What do we know (and what we dont) about the computational complexity of Reaction Systems

Luca Manzoni

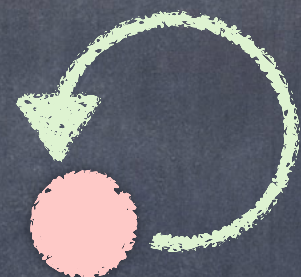
Antonio E. Porreca

Outline

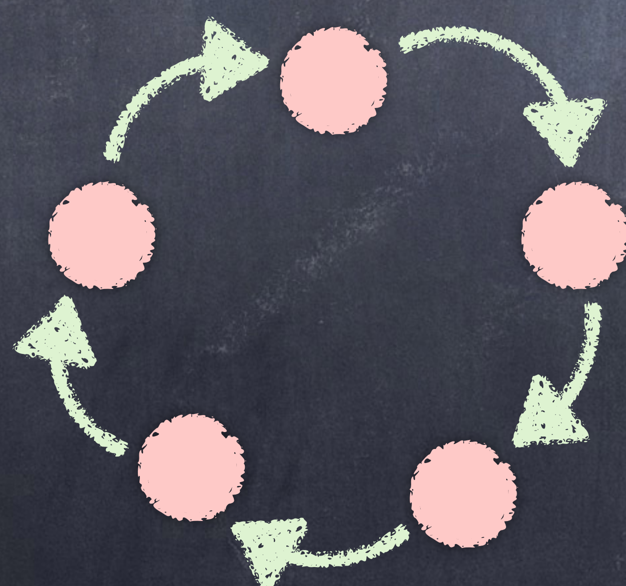
- Dynamical behaviours
- Detection of behaviours
- Using RS for computing

