\mathbf{P}^{\#\mathbf{P}}$. This consists of all the problems solved by polynomial-time deterministic Turing machines with polynomial-time counting oracles. In this paper

#P

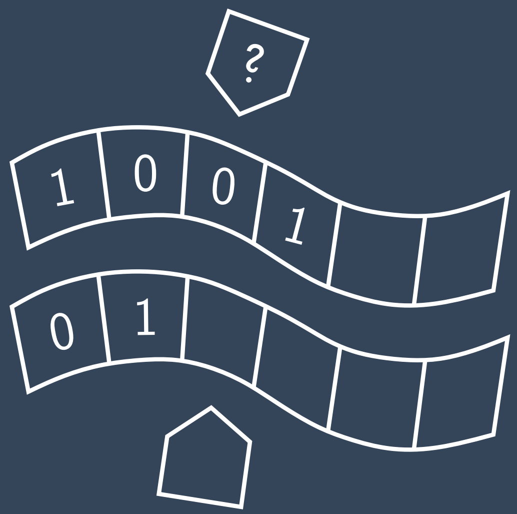


#P

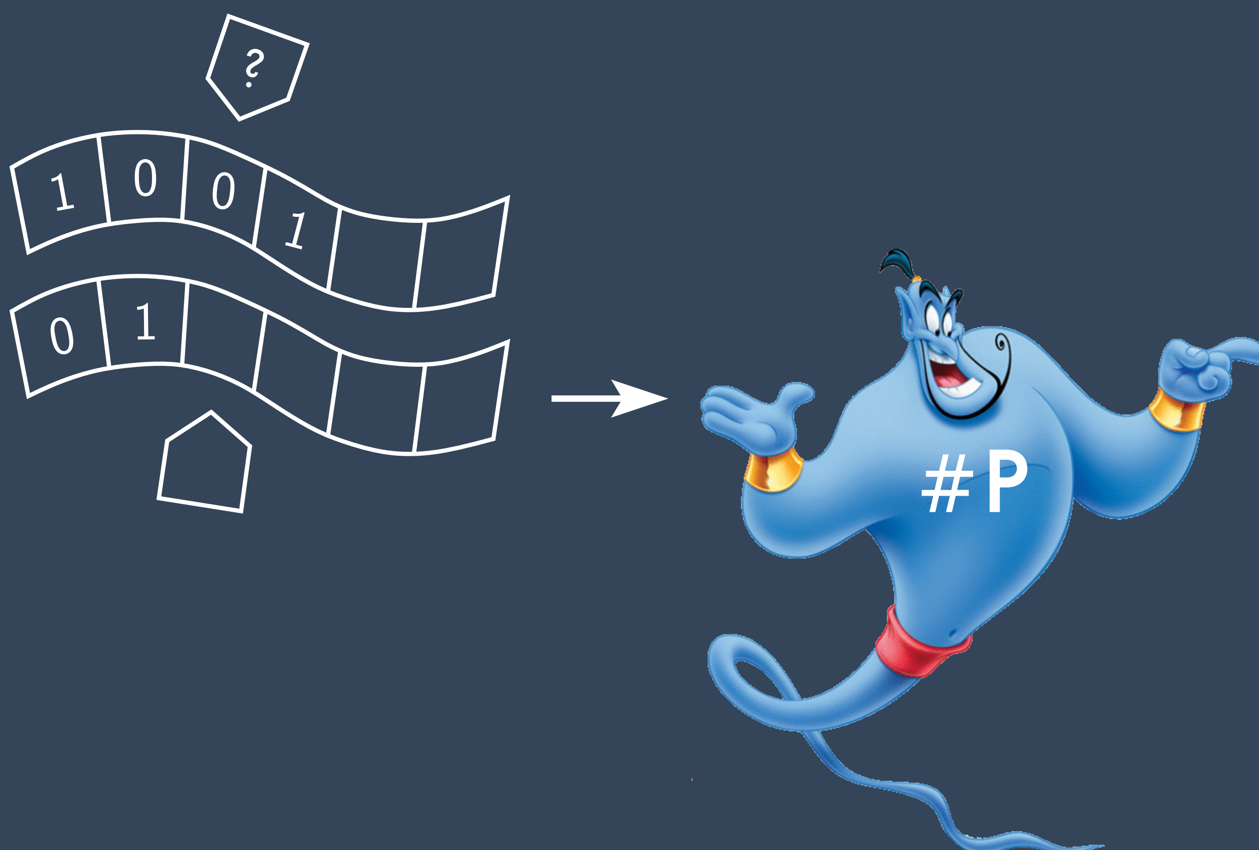


$$1 + 0 + 0 + 1 + 1 + 0 + 1 + 1 = f(x)$$

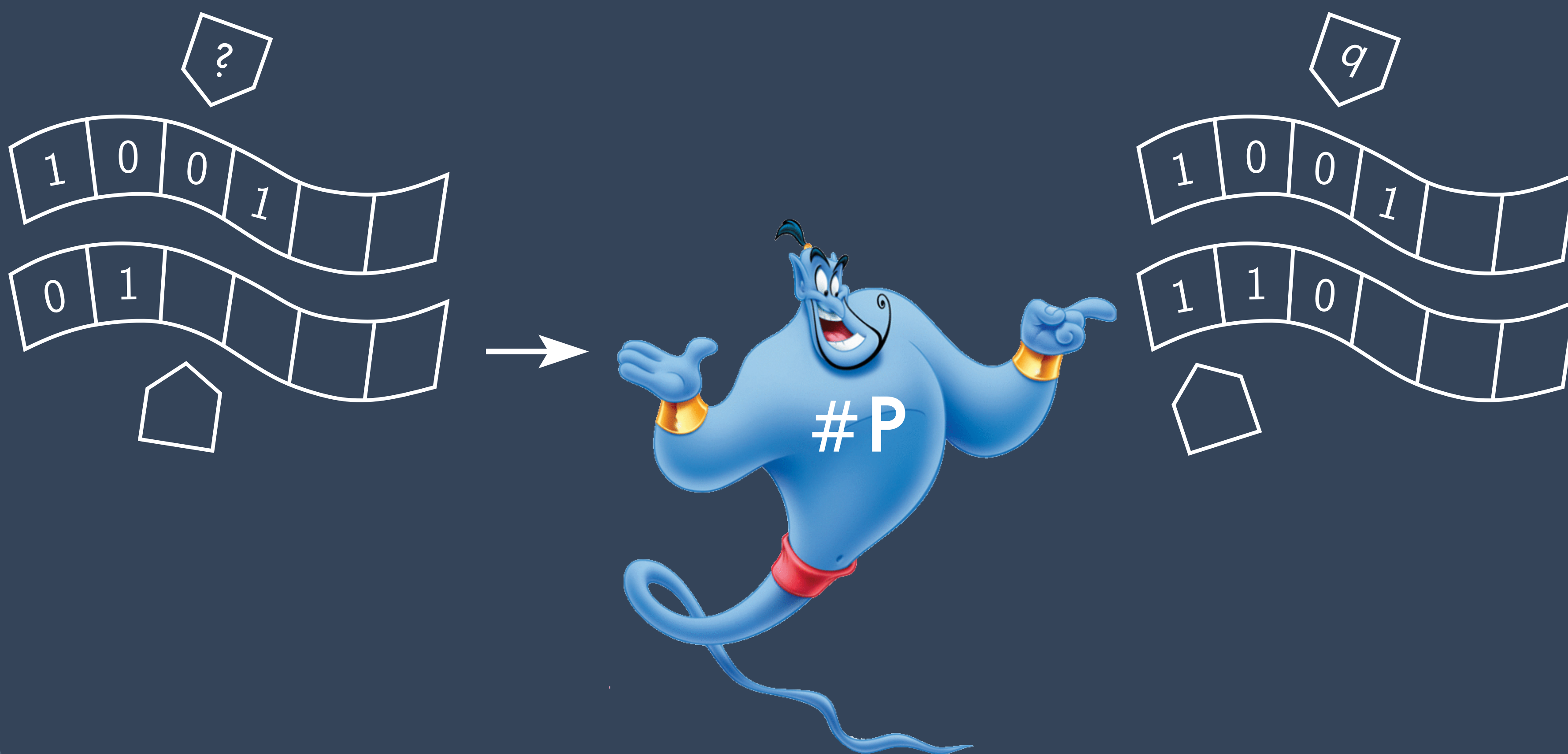
$P^{\#P}$



$P^{\#P}$



$P^{\#P}$



Milano Theorem

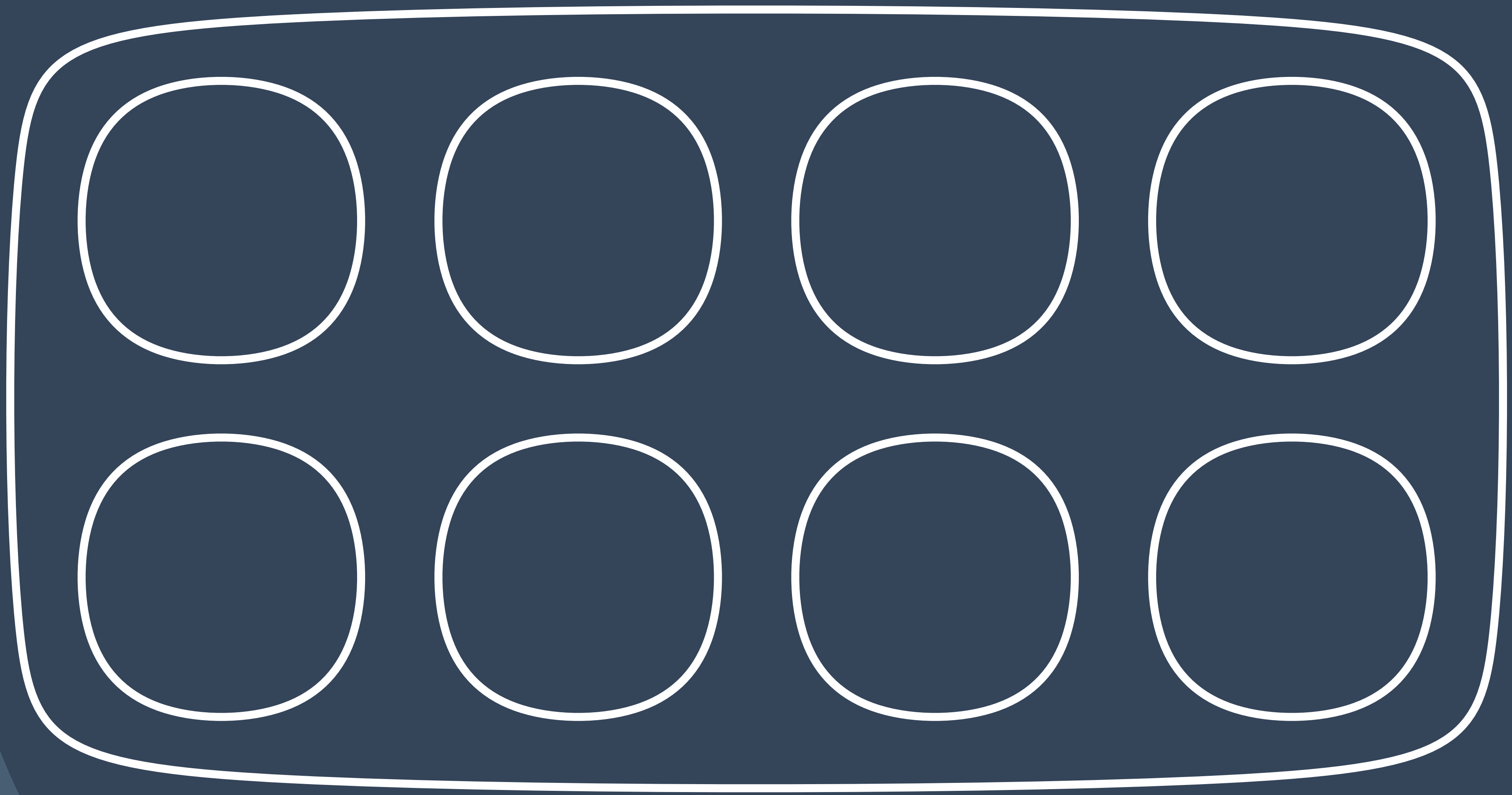
$$\mathbf{PMC}^{\star}_{\mathcal{AM}(-e,-n)} \subseteq \mathbf{P}$$

