Introduction à l’informatique CM5

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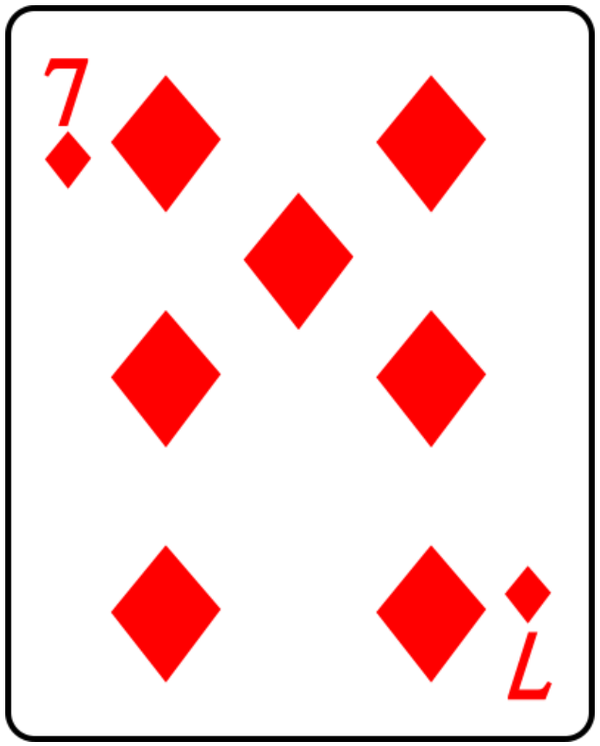
aeporreca.org/introinfo

⚠️ Rappel ⚠️

Vendredi 22 octobre (normalement à 14h) on a le partiel !

Tri par insertion

Tri par insertion



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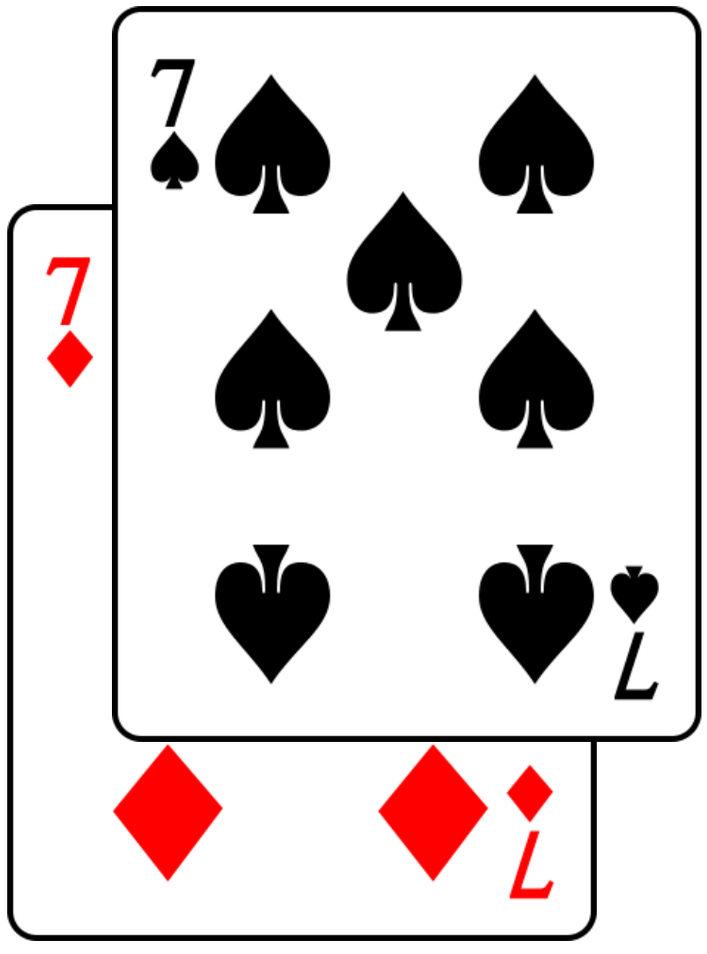

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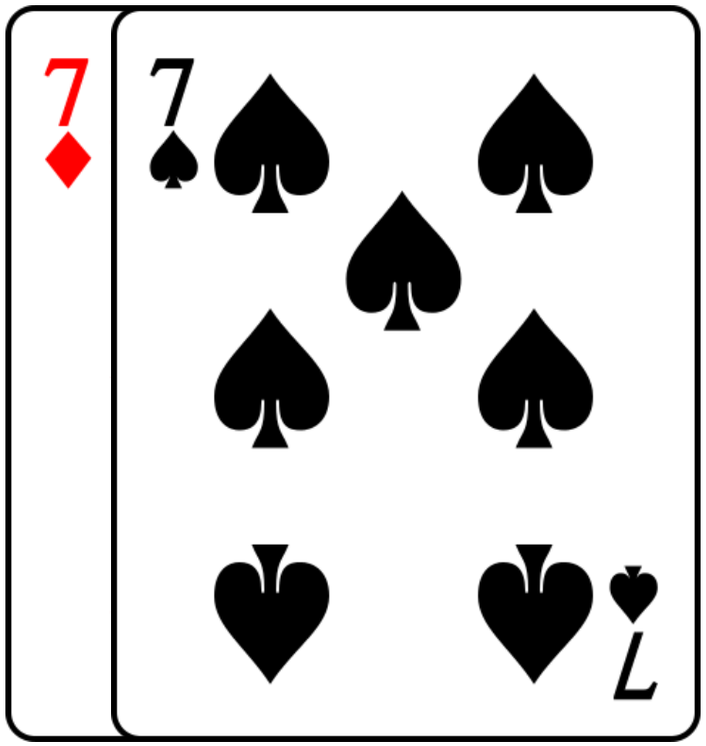

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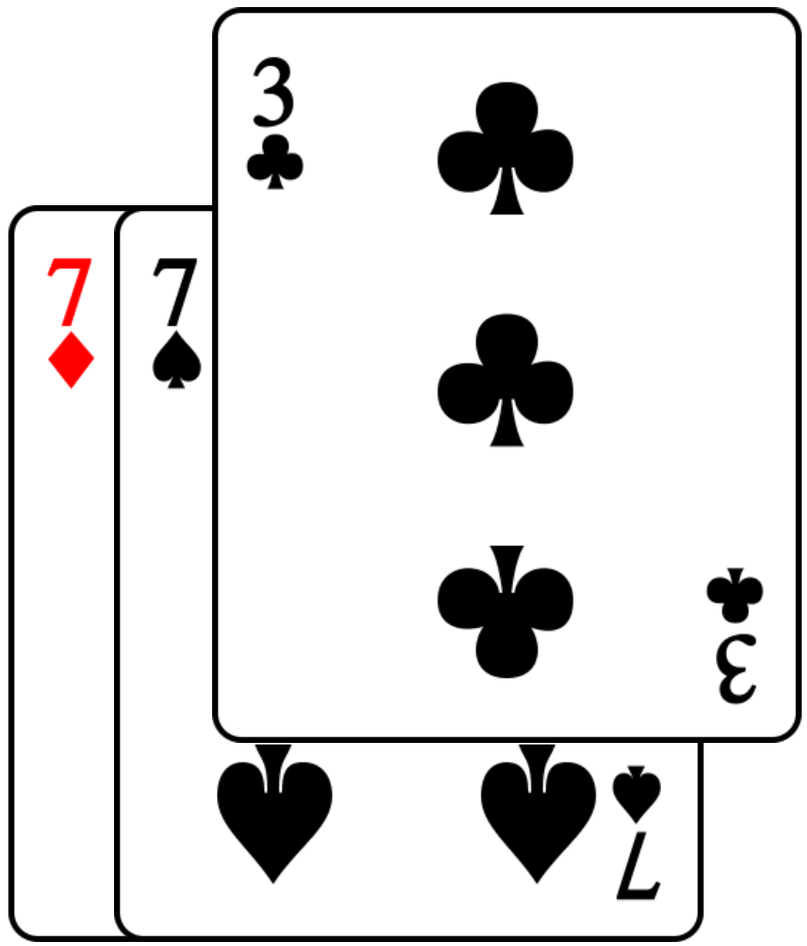

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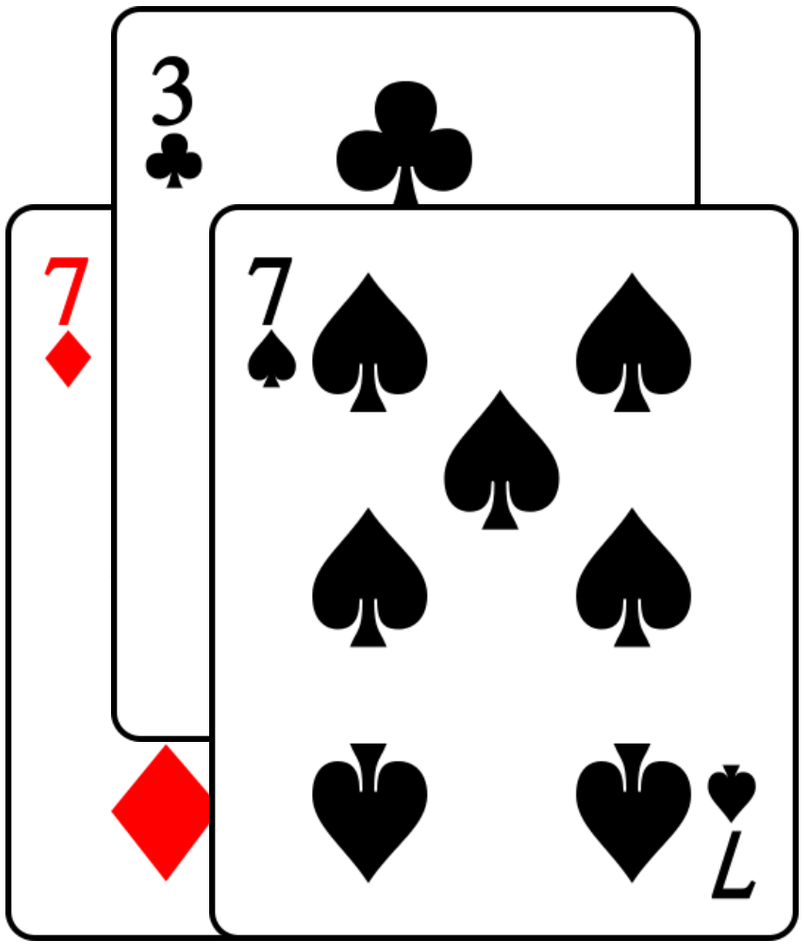

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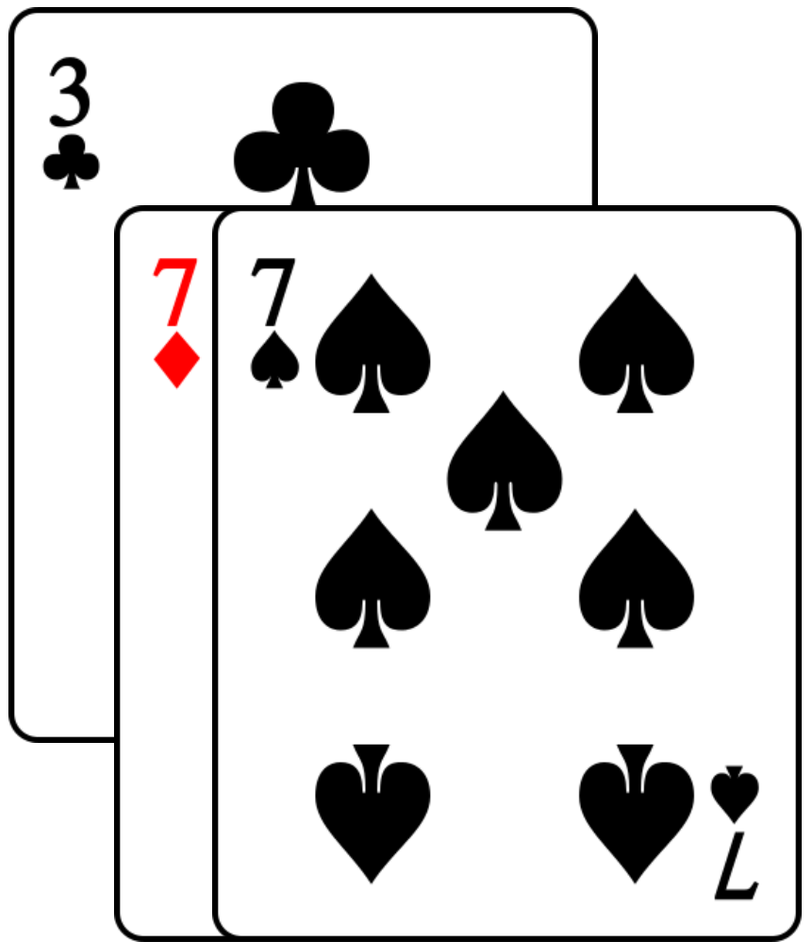

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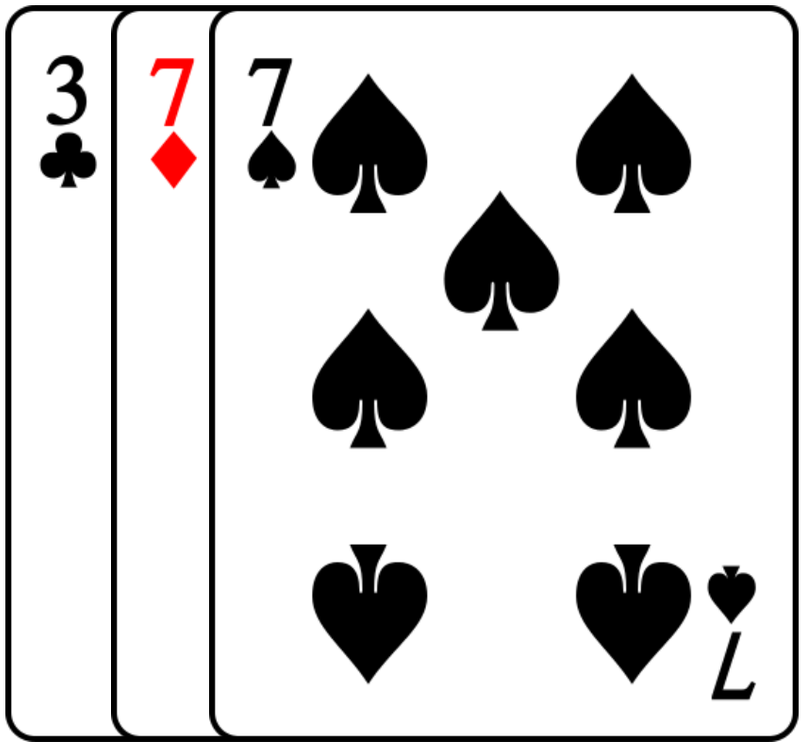

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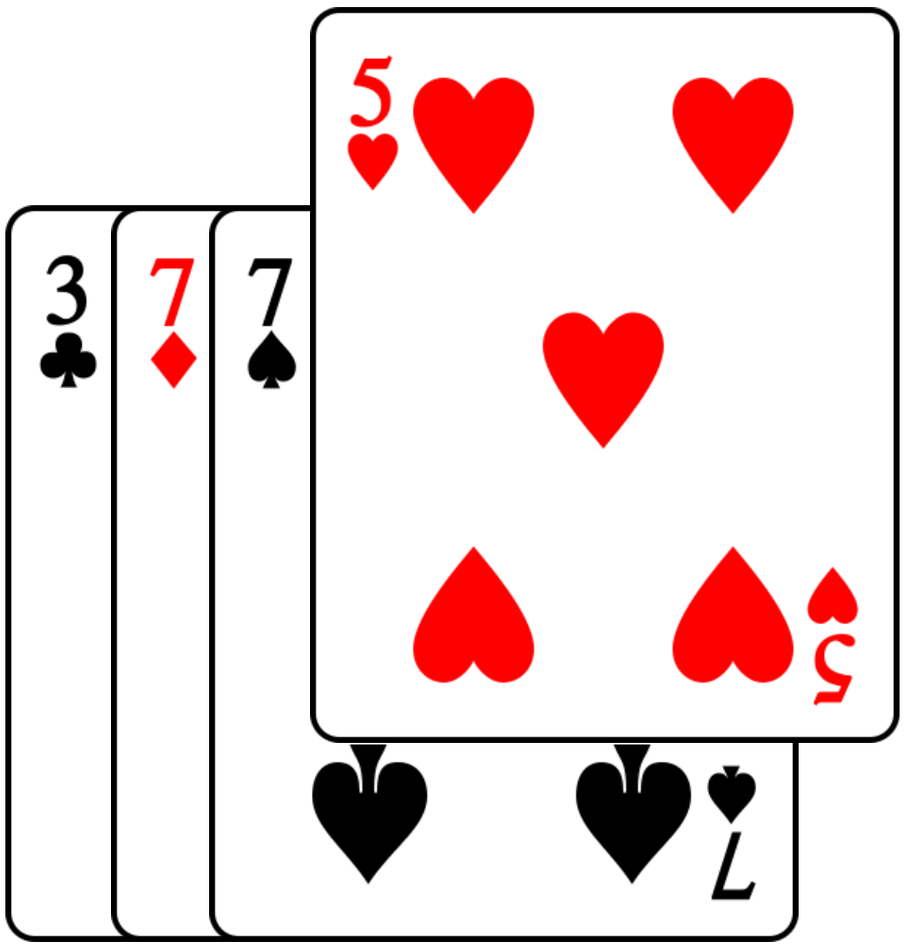

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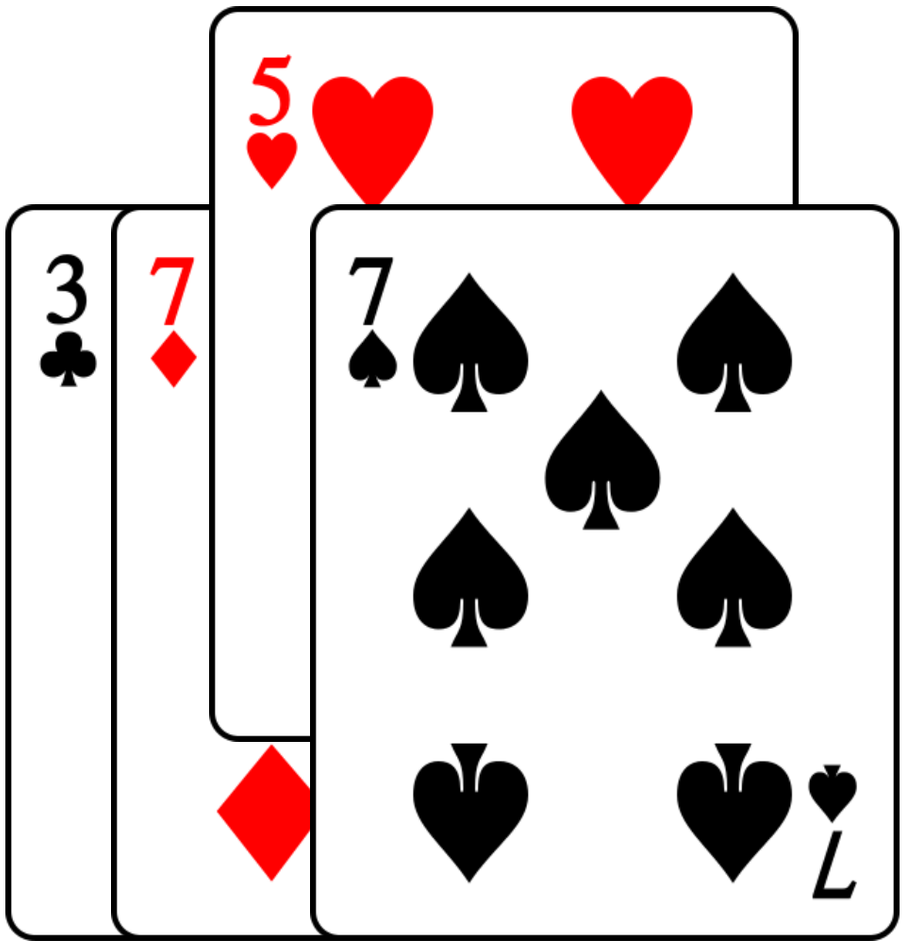

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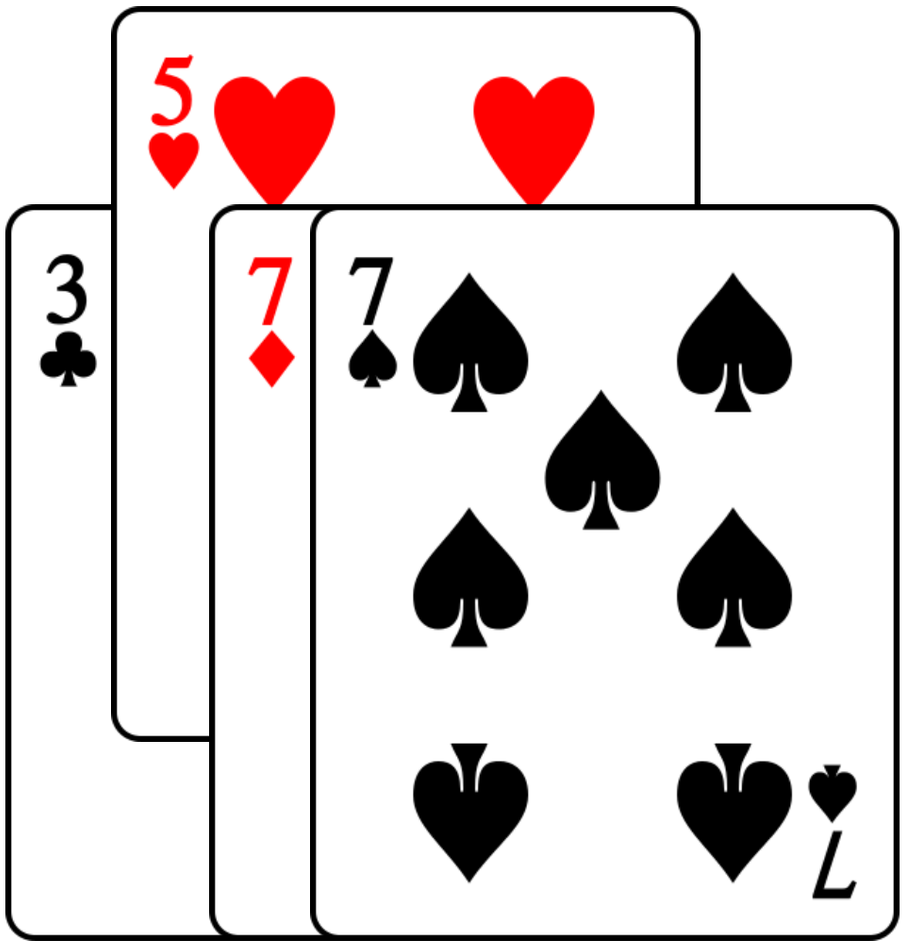

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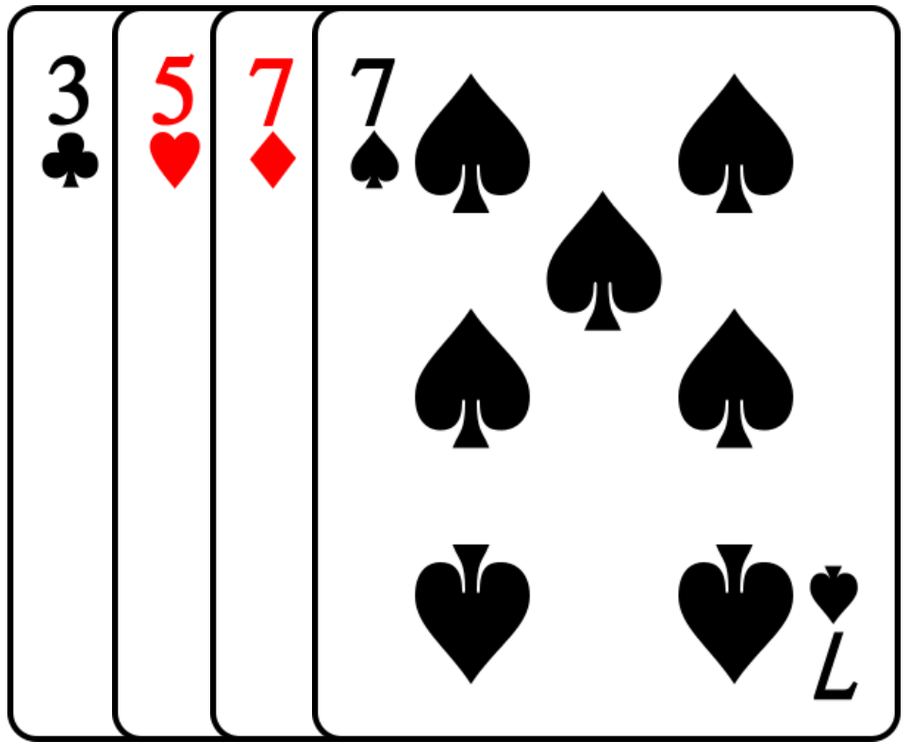