Dynamical behaviours

Dynamical Behaviours

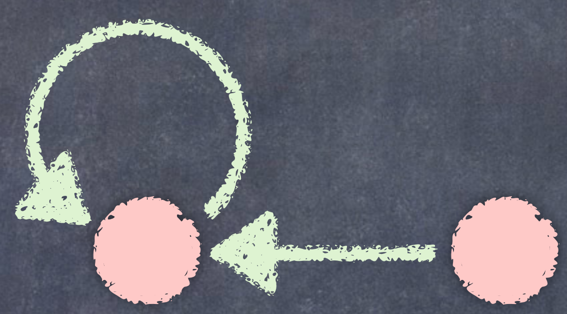


Fixed point

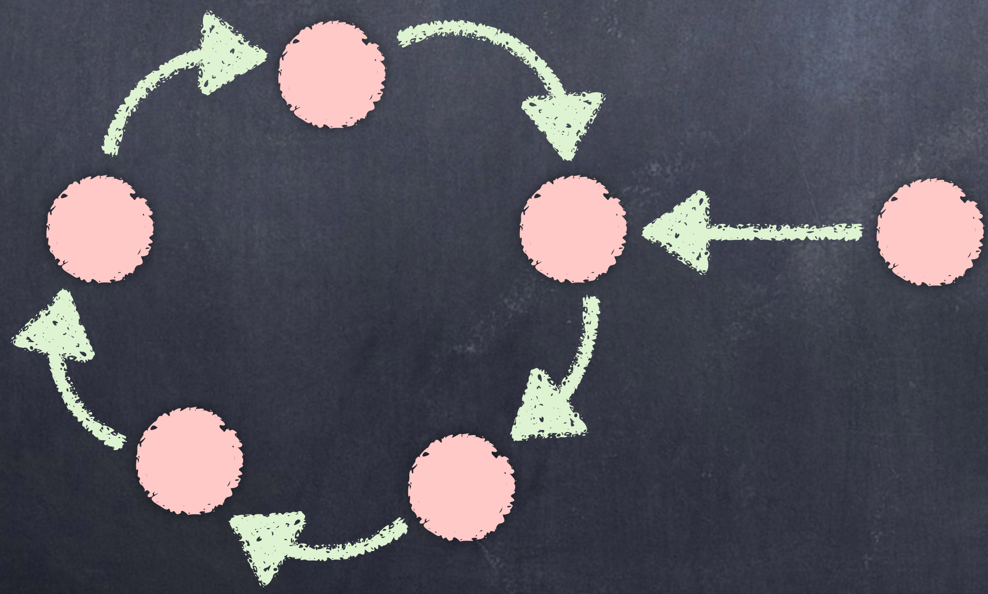


Cycle

Dynamical Behaviours

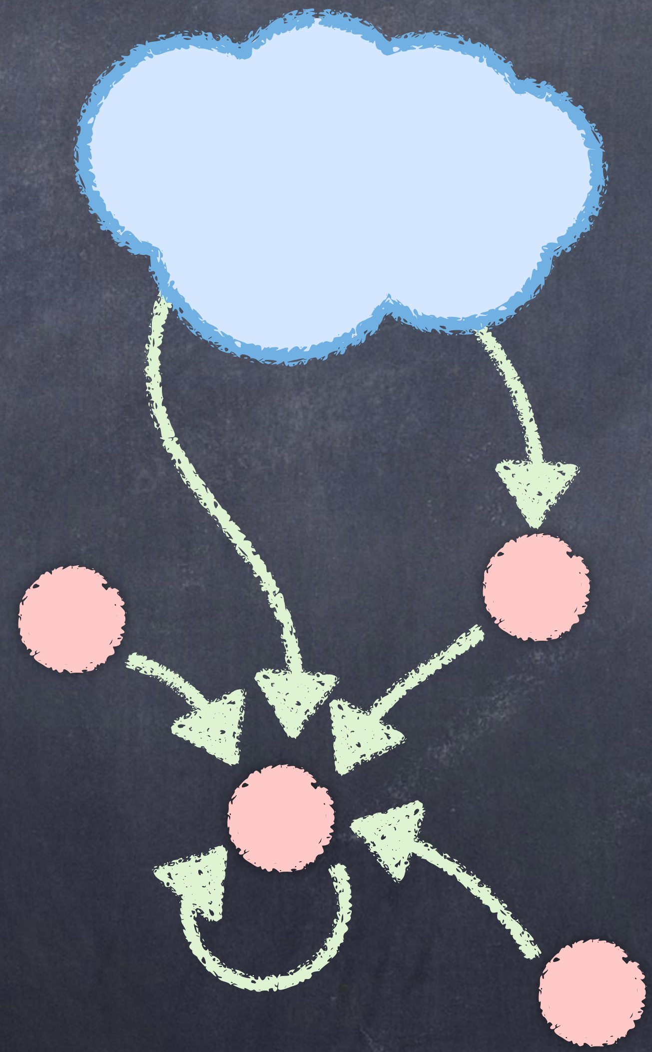


Fixed point attractor



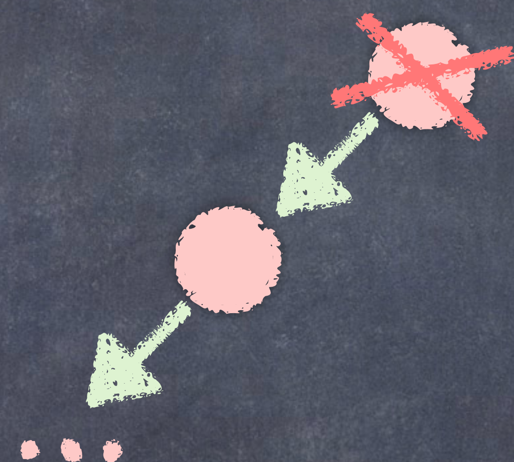
Attractor Cycle

Dynamical Behaviours

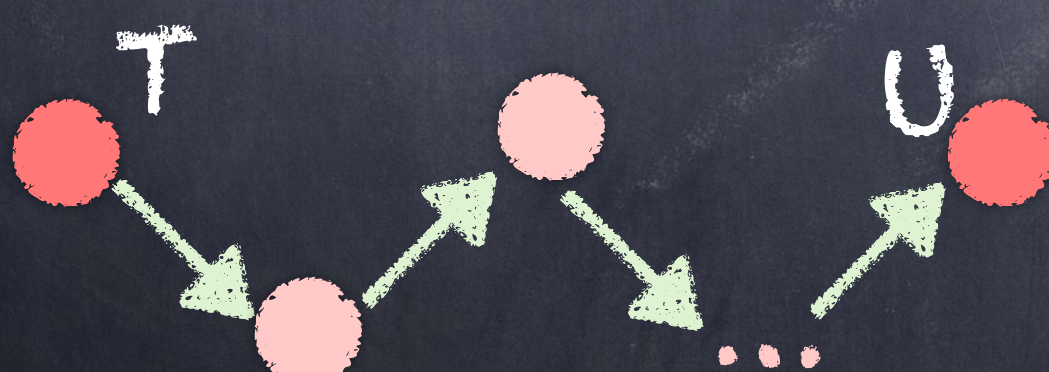


(Fixed point) global attractor

Dynamical Behaviours