Elementary division is hard

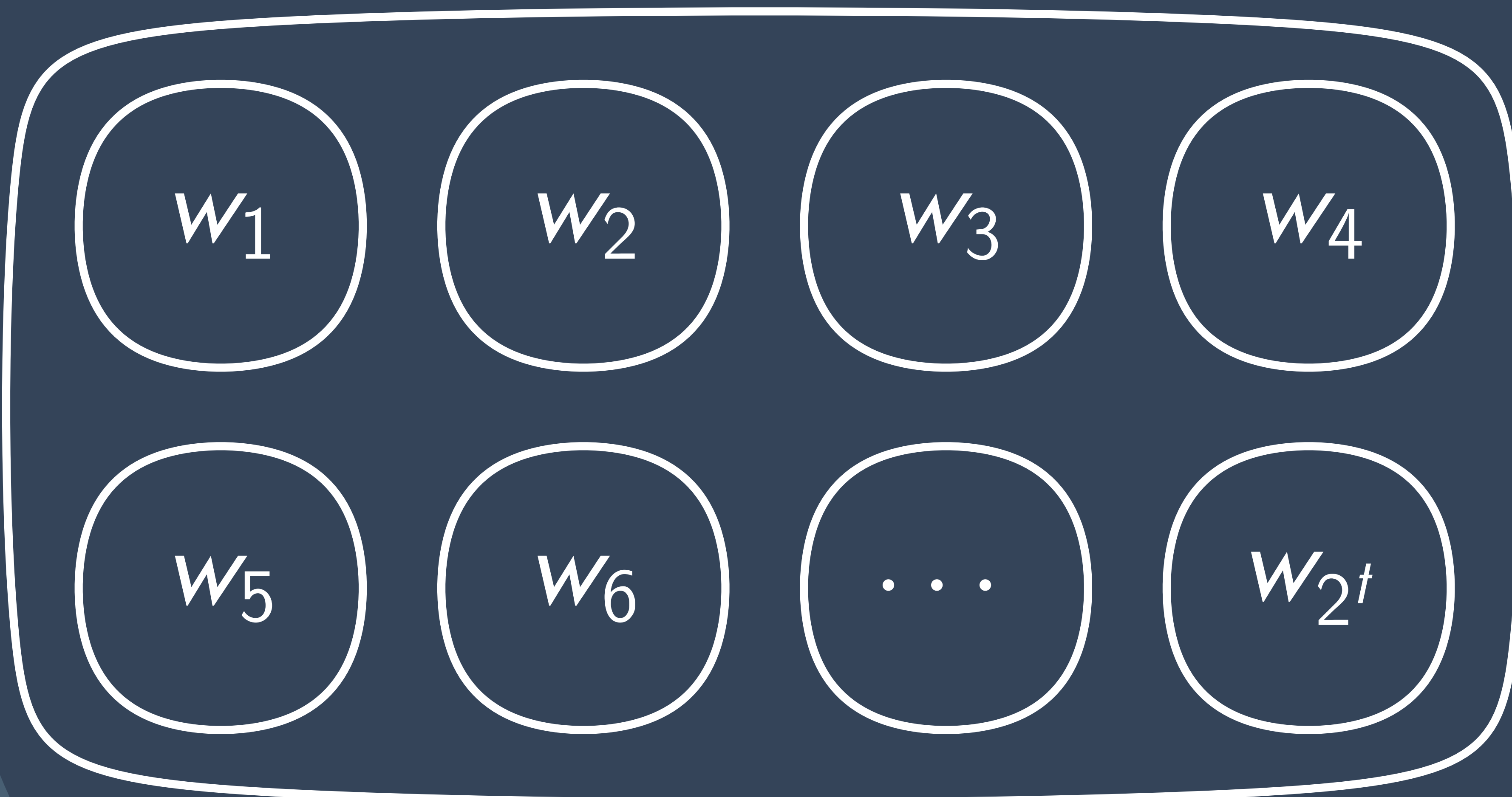
Elementary division is hard

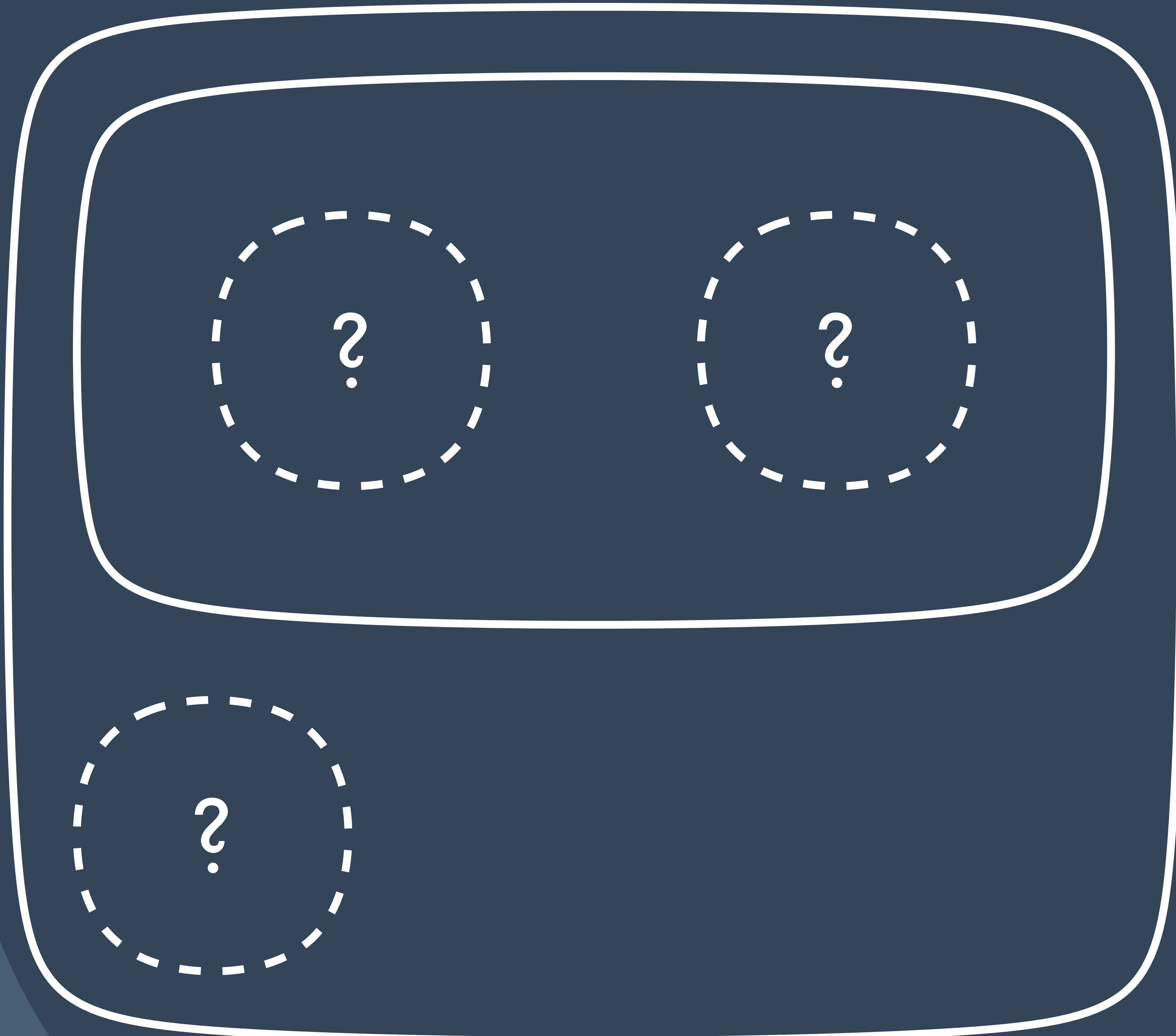
Elementary division is hard

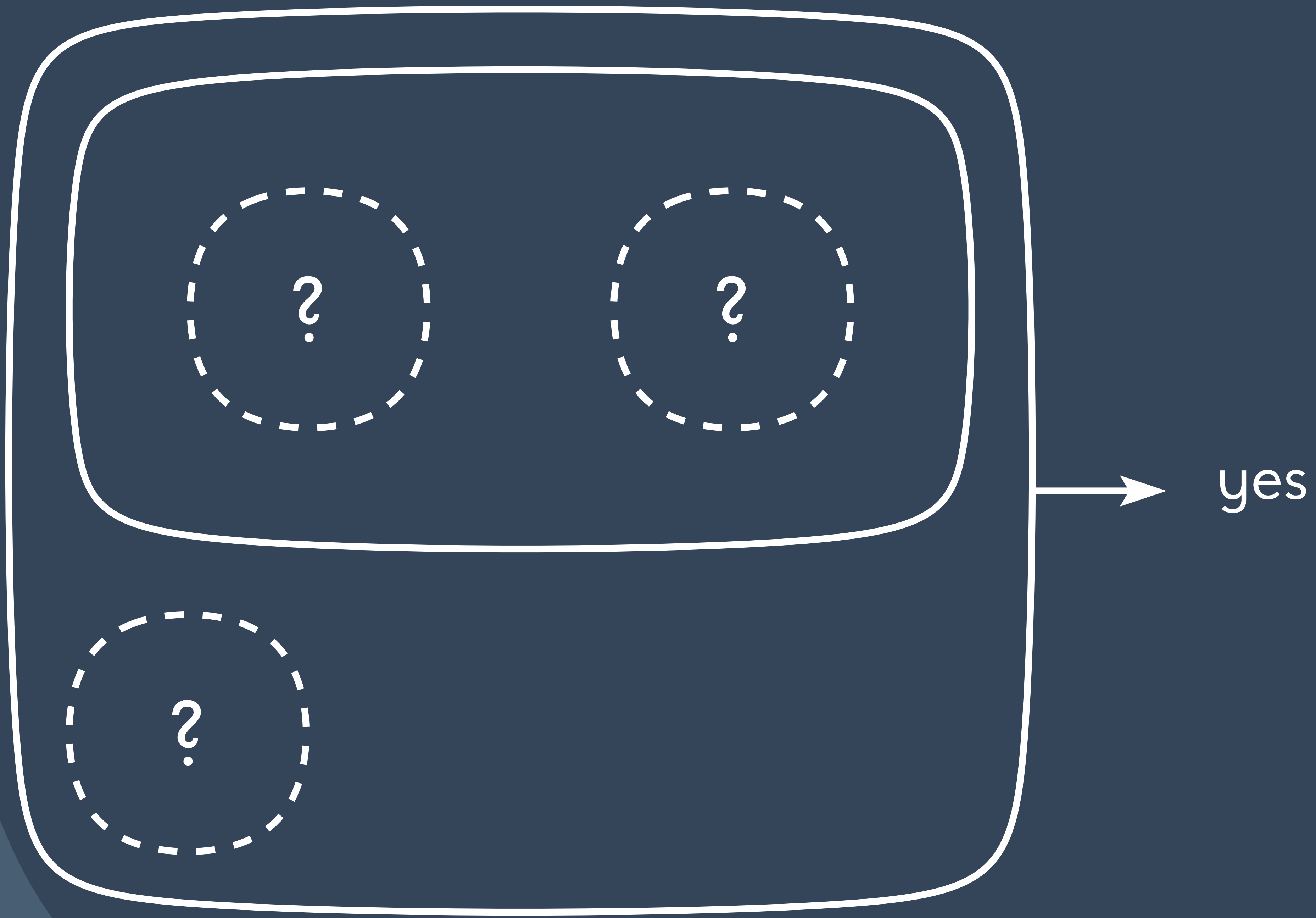
Elementary division is hard