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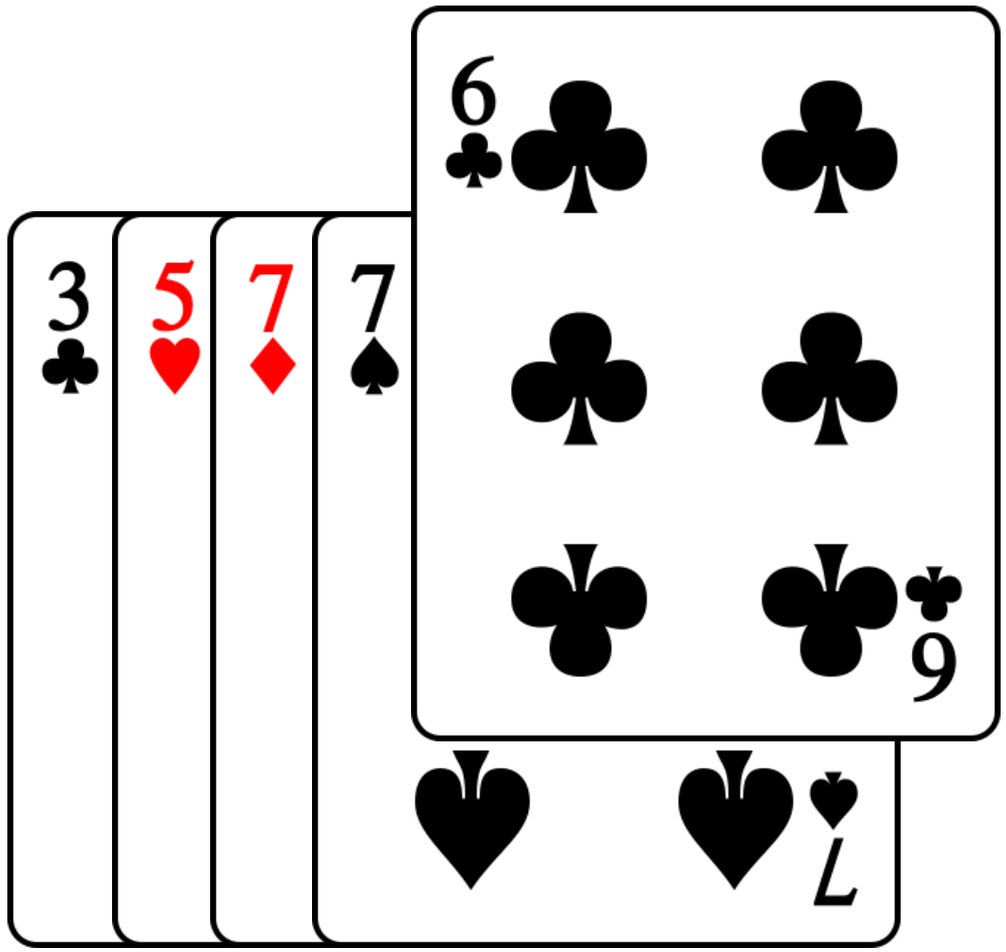

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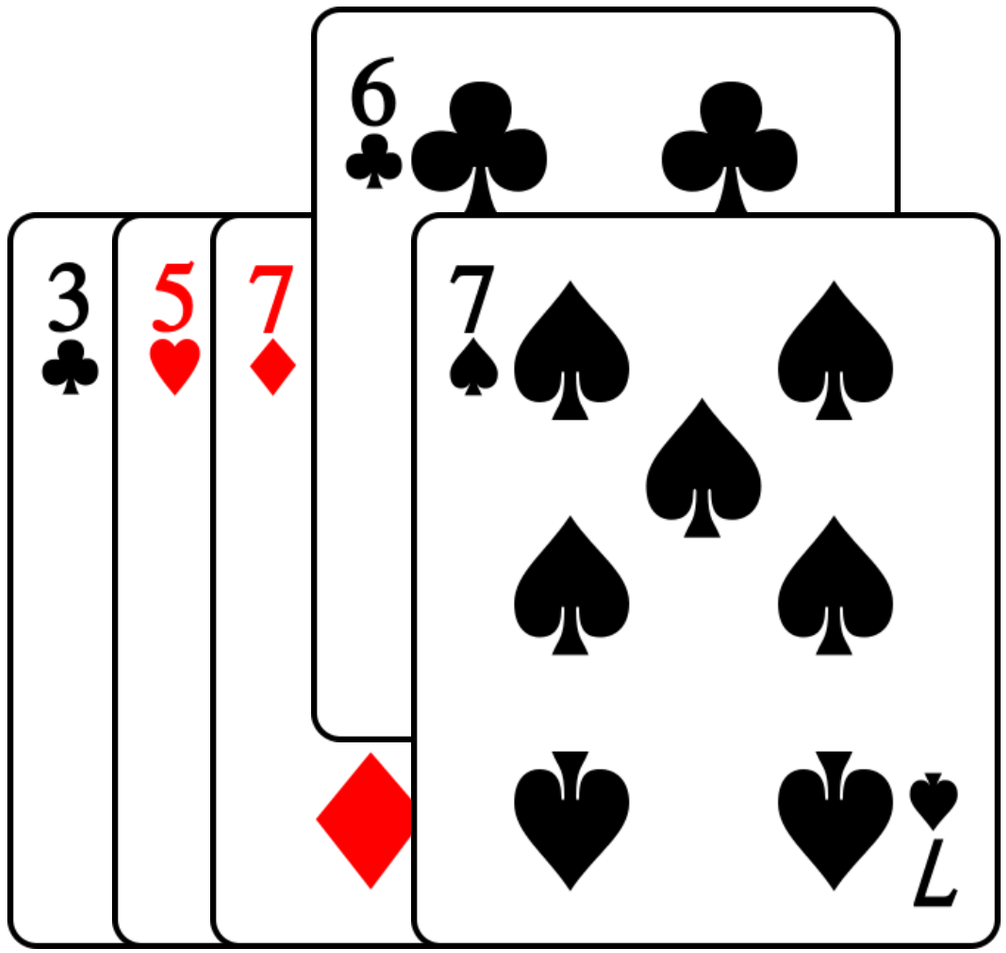

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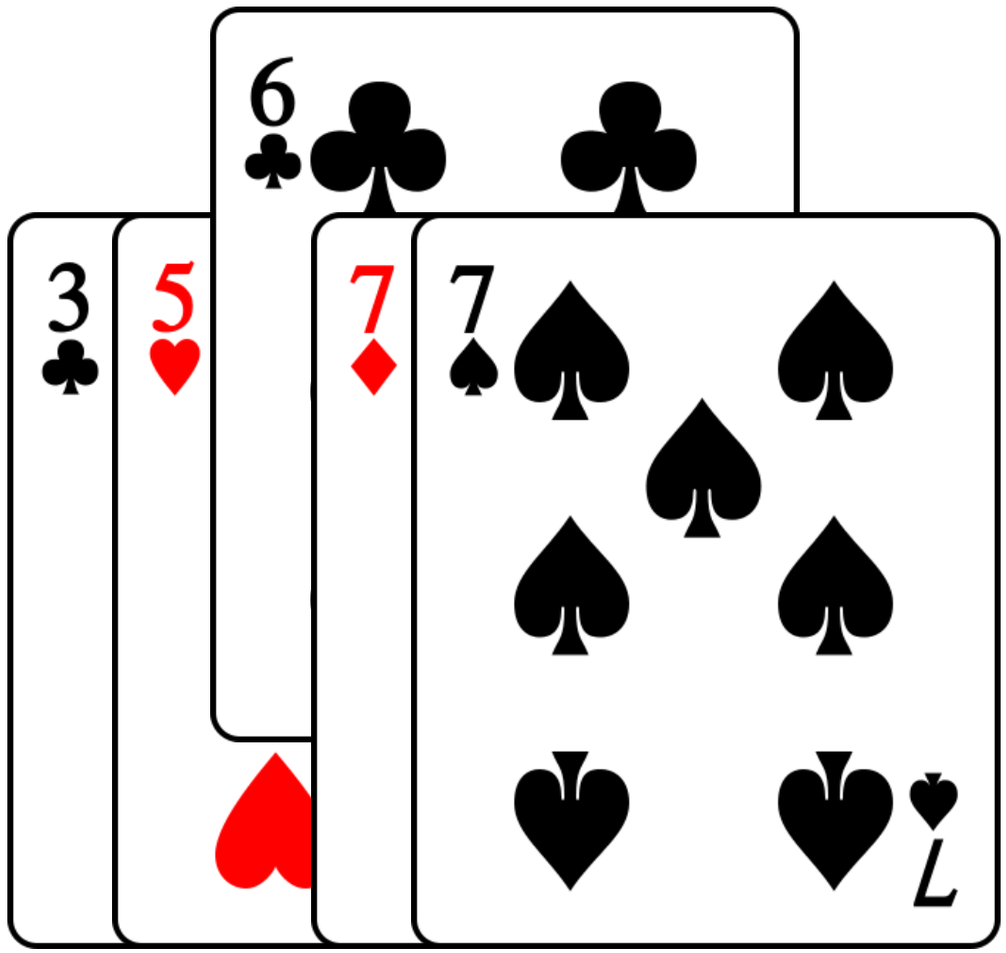

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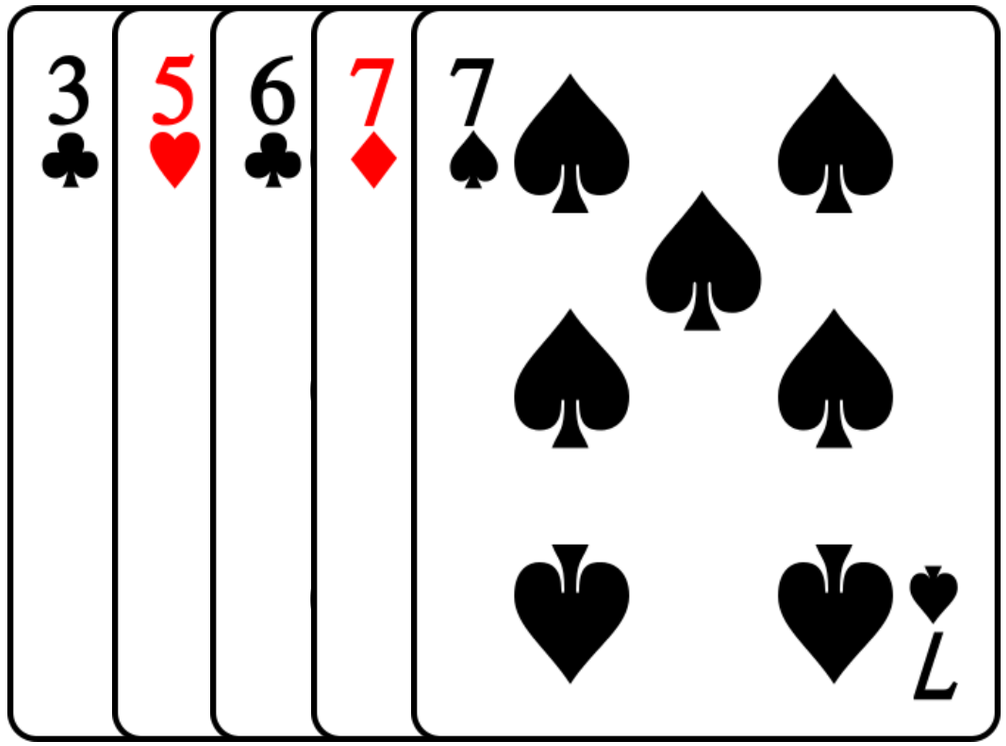
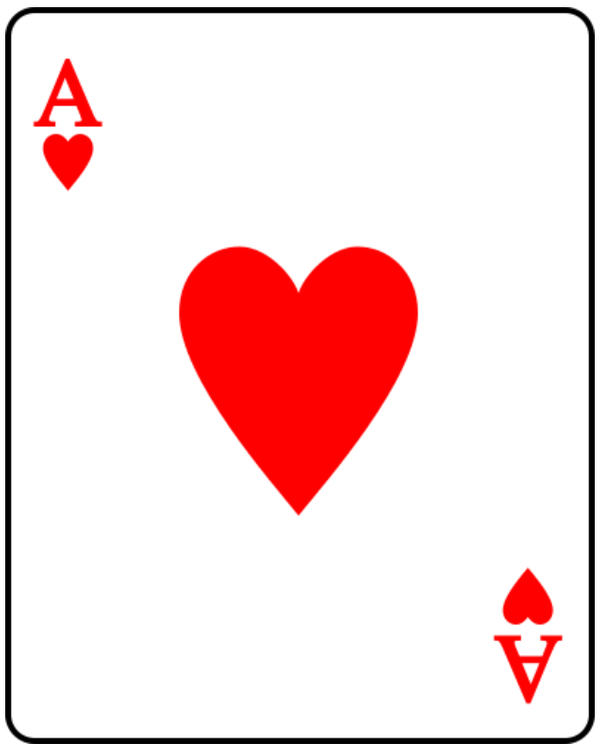
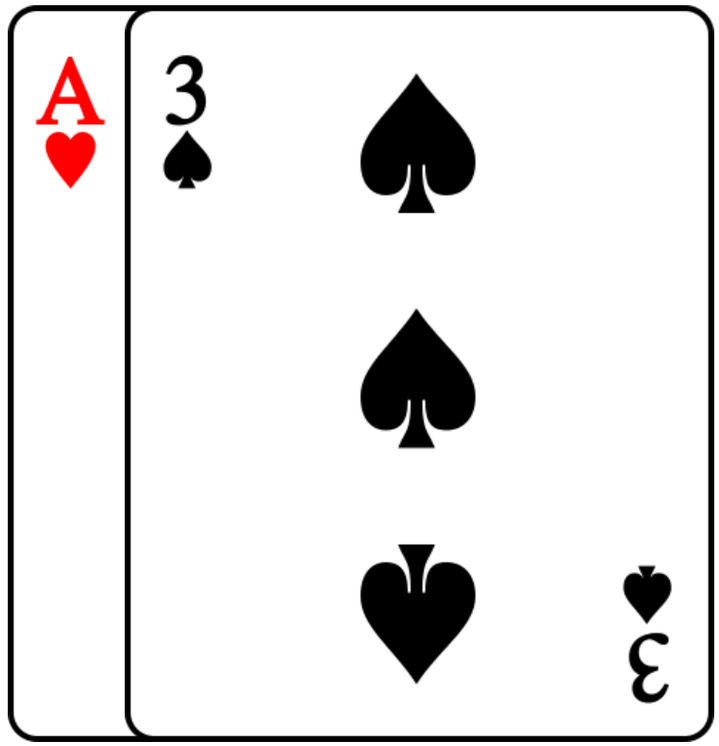


Le meilleur des cas



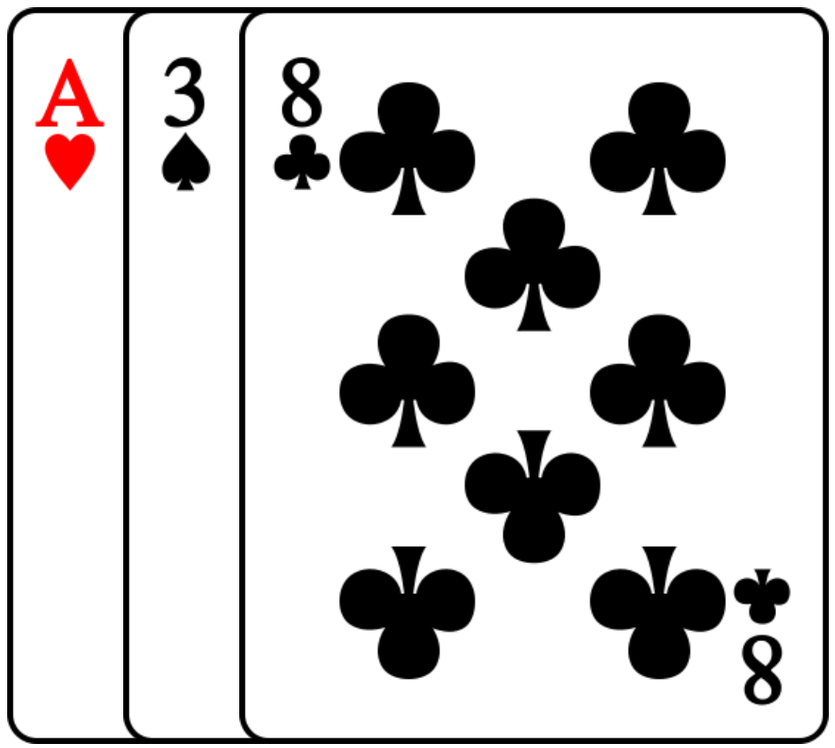
№ operations = 1

Le meilleur des cas



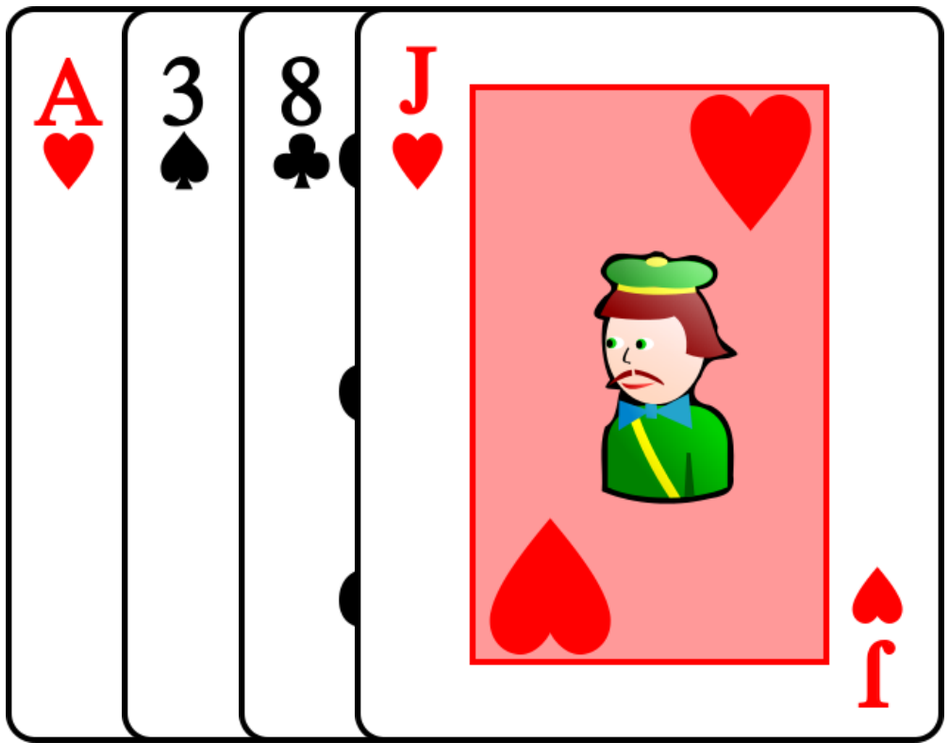
№ operations = 2

Le meilleur des cas



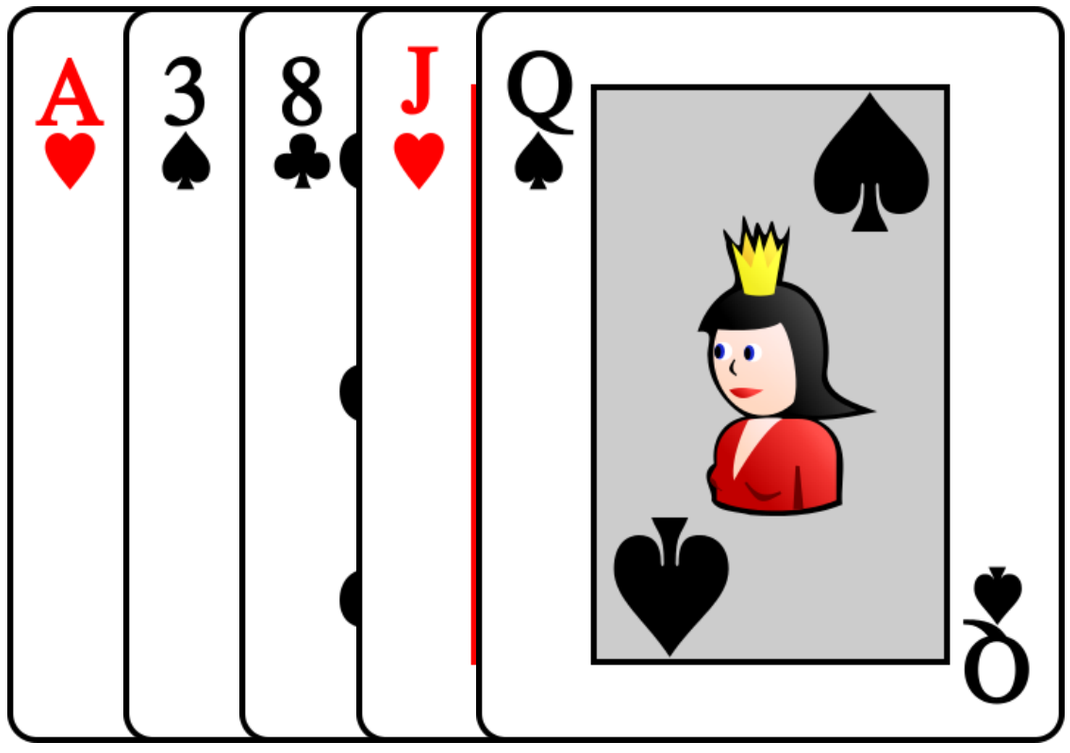
№ operations = 3

Le meilleur des cas



№ operations = 4

Le meilleur des cas



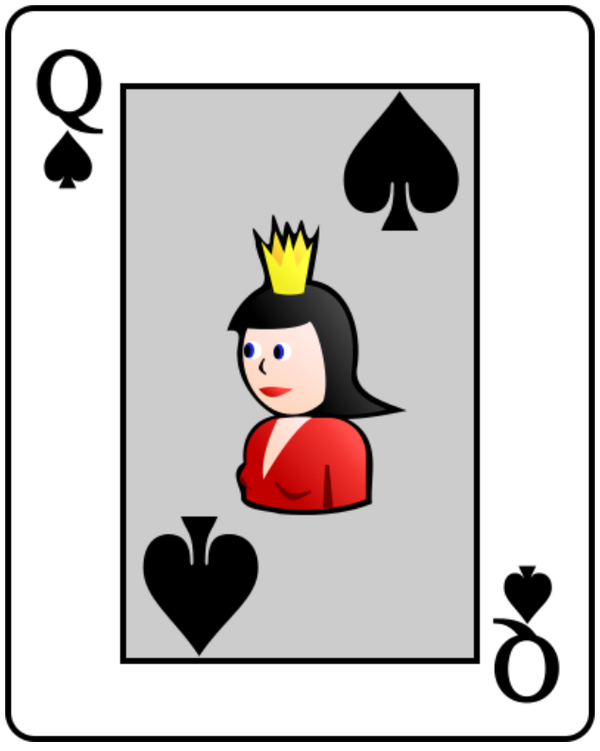
№ operations = 5

Le meilleur des cas

- Les cartes arrivent déjà triées

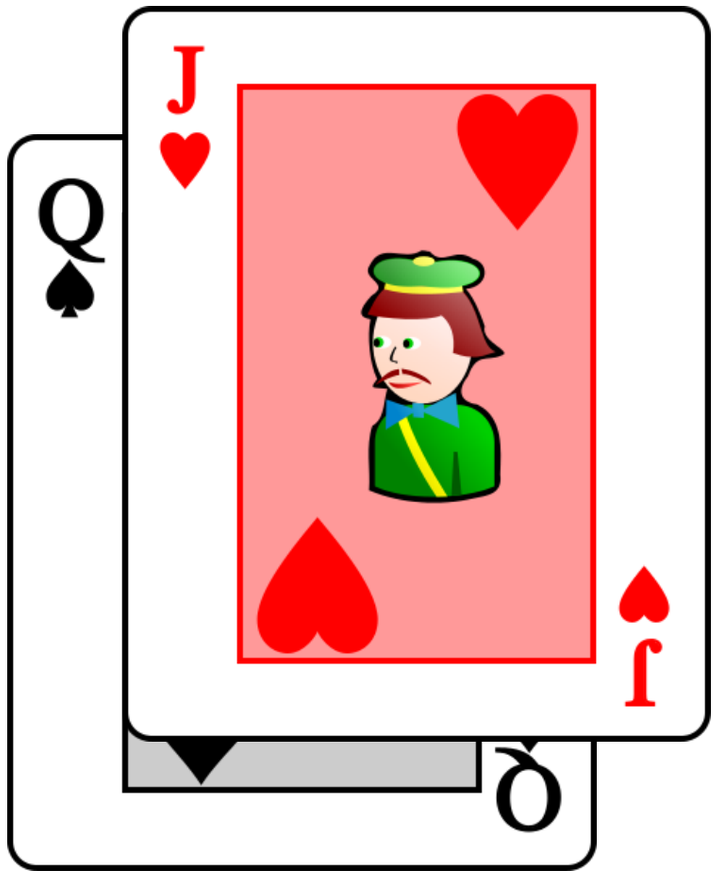
- On fait n opérations (déplacements de cartes)

