Space complexity for P systems

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BWMC7 – February 5, 2009

Size of a configuration (1)

Suppose we manage to build a real P system with active membranes. How large is it?

Definition

Let Π be a P system with active membranes. The size $|\mathcal{C}|$ of a configuration $\mathcal{C}$ of Π is the sum of the number of membranes and the total number of objects inside its regions.

Size of a configuration (2)

…or maybe we just want to simulate P systems *in silico*?

Definition

Let Π be a P system with active membranes. The size $|\mathcal{C}|$ of a configuration $\mathcal{C}$ of Π is the sum of the number of membranes and the number of bits required to store the number of objects inside each region.

Size of a configuration (3)

Problem

Which definition do we choose? Does our choice change the theory in any important way, i.e. is the number of objects as important as the number of membranes?

On the number of membranes and objects

- Let Π be a recogniser P system with active membranes
- Let m be the maximum number of membranes in Π (considering all computations and all configurations)
- Let o be the maximum number of objects inside Π

Problem

Can we always modify Π (without changing its behaviour) in such a way that $o \leq \text{poly}(m)$?

Size of a computation

Meanwhile, assume the first definition of "size of a configuration".

Definition

Let Π be a P system with active membranes, and let $\vec{\mathcal{C}} = (\mathcal{C}_0, \ldots, \mathcal{C}_n)$ be a halting computation of Π. The size of $\vec{\mathcal{C}}$ is defined as

$$|\vec{\mathcal{C}}| = \max\{|\mathcal{C}_0|, \ldots, |\mathcal{C}_n|\}$$

Space bound for a P system

Definition

Let Π be a (confluent or non-confluent) P system with active membranes such that every computation of Π halts. The size of Π is defined as

$$|\Pi| = \max\{|\vec{\mathcal{C}}| : \vec{\mathcal{C}} \text{ is a computation of } \Pi\}$$

If $|\Pi| \leq n$ for some $n \in \mathbf{N}$ then Π is said to operate within space n.

Space bound for a family of P systems

Definition

Let $\mathbf{\Pi} = \{\Pi_x : x \in \Sigma^\star\}$ be a polynomially semi-uniform family of recogniser P systems, and let $f : \mathbf{N} \rightarrow \mathbf{N}$. Then $\mathbf{\Pi}$ is said to operate within space bound f iff $|\Pi_x| \leq f(|x|)$ for each $x \in \Sigma^\star$.

Space complexity classes

Definition

Let $\mathcal{D}$ be a class of P systems (e.g. with active membranes but without division rules), $f : \mathbf{N} \to \mathbf{N}$ and $L \subseteq \Sigma^\star$. Then $L \in \mathbf{MCSPACE}^\star_{\mathcal{D}}(f)$ iff $L = L(\mathbf{\Pi})$ for some polynomially semi-uniform family $\mathbf{\Pi} \subseteq \mathcal{D}$ of confluent recogniser P systems operating within space bound f.

Definition

The corresponding class for non-confluent P systems is $\mathbf{NMCSPACE}^\star_{\mathcal{D}}(f)$.

Abbreviations

Definition

$$
\begin{aligned}
\mathbf{PMCSPACE}^{\star}_{\mathcal{D}} &= \mathbf{MCSPACE}^{\star}_{\mathcal{D}}(\mathsf{poly}) \\
\mathbf{NPMCSPACE}^{\star}_{\mathcal{D}} &= \mathbf{NMCSPACE}^{\star}_{\mathcal{D}}(\mathsf{poly}) \\
\mathbf{EXPMCSPACE}^{\star}_{\mathcal{D}} &= \mathbf{MCSPACE}^{\star}_{\mathcal{D}}(2^{\mathsf{poly}}) \\
\mathbf{NEXPMCSPACE}^{\star}_{\mathcal{D}} &= \mathbf{NMCSPACE}^{\star}_{\mathcal{D}}(2^{\mathsf{poly}})
\end{aligned}
$$

. . . and so on.

Preliminary results (1)

Proposition

$\mathbf{MCSPACE}^{\star}_{\mathcal{D}}(f) \subseteq \mathbf{NMCSPACE}^{\star}_{\mathcal{D}}(f)$ *for all* $f : \mathbf{N} \to \mathbf{N}$.

$$\mathbf{PMCSPACE}^{\star}_{\mathcal{D}} \subseteq \mathbf{NPMCSPACE}^{\star}_{\mathcal{D}}$$
$$\mathbf{EXPMCSPACE}^{\star}_{\mathcal{D}} \subseteq \mathbf{NEXPMCSPACE}^{\star}_{\mathcal{D}}$$
$$\mathbf{PMCSPACE}^{\star}_{\mathcal{D}} \subseteq \mathbf{EXPMCSPACE}^{\star}_{\mathcal{D}}$$
$$\mathbf{NPMCSPACE}^{\star}_{\mathcal{D}} \subseteq \mathbf{NEXPMCSPACE}^{\star}_{\mathcal{D}}$$

Preliminary results (2)

Proposition

$\mathbf{PMCSPACE}^{\star}_{\mathcal{D}}$, $\mathbf{NPMCSPACE}^{\star}_{\mathcal{D}}$, $\mathbf{EXPMCSPACE}^{\star}_{\mathcal{D}}$ *and* $\mathbf{NEXPMCSPACE}^{\star}_{\mathcal{D}}$ *are all closed under polytime reductions.*

Proposition

$\mathbf{MCSPACE}^{\star}_{\mathcal{D}}(f)$ *is closed under complementation for each* $f : \mathbf{N} \to \mathbf{N}$.

Preliminary results (3)

Proposition

$\mathbf{NPMC}^{\star}_{\mathcal{NAM}} \subseteq \mathbf{EXPMCSPACE}^{\star}_{\mathcal{EAM}}$.

Proof.

$\mathbf{NPMC}^{\star}_{\mathcal{NAM}} = \mathbf{NP}$ and $\mathrm{SAT} \in \mathbf{EXPMCSPACE}^{\star}_{\mathcal{EAM}}$, which is closed under polytime reductions. $\square$

Problems (1)

Problem

$\textbf{PMCSPACE}^{\star}_{\mathcal{NAM}} = \textbf{PMCSPACE}^{\star}_{\mathcal{EAM}} = \textbf{PMCSPACE}^{\star}_{\mathcal{AM}}$.

Ideas.

Can we resolve all membrane divisions at construction time (by precomputing the final membrane structure)?

What about "conditional" divisions and "cyclic" behaviour (divide $\to$ dissolve $\to$ divide $\to \cdots$)? $\square$

Problems (2)

Problem

Is $\mathbf{PMC}^{\star}_{\mathcal{NAM}} = \mathbf{PMCSPACE}^{\star}_{\mathcal{NAM}}$*?*
Is $\mathbf{NPMC}^{\star}_{\mathcal{NAM}} = \mathbf{NPMCSPACE}^{\star}_{\mathcal{NAM}}$*?*

Idea.

Maybe when the size of the P system is polynomial there is no need to compute for a superpolynomial amount of time. □

Problems (3)

Problem

Does any of the following hold?

$$\mathbf{PMC}^{\star}_{\mathcal{EAM}} \neq \mathbf{PMCSPACE}^{\star}_{\mathcal{EAM}}$$
$$\mathbf{PMC}^{\star}_{\mathcal{AM}} \neq \mathbf{PMCSPACE}^{\star}_{\mathcal{AM}}$$

Problems (4)

And, of course. . .

Problem

What are the relations between these new complexity classes and traditional ones like **P**, **NP**, **PSPACE**, **EXP**, **EXPSPACE**?