Introduction à l’informatique CM4

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Opérations sur les tableaux

Ajouter des éléments au fond d'un tableau

```
def copier(A):
    n = len(A)
    B = []
    for i in range(n):
        B.append(A[i])
    return B
```

Ajouter des éléments au fond d'un tableau

```
def copier(A):
    n = len(A)
    B = []
    for i in range(n):
        B.append(A[i])
    return B
```

Parcourir un tableau au contraire

```
def inverser(A):
    n = len(A)
    B = []
    for i in reversed(range(n)):
        B.append(A[i])
    return B
```

Parcourir un tableau au contraire

```
def inverser(A):
    n = len(A)
    B = []
    for i in reversed(range(n)):
        B.append(A[i])
    return B
```

intervalle
n-1, n-2, …, 0

Créer un tableau de taille donnée

```
def intervalle(n):
    A = [0 for i in range(n)]
    for i in range(n):
        A[i] = i
    return A
```

Créer un tableau de taille donnée

```
def intervalle(n):
    A = [0 for i in range(n)]
    for i in range(n):
        A[i] = i
    return A
```

tableau de longueur `n` rempli de `0`

Matrices

```
M = [[1,  2,  3,  4],
     [5,  6,  7,  8],
     [9, 10, 11, 12]]
```

```
>>> M[0]
[1, 2, 3, 4]
>>> M[1]
[5, 6, 7, 8]
>>> M[1][2]
7
>>> M[0][3]
4
```

Matrices

une matrice
est un tableau
de tableaux

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M = [[1,  2,  3,  4],
     [5,  6,  7,  8],
     [9, 10, 11, 12]]
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Matrices

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M = [[1,  2,  3,  4],
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une matrice est un tableau de tableaux

accéder à une ligne de la matrice

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Matrices

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une matrice est un tableau de tableaux