Garden of Eden



Path from T to U

Detection of behaviours

Fill the blanks

Does [Reaction System]

given [additional parameters]

exhibit [dynamical behaviour] ?

Fixed Point attractor

Does $\mathcal{A}=(S,A)$

given a fixed point T

exhibit a state $U \neq T$
such that $res_{\mathcal{A}}(U) = T$?

Equality of RS

Does $\mathcal{A}=(S,A)$ $\mathcal{B}=(S,B)$

given [nothing more]

exhibit the same result function ?

Reachability

Does $\mathcal{A}=(S,A)$

given T, U subsets of S

exhibit a path from T to U ?

Fixed Point attractor

Reduction from SAT in CNF

$$f = \underbrace{(x \vee y)}_{C_1} \,\&\, \underbrace{(x \vee \neg z)}_{C_2}$$

NP-complete

$$f = (x \vee y) \,\&\, (x \vee \neg z)$$

$$S = \{x, y, z, C_1, C_2\}$$

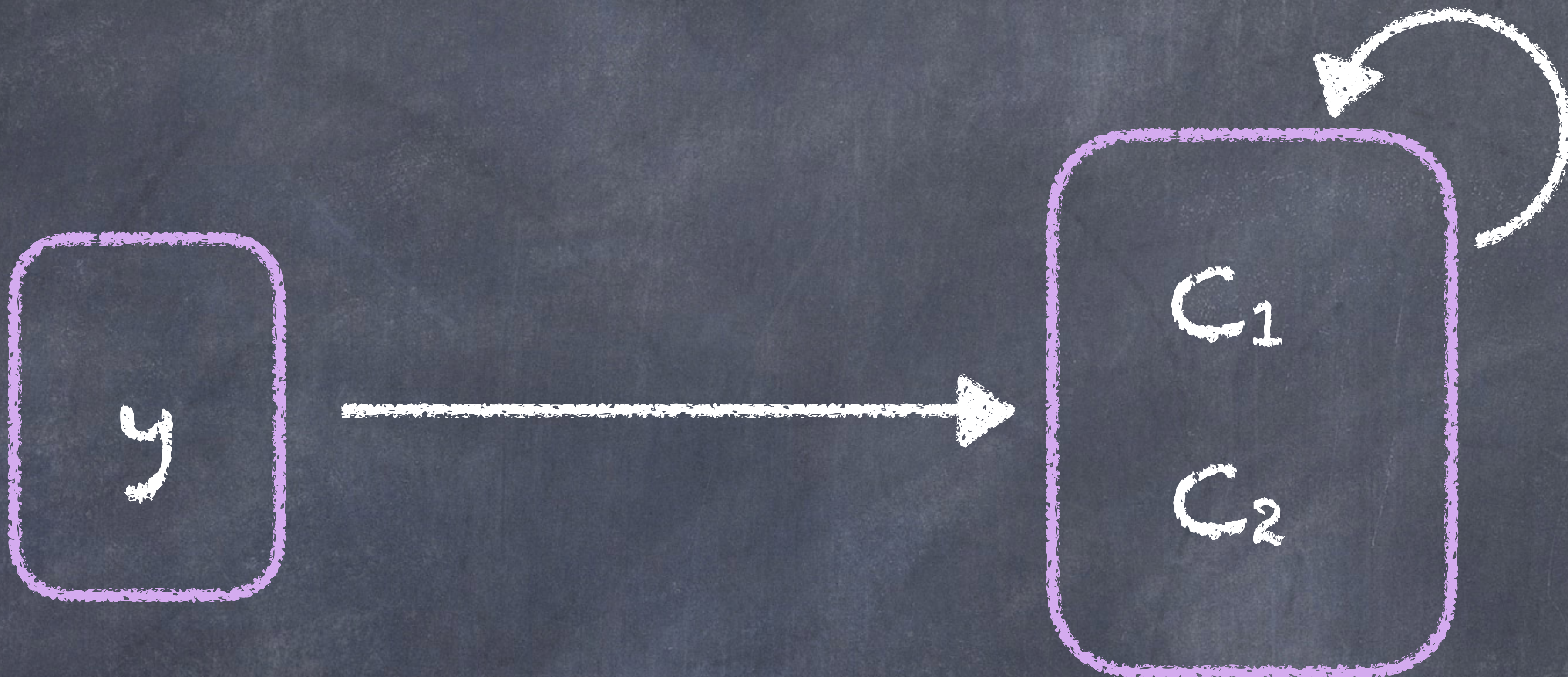
$$(\{C_1, C_2\}, \{x, y, z\}, \{C_1, C_2\})$$