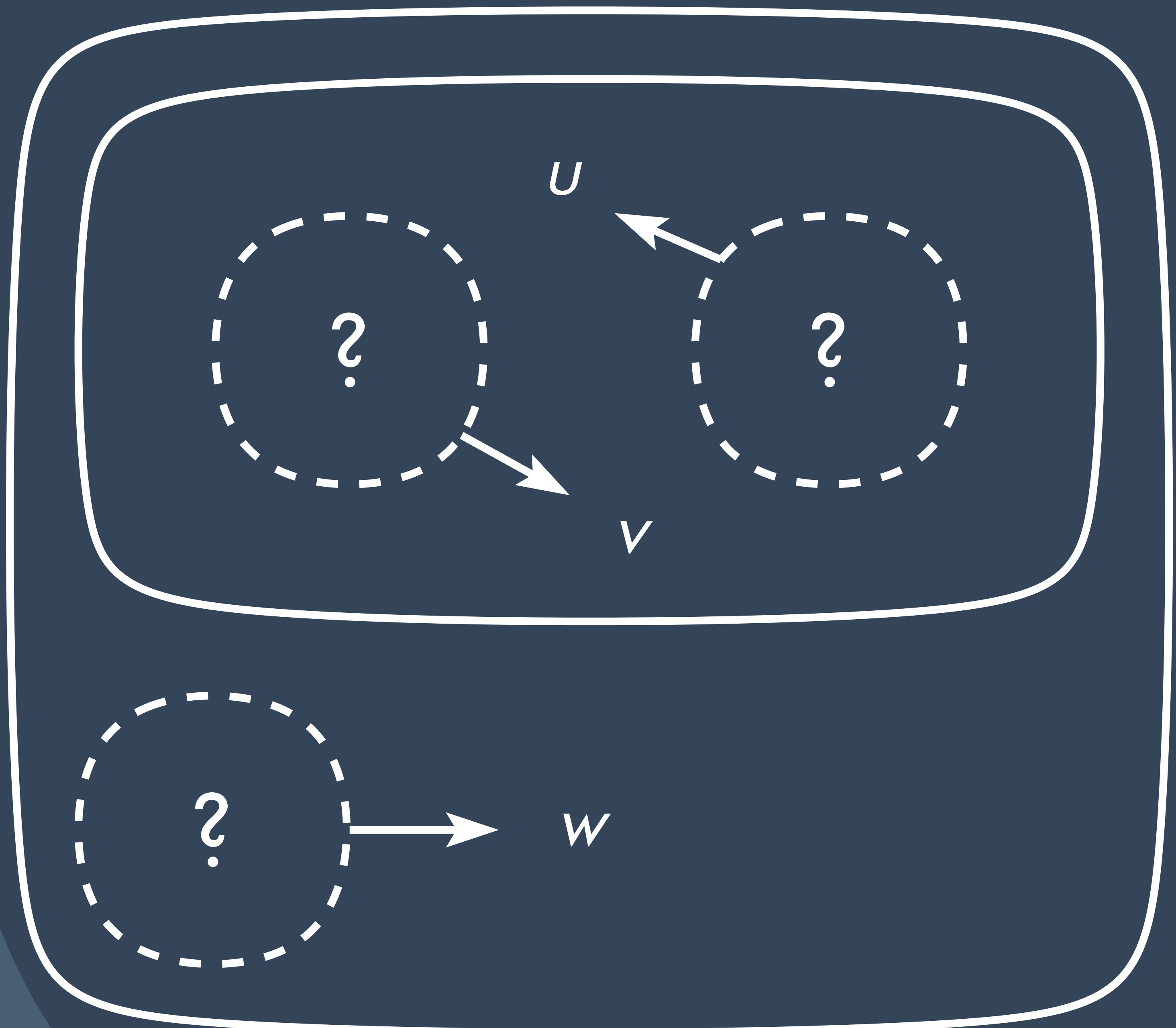


Elementary division is hard






?
?
?
yes


U
?
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V
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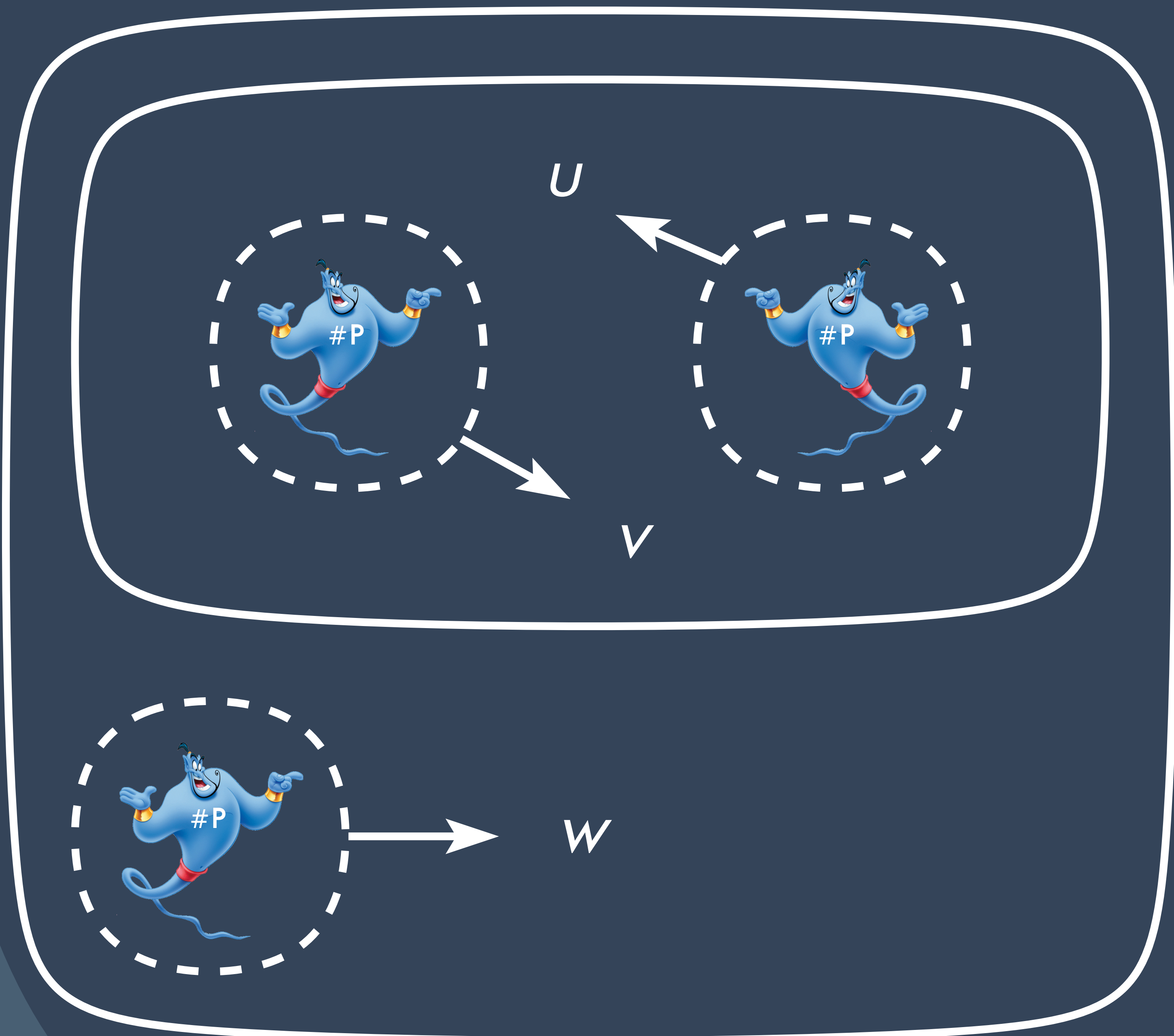
W