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>>> M[0]
[1, 2, 3, 4]
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7
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4
```

accéder à une ligne de la matrice

accéder à un élément spécifique

Algorithmes de tri

Algorithmes de tri pour accélérer la recherche dans un tableau

- La recherche dans un tableau non trié prend temps $O(n)$ avec la recherche séquentielle (ou linéaire)

- Par contre, on peut faire une recherche dichotomique dans un tableau trié en temps $O(\log_2 n)$

- Donc ça vaut la peine de trier le tableau si on a beaucoup de recherches à faire

Algorithmes de tri dans le commerce électronique

amazonie.fr

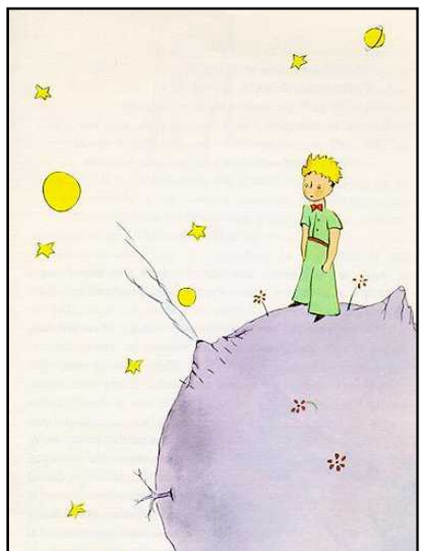
amazonie

Chercher : Le Petit Prince

Résultats 1–20 sur 928572785 pour « Le Petit Prince »

Trier par :

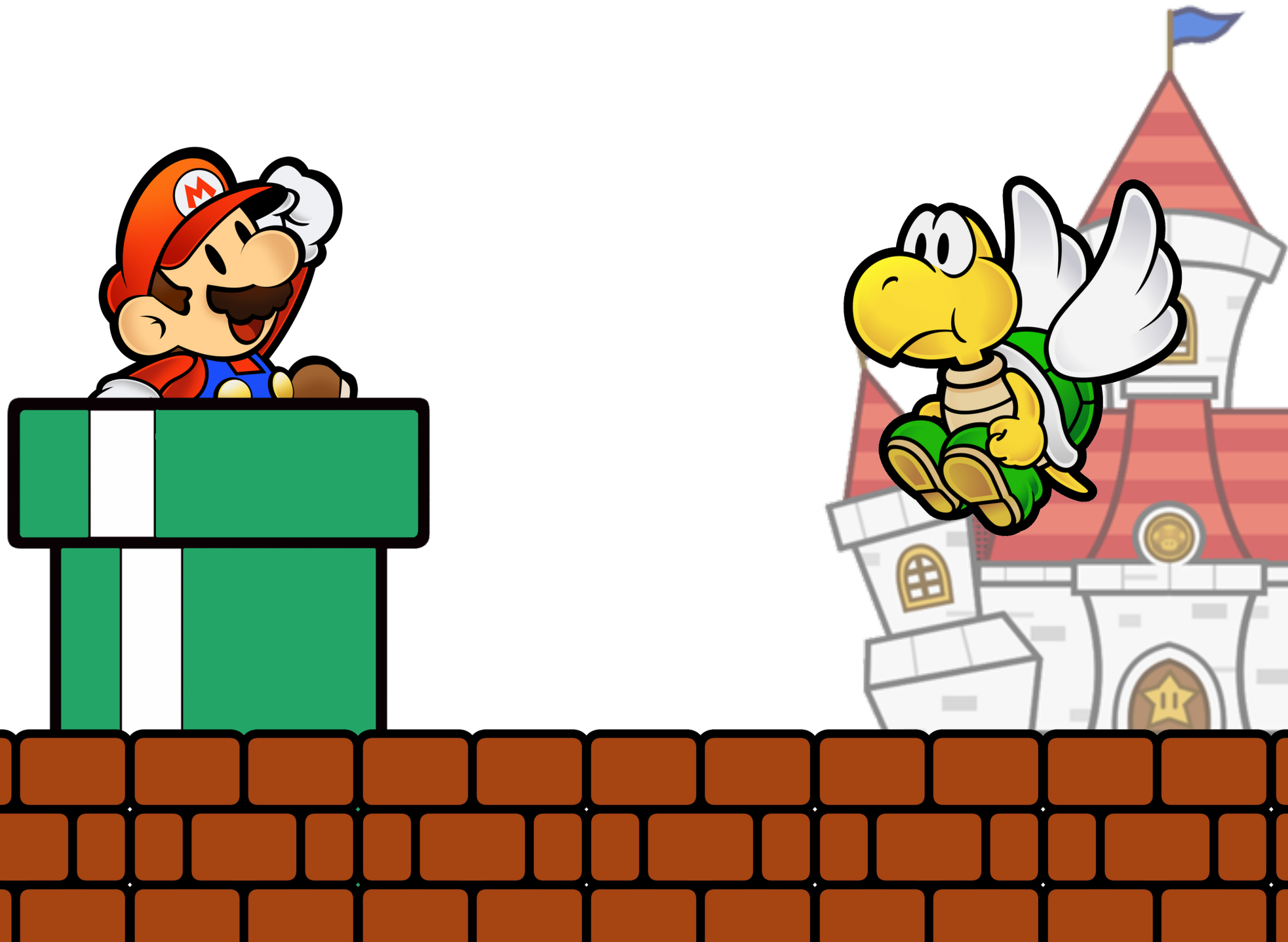
- prix croissant
- prix décroissant
- note moyenne
- nouveauté



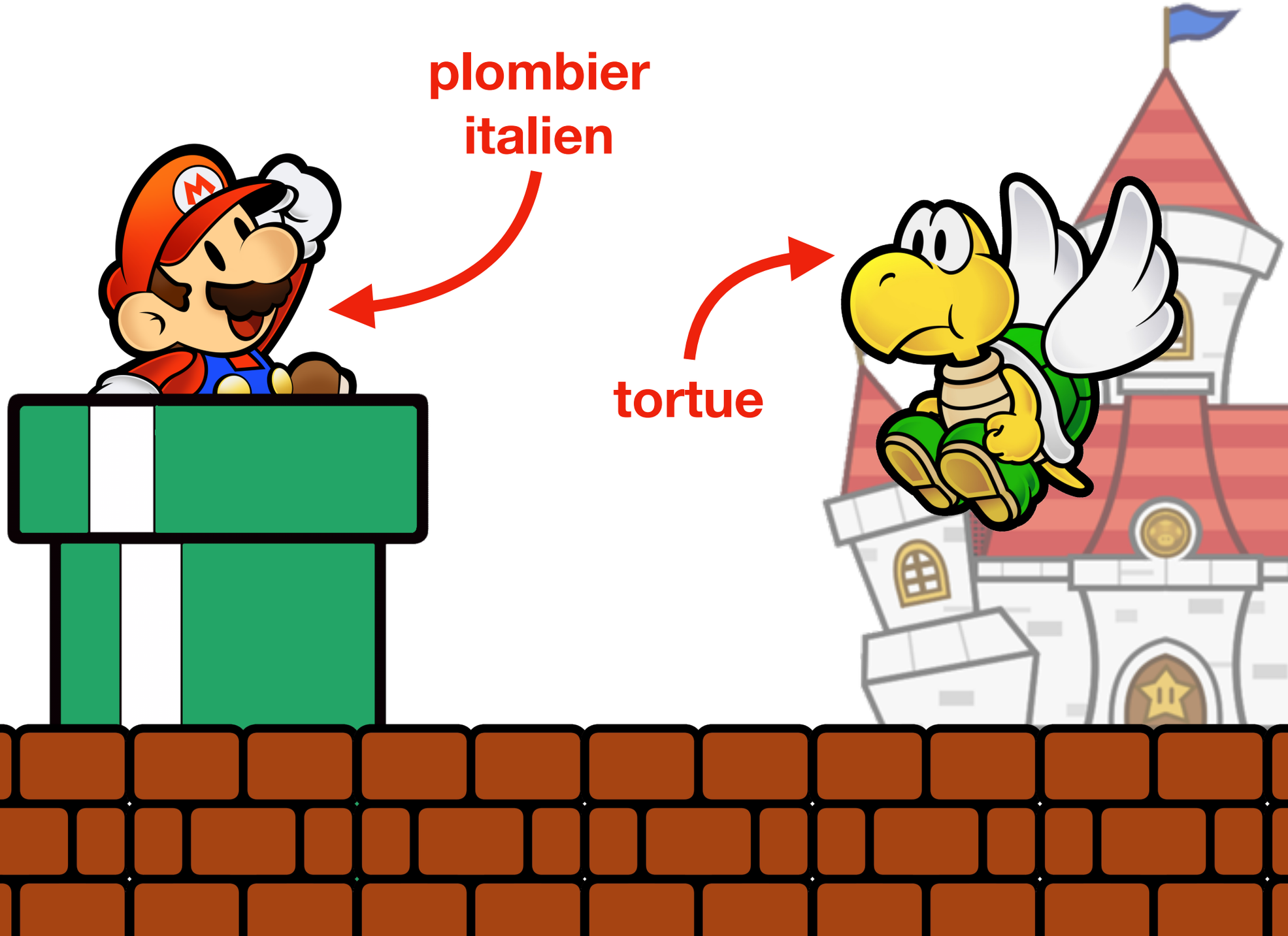