$$(\{x\}, \{C_1, C_2\}, \{C_1, C_2\})$$

$$(\{y\}, \{C_1, C_2\}, \{C_1\})$$

$$(\emptyset, \{z, C_1, C_2\}, \{C_2\})$$

$$f = (x \vee y) \,\&\, (x \vee \neg z)$$

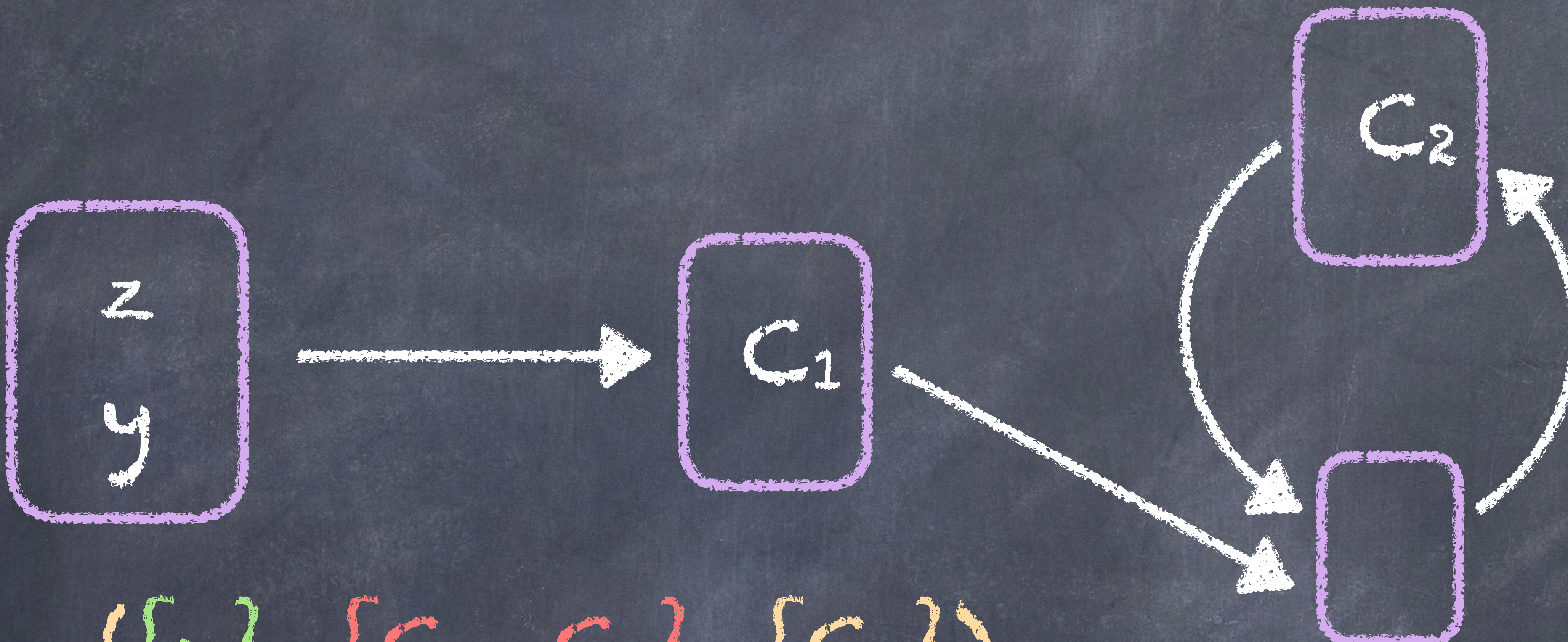