U
#P
#P
V
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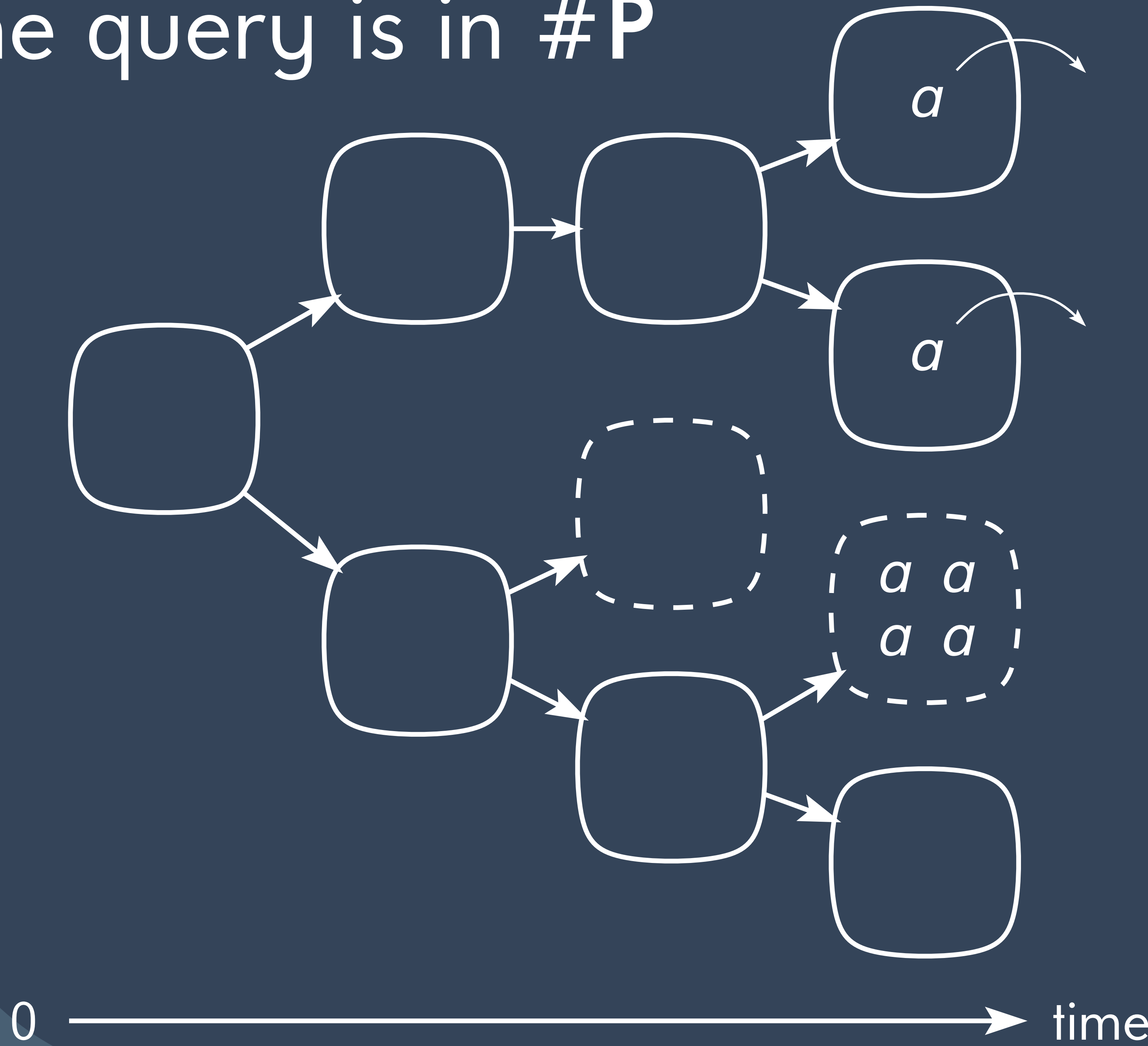
W

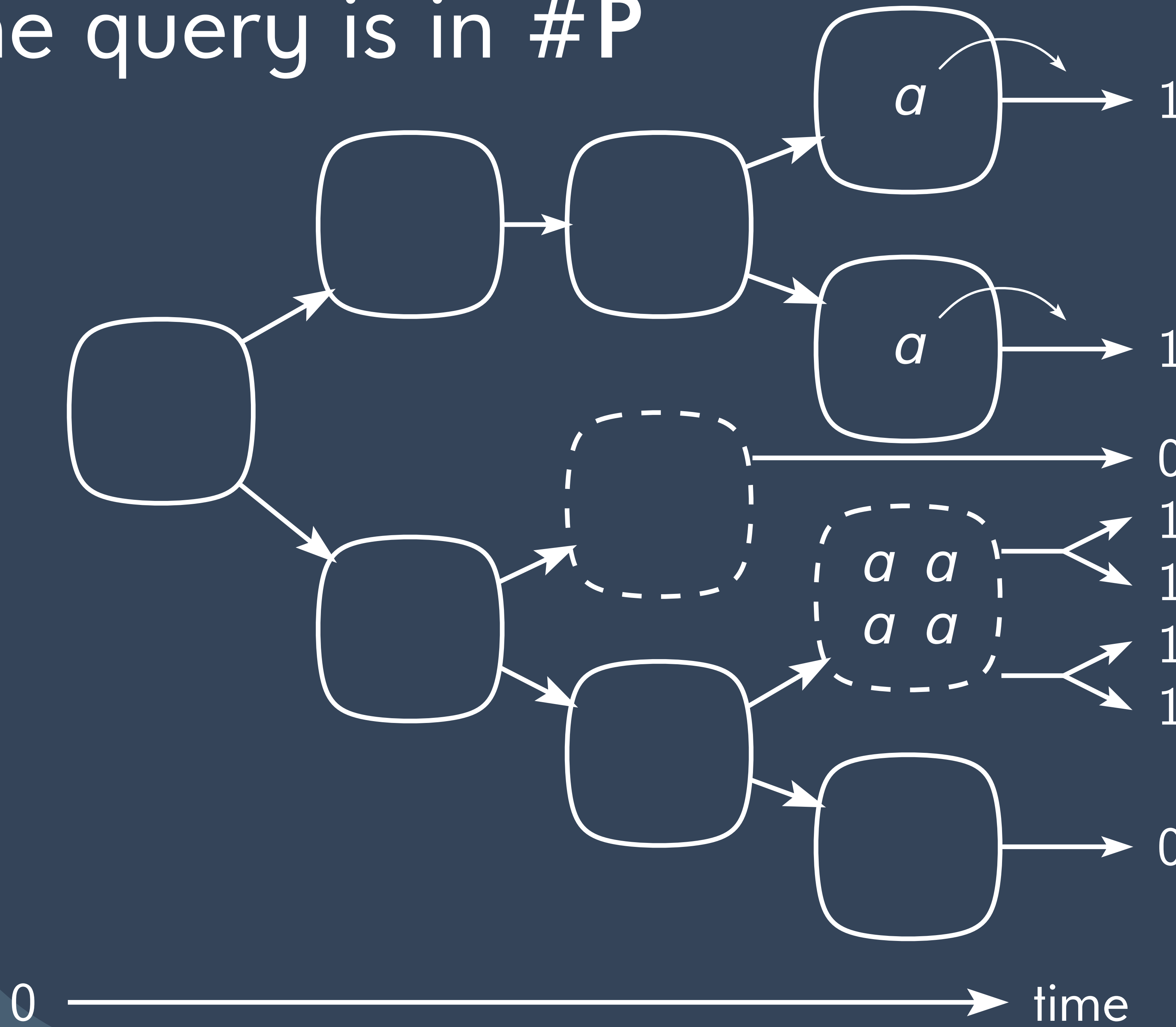
The query

Given the initial configuration of an elementary membrane h, how many copies of object a are sent out by membranes with label h at time t?