Le Petit Prince
de Antoine de Saint-Exu

Format poche 6,90 €
Format Kinder 6,49 €

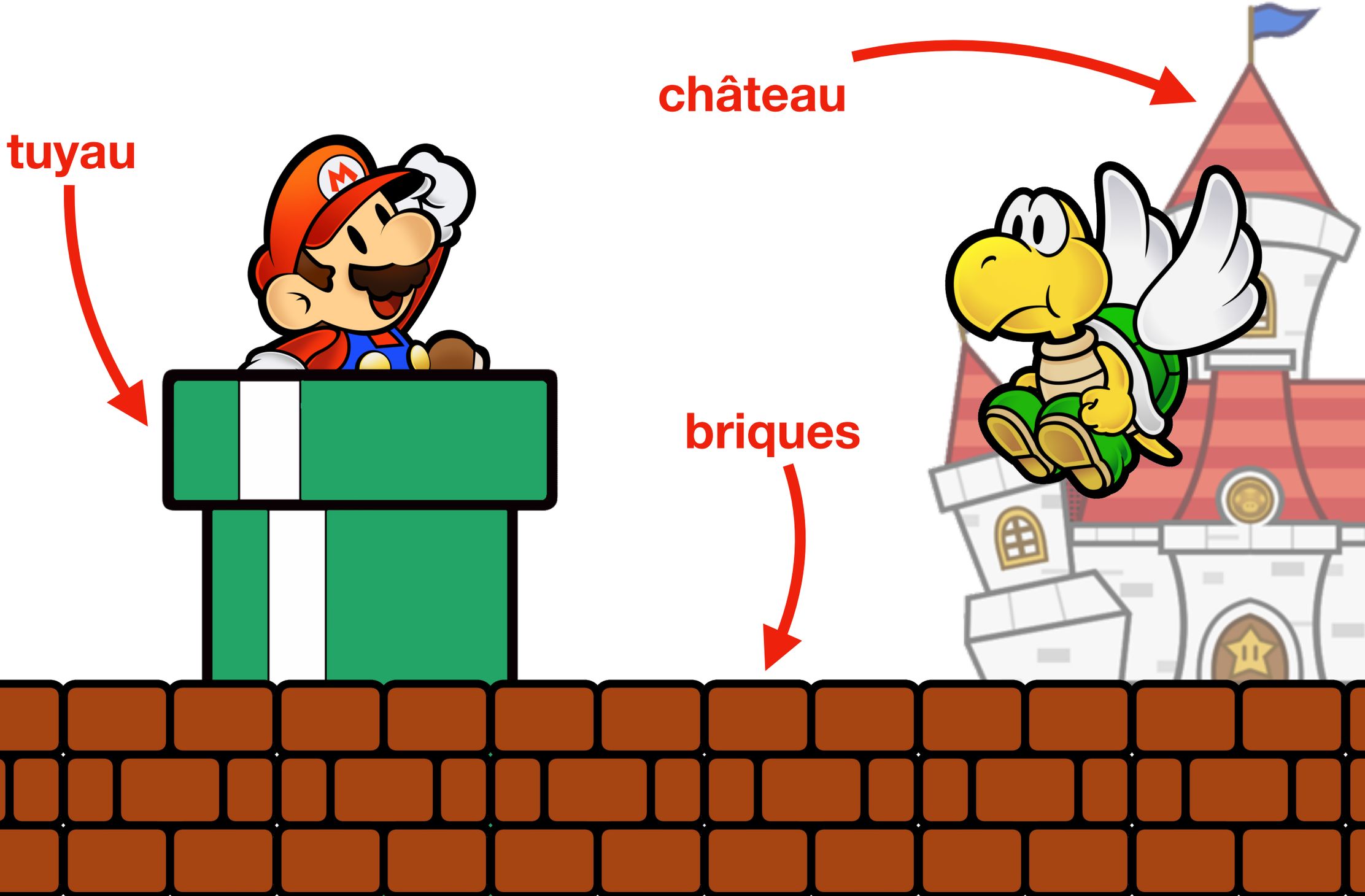
Algos de tri dans le jeux vidéo « Super Plombiers Italiens »



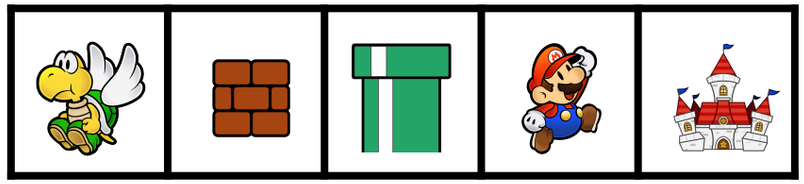
Algos de tri dans le jeux vidéo « Super Plombiers Italiens »



Algos de tri dans le jeux vidéo « Super Plombiers Italiens »



Affichage des objets dans le mauvais ordre



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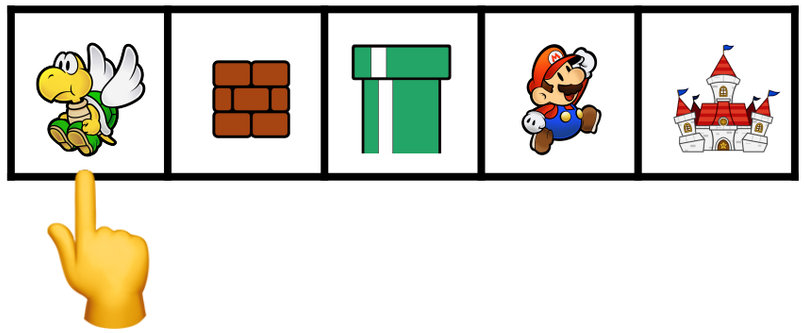

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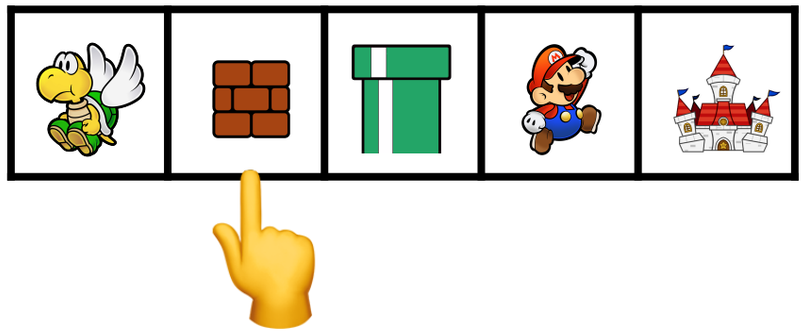

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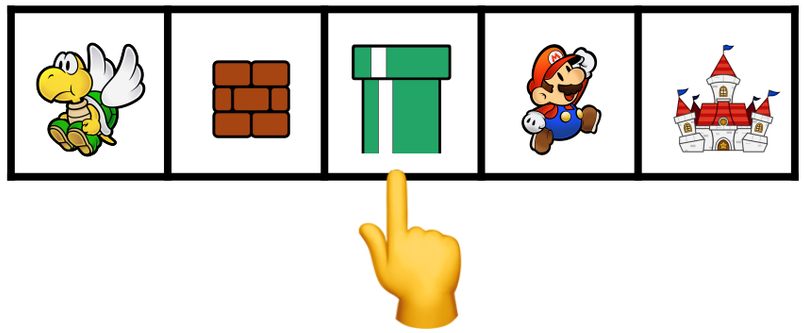

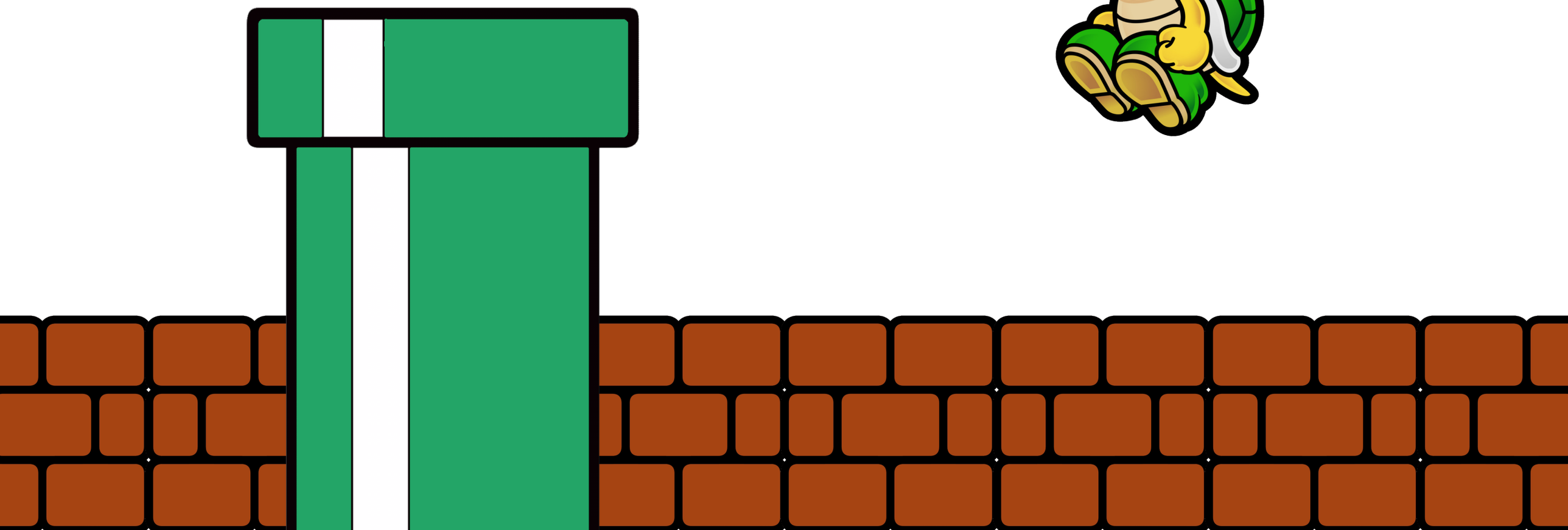

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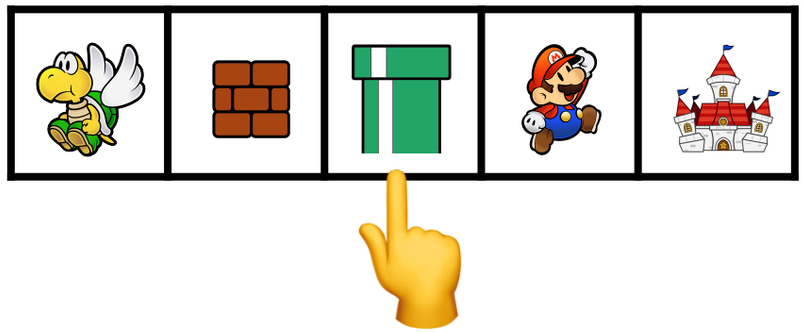

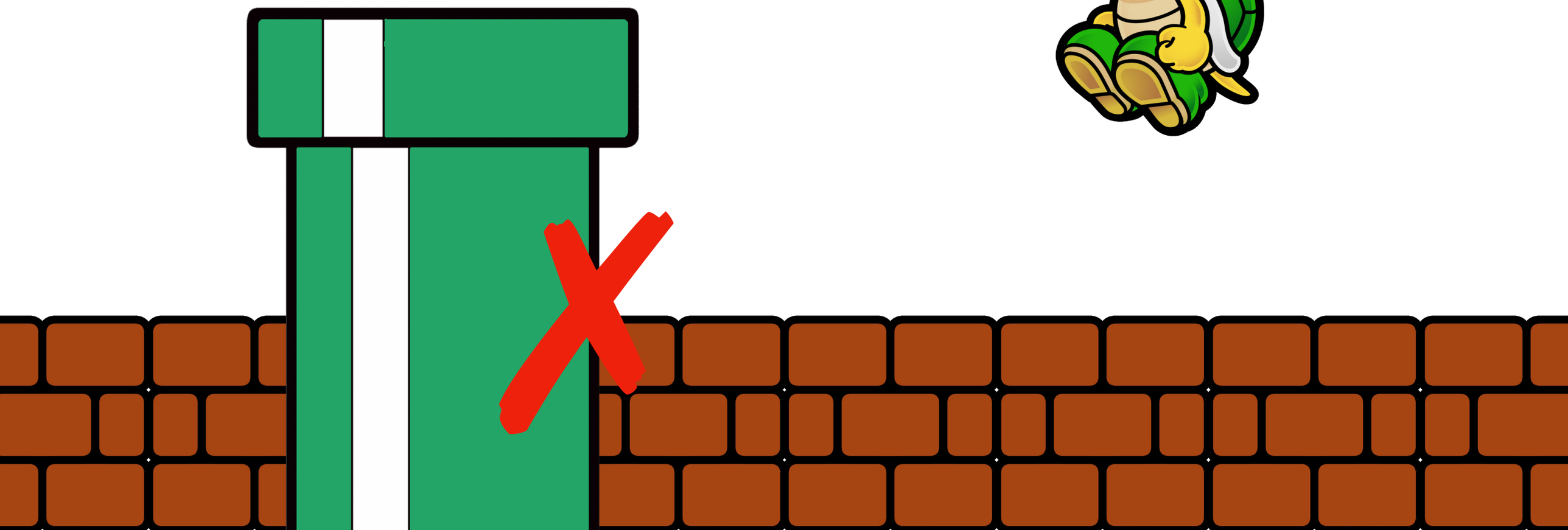

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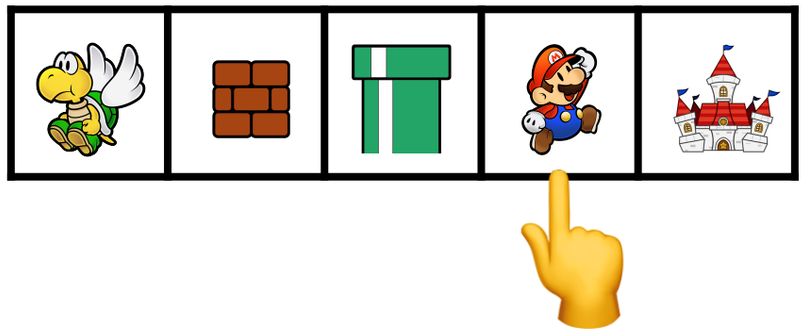

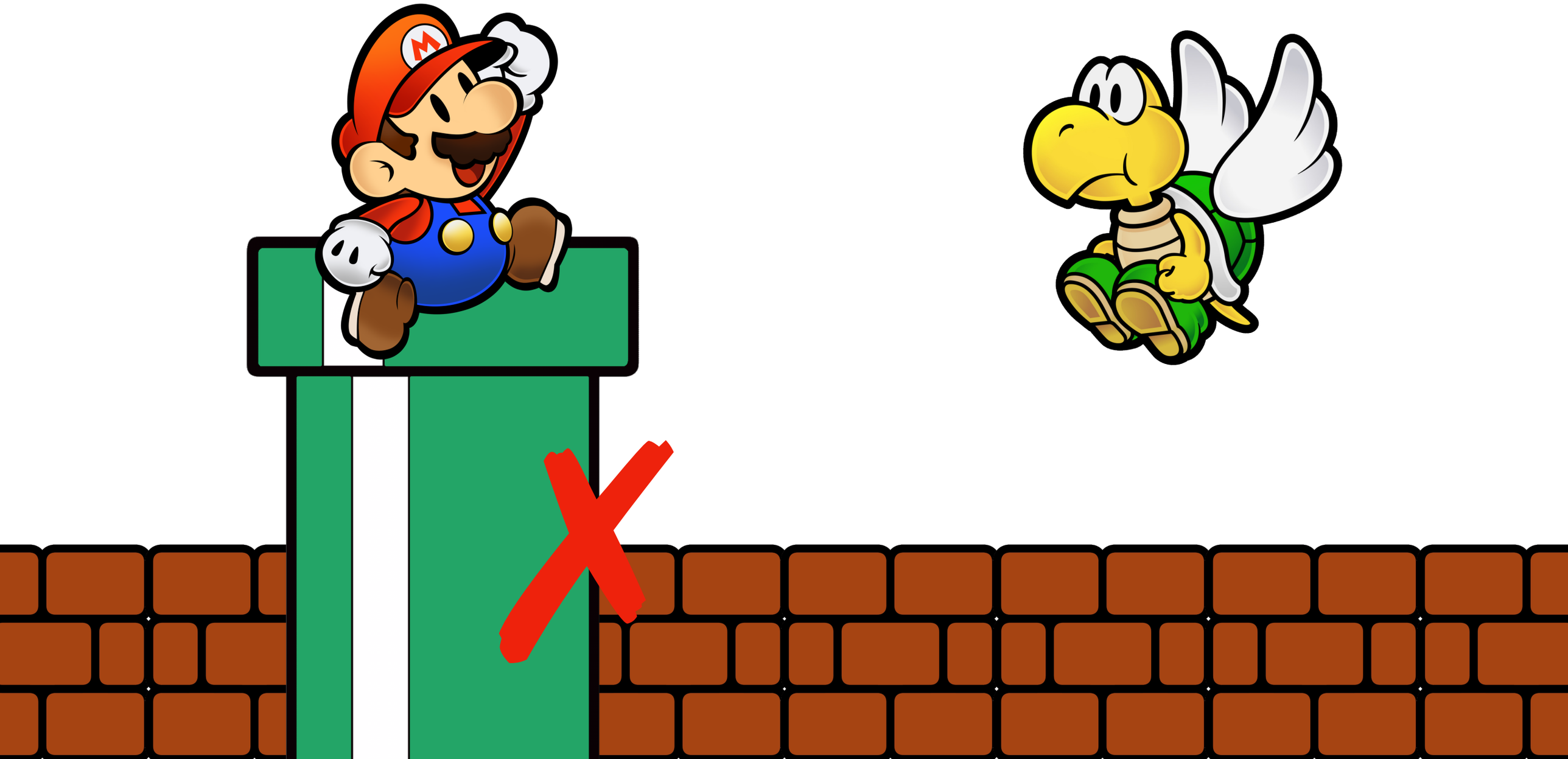

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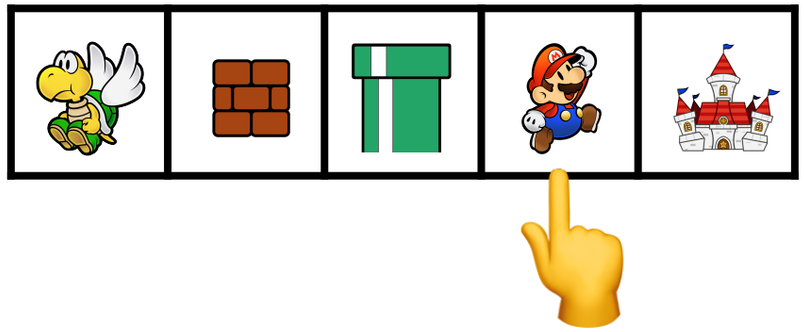

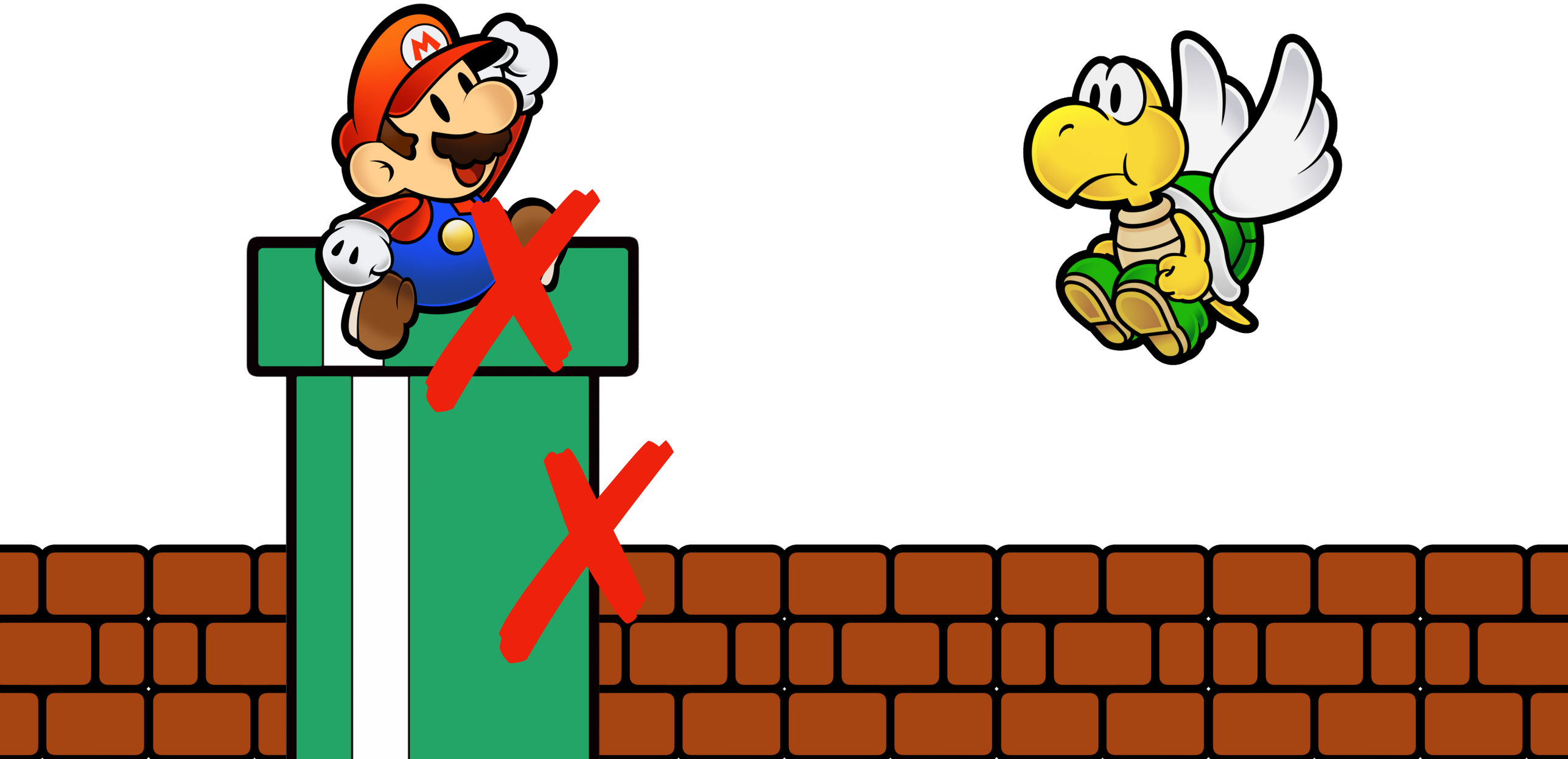

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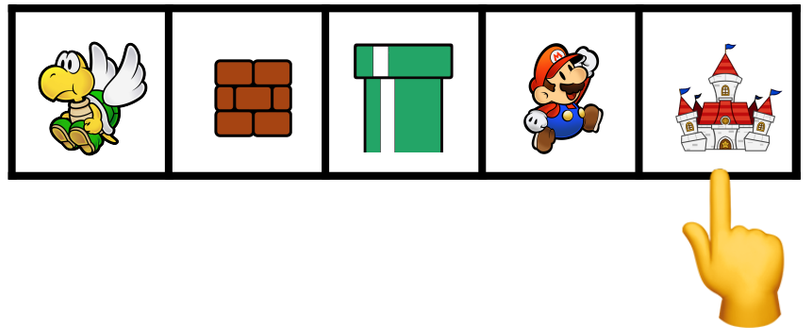

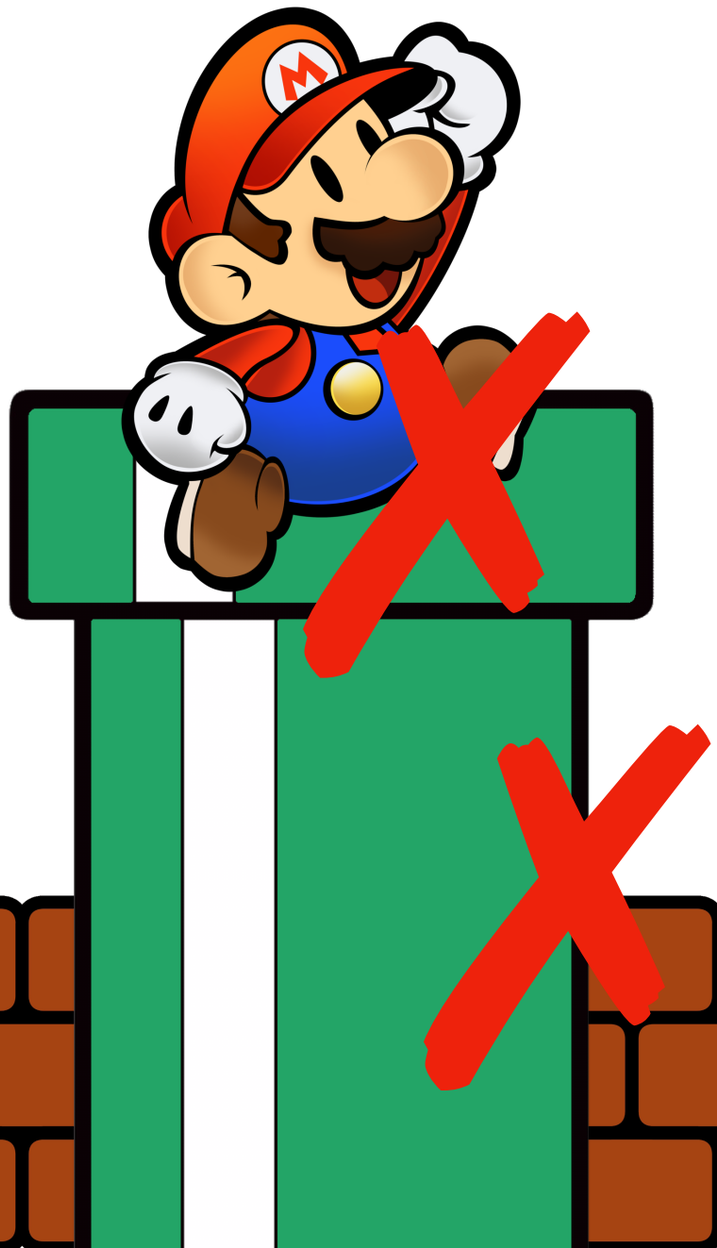

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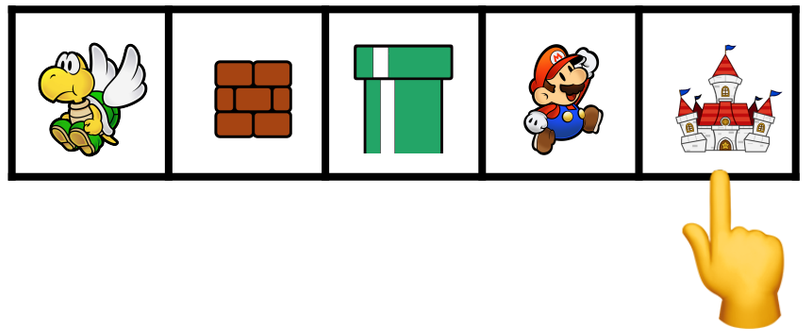

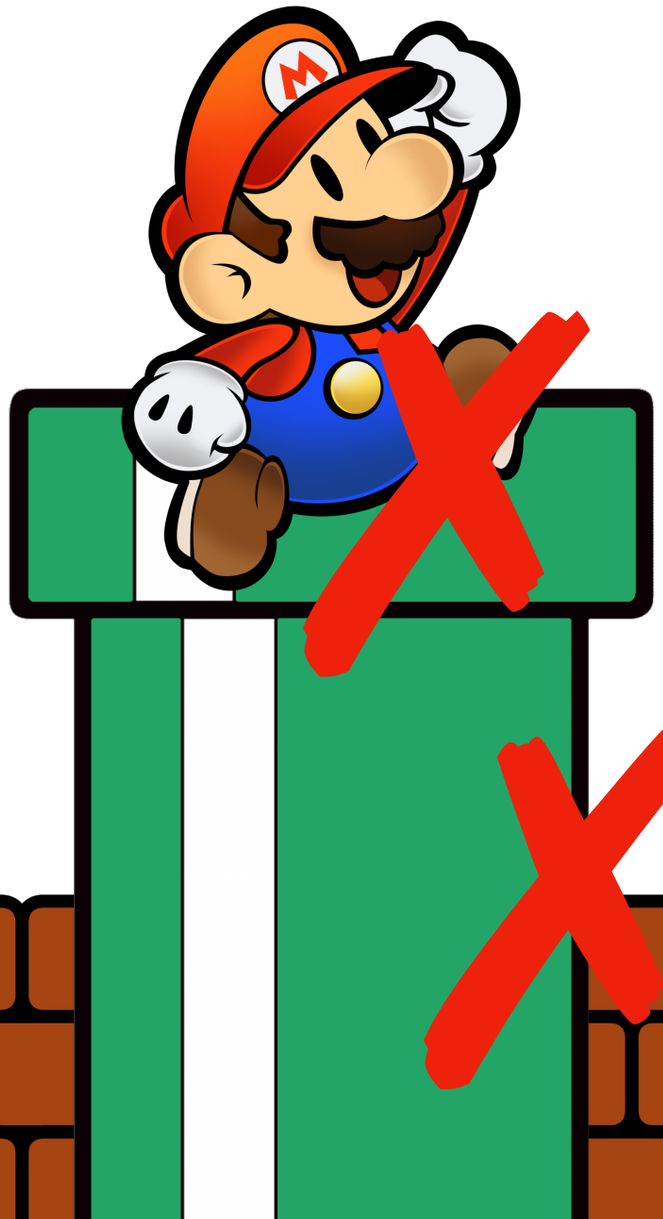

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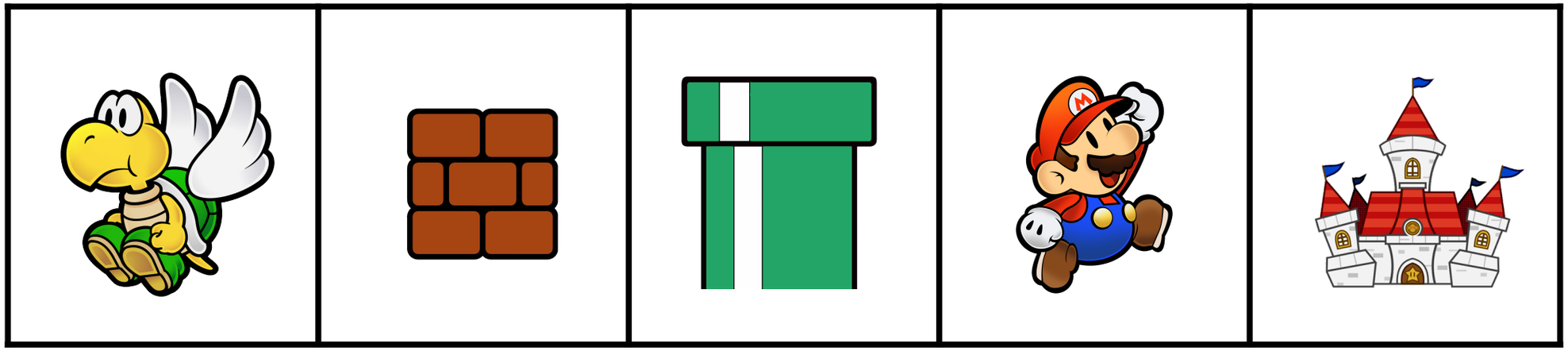
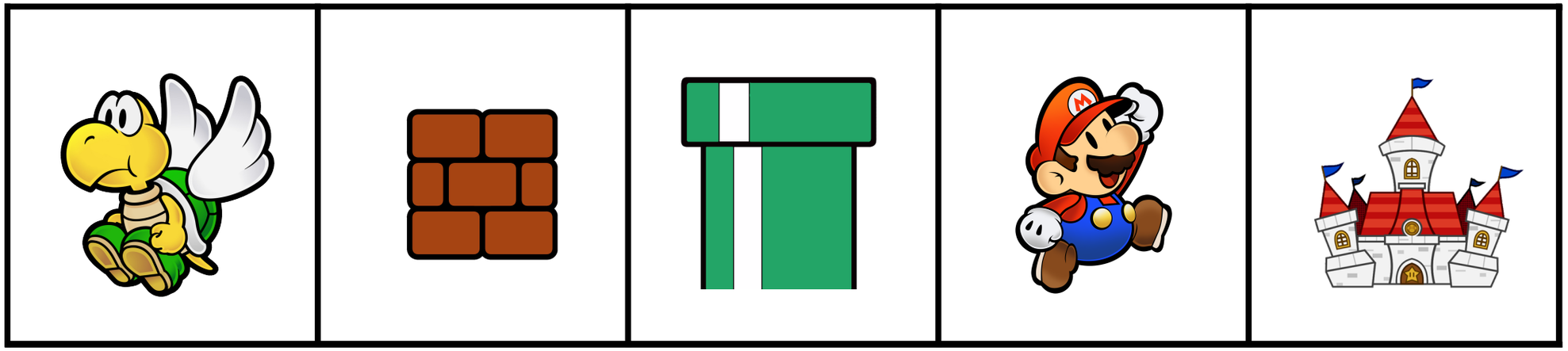


0 1 2 3 4

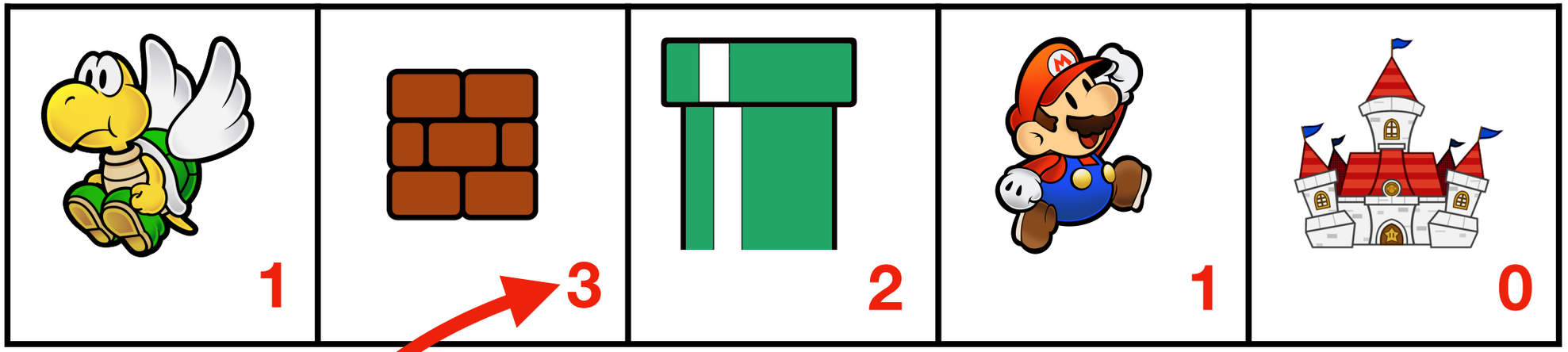
Affichage des objets dans le mauvais ordre



0 1 2 3 4

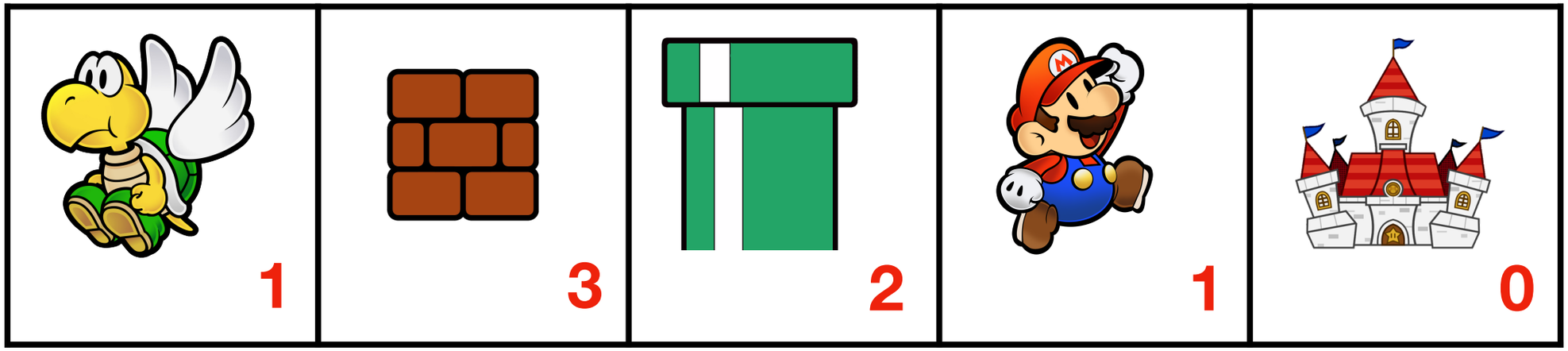
ordre d’affichage

Affichage des objets dans le mauvais ordre



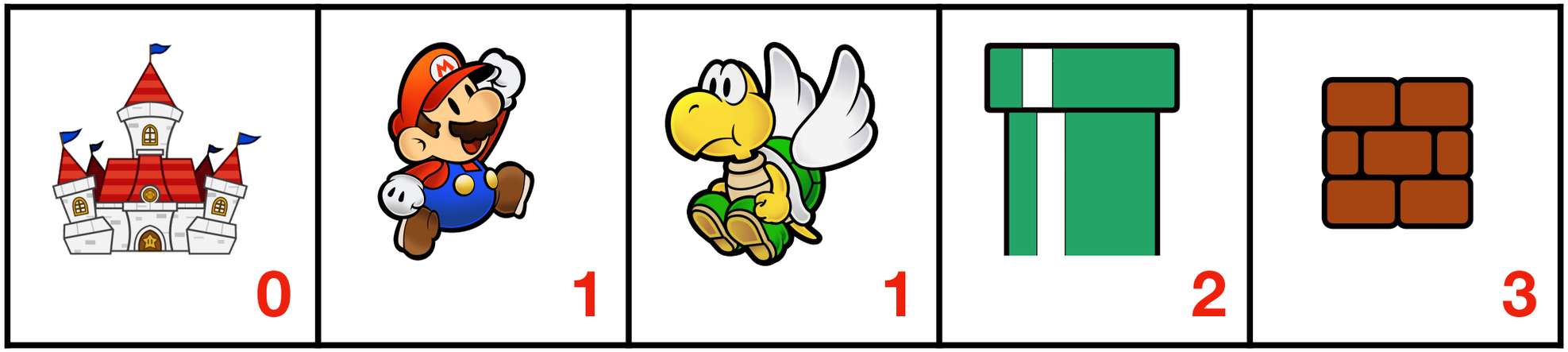
Tri ! 

Affichage des objets dans le mauvais ordre



0 1 2 3 4

Affichage des objets dans le bon ordre



0 1 2 3 4

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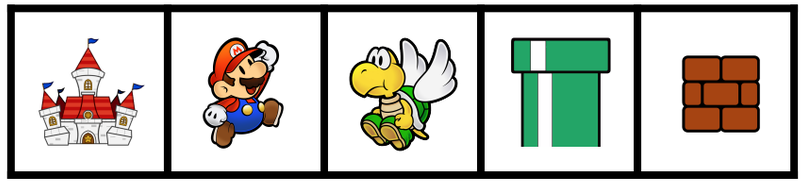

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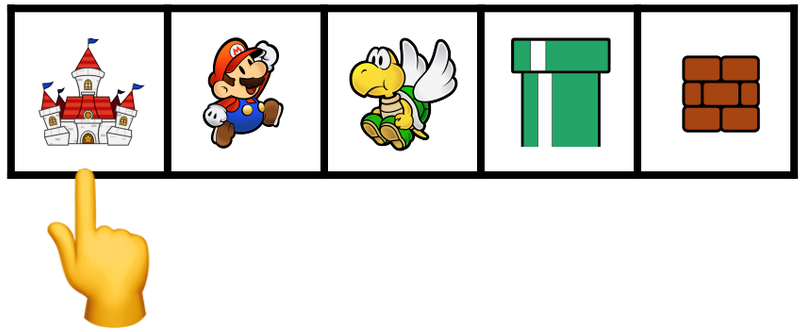

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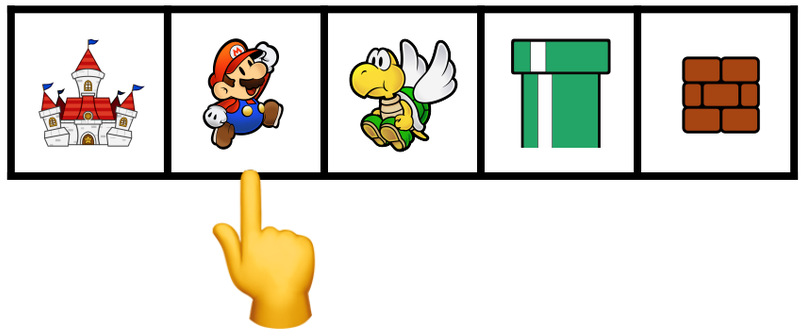

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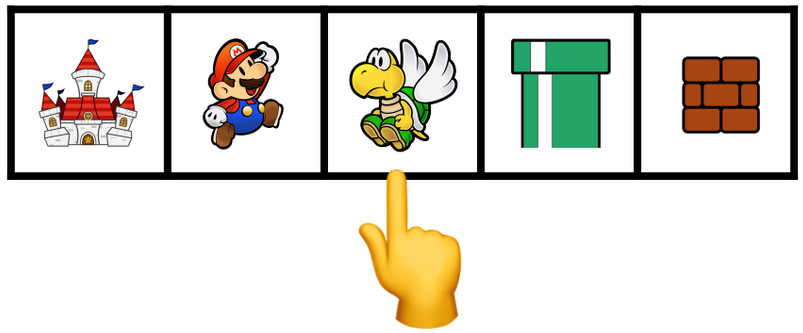

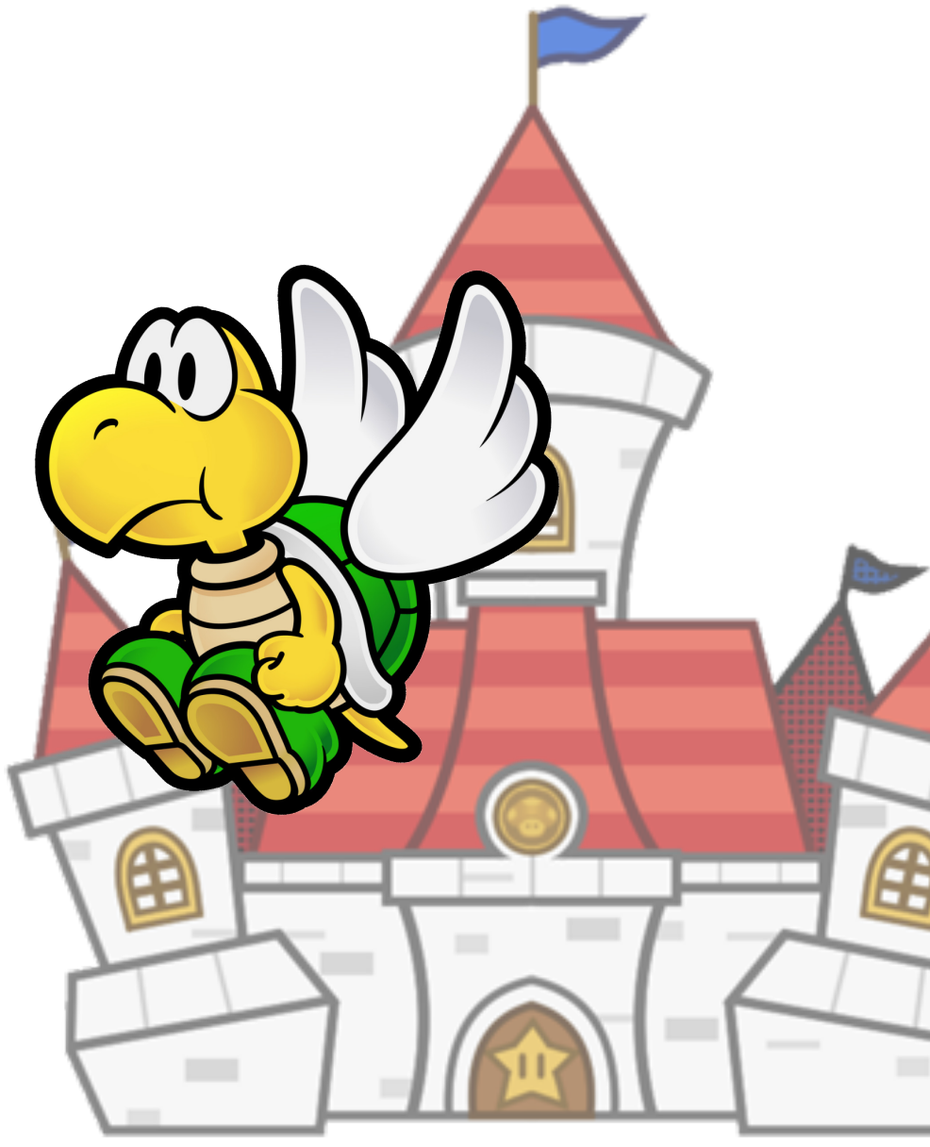

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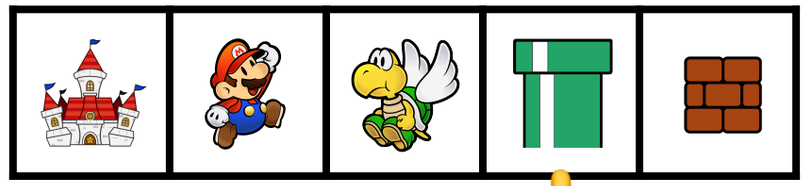