$$(\{y\}, \{C_1, C_2\}, \{C_1\})$$

$$(\phi, \{z, C_1, C_2\}, \{C_2\})$$

$$(\{C_1, C_2\}, \{x, y, z\}, \{C_1, C_2\})$$

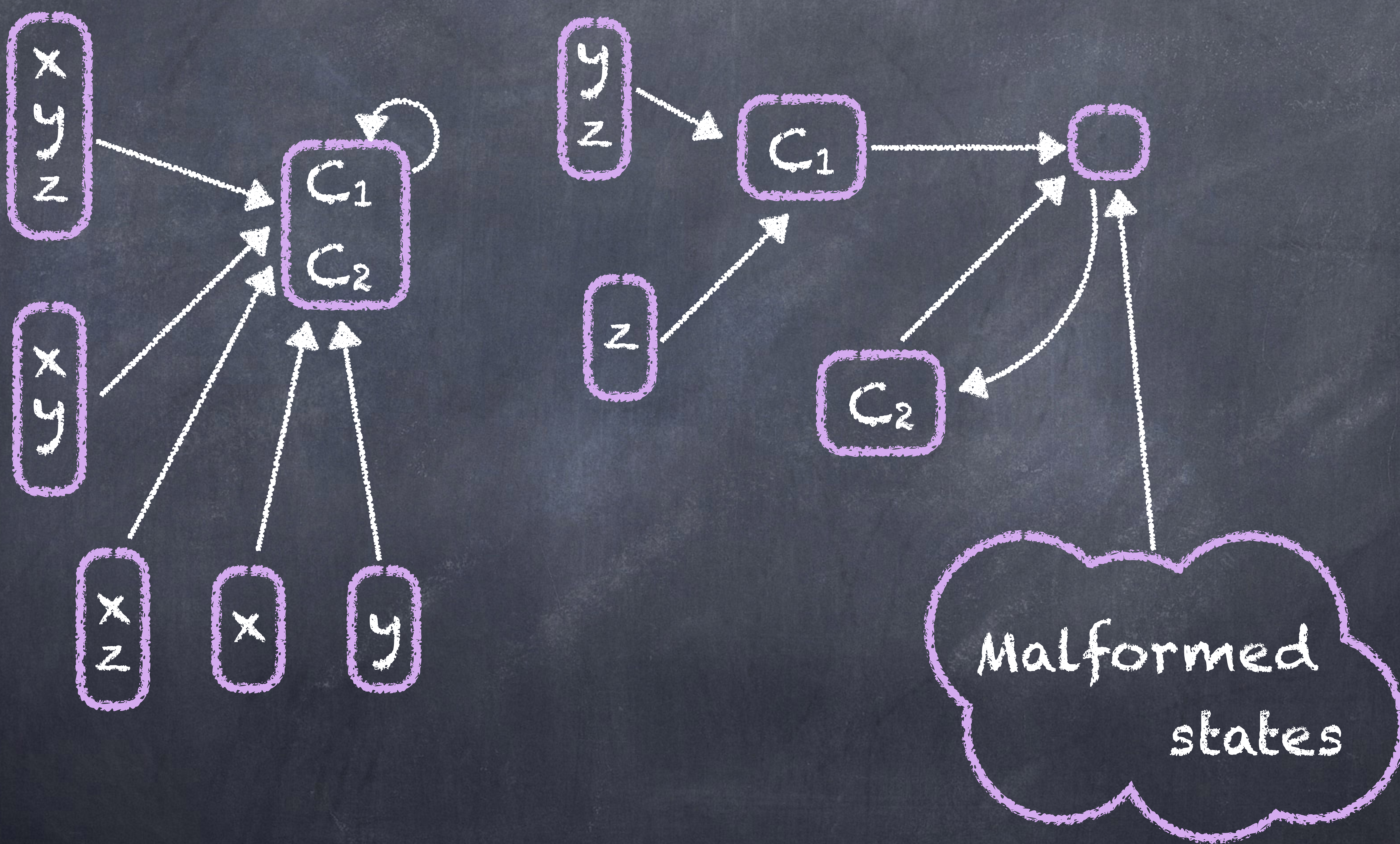
$$f = (x \vee y) \,\&\, (x \vee \neg z)$$