The query is in #P



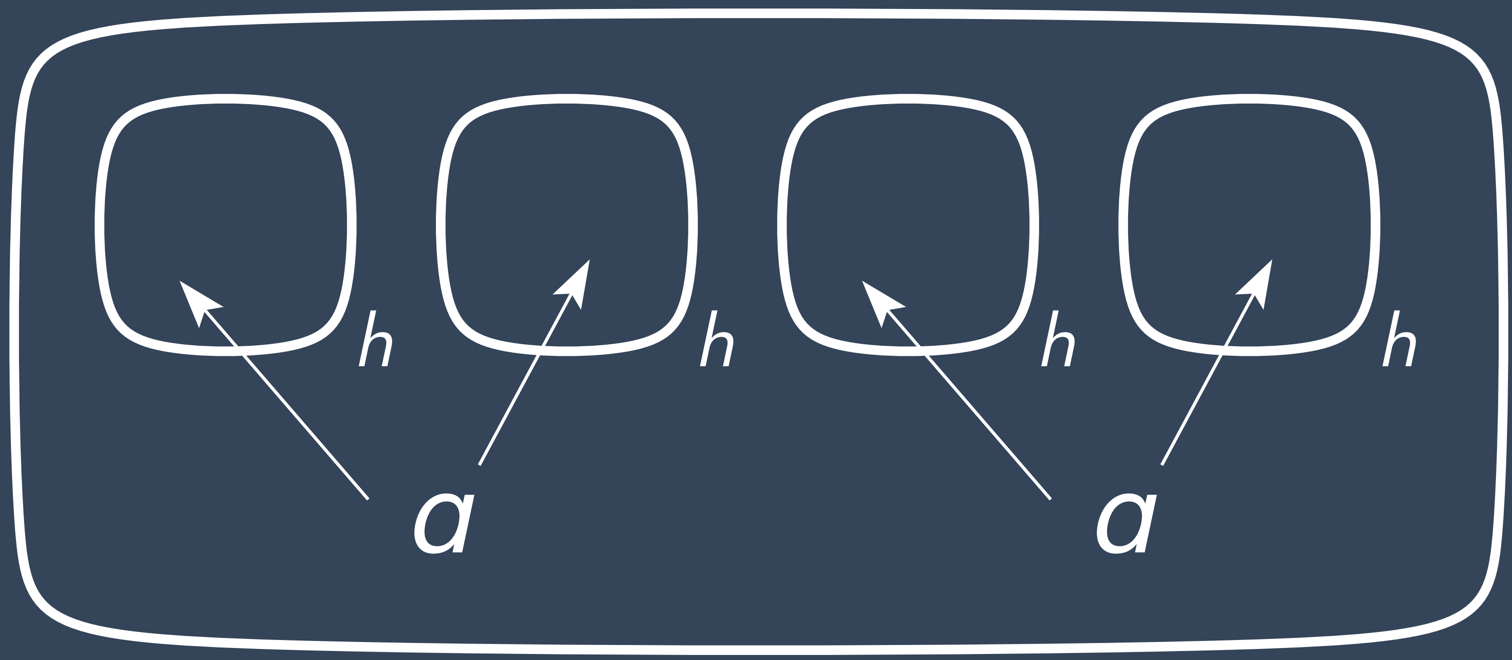
The query is in #P



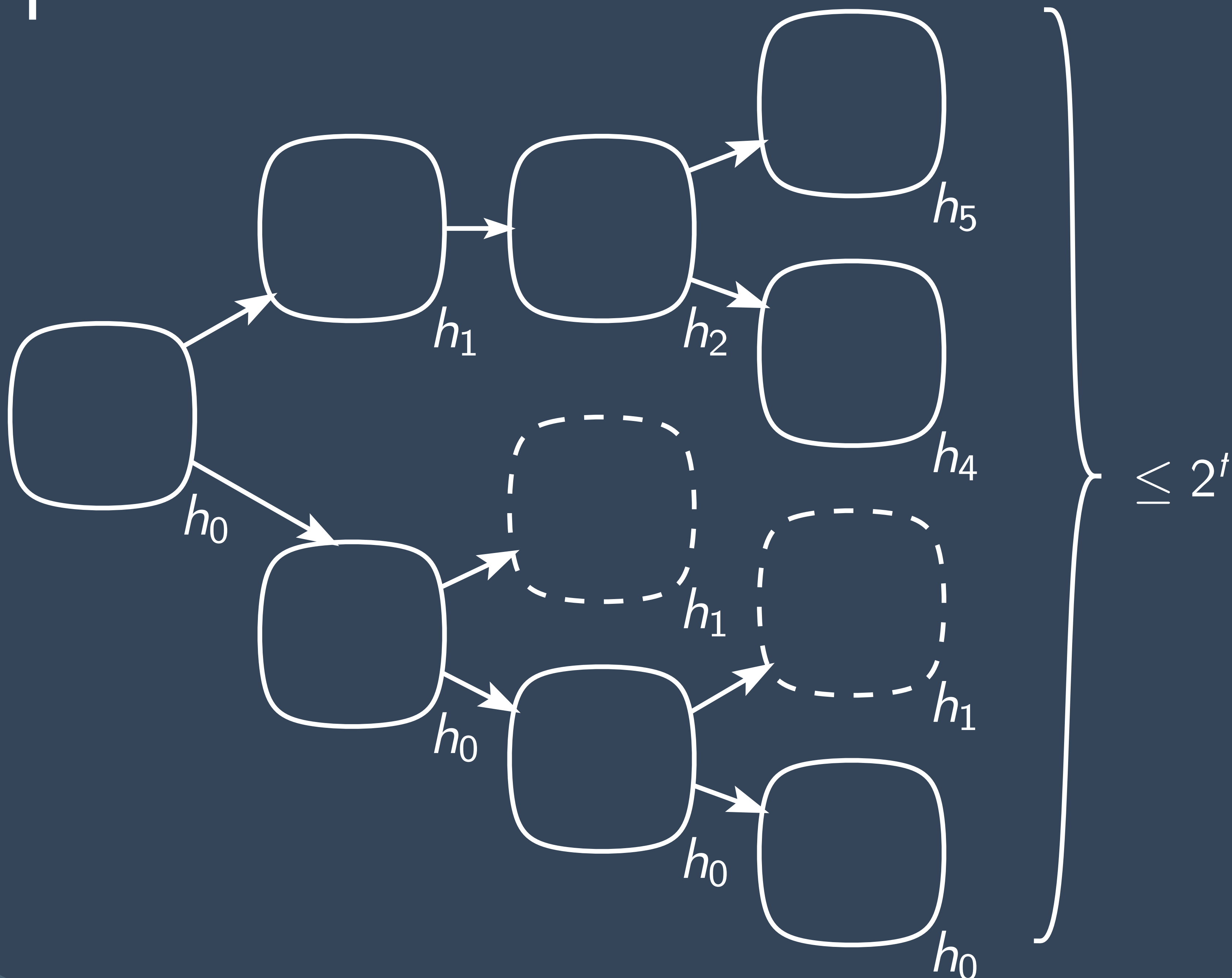
Preliminary result

$$\mathbf{PMC}^{\star}_{\mathcal{AM}(-i,-n)} \subseteq \mathbf{P}^{\#\mathbf{P}}$$

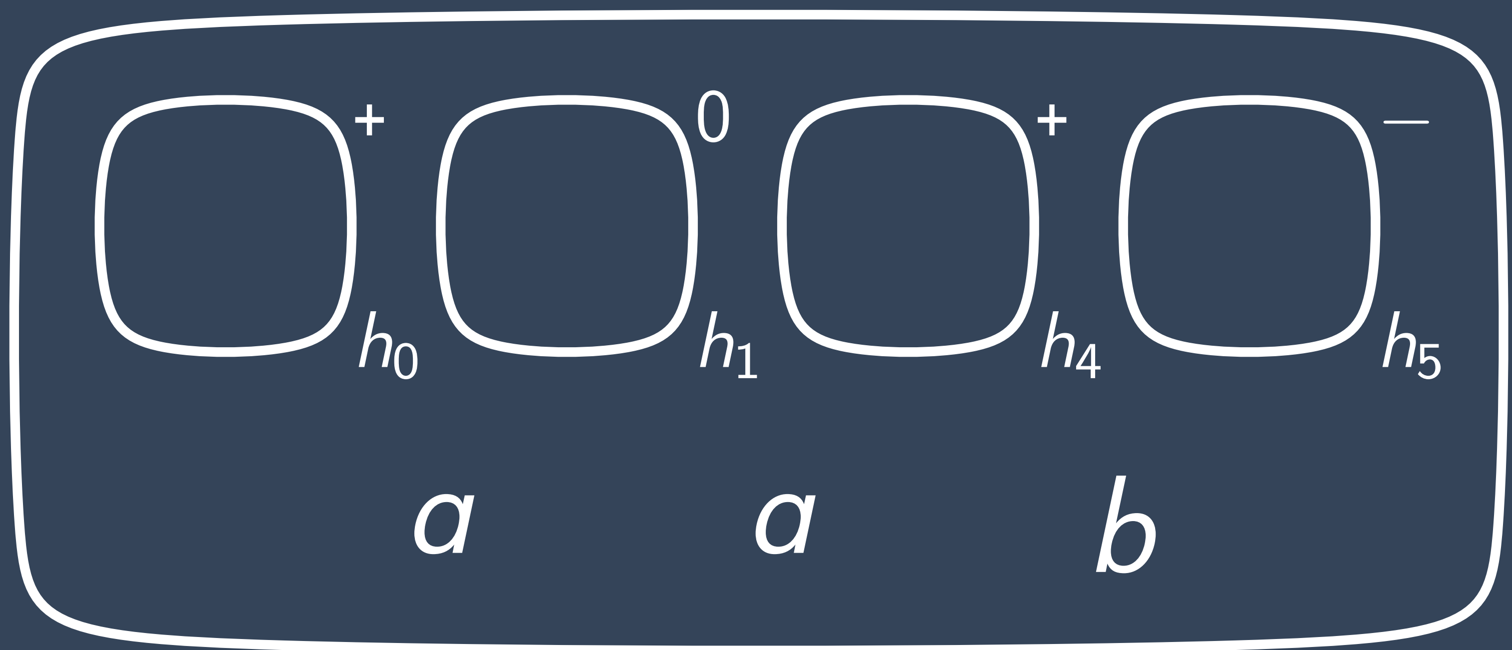
What about send-in rules?



