What do we know
(and what we don’t)
about the
computational complexity
of Reaction Systems

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Outline

- Dynamical behaviours
- Detection of behaviours
- Using RS for computing
Dynamical behaviours
Dynamical Behaviours

Fixed point

Cycle
Dynamical Behaviours

Fixed point attractor

Attractor Cycle
Dynamical Behaviours

(Fixed point)

global attractor
Dynamical Behaviours

Garden of Eden

Path from $T$ to $U$
Detection of behaviours
Fill the blanks

Does [Reaction System] given [additional parameters] exhibit [dynamical behaviour]?
Fixed Point attractor

Does \( A = (S,A) \)
given a fixed point \( T \)
exhibit a state \( U \neq T \)
such that \( res_A(U) = T \)?
Equality of RS

Does $A = (S, A)$ and $B = (S, B)$ given [nothing more] exhibit the same result function?
Reachability

Does \( A = (S, A) \)

given \( T, U \) subsets of \( S \)

exhibit a path from \( T \) to \( U \) ?
Fixed Point attractor

Reduction from SAT in CNF

\[ f = (x \lor y) \land (x \lor \neg z) \]

\( C_1 \quad \text{and} \quad C_2 \)

NP-complete
\[ f = (x \lor y) \land (x \lor \neg z) \]

\[ S = \{x, y, z, C_1, C_2\} \]

\[ (\{C_1, C_2\}, \{x, y, z\}, \{C_1, C_2\}) \]

\[ (\{x\}, \{C_1, C_2\}, \{C_1, C_2\}) \]

\[ (\{y\}, \{C_1, C_2\}, \{C_1\}) \]

\[ (\emptyset, \{z, C_1, C_2\}, \{C_2\}) \]
\[ f = (x \lor y) \land (x \lor \neg z) \]

\[
(\{y\}, \{C_1, C_2\}, \{C_1\})
\]

\[
(\emptyset, \{z, C_1, C_2\}, \{C_2\})
\]

\[
(\{C_1, C_2\}, \{x, y, z\}, \{C_1, C_2\})
\]
$f = (x \lor y) \land (x \lor \neg z)$

$\{y\}, \{C_1, C_2\}, \{C_1\}$

$\emptyset, \{z, C_1, C_2\}, \{C_2\}$
\[ f = (x \lor y) \land (x \lor \neg z) \]

Diagram with nodes and edges labeled with variables and logical operators. A cloud labeled "Malformed states" is connected to the diagram.
A diagram illustrating the logical expression:

\[ f = (x \lor y) \land (x \lor \neg z) \]

The diagram shows satisfying assignments and malformed states.
Equality of res of two RS

Reduction from VALIDITY in DNF

\[ f = (x \land y) \lor (x \land \neg z) \lor z \]

coNP-complete
\[ f = (x \land y) \lor (x \land \neg z) \lor z \]

**First RS**

\[ (\emptyset, \emptyset, \{\text{True}\}) \]

\[ S = \{x, y, z, \text{True}\} \]
\[ f = (x \land y) \lor (x \land \neg z) \lor z \]

Second RS

\[
\begin{align*}
\{x, y\}, \emptyset, \{\text{True}\} \\
\{x\}, \{z\}, \{\text{True}\} \\
\{z\}, \emptyset, \{\text{True}\}
\end{align*}
\]
\[ f = (x \land y) \lor (x \land \neg z) \lor z \]

**Second RS**

\[
\begin{align*}
\{z\}, \emptyset, \{\text{True}\} & \quad \{x, y\}, \emptyset, \{\text{True}\} \\
\{x\}, \{z\}, \{\text{True}\} &
\end{align*}
\]
Since $f$ is valid, the two systems describe the same result function.
Reachability

We have already seen how to simulate bounded-tape TM

Reachability for bounded-tape TM is \textit{PSPACE}-complete...
...and also for RS
Existence of a fixed point
Existence of a fixed point attractor

Equality of result functions
Existence of a Garden of Eden

Reachability
Existence of a global attractor
RS for computing
Uniform Families of RS

Input $x$ of length $n$

Input $x$ of length $n$

$T \subset S_n$
Uniform Families of RS

- RS can be simulated by TM with polynomial slowdown (and vice versa)...

- ...hence, we need to select two very weak TM for the uniformity condition
Uniform Families of RS

- We need to take advantage of parallelism in RS
- What can they do in sublinear time?
- Explore the relation with languages recognised by real-time CA
Thank you for your attention!

Dziękuję za uwagę!

Questions?