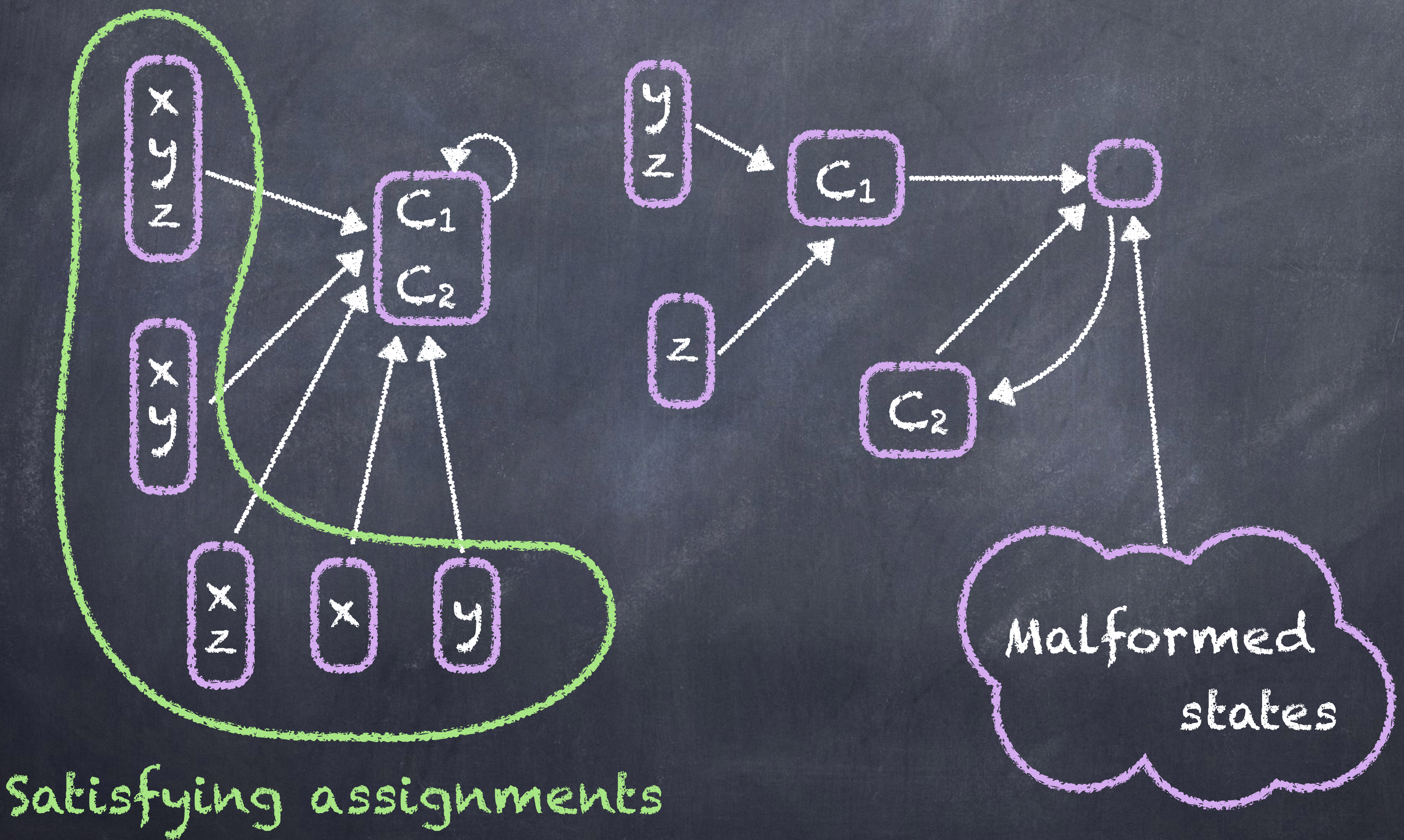
$$(\{y\}, \{C_1, C_2\}, \{C_1\})$$

$$(\emptyset, \{z, C_1, C_2\}, \{C_2\})$$

$$f = (x \vee y) \,\&\, (x \vee \neg z)$$



$$f = (x \lor y) \,\&\, (x \lor \neg z)$$



Equality of res of two RS

Reduction from VALIDITY in DNF

$$f = \underbrace{(x \;\&\; y)}_{} \lor \underbrace{(x \;\&\; \neg z)}_{} \lor \underbrace{z}_{}$$

coNP-complete

$$f = (x \,\&\, y) \vee (x \,\&\, \neg z) \vee z$$

First RS

$(\emptyset, \emptyset, \{True\})$

$S = \{x, y, z, True\}$

$$f = (x \,\&\, y) \lor (x \,\&\, \neg z) \lor z$$

Second RS

$(\{x, y\}, \emptyset, \{True\})$

$(\{x\}, \{z\}, \{True\})$

$(\{z\}, \emptyset, \{True\})$

$$f = (x \,\&\, y) \lor (x \,\&\, \neg z) \lor z$$

Second RS

$$\boxed{\begin{matrix} z \\ y \end{matrix}} \longrightarrow \boxed{\text{True}} \longleftarrow \boxed{\begin{matrix} x \\ y \end{matrix}}$$

$(\{z\}, \phi, \{\text{True}\})$

$(\{x, y\}, \phi, \{\text{True}\})$

$(\{x\}, \{z\}, \{\text{True}\})$

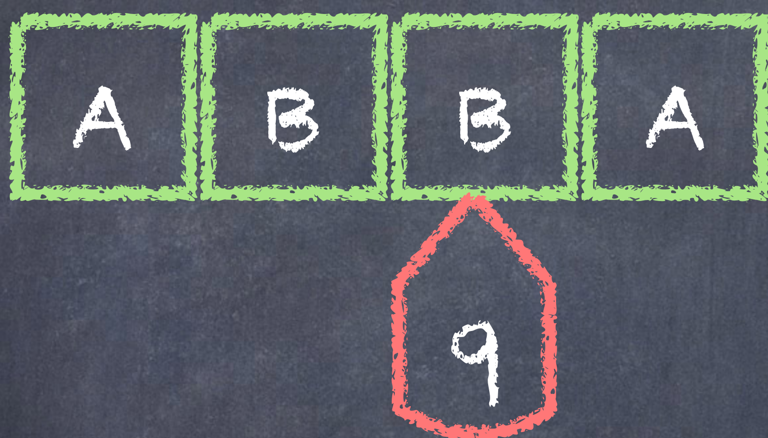
$$f = (x \,\&\, y) \lor (x \,\&\, \neg z) \lor z$$