Unique membrane ids

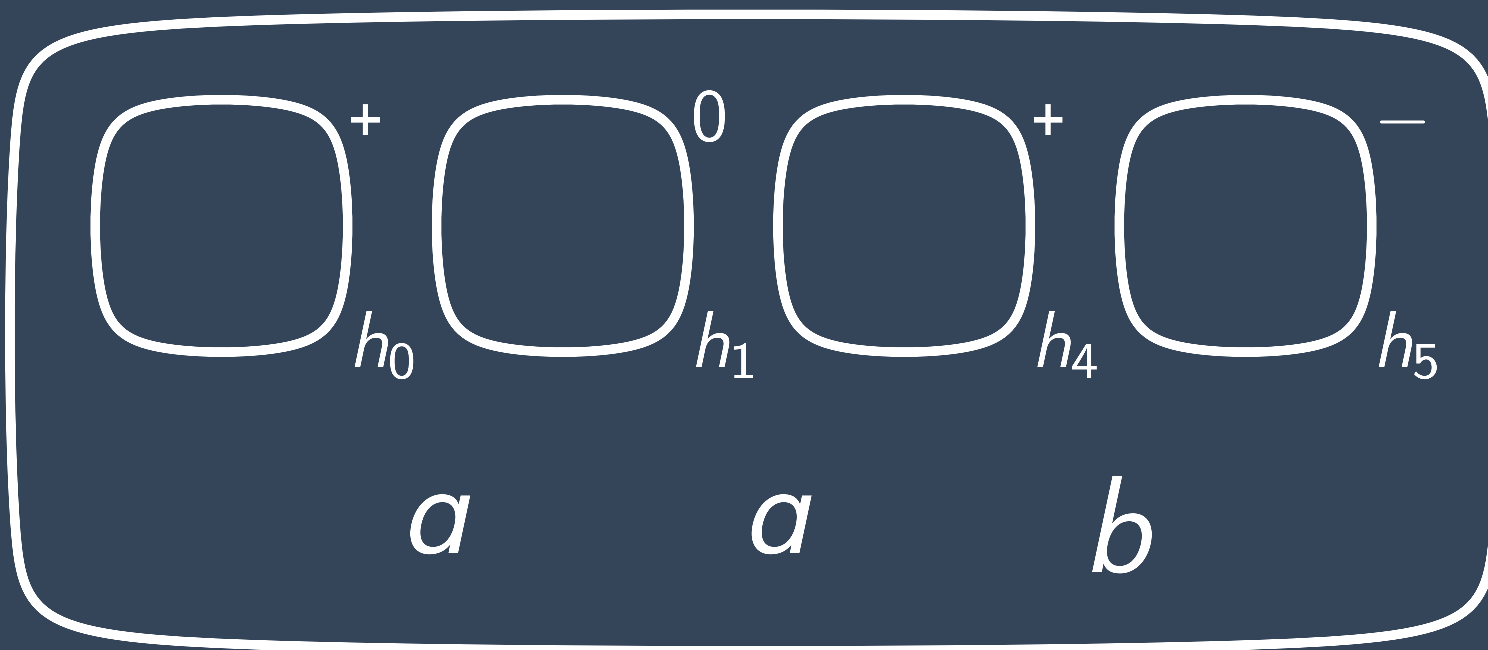


Magic table



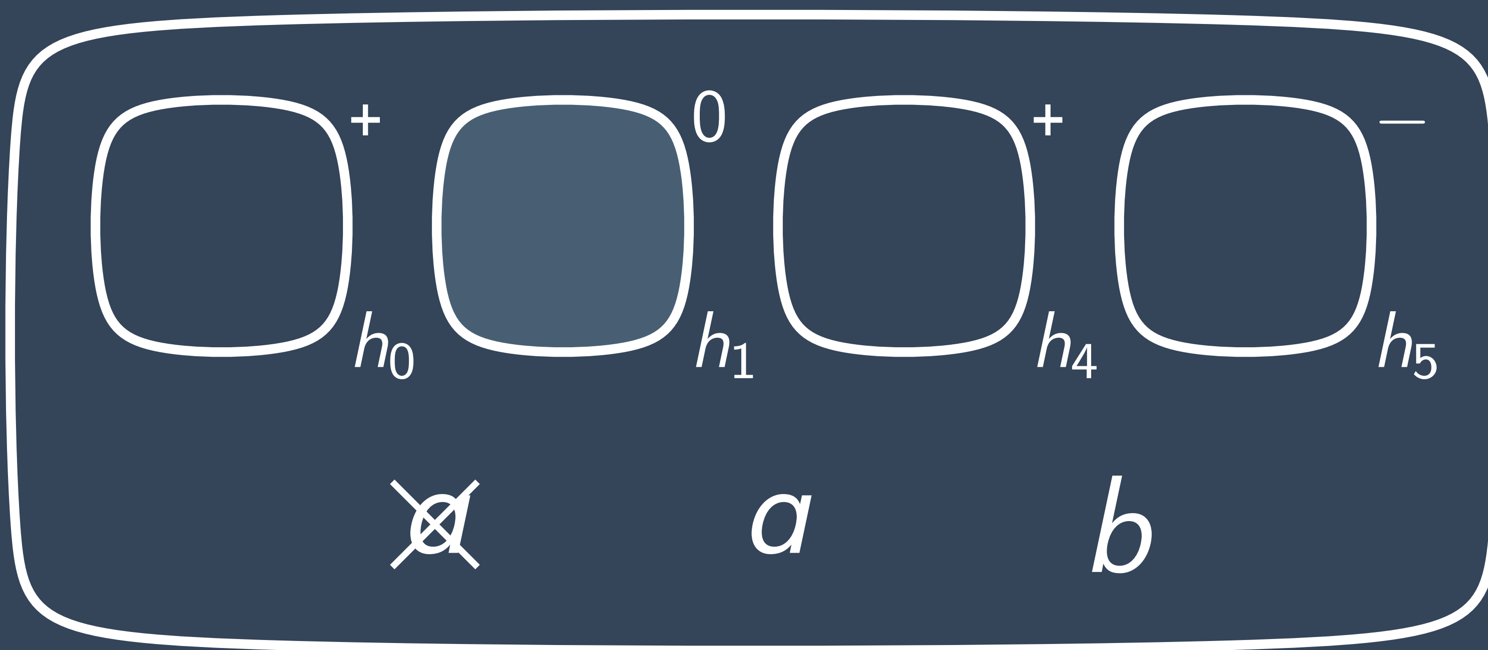
time	$a\,[\ ]_h^0 \rightarrow [b]_h^0$	$a\,[\ ]_h^+ \rightarrow [c]_h^+$	$b\,[\ ]_h^+ \rightarrow [d]_h^+$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
t	0–7	0–3	4–7

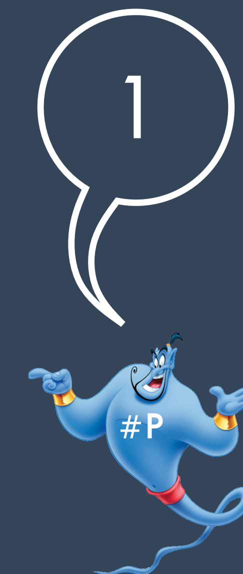
Filling the table



time	$a\,[\;]_h^0 \to [b]_h^0$	$a\,[\;]_h^+ \to [c]_h^+$	$b\,[\;]_h^+ \to [d]_h^+$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
t	0–7	?	?