Le pire des cas



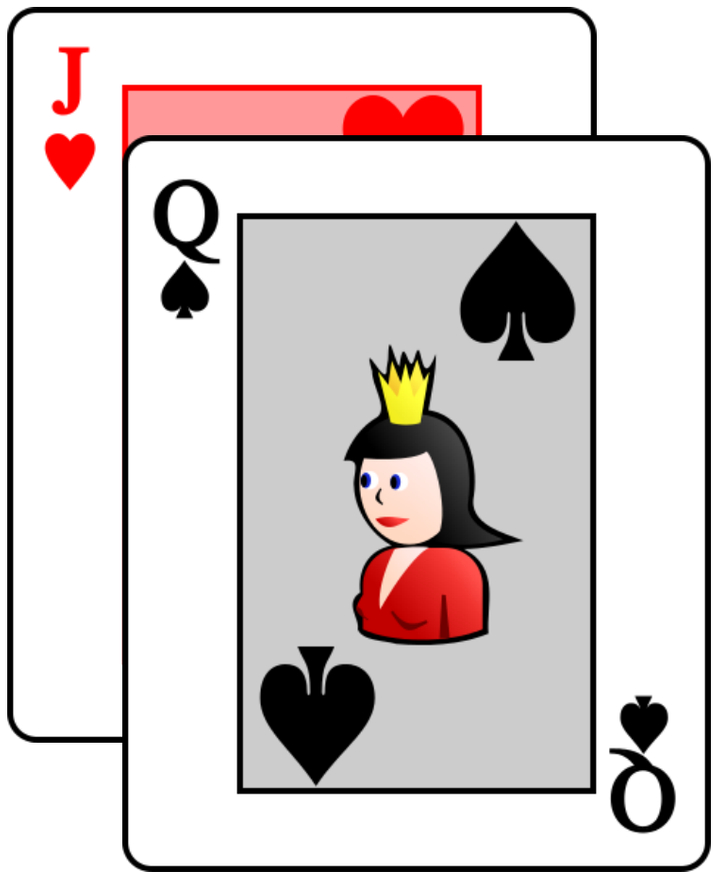
№ operations = 1

Le pire des cas



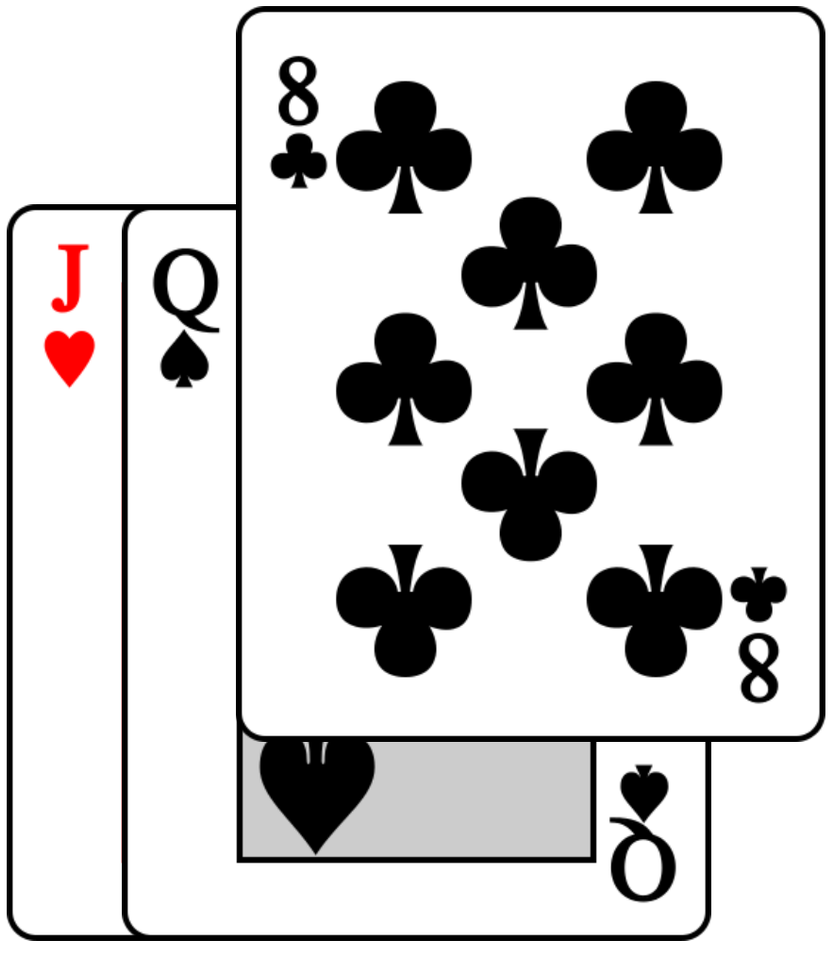
№ operations = 1 + 1

Le pire des cas



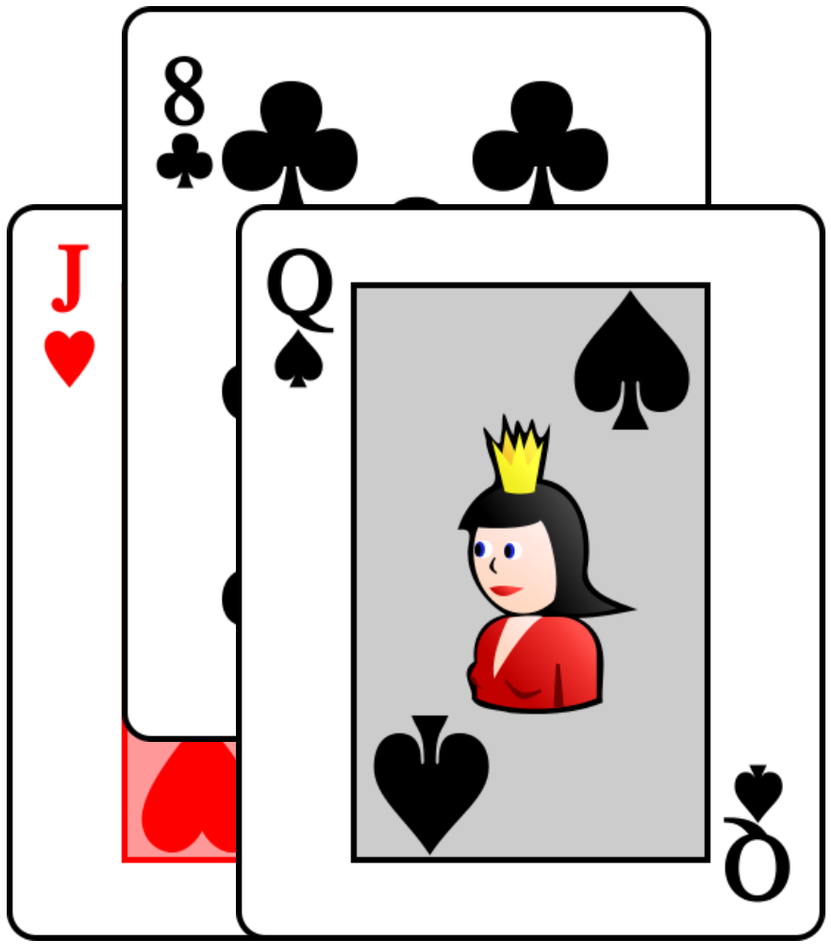
№ operations = 1 + 2

Le pire des cas



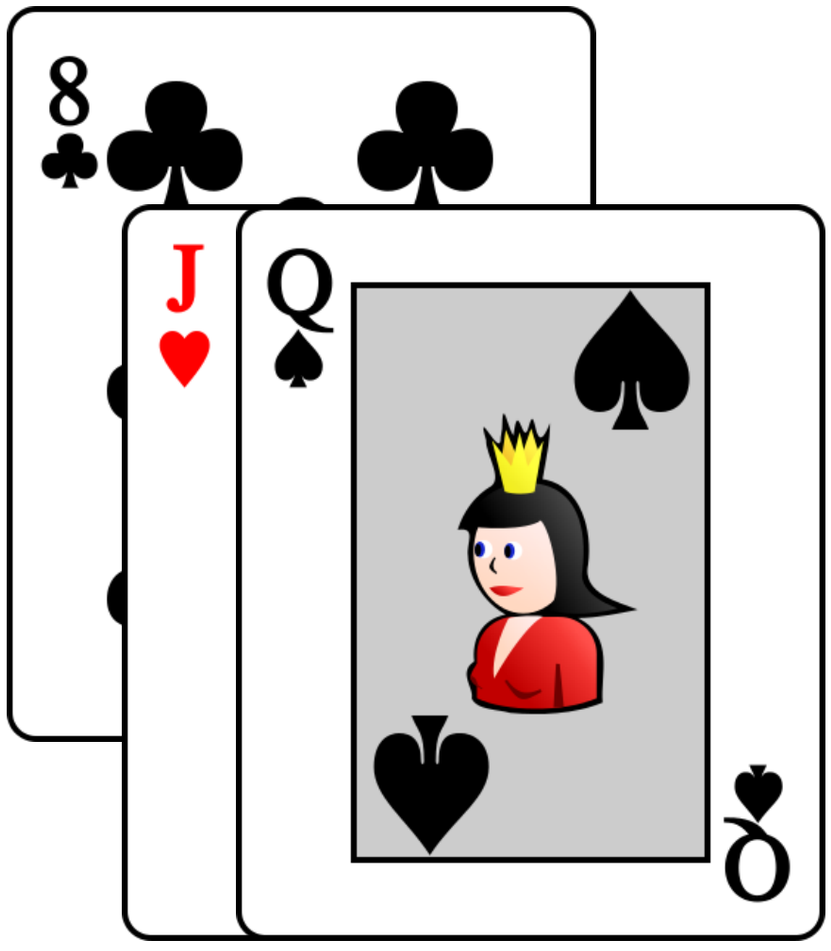
№ operations = 1 + 2 + 1

Le pire des cas



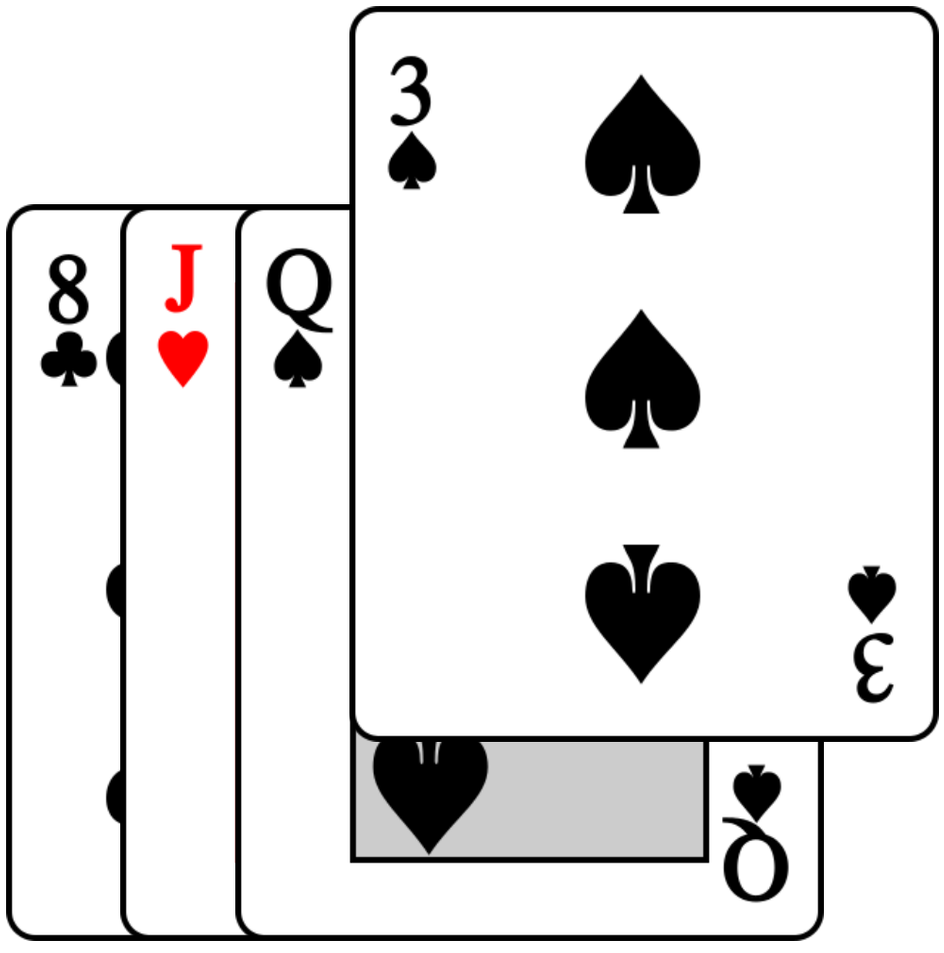
№ operations = 1 + 2 + 2

Le pire des cas



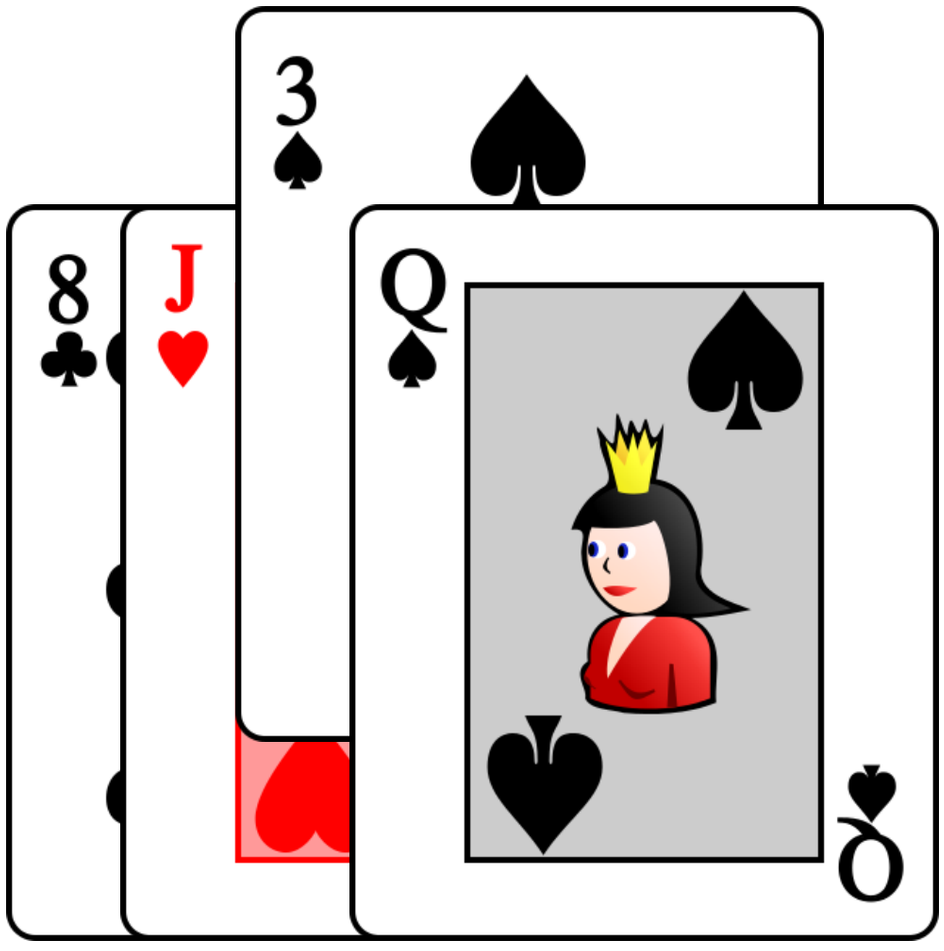
№ operations = 1 + 2 + 3

Le pire des cas



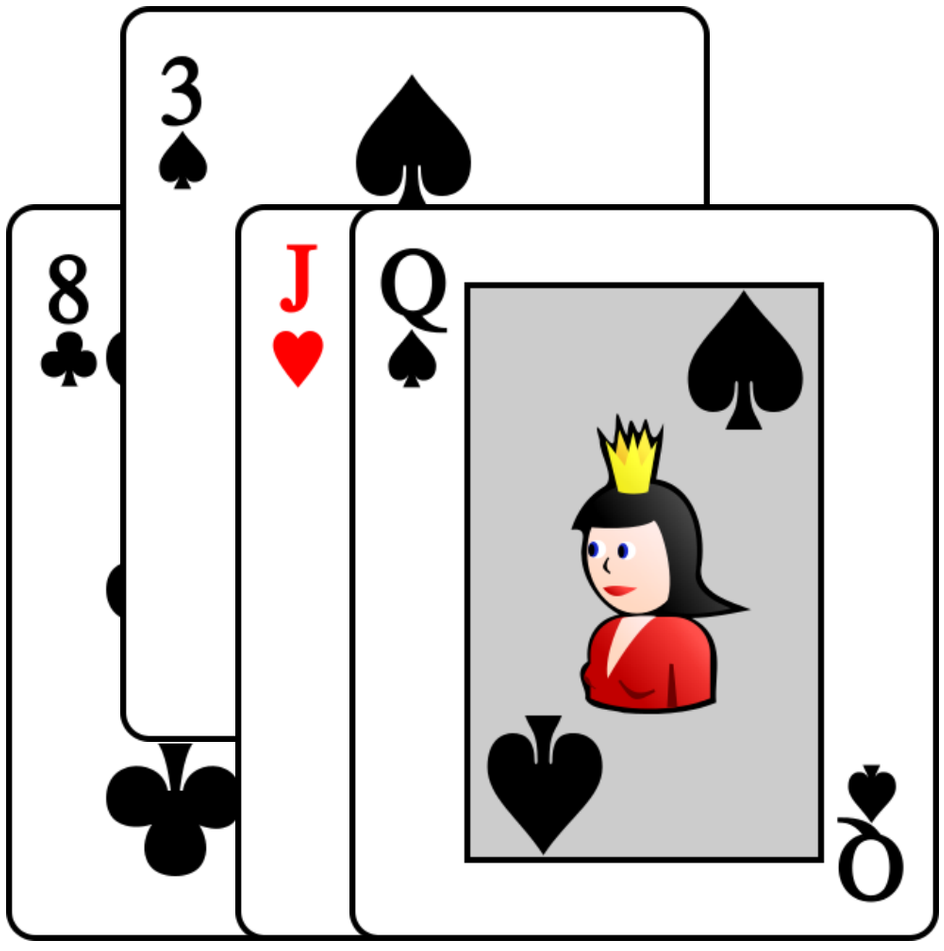
№ operations = 1 + 2 + 3 + 1

Le pire des cas



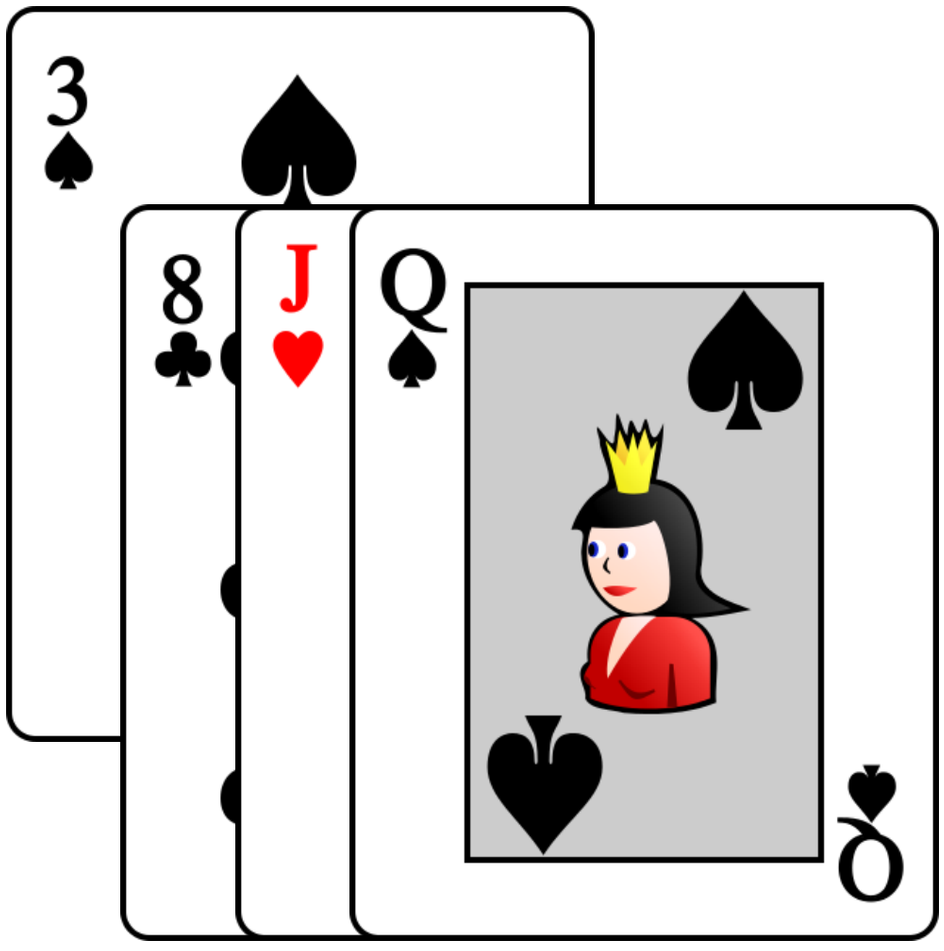
№ operations = 1 + 2 + 3 + 2

Le pire des cas



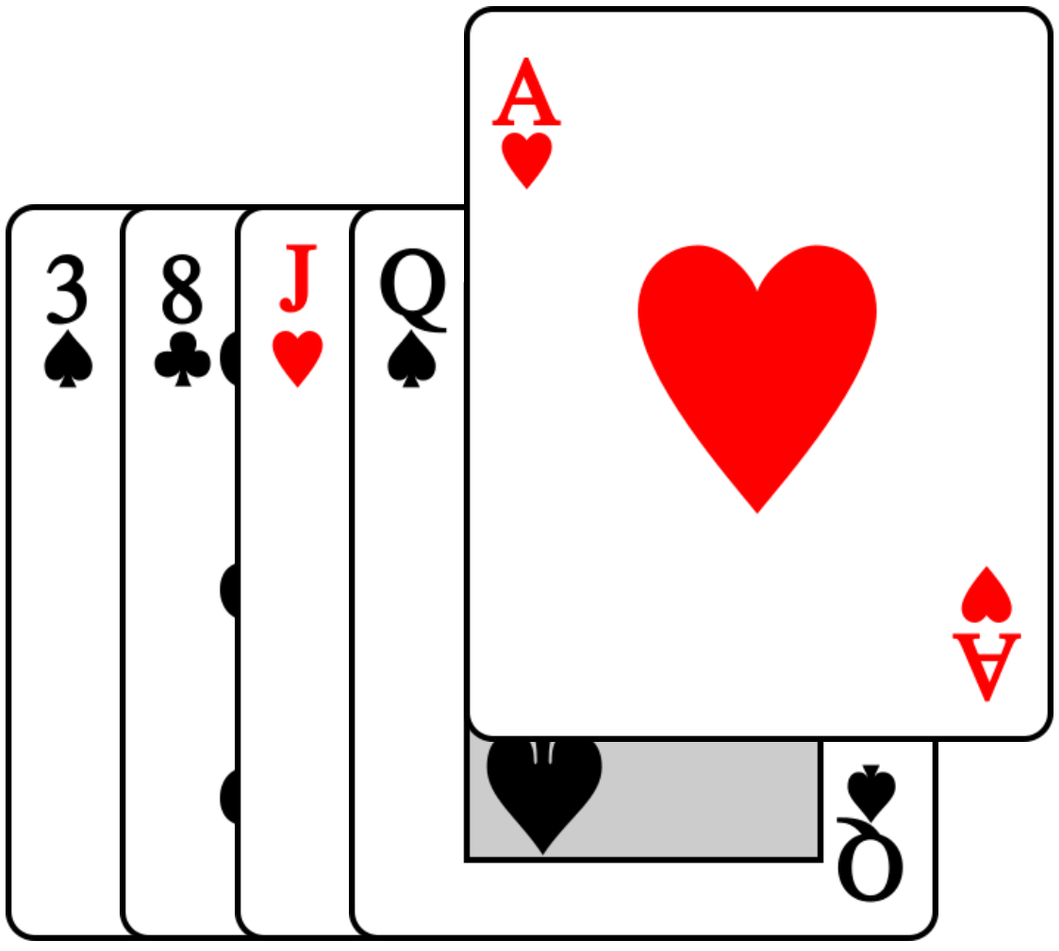
№ operations = 1 + 2 + 3 + 3

Le pire des cas



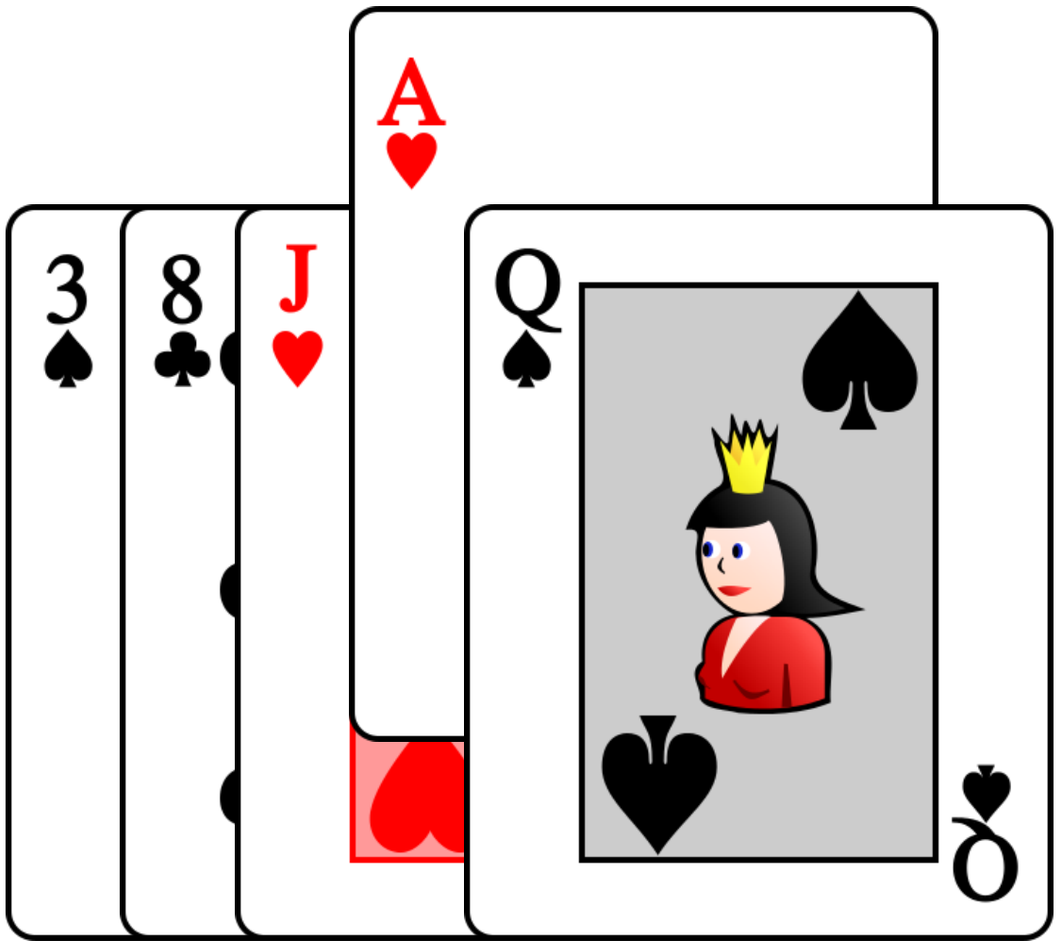
№ operations = 1 + 2 + 3 + 4

Le pire des cas



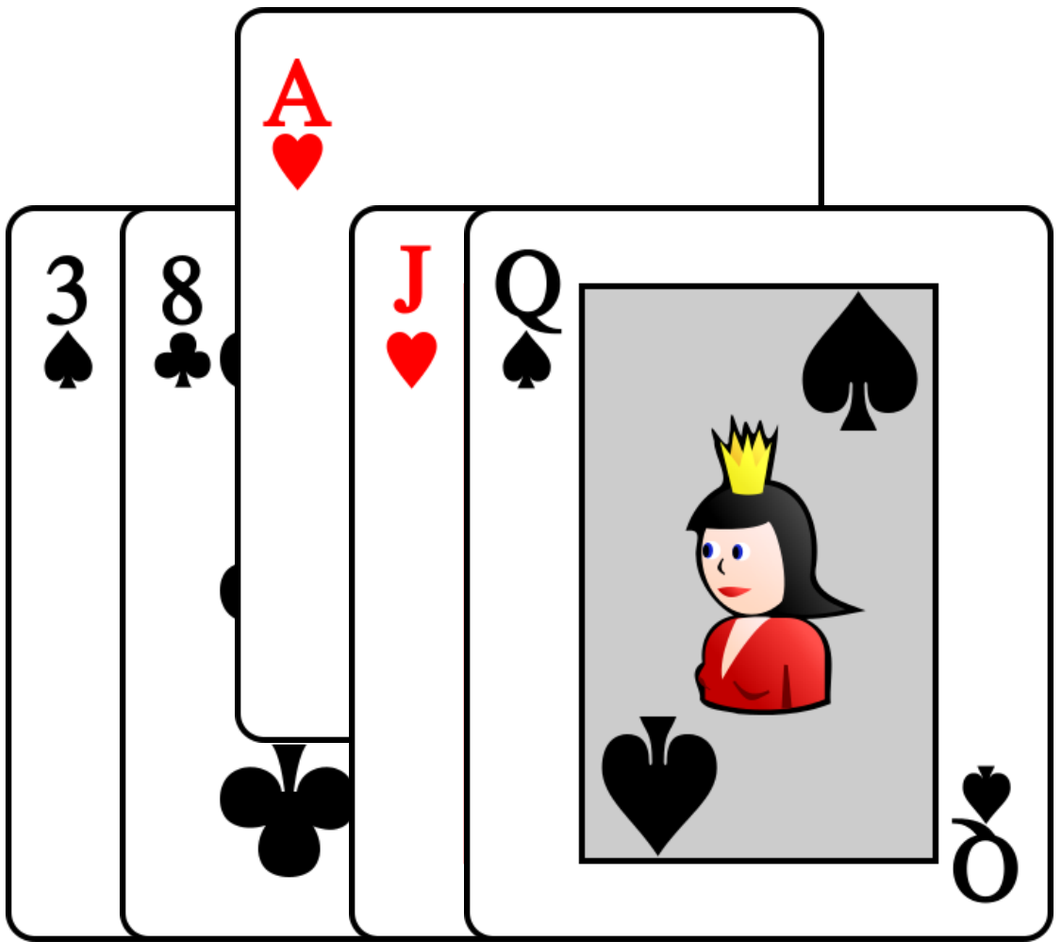
№ operations = 1 + 2 + 3 + 4 + 1

Le pire des cas