First RS

Second RS

Since f is valid,
the two systems describe
the same result function

Reachability



We have already seen how to simulate bounded-tape TM

Reachability for bounded-tape TM is PSPACE-complete...
...and also for RS

NP-complete

- Existence of a fixed point
- Existence of a fixed point attractor

coNP-complete

- Equality of result functions
- Existence of a Garden of Eden

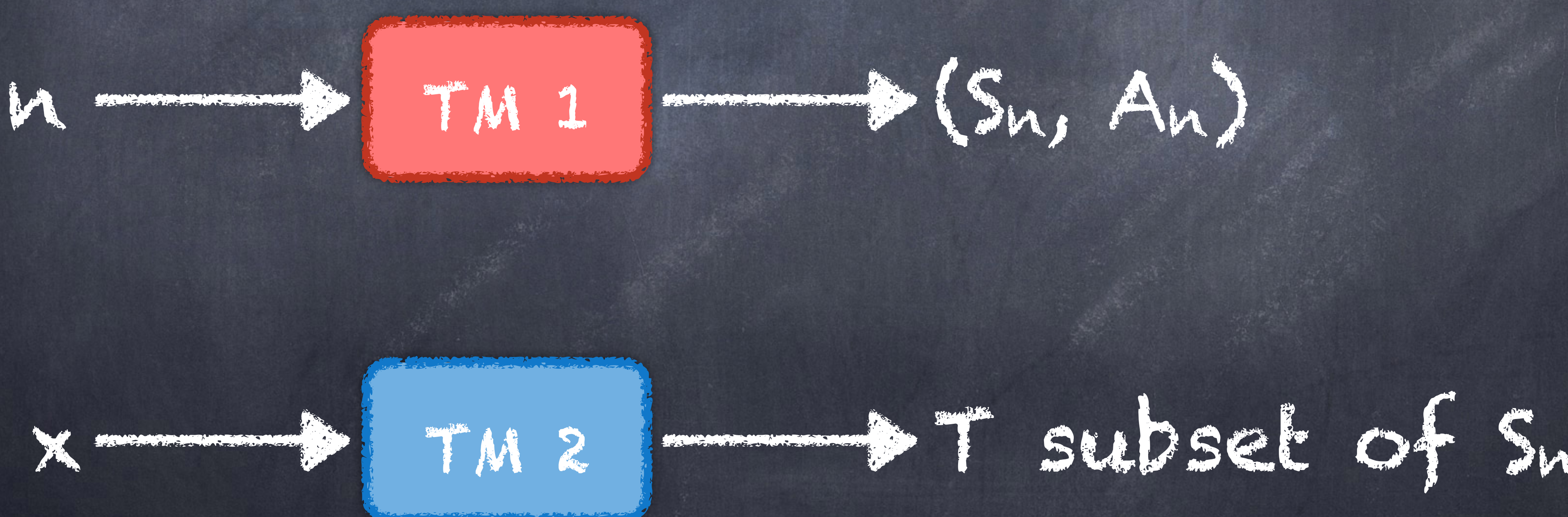
PSPACE-complete

- Reachability
- Existence of a global attractor

RS for computing

Uniform Families of RS

Input x of length n



Uniform Families of RS

- RS can be simulated by TM with polynomial slowdown (and viceversa)...
- ...hence, we need to select two very weak TM for the uniformity condition

Uniform Families of RS

- We need to take advantage of parallelism in RS
- What can they do in sublinear time?
- Explore the relation with languages recognised by real-time CA

Thank you for your attention!

Dziękuje za uwagę!

Questions?