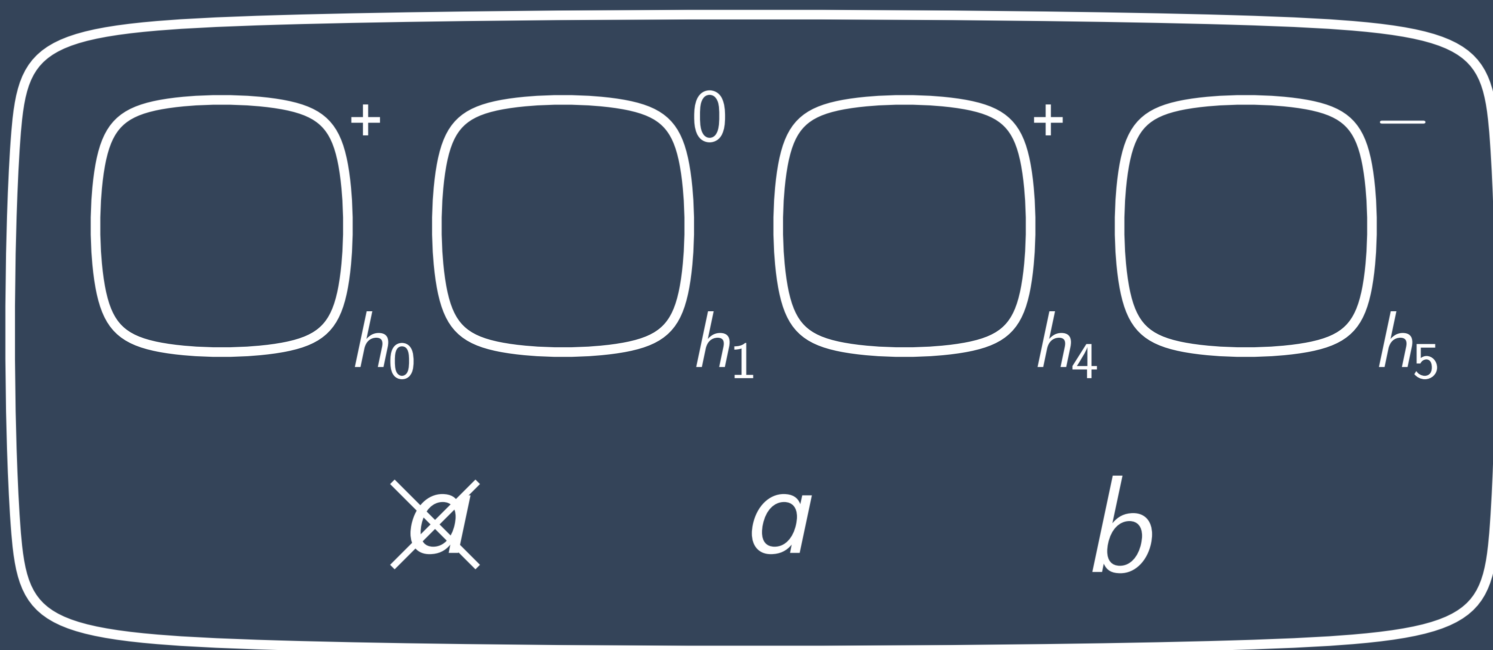
Filling the table





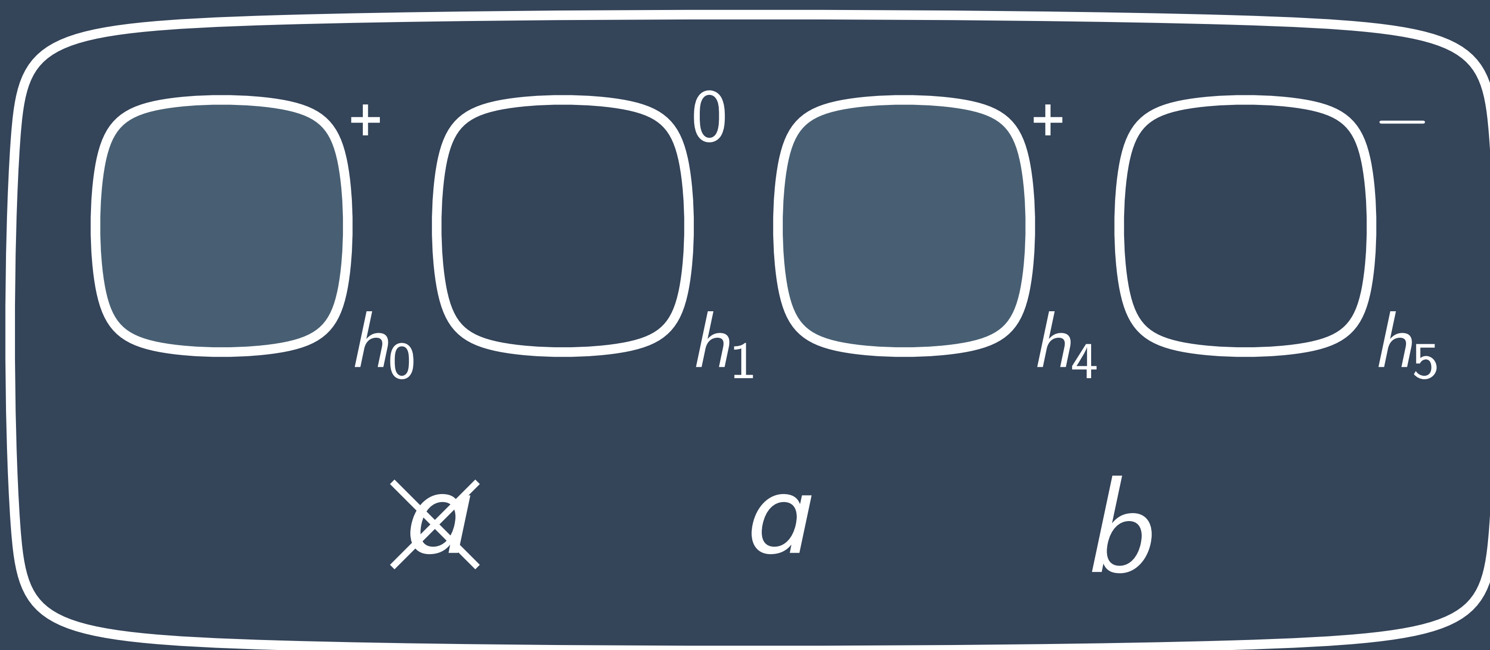
time	$a\,[\;]_h^0 \to [b]_h^0$	$a\,[\;]_h^+ \to [c]_h^+$	$b\,[\;]_h^+ \to [d]_h^+$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
t	0–7	?	?

Filling the table



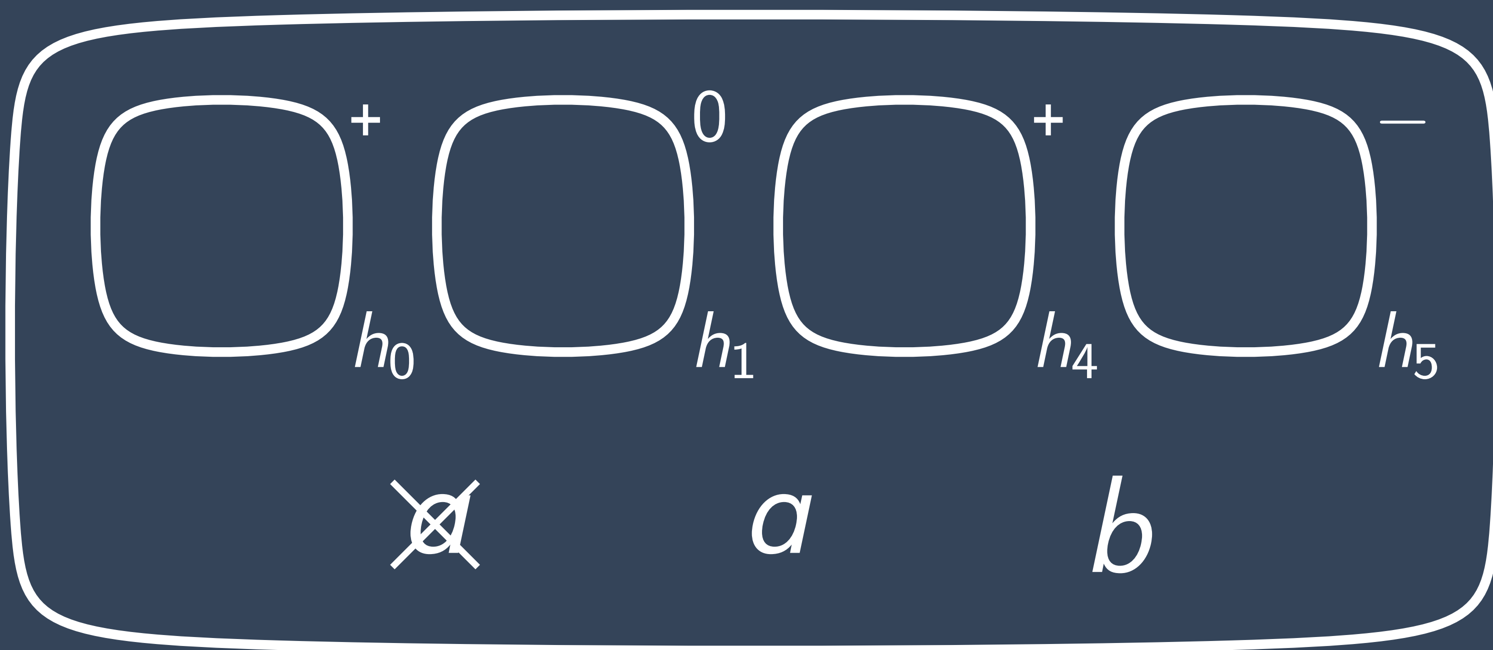
time	$a\,[\;]_h^0 \to [b]_h^0$	$a\,[\;]_h^+ \to [c]_h^+$	$b\,[\;]_h^+ \to [d]_h^+$
⋮	⋮	⋮	⋮
t	0–7	0–7	?

Filling the table



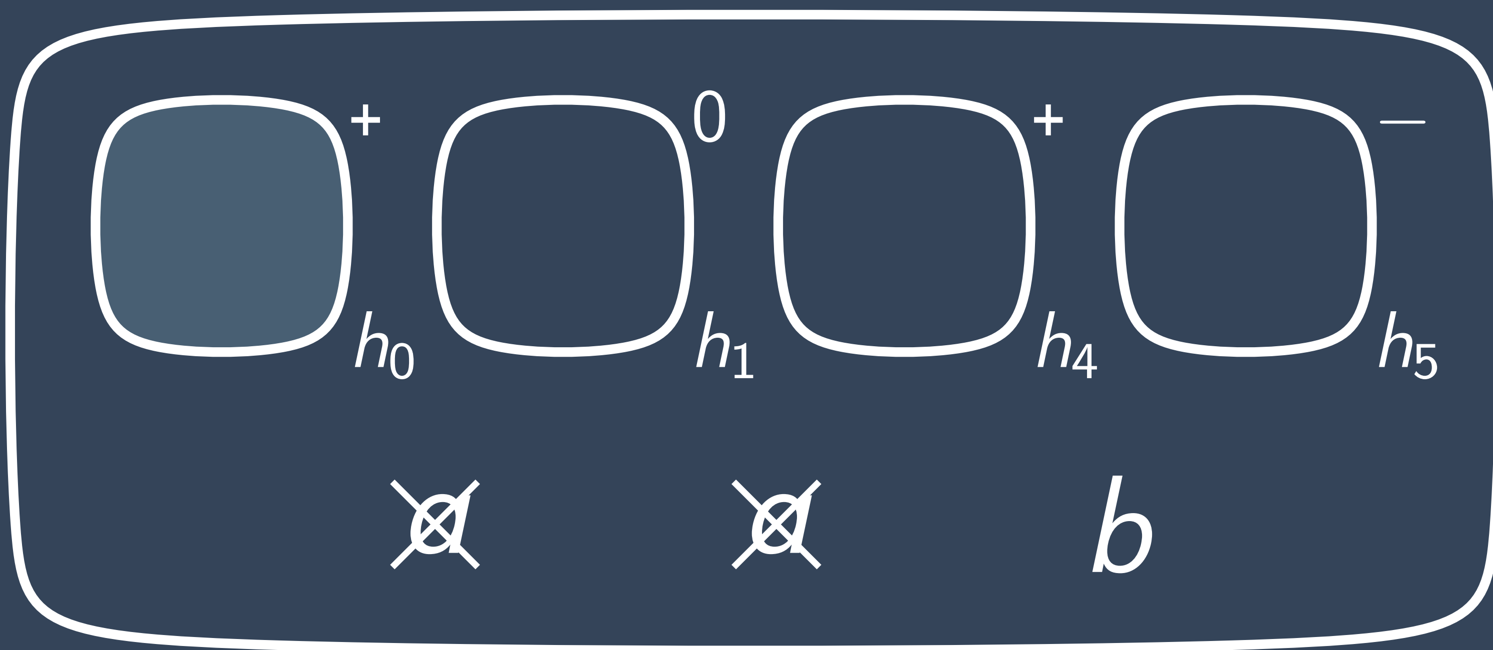
time	$a\,[\ ]_h^0 \to [b]_h^0$	$a\,[\ ]_h^+ \to [c]_h^+$	$b\,[\ ]_h^+ \to [d]_h^+$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
t	0–7	0–7	?