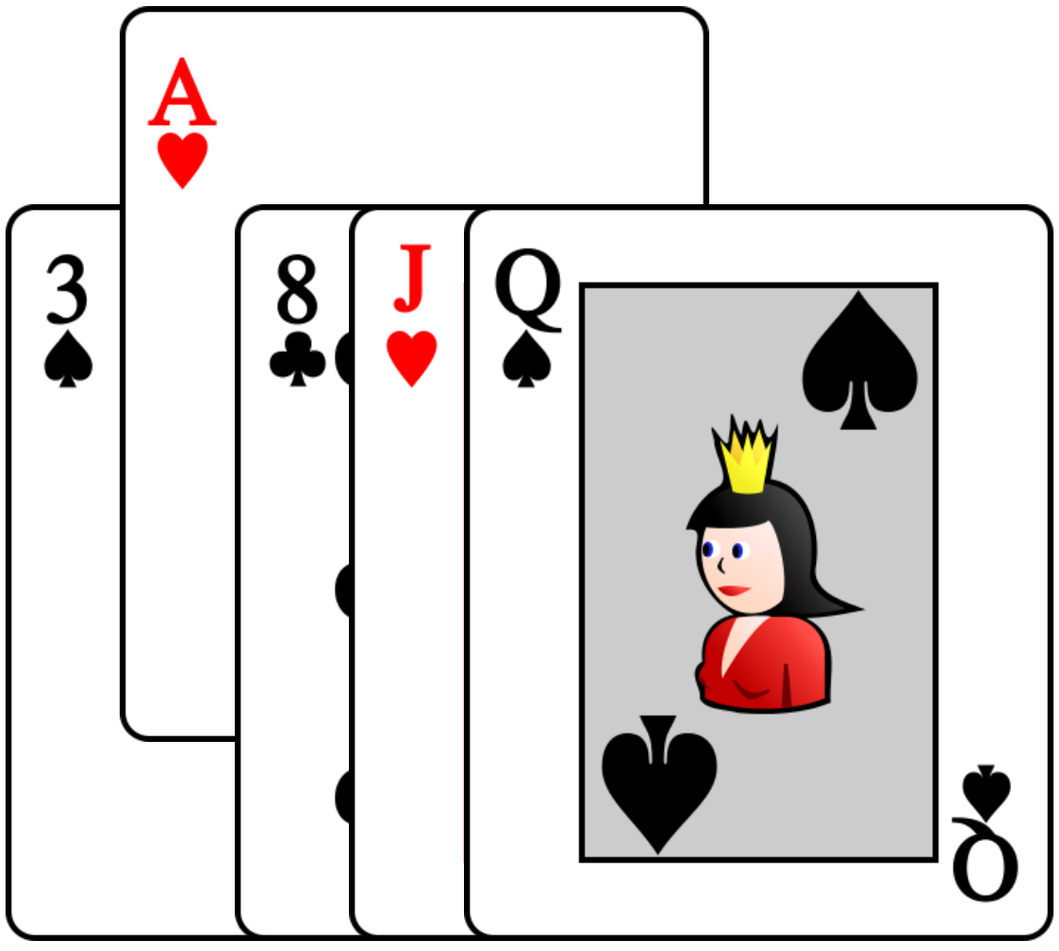
№ operations = 1 + 2 + 3 + 4 + 2

Le pire des cas



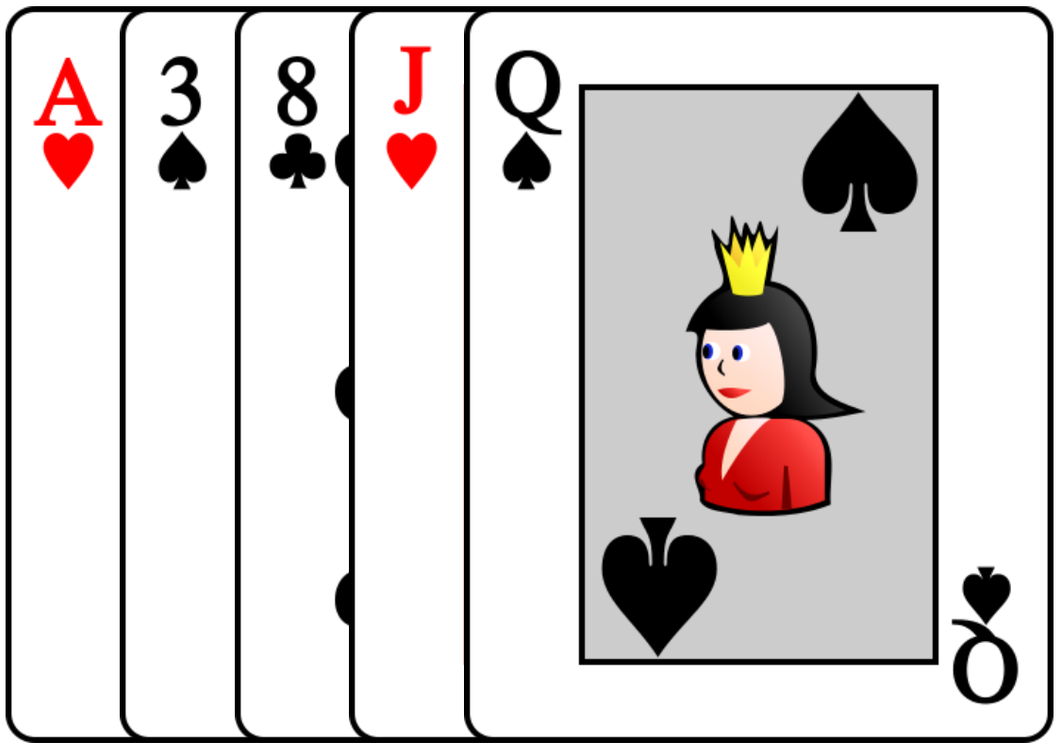
№ operations = 1 + 2 + 3 + 4 + 3

Le pire des cas



№ operations = 1 + 2 + 3 + 4 + 4

Le pire des cas



№ operations = 1 + 2 + 3 + 4 + 5

Le pire des cas

- Les cartes arrivent en ordre décroissant
- On fait i opérations pour la i-ème carte
- Le nombre totale est 1 + 2 + 3 + ⋯ + n

$$\sum_{i=1}^{n} i = \frac{1}{2}n(n+1) = \frac{1}{2}(n^2+n) = \frac{1}{2}n^2 + \frac{1}{2}n \in O(n^2)$$

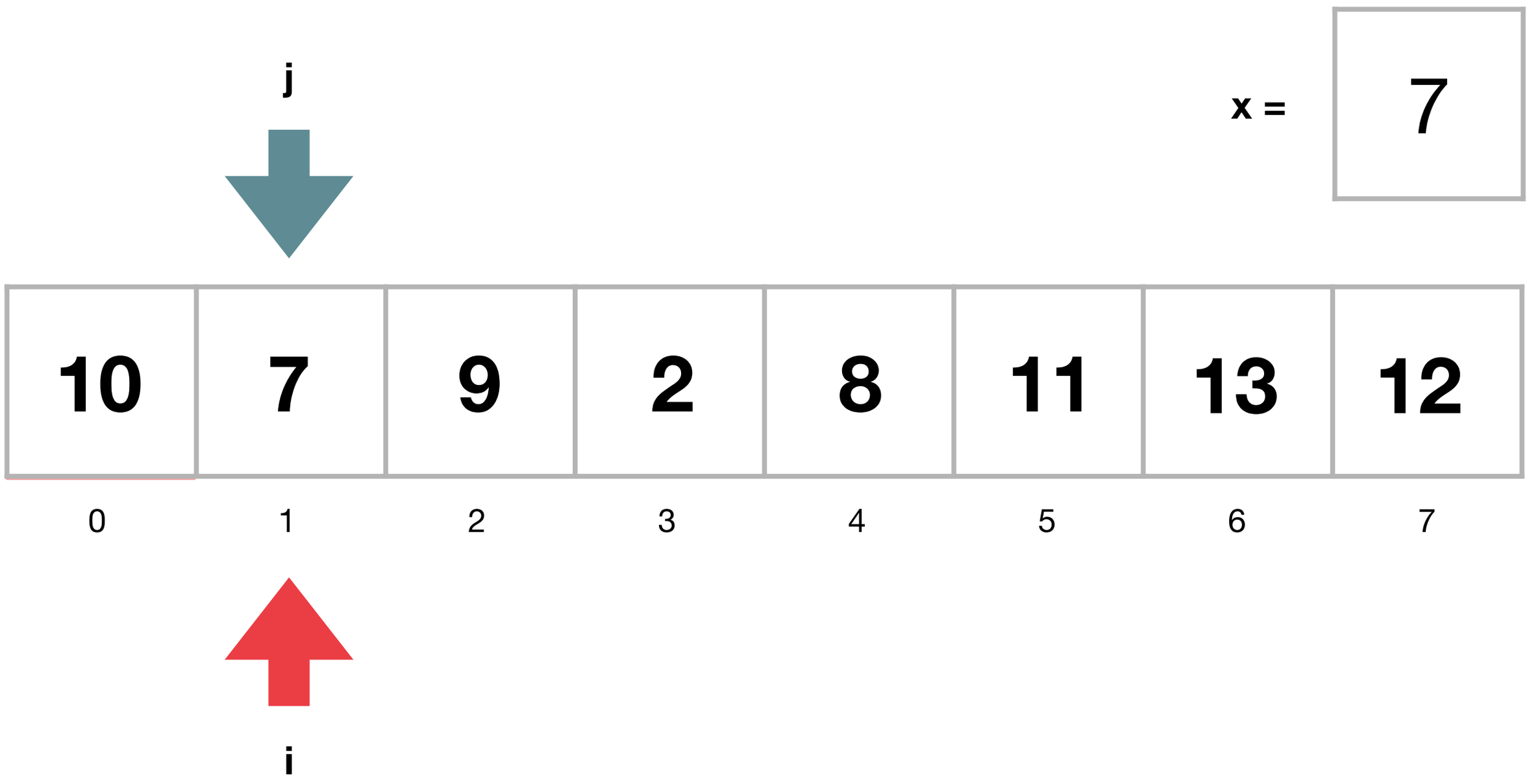
Tri d’un tableau par insertion

```
procedure trier-par-insertion(T)
    n ≔ longueur(T)
    pour i ≔ 1 à n – 1 faire (on saute le 0)
        x ≔ T[i]
        j ≔ i
        tant que j > 0 et x < T[j – 1] faire
            (décaler d’un élément)
            T[j] ≔ T[j – 1]
            j ≔ j – 1
        fin tant que
        (ici x ≥ T[j – 1] ou bien j = 0)
        T[j] ≔ x
    fin pour
fin procedure
```

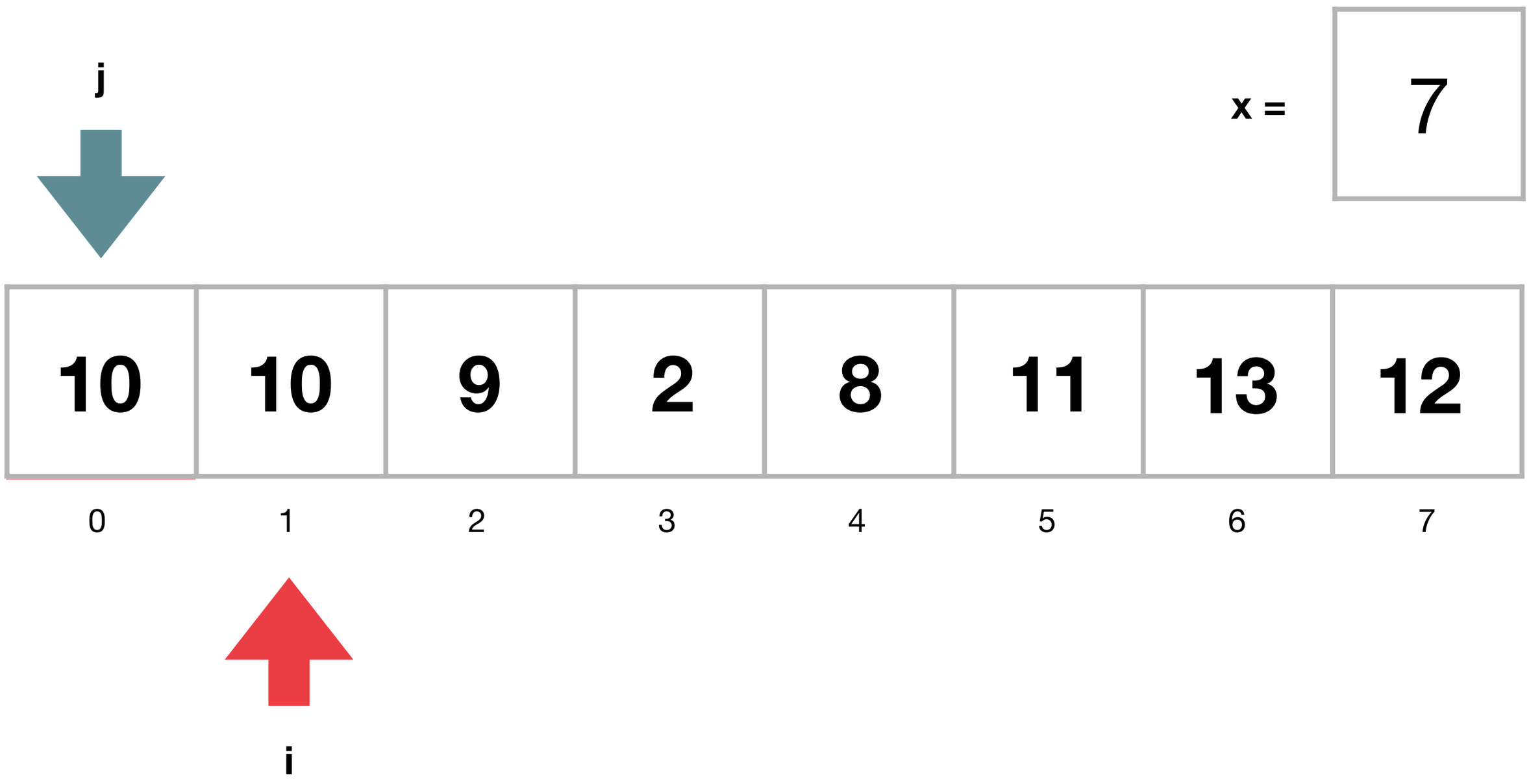
Tri par insertion

10	7	9	2	8	11	13	12
0	1	2	3	4	5	6	7

Tri par insertion



Tri par insertion



Tri par insertion

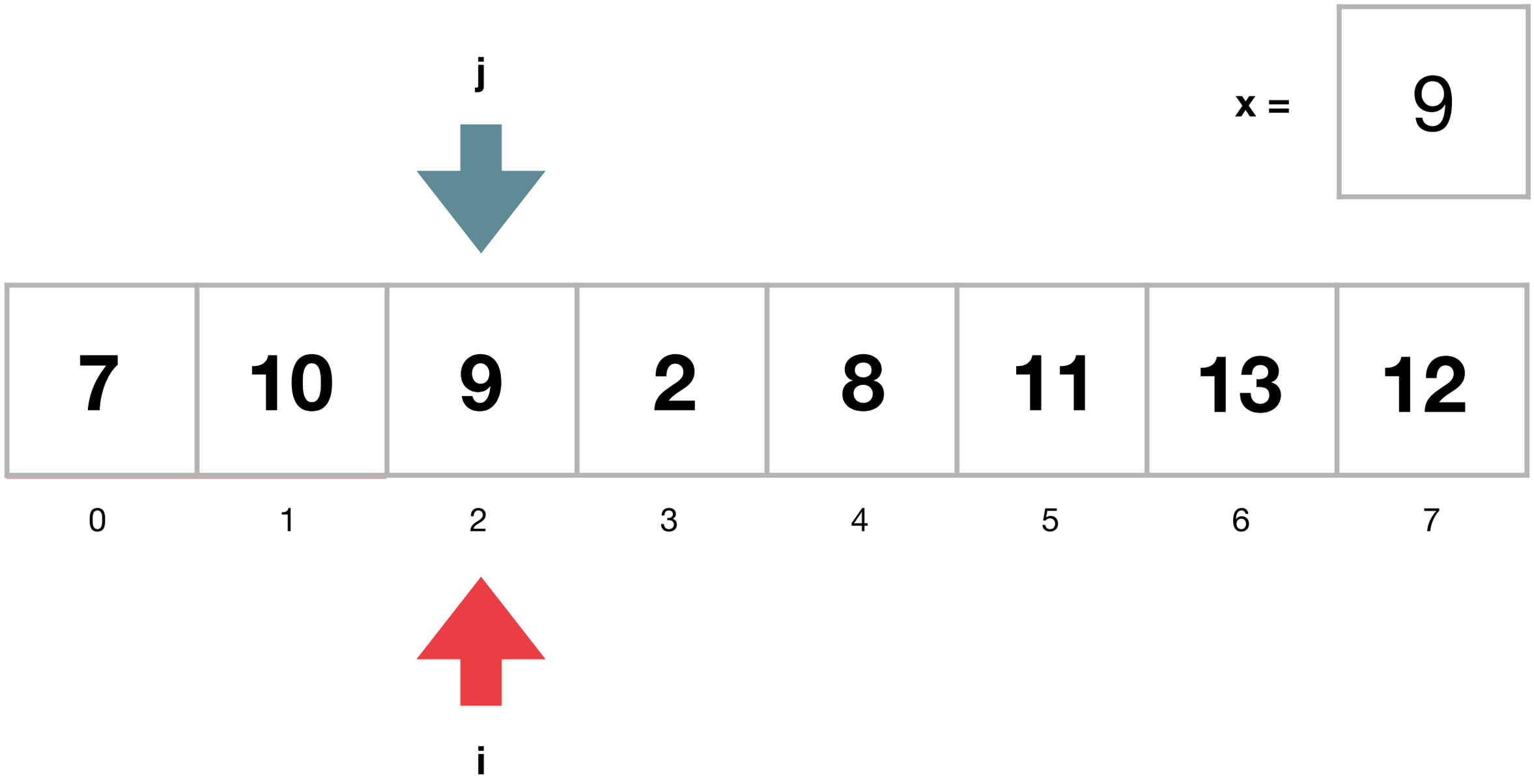
j

x = 7

7	10	9	2	8	11	13	12
0	1	2	3	4	5	6	7

i

Tri par insertion



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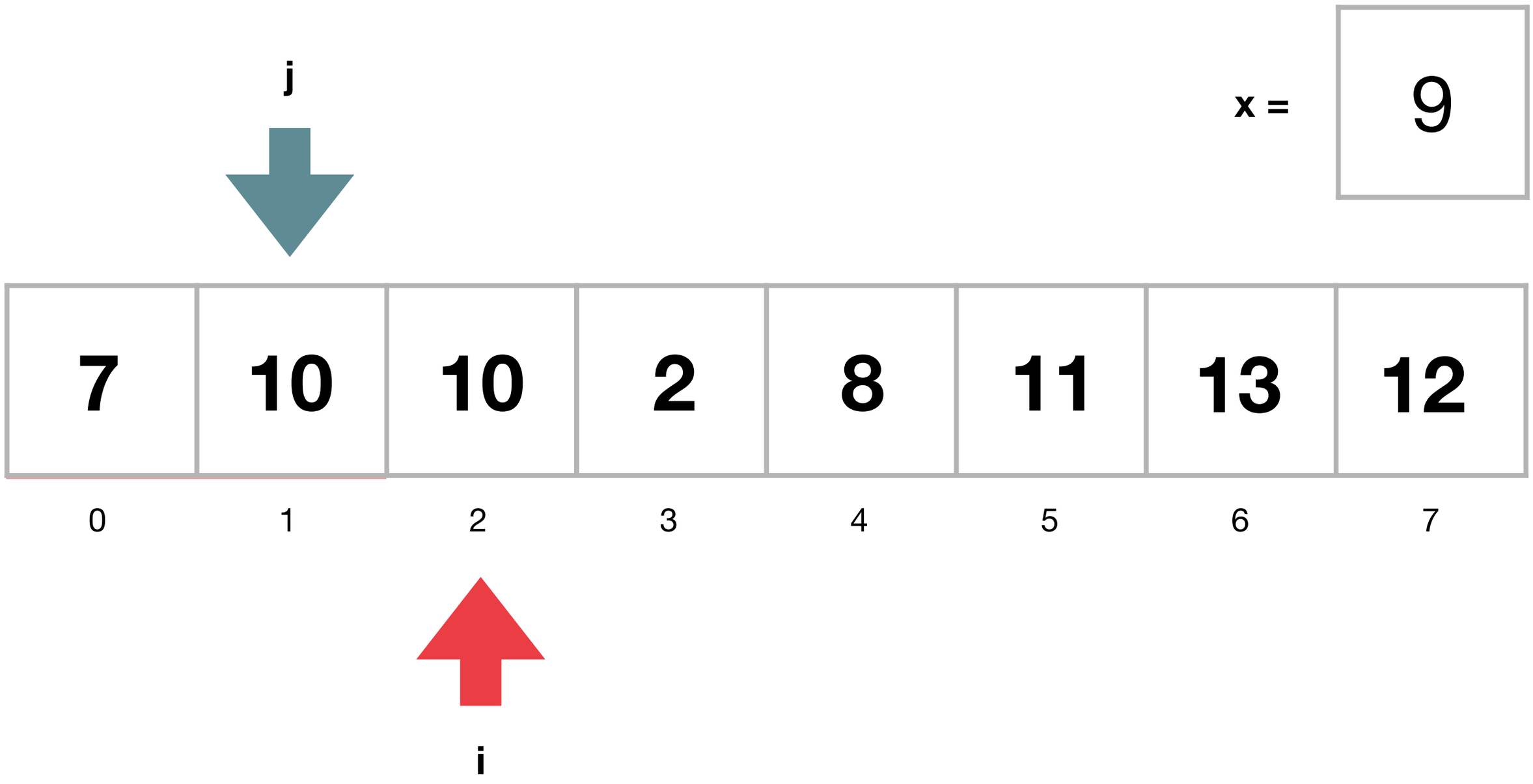


Tri par insertion

j

x = 9

7	9	10	2	8	11	13	12
0	1	2	3	4	5	6	7

i

Tri par insertion

j

x = 2

7	9	10	2	8	11	13	12
0	1	2	3	4	5	6	7

i

Tri par insertion

j

x = 2

7	9	10	10	8	11	13	12
0	1	2	3	4	5	6	7

i

Tri par insertion

j

x = 2

7	9	9	10	8	11	13	12
0	1	2	3	4	5	6	7

i

Tri par insertion

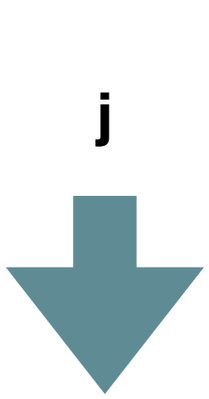
j

x = 2

7	7	9	10	8	11	13	12
0	1	2	3	4	5	6	7

i

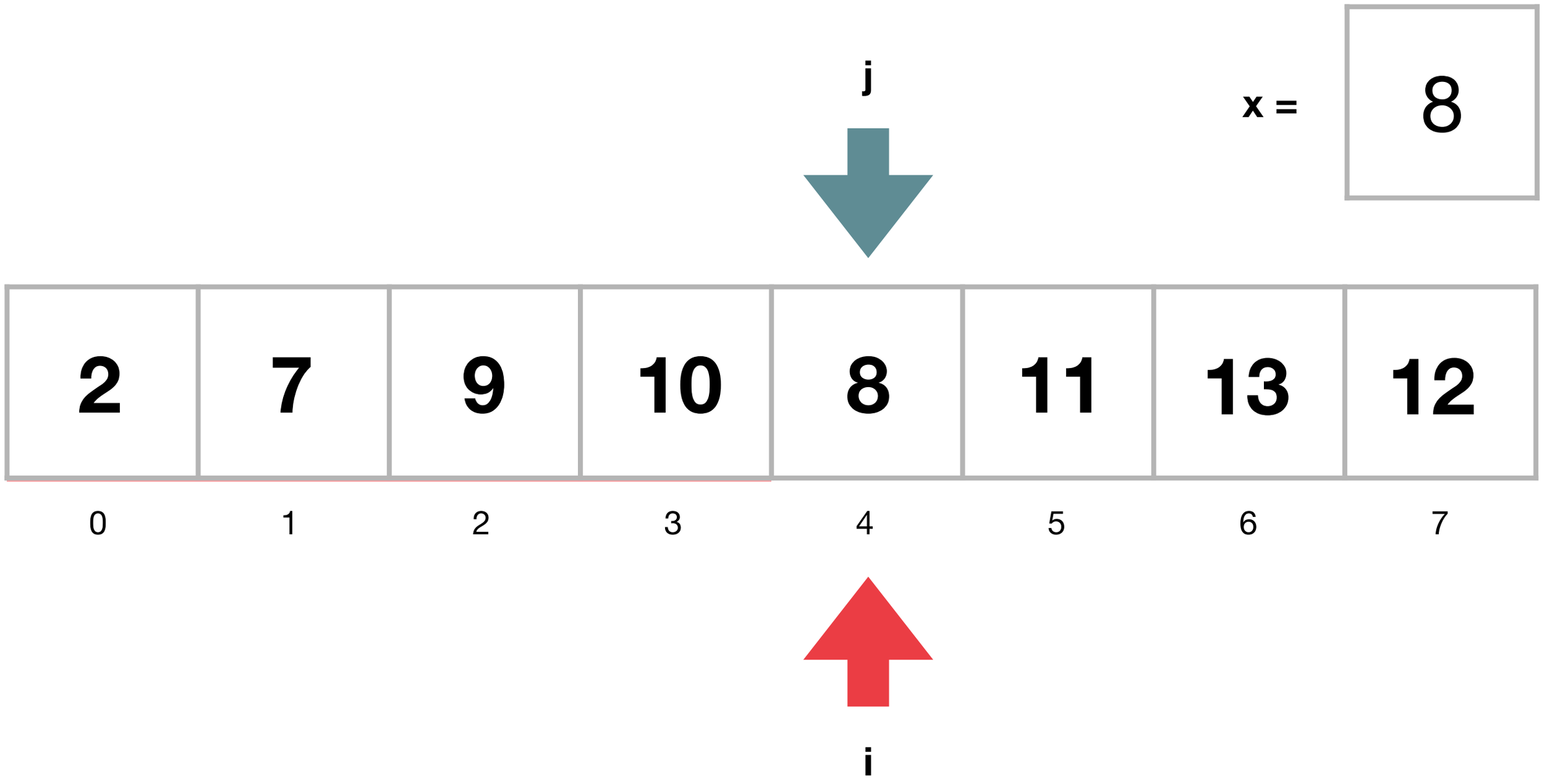
Tri par insertion



x = 2

2	7	9	10	8	11	13	12
0	1	2	3	4	5	6	7

Tri par insertion



Tri par insertion

j

x = 8

2	7	9	10	10	11	13	12
0	1	2	3	4	5	6	7

i

Tri par insertion

j

x = 8

2	7	9	9	10	11	13	12
0	1	2	3	4	5	6	7

i

Tri par insertion

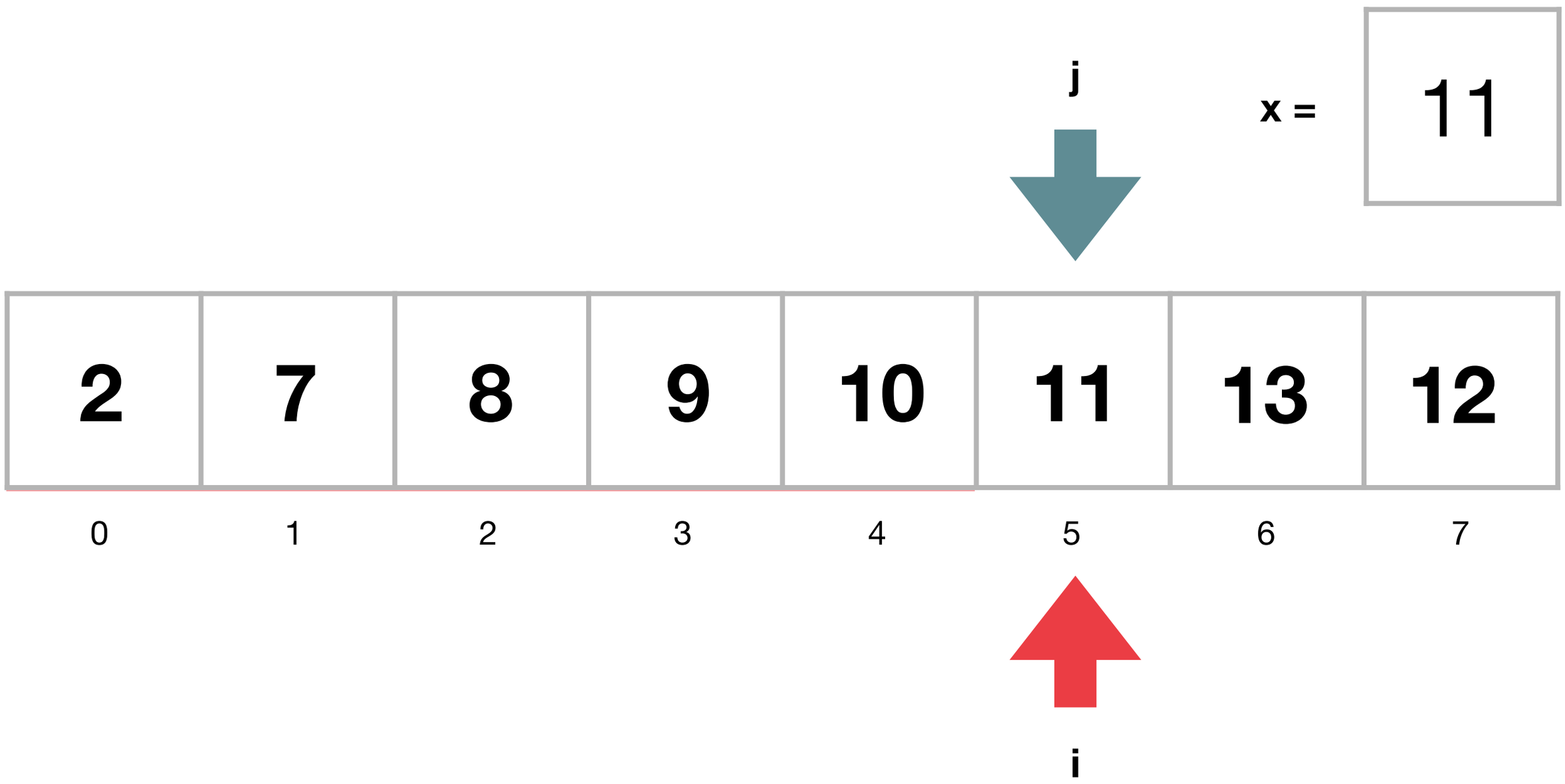
j

x = 8

2	7	8	9	10	11	13	12
0	1	2	3	4	5	6	7

i

Tri par insertion



Tri par insertion

x = 13

j

2	7	8	9	10	11	13	12
0	1	2	3	4	5	6	7

i

Tri par insertion

x = 12

j

2	7	8	9	10	11	13	12
0	1	2	3	4	5	6	7

i

Tri par insertion

x = 12

j

2	7	8	9	10	11	13	13
0	1	2	3	4	5	6	7

i

Tri par insertion

x = 12

j

2	7	8	9	10	11	12	13
0	1	2	3	4	5	6	7

i

Tri par insertion

x = 12

j

2	7	8	9	10	11	12	13
0	1	2	3	4	5	6	7

i

Terminaison

```
procedure trier-par-insertion(T)
    n ≔ longueur(T)
    pour i ≔ 1 à n – 1 faire (on saute le 0)
        x ≔ T[i]
        j ≔ i
        tant que j > 0 et x < T[j – 1] faire
            (décaler d'un élément)
            T[j] ≔ T[j – 1]
            j ≔ j – 1
        fin tant que
        (ici x ≥ T[j – 1] ou bien j = 0)
        T[j] ≔ x
    fin pour
fin procedure
```

- La boucle **pour** termine toujours
- La boucle **tant que** termine (au pire) quand j = 0

Correction

```
procedure trier-par-insertion(T)
    n ≔ longueur(T)
    pour i ≔ 1 à n – 1 faire (on saute le 0)
        x ≔ T[i]
        j ≔ i
        tant que j > 0 et x < T[j – 1] faire
            (décaler d'un élément)
            T[j] ≔ T[j – 1]
            j ≔ j – 1
        fin tant que
        (ici x ≥ T[j – 1] ou bien j = 0)
        T[j] ≔ x
    fin pour
fin procedure
```

- Le sous-tableau T[0, …, i – 1] est trié au début de la boucle **pour**

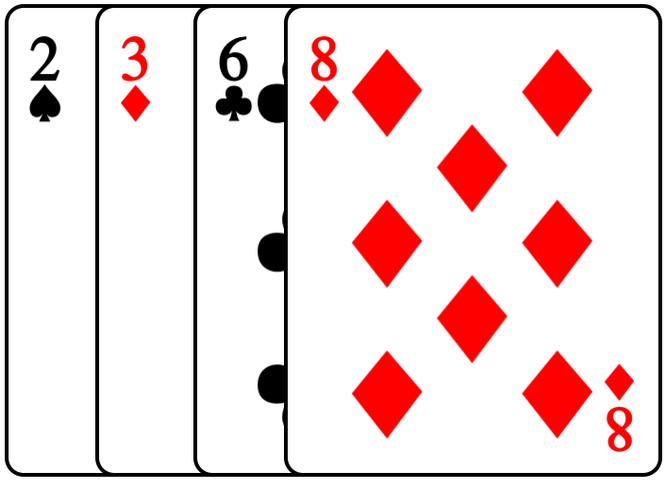
Efficacité

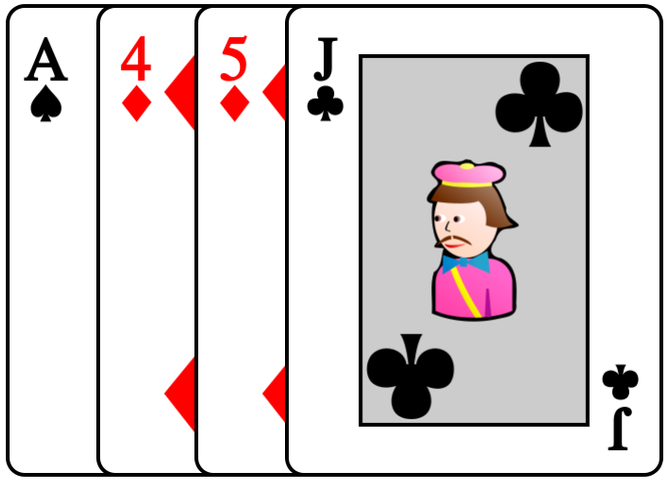
```
procedure trier-par-insertion(T)
    n ≔ longueur(T)
    pour i ≔ 1 à n – 1 faire (on saute le 0)
        x ≔ T[i]
        j ≔ i
        tant que j > 0 et x < T[j – 1] faire
            (décaler d'un élément)
            T[j] ≔ T[j – 1]
            j ≔ j – 1
        fin tant que
        (ici x ≥ T[j – 1] ou bien j = 0)
        T[j] ≔ x
    fin pour
fin procedure
```

- O(n) opérations dans le meilleur des cas
- $O(n^2)$ opérations dans le pire des cas

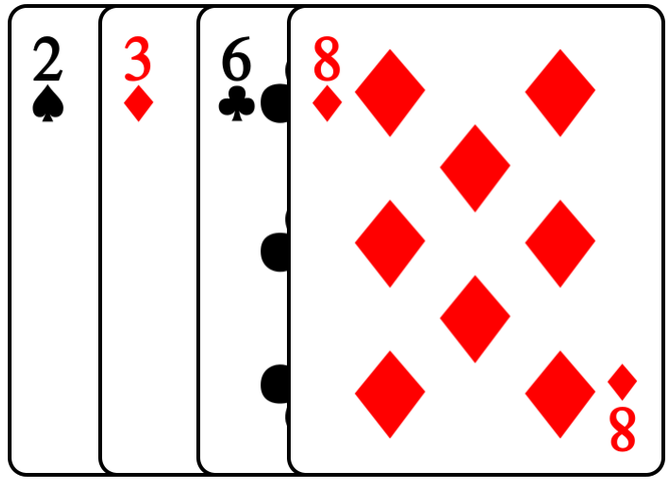
Est-ce qu’on peut faire mieux que ça ?

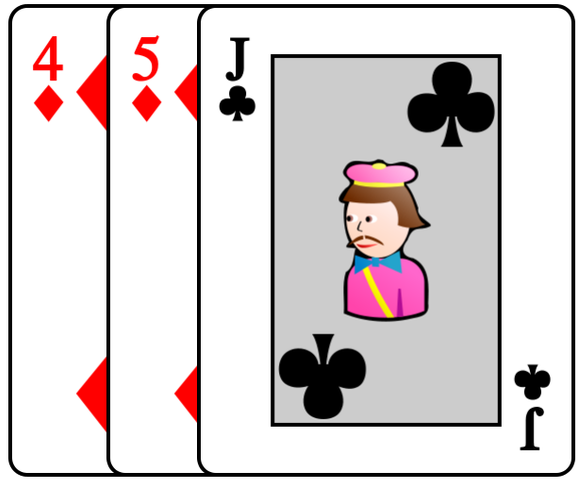
Fusion de tableaux triés

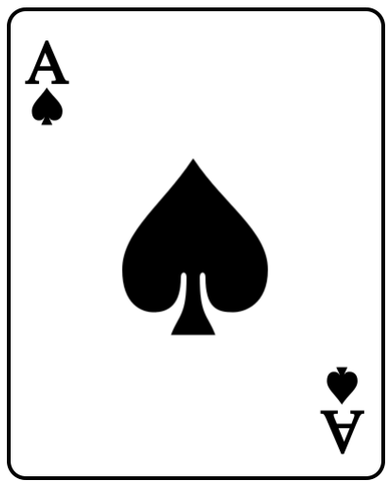




Fusion de tableaux triés







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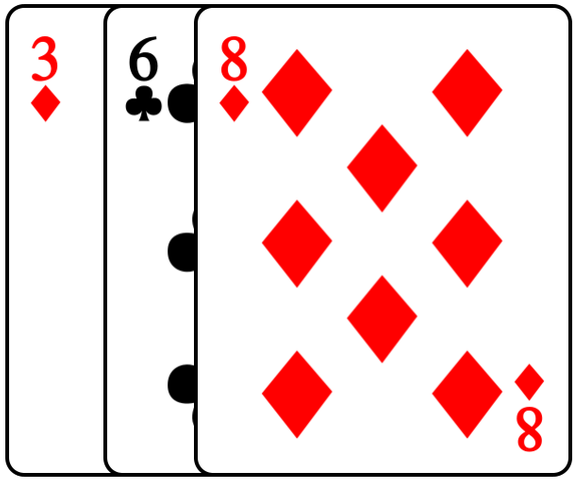

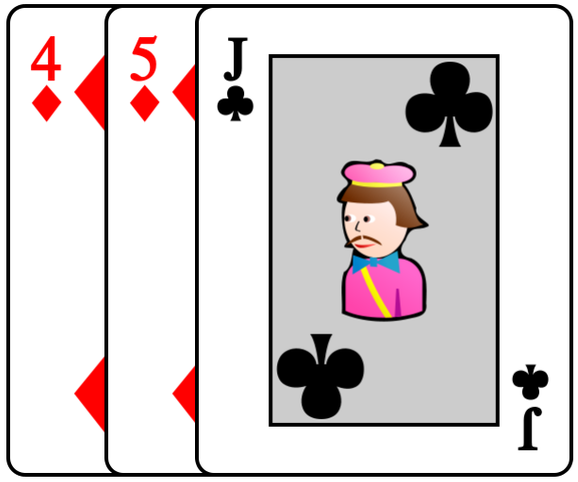

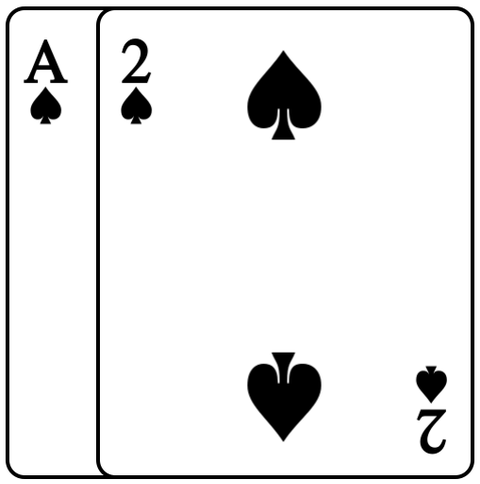

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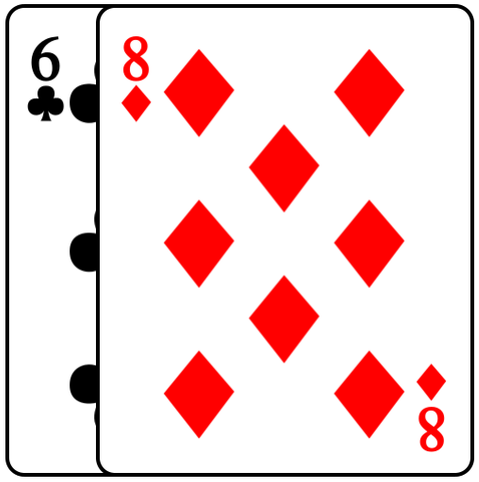

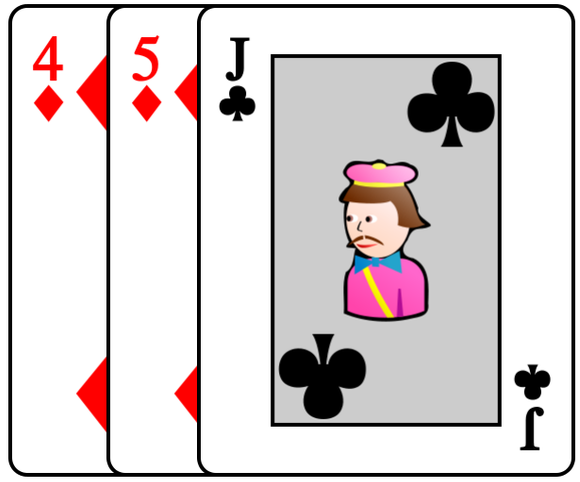

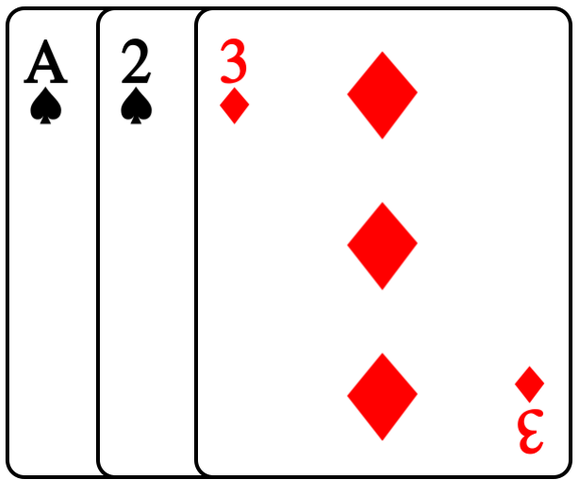

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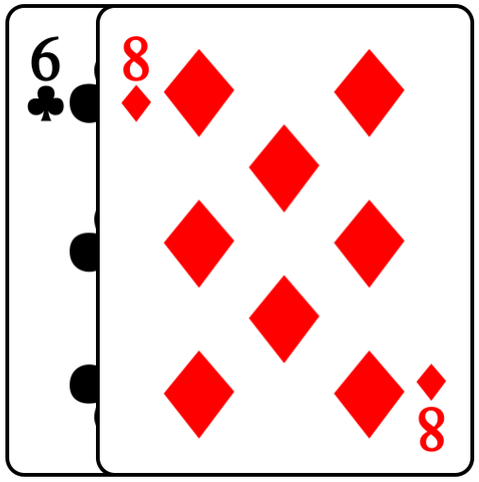

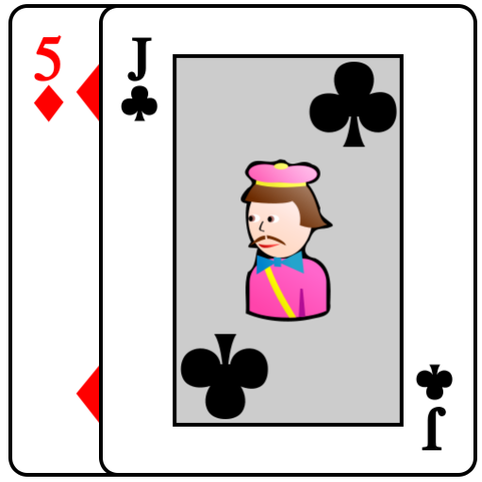

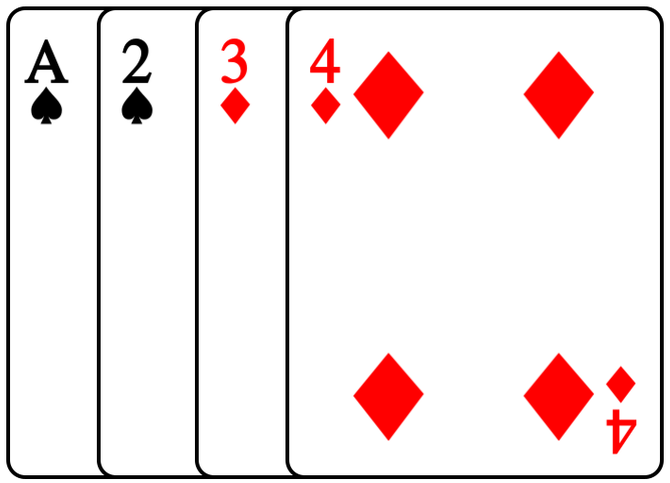

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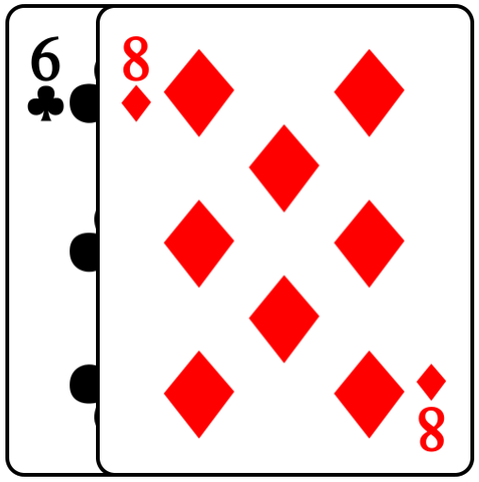

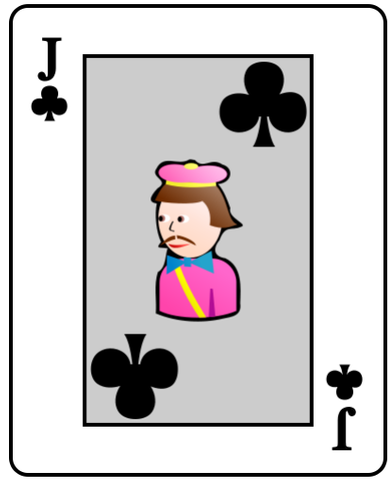

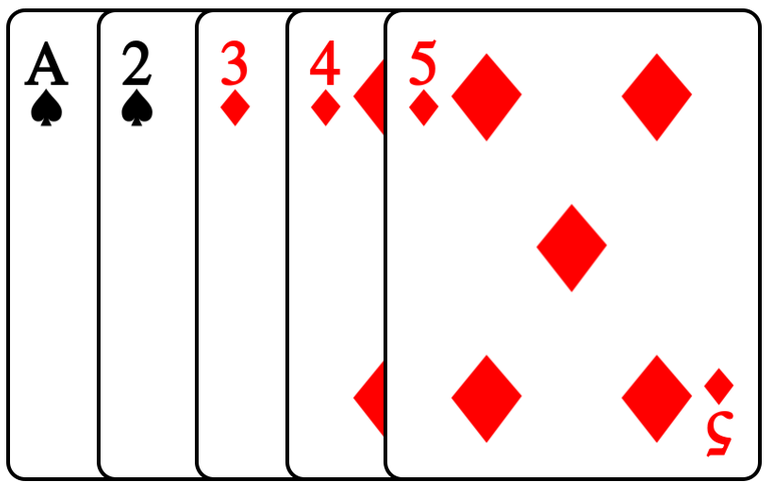

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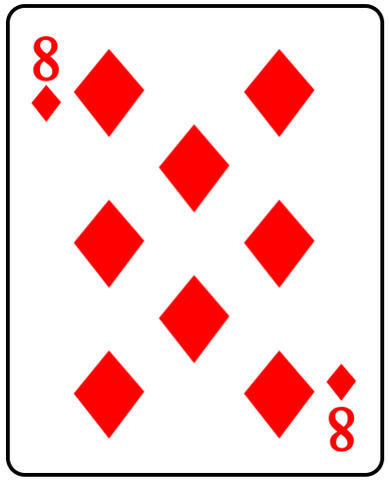

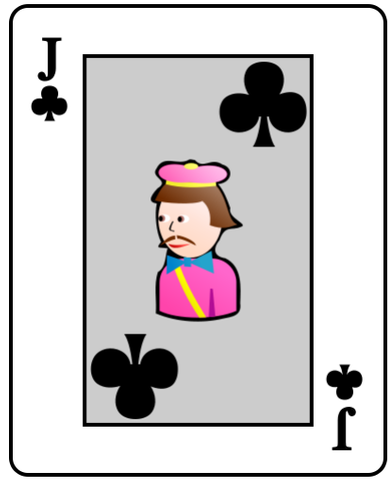

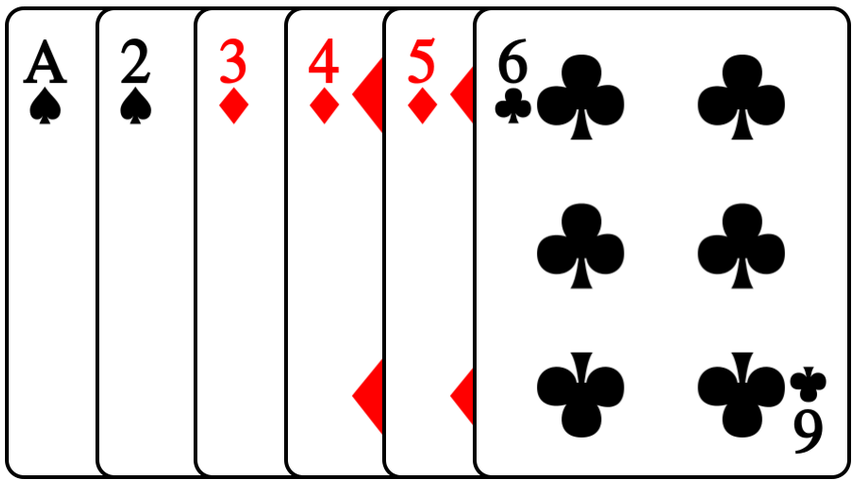

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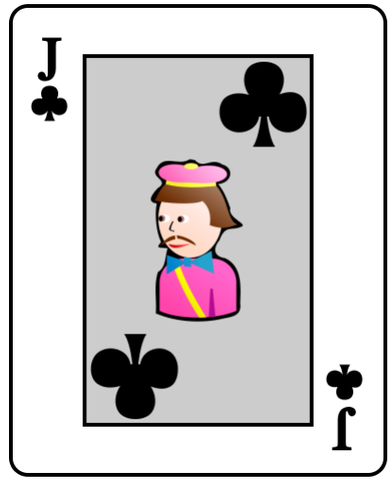

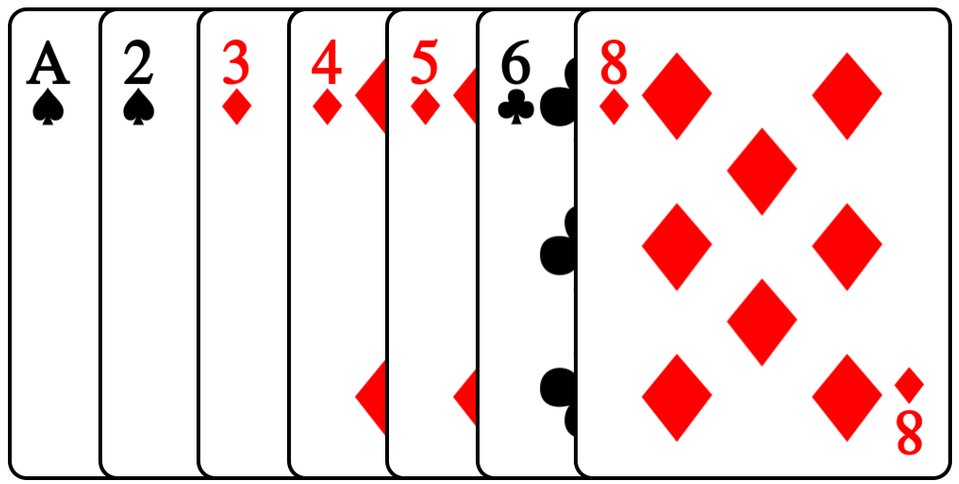

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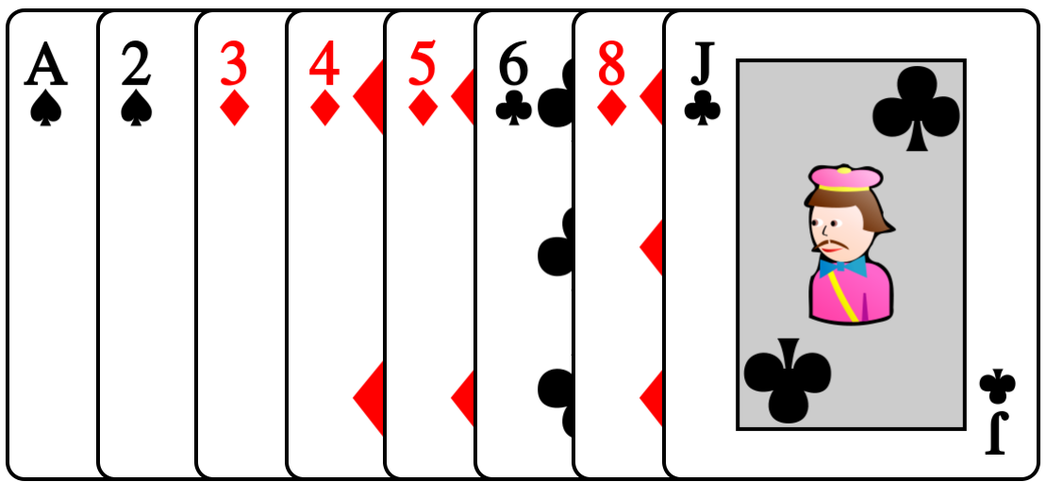

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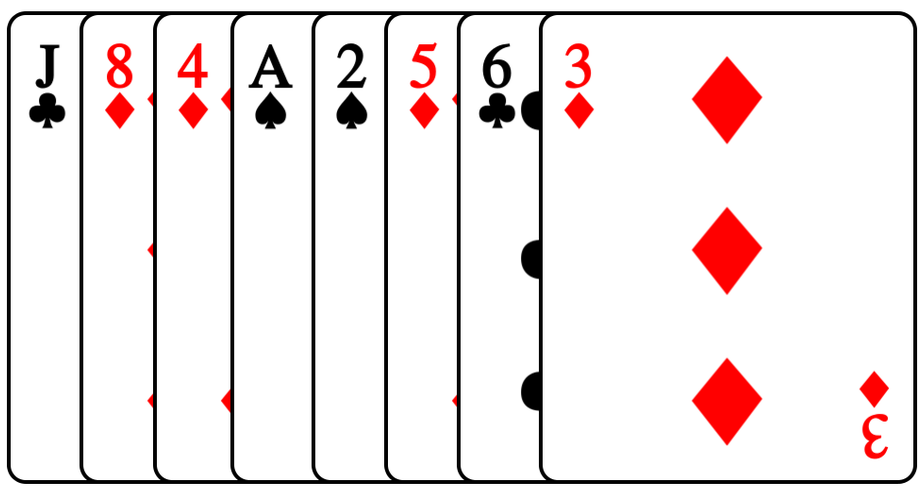

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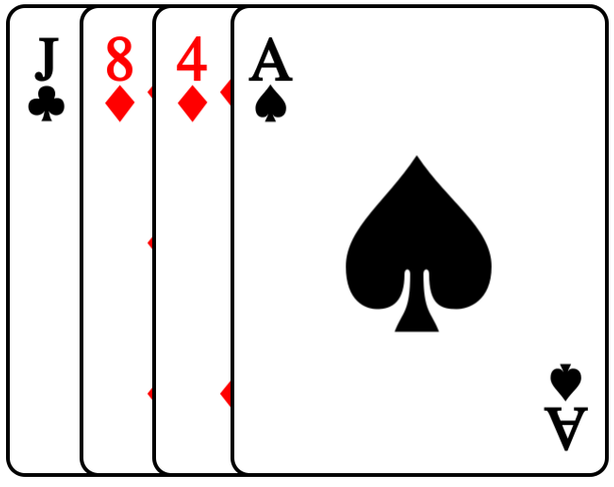

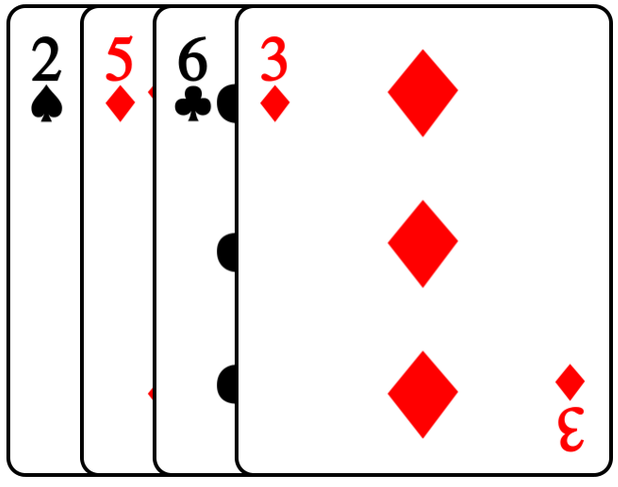

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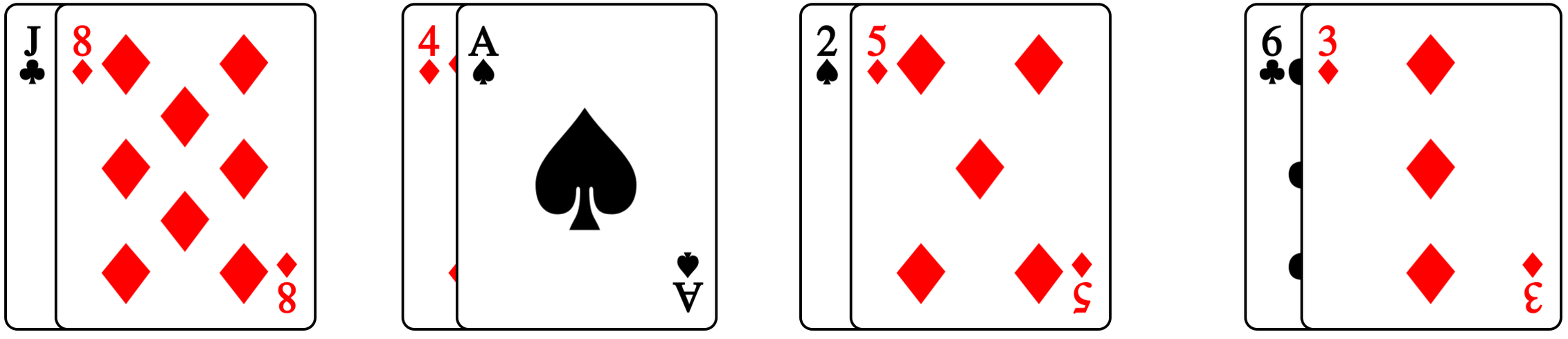

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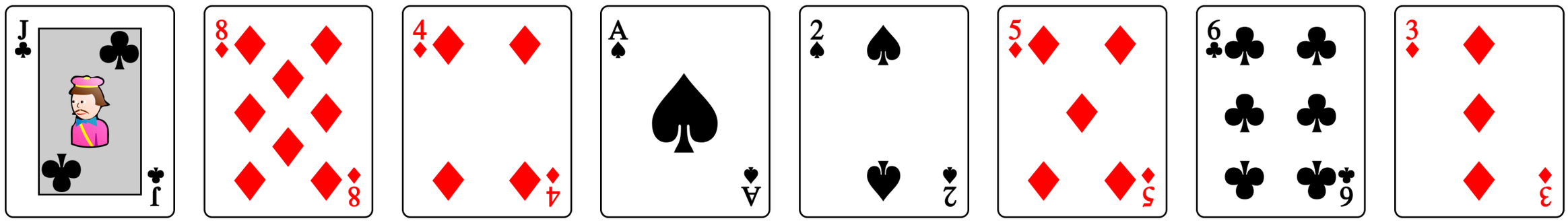

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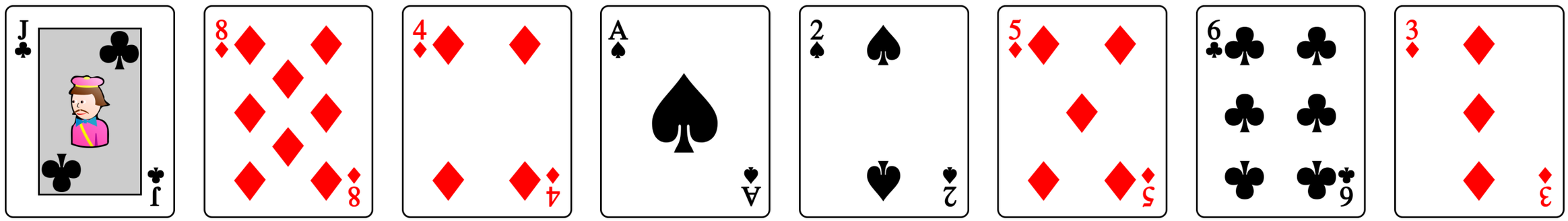

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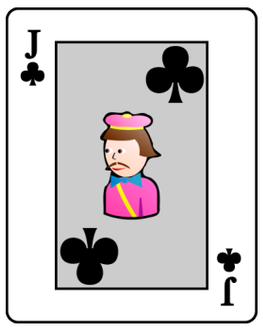
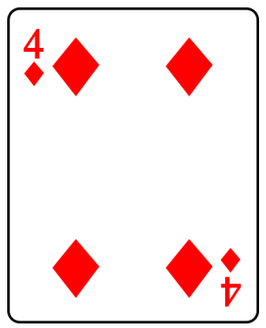
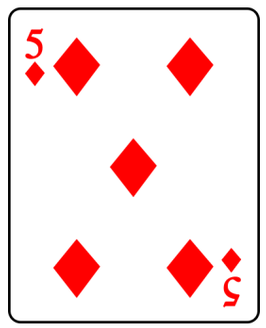
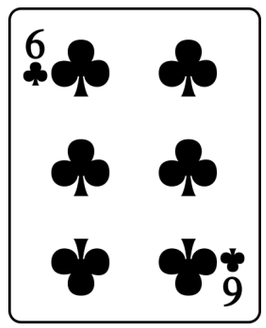
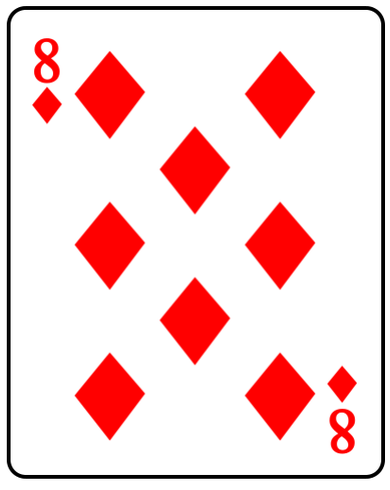
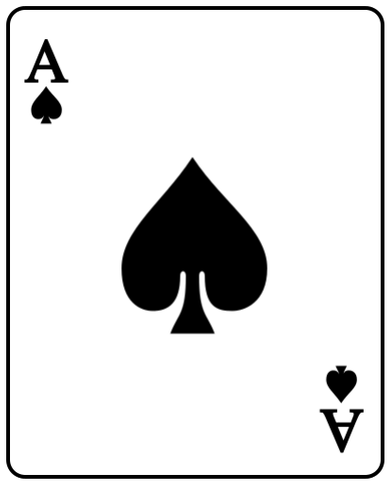
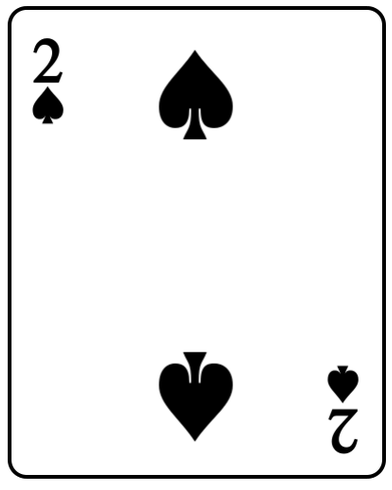
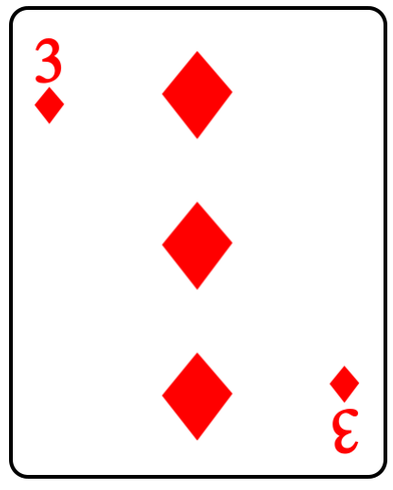

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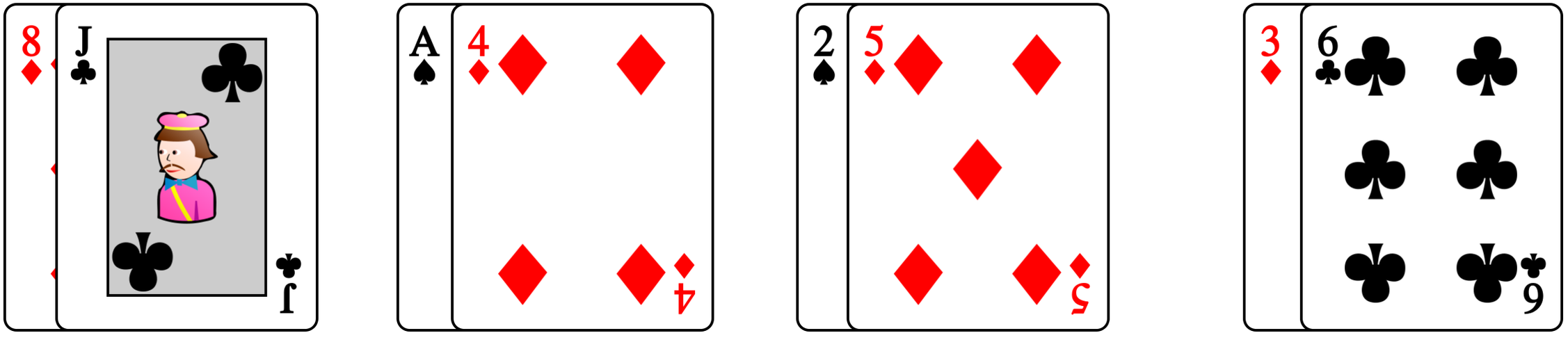

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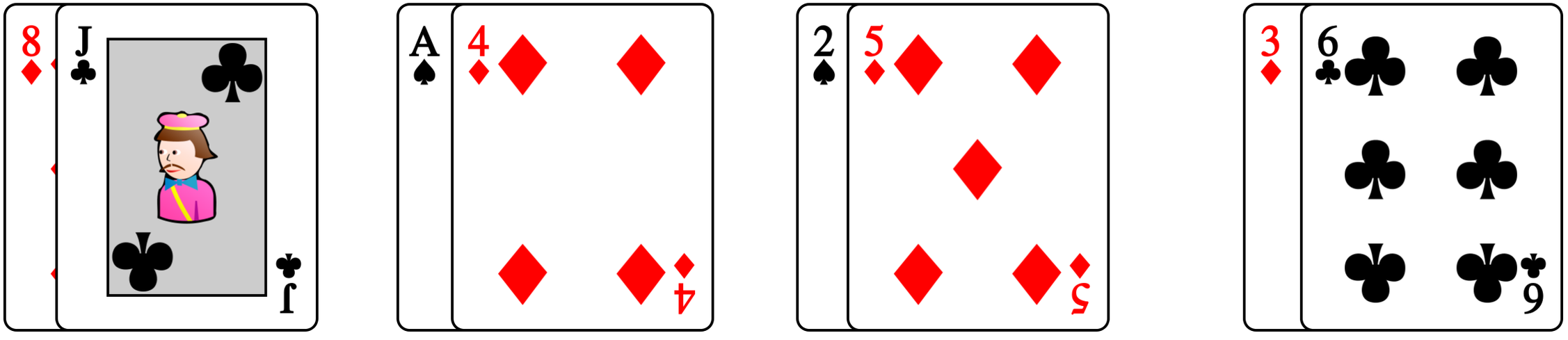

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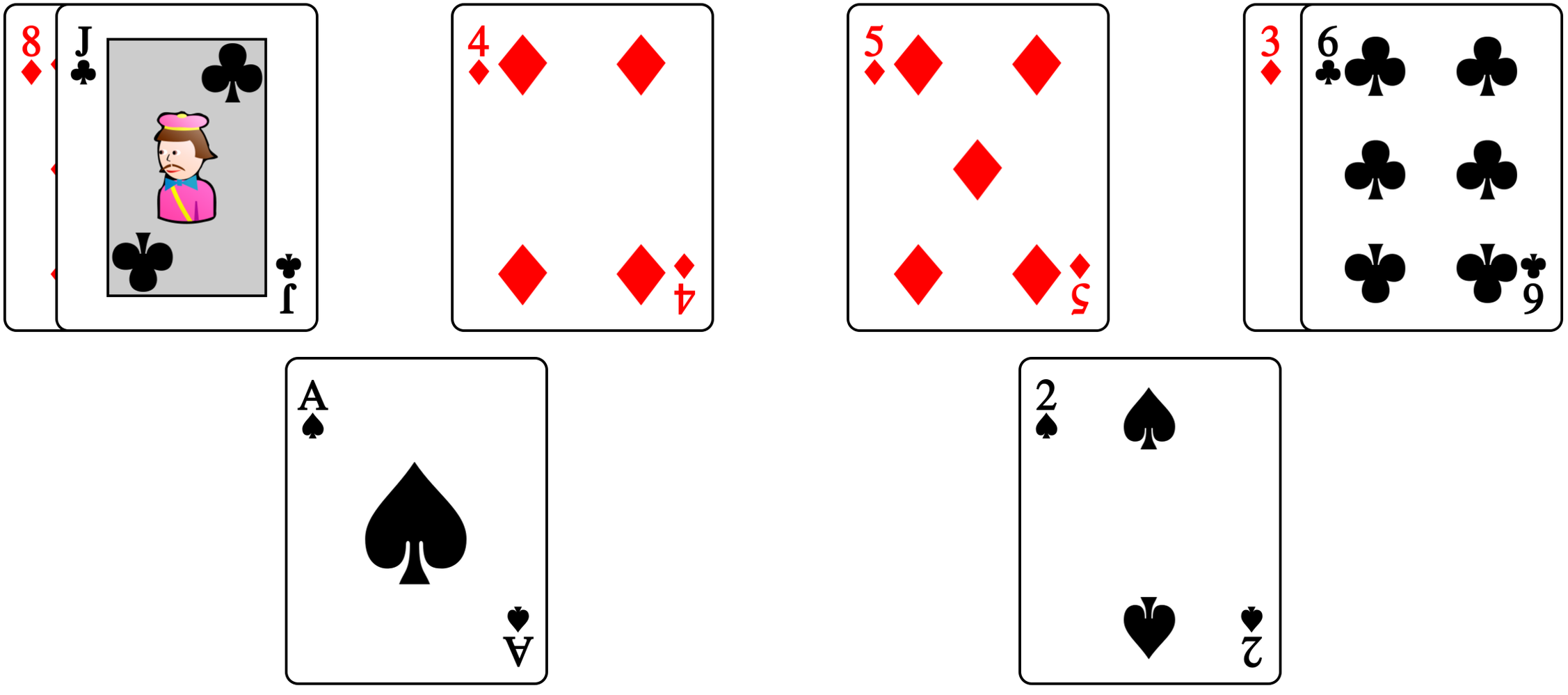

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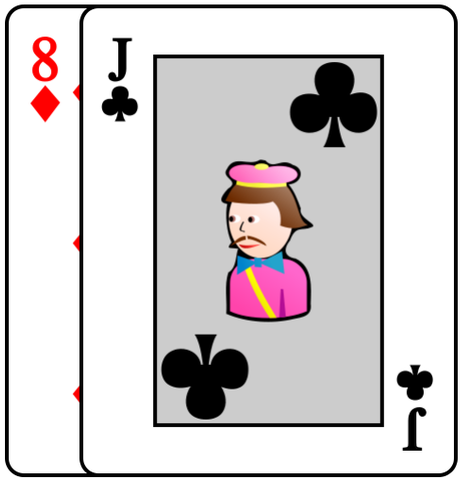

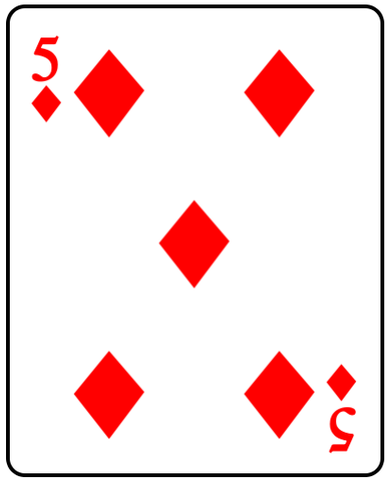

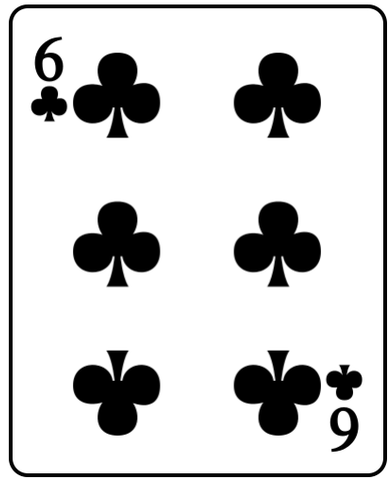

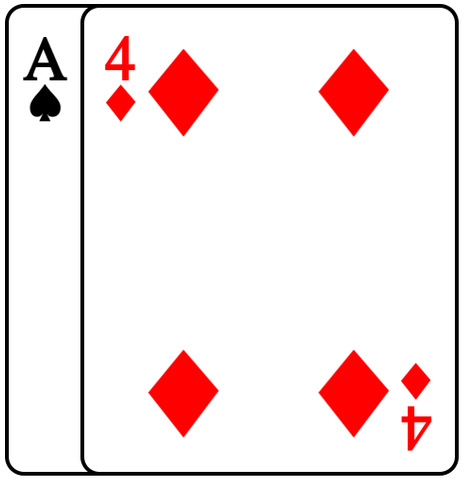

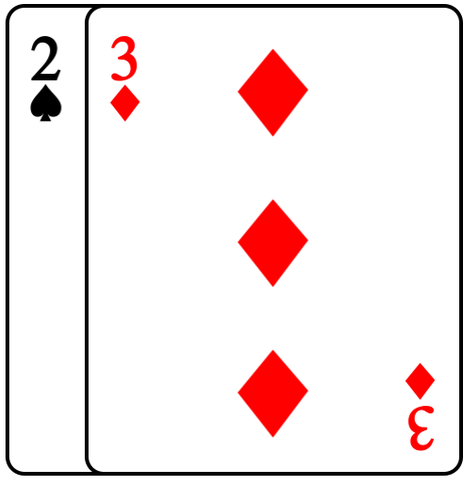

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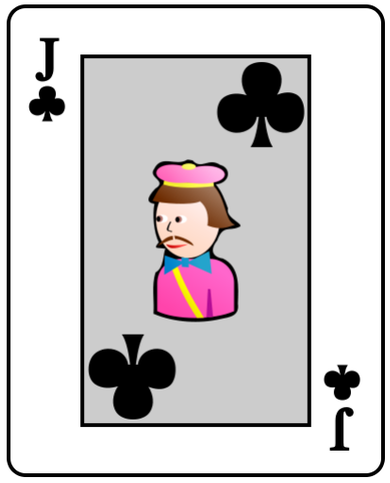

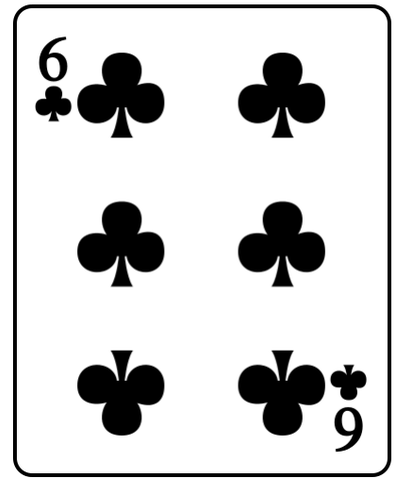

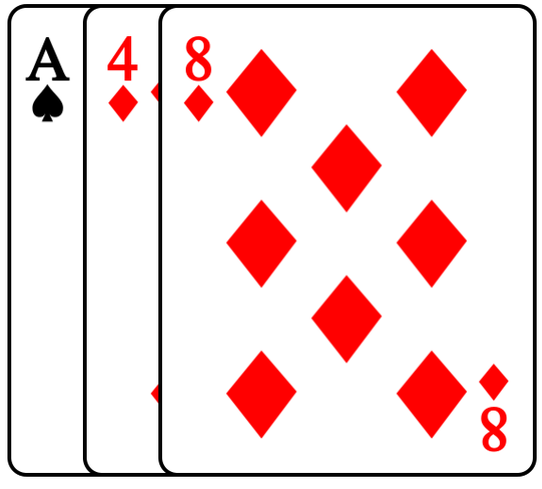

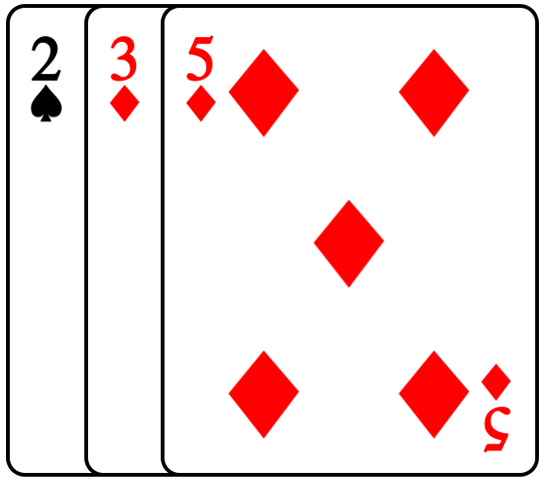

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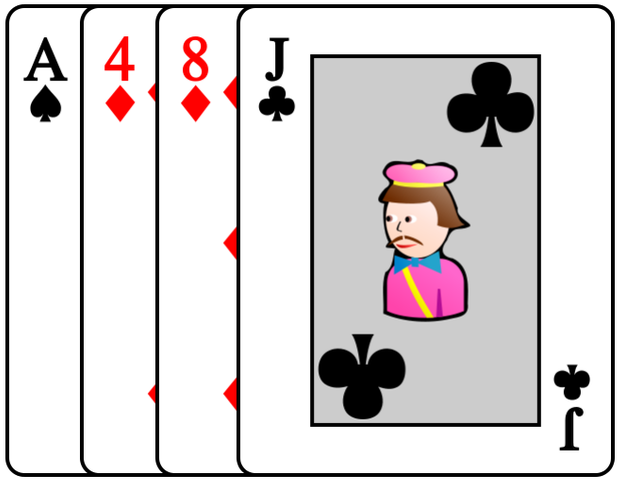

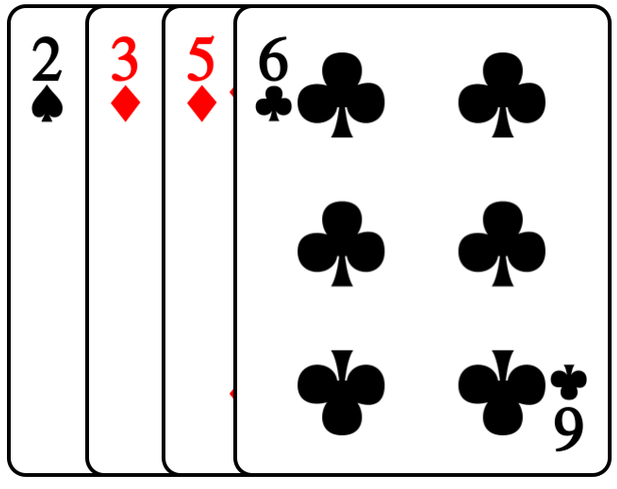

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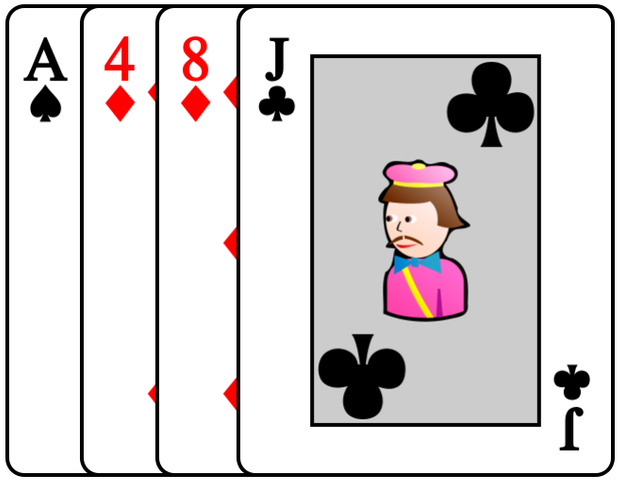

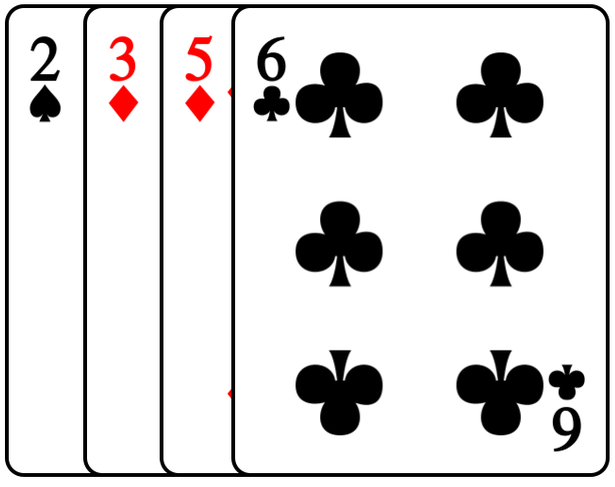

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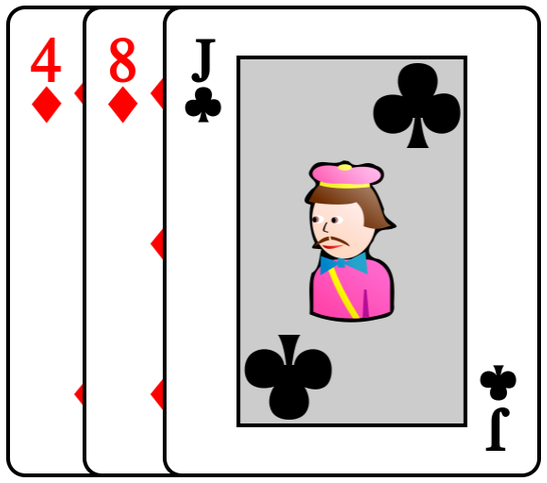

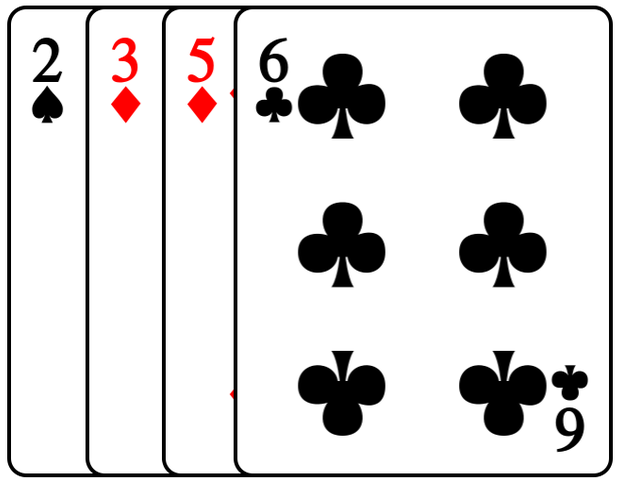

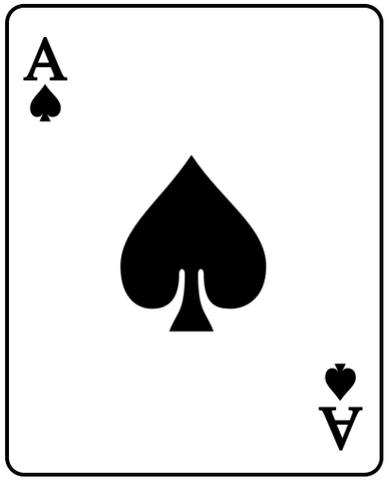

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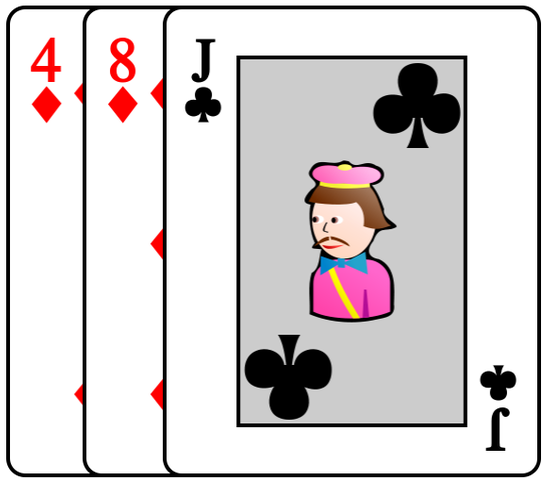

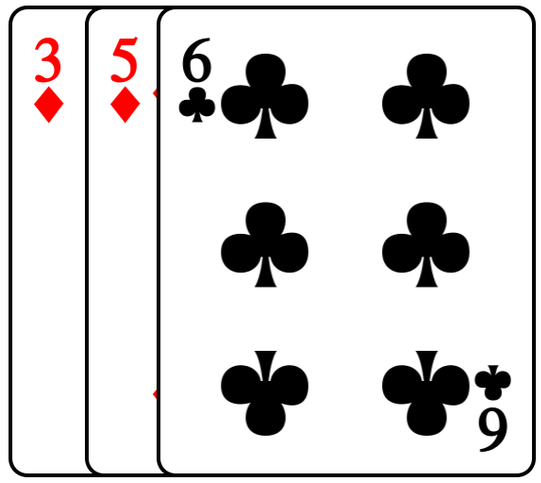

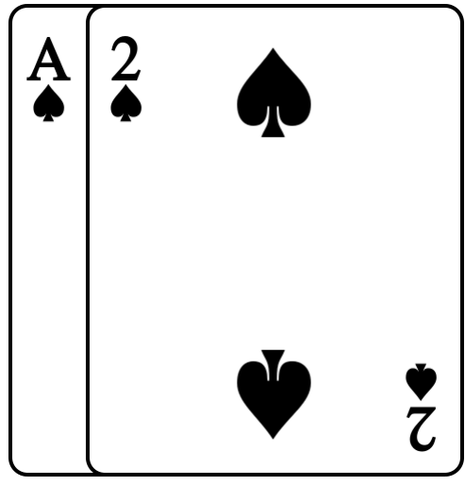

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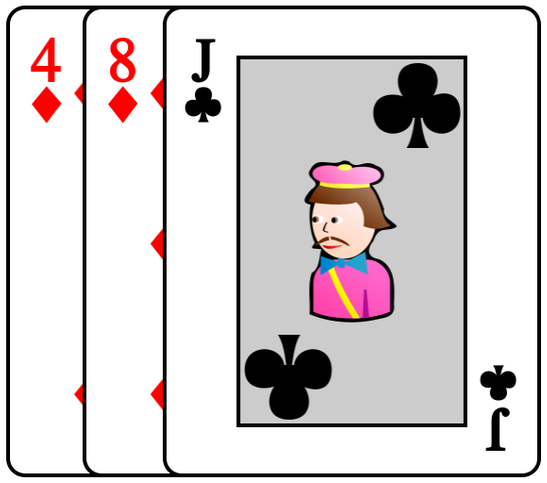

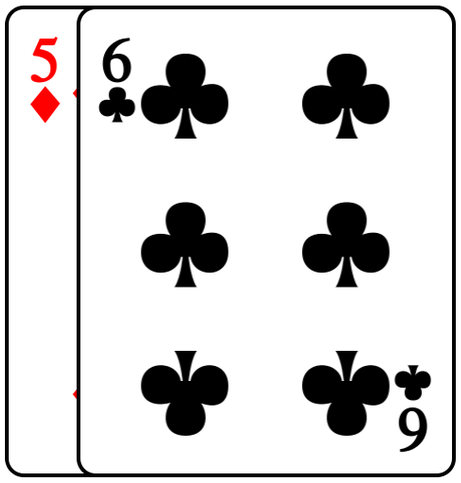

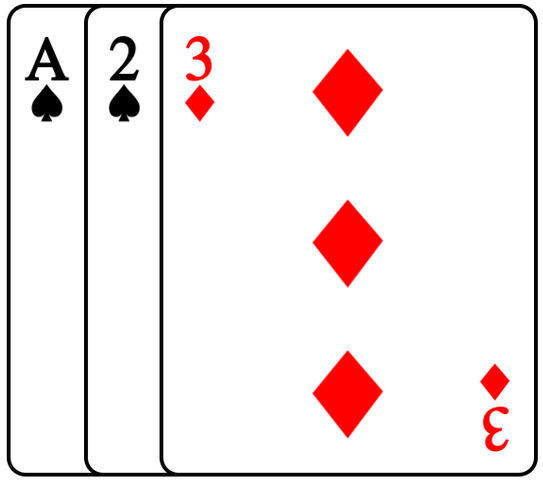

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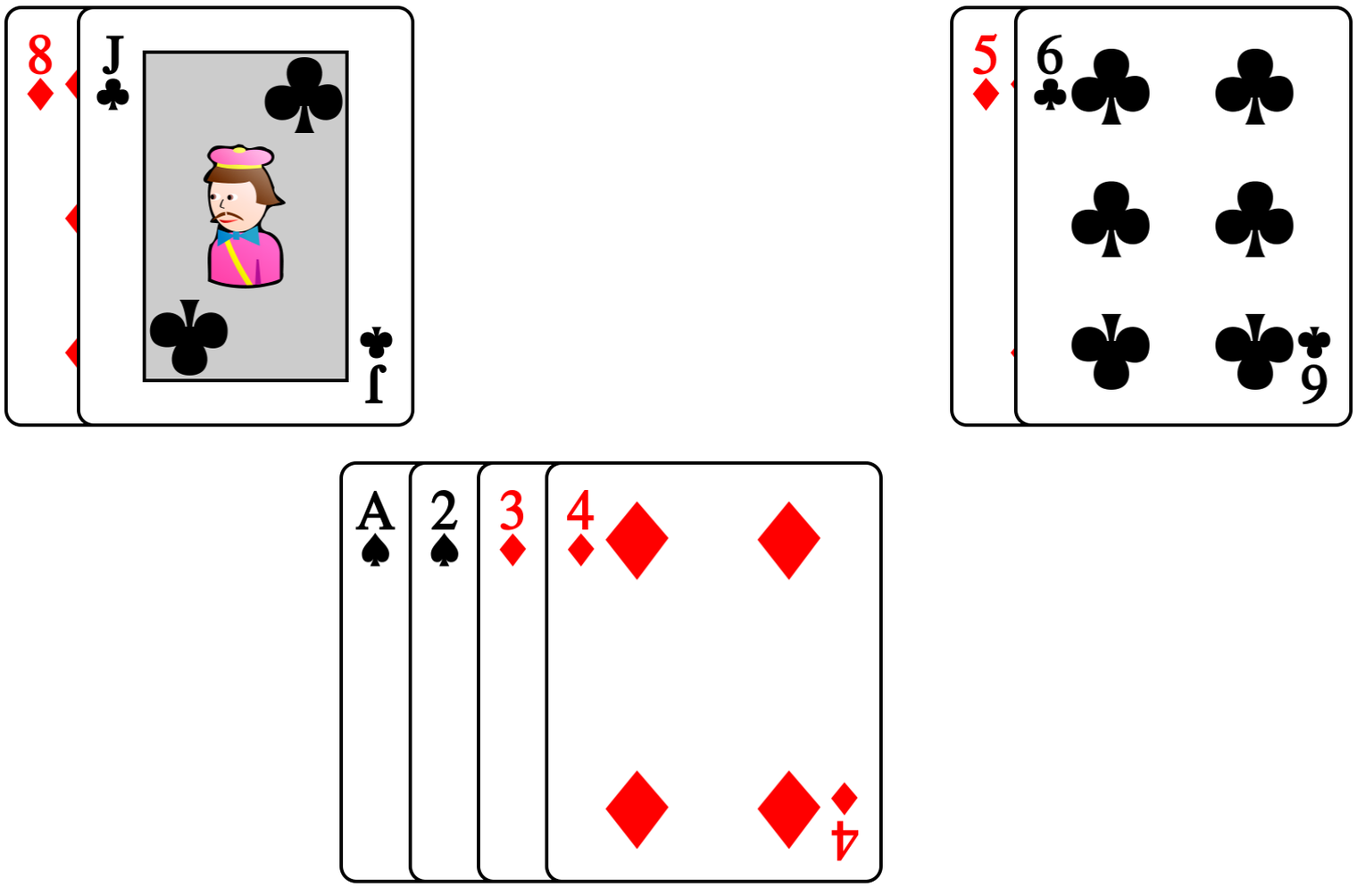
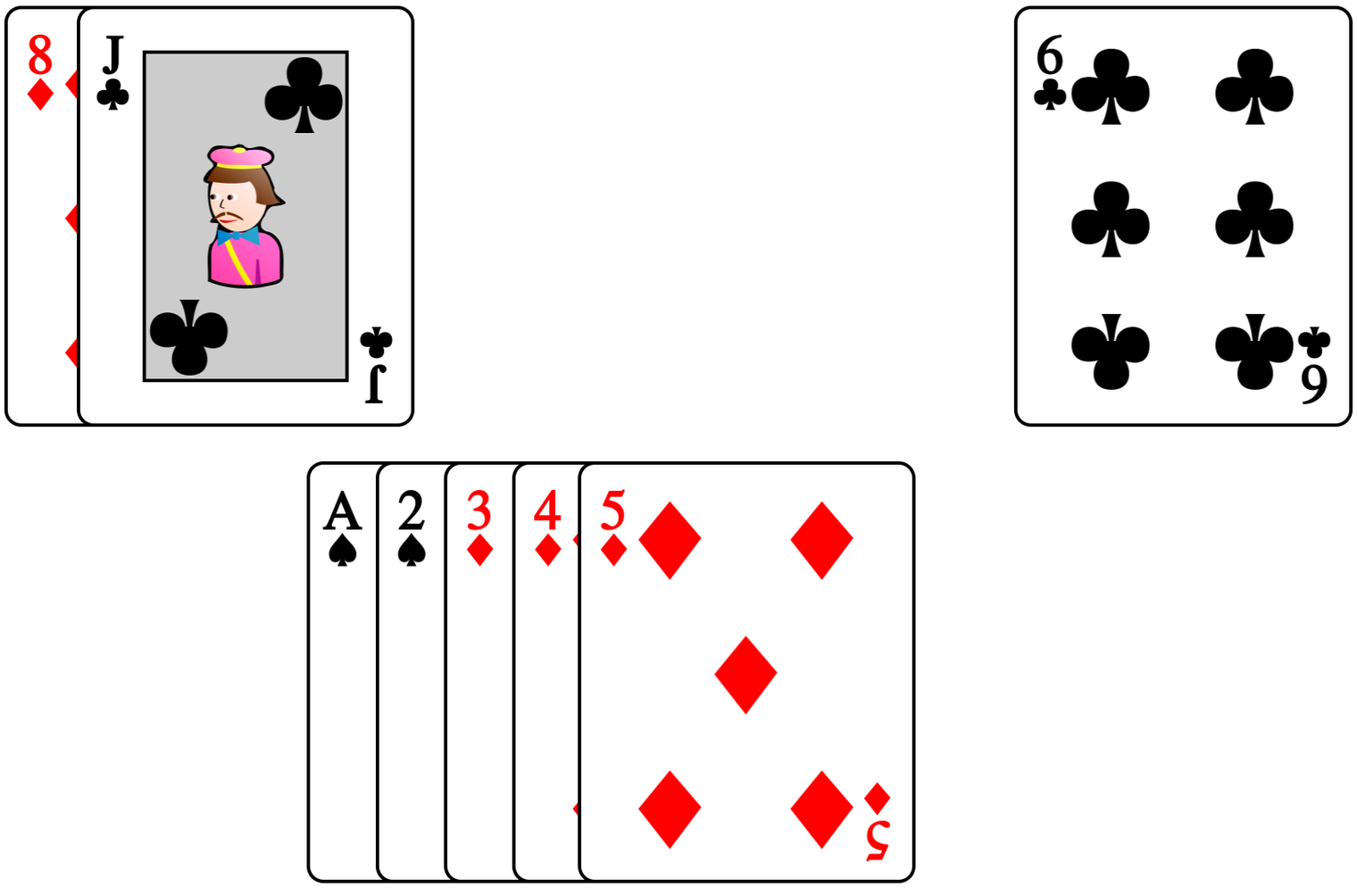
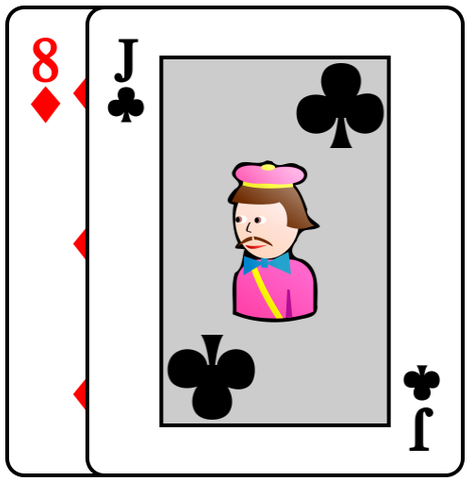
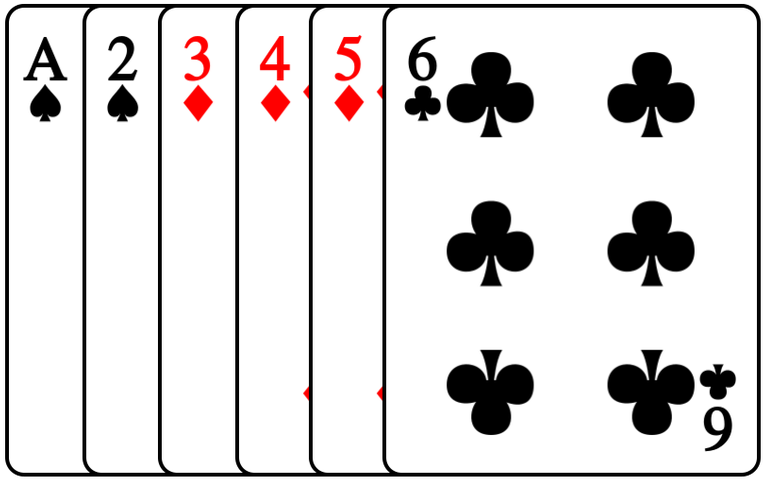


Fusionner

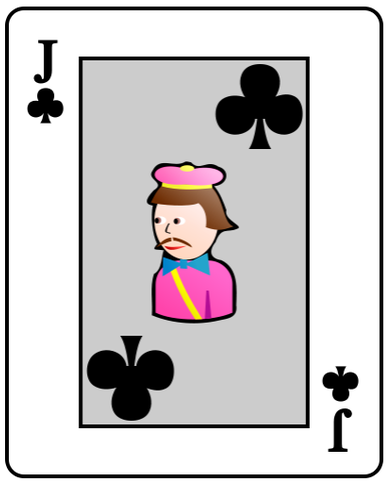


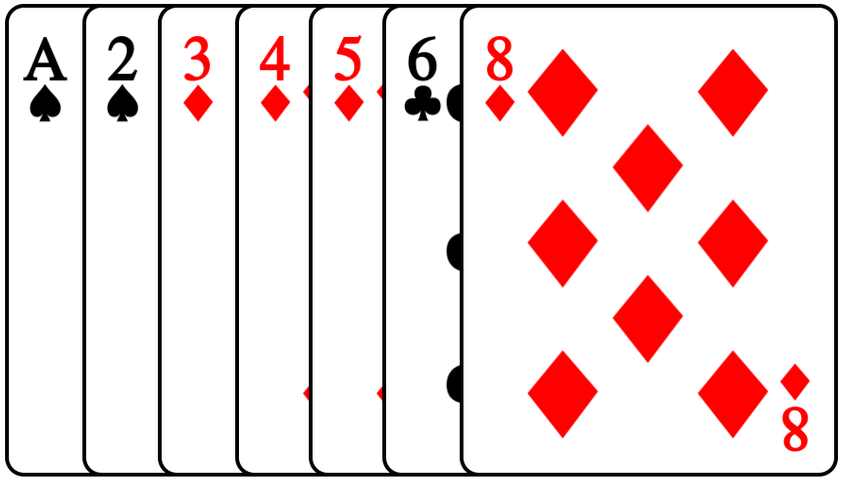
Fusionner



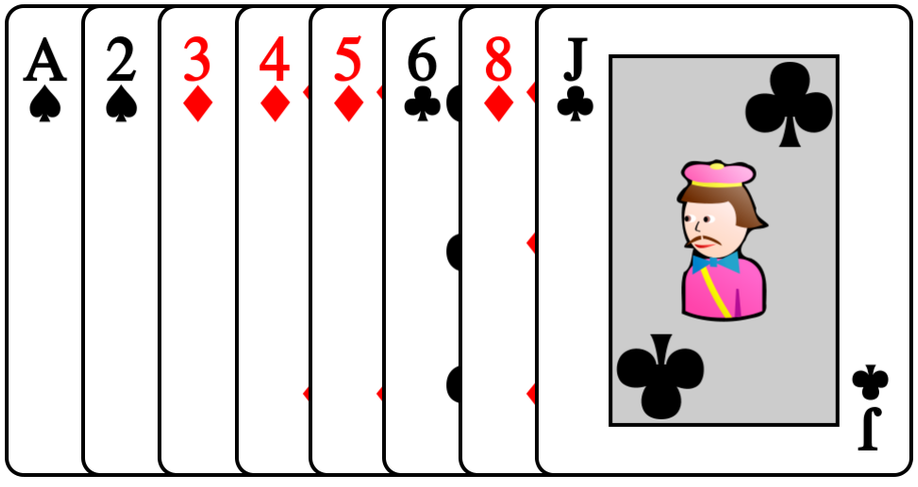


Fusionner

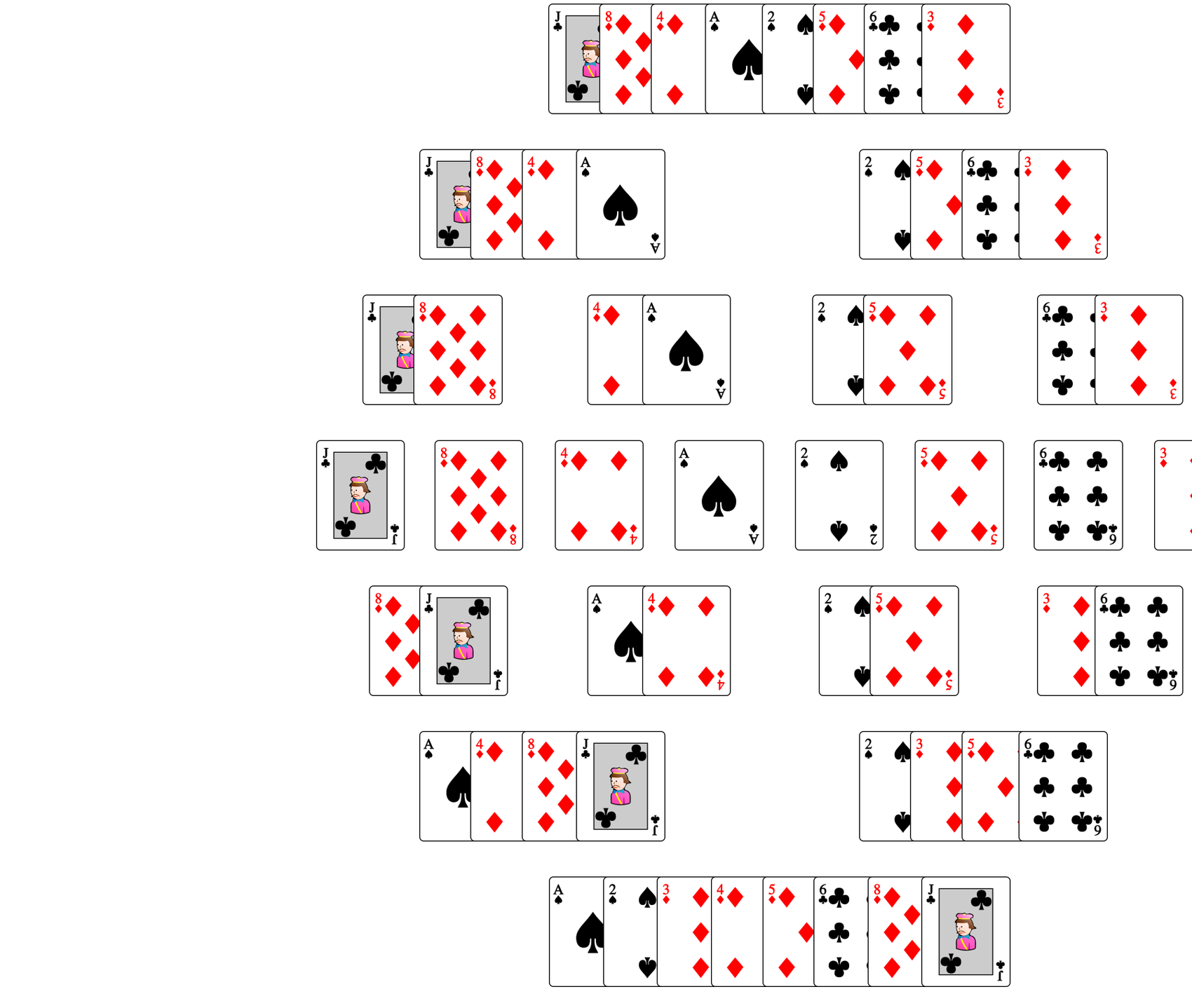




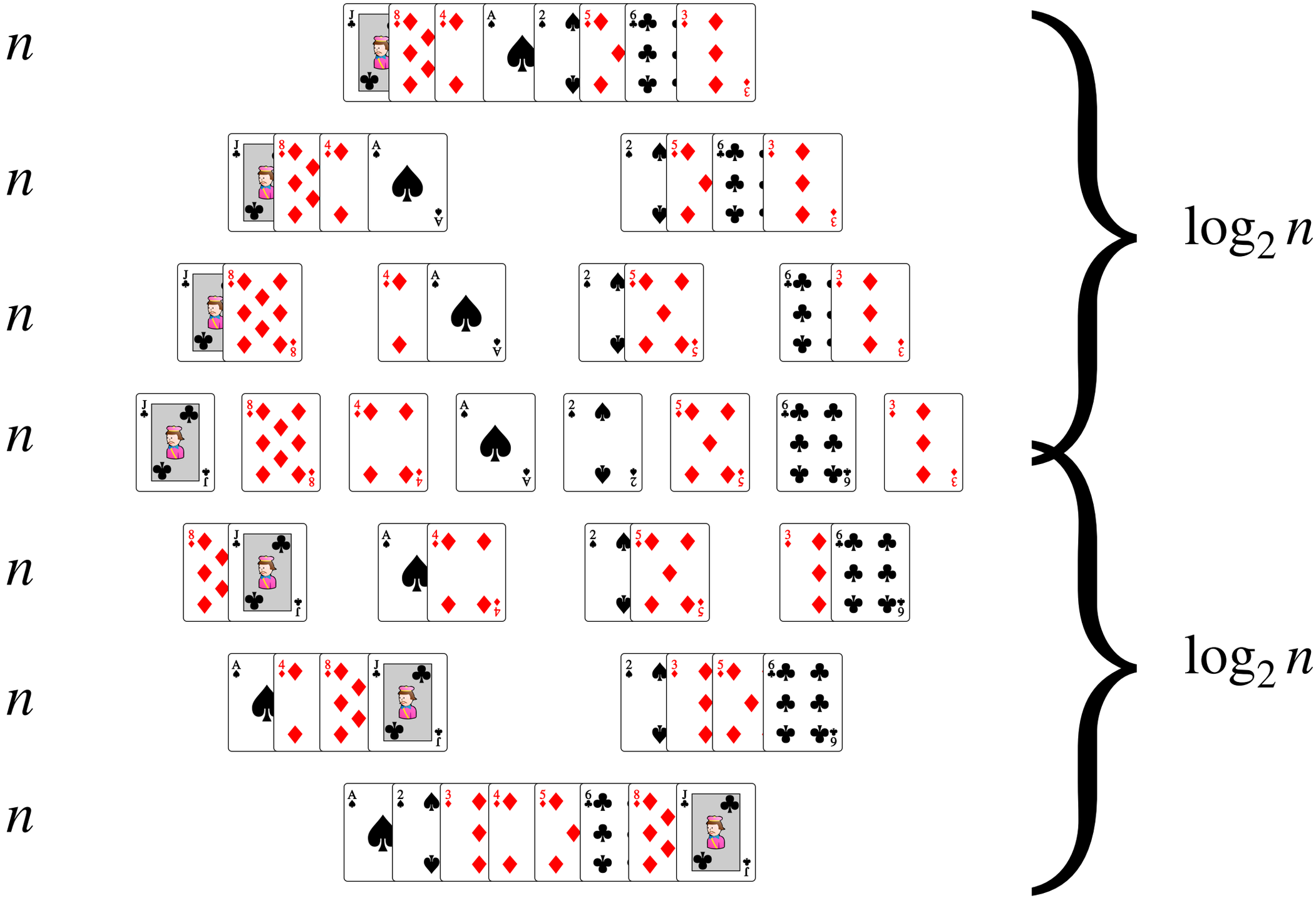
Le jeu est trié !



Efficacité du tri fusion



Efficacité du tri fusion



La complexité du tri fusion est

$$O(n \log_2 n)$$

Fusion

```
fonction fusionner(A, B)
    n ≔ longueur(A)
    m ≔ longueur(B)
    C ≔ tableau de longueur m + n
    i ≔ j ≔ k ≔ 0
    tant que i < n et j < m faire
        si A[i] < B[j] alors
            C[k] ≔ A[i]
            i ≔ i + 1
        sinon
            C[k] ≔ B[j]
            j ≔ j + 1
        fin si
        k ≔ k + 1
    fin tant que
    tant que i < n faire
        C[k] ≔ A[i]
        i ≔ i + 1
        k ≔ k + 1
    fin tant que
    tant que j < m faire
        C[k] ≔ B[j]
        j ≔ j + 1
        k ≔ k + 1
    fin tant que
    retourner C
fin fonction
```

encore d'éléments dans A et B

encore d'éléments dans A, mais B épuise

encore d'éléments dans B, mais A épuisé

A et B épuisés

Fusion

```
fonction fusionner(A, B)
    n ≔ longueur(A)
    m ≔ longueur(B)
    C ≔ tableau de longueur m + n
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        sinon
            C[k] ≔ B[j]
            j ≔ j + 1
```

encore d'éléments dans A et B

Fusion

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        fin si
        k ≔ k + 1
    fin tant que
    tant que i < n faire
        C[k] ≔ A[i]
        i ≔ i + 1
        k ≔ k + 1
    fin tant que
```

encore d'éléments dans A et B

encore d'éléments dans A, mais B épuise

```
        i ≔ i + 1
    sinon
        C[k] ≔ B[j]
        j ≔ j + 1
    fin si
    k ≔ k + 1
fin tant que
tant que i < n faire
    C[k] ≔ A[i]
    i ≔ i + 1
    k ≔ k + 1
fin tant que
tant que j < m faire
    C[k] ≔ B[j]
    j ≔ j + 1
    k ≔ k + 1
fin tant que
retourner C
fin fonction
```

encore d'éléments dans A et B

encore d'éléments dans A, mais B épuise

encore d'éléments dans B, mais A épuise

A et B épuisés

```
        k ≔ k + 1
    fin tant que
    tant que i < n faire
        C[k] ≔ A[i]
        i ≔ i + 1
        k ≔ k + 1
    fin tant que
    tant que j < m faire
        C[k] ≔ B[j]
        j ≔ j + 1
        k ≔ k + 1
    fin tant que
    retourner C
fin fonction
```

encore d'éléments dans A, mais B épuise

encore d'éléments dans B, mais A épuise

A et B épuisés

```
tant que i < n faire
    C[k] ≔ A[i]
    i ≔ i + 1
    k ≔ k + 1
fin tant que
tant que j < m faire
    C[k] ≔ B[j]
    j ≔ j + 1
    k ≔ k + 1
fin tant que
    retourner C
fin fonction
```

encore d'éléments dans A, mais B épuise

encore d'éléments dans A, mais B épuise

A et B épuisés

Fusion

```
fonction fusionner(A, B)
    n ≔ longueur(A)
    m ≔ longueur(B)
    i ≔ j ≔ k ≔ 0
    tant que i < n et j < m faire
        si A[i] < B[j] alors
            C[k] ≔ A[i]
            i ≔ i + 1
        sinon
            C[k] ≔ B[j]
            j ≔ j + 1
        fin si
        k ≔ k + 1
    fin tant que
    tant que i < n faire
        C[k] ≔ A[i]
        i ≔ i + 1
        k ≔ k + 1
    fin tant que
    tant que j < m faire
        C[k] ≔ B[j]
        j ≔ j + 1
        k ≔ k + 1
    fin tant que
    retourner C
fin fonction
```

encore d'éléments dans A et B

encore d'éléments dans A, mais B épuise

encore d'éléments dans B, mais A épuisé

A et B épuisés

Tri fusion

```
fonction tri-fusion(A)
    n ≔ longueur(A)
    si n > 1 alors
        m ≔ ⌊n ÷ 2⌋
        B ≔ A[0, …, m–1]
        C ≔ A[m, …, n–1]
        B’ ≔ tri-fusion(B)
        C’ ≔ tri-fusion(C)
        retourner fusionner(B’, C’)
    sinon
        retourner A
    fin si
fin fonction
```