Filling the table



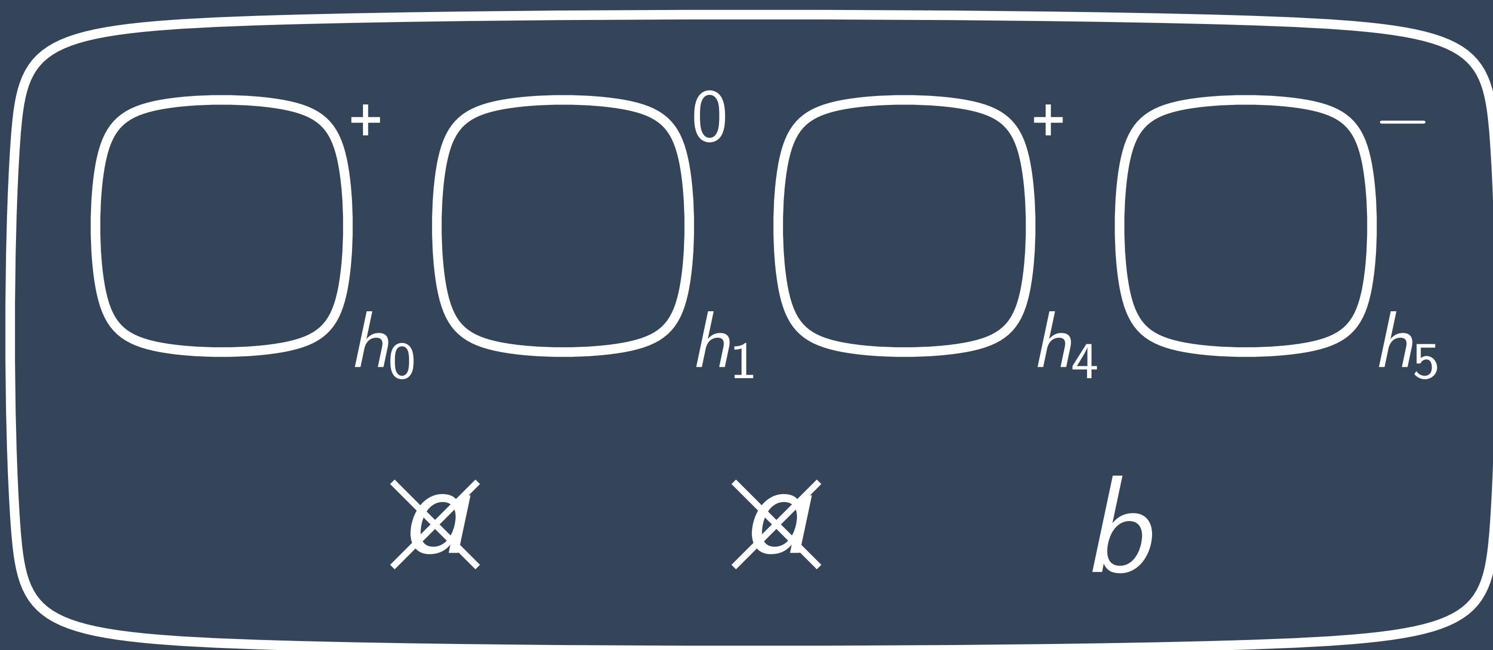
time	$a\,[\;]_h^0 \rightarrow [b]_h^0$	$a\,[\;]_h^+ \rightarrow [c]_h^+$	$b\,[\;]_h^+ \rightarrow [d]_h^+$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
t	0–7	0–3	?

Filling the table



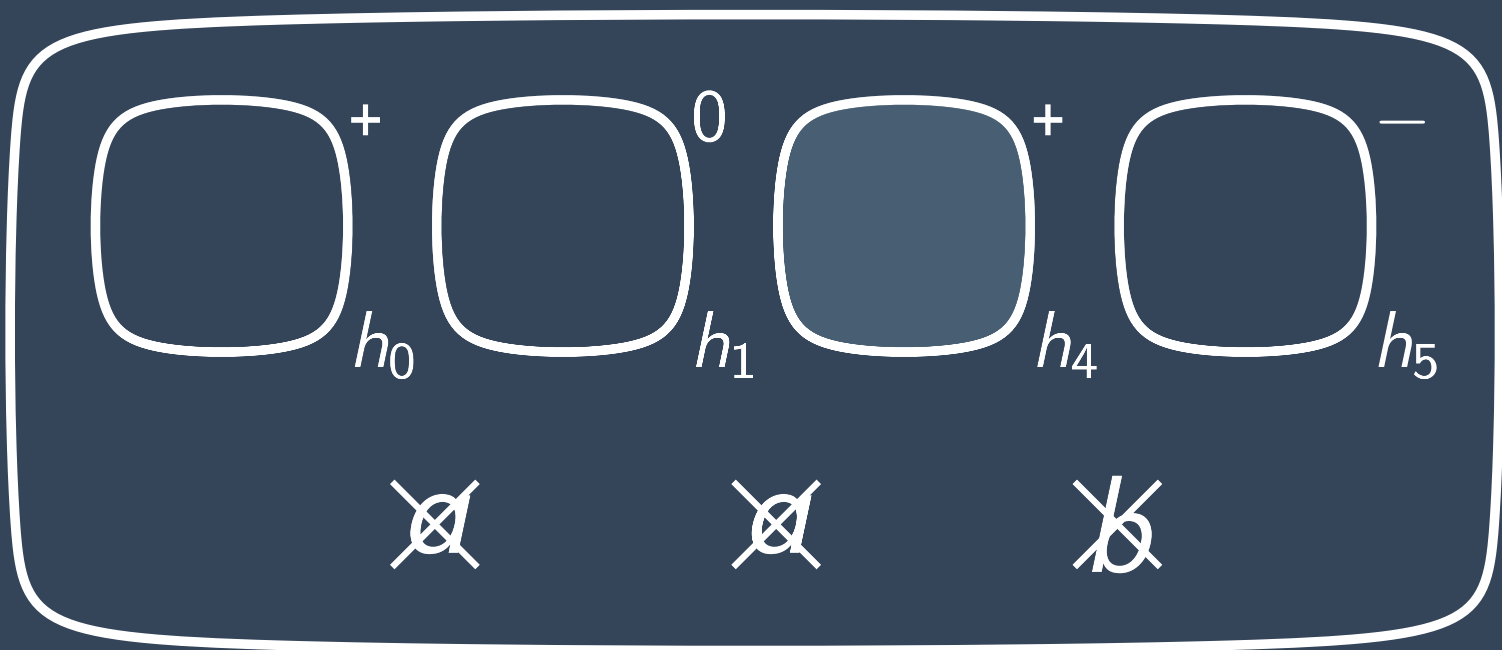
time	$a\,[\,]_h^0 \rightarrow [b]_h^0$	$a\,[\,]_h^+ \rightarrow [c]_h^+$	$b\,[\,]_h^+ \rightarrow [d]_h^+$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
t	0–7	0–3	?

Filling the table



time	$a\ [\ ]_h^0 \rightarrow [b]_h^0$	$a\ [\ ]_h^+ \rightarrow [c]_h^+$	$b\ [\ ]_h^+ \rightarrow [d]_h^+$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
t	0–7	0–3	4–7

Filling the table



time	$a\,[\ ]_h^0 \to [b]_h^0$	$a\,[\ ]_h^+ \to [c]_h^+$	$b\,[\ ]_h^+ \to [d]_h^+$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
t	0–7	0–3	4–7

Main result

$$\mathbf{PMC}^{\star}_{\mathsf{AM}(-n)} \subseteq \mathbf{P}^{\#\mathbf{P}}$$

$$\begin{tikzcd}
 & \mathbf{P}^{\#\mathbf{P}} & \\
 & \mathbf{PMC}^{\star}_{AM(-n)} \arrow[u, phantom, "\subseteq" sloped] & \\
\mathbf{PMC}_{AM(-n)} \arrow[ur, phantom, "\subseteq" sloped] & & \mathbf{PMC}^{\star}_{AM(-d,-n)} \arrow[ul, phantom, "\subseteq" sloped] \\
 & \mathbf{PMC}_{AM(-d,-n)} \arrow[ul, phantom, "\subseteq" sloped] \arrow[ur, phantom, "\subseteq" sloped] & \\
 & \mathbf{P}^{\#\mathbf{P}} \arrow[u, phantom, "\subseteq" sloped] &
\end{tikzcd}$$

Application to the P conjecture

$$\mathbf{PMC}^{\star}_{\mathsf{AM}^0(-n)} \subseteq \mathbf{P}^{\#\mathbf{P}}$$

(previously: **PSPACE**)

Open problems

- Constant depth vs the counting hierarchy
- Non-confluent case
- Constant depth without charges
- (P conjecture)

Děkuji vám za pozornost!
Thanks for your attention!

Any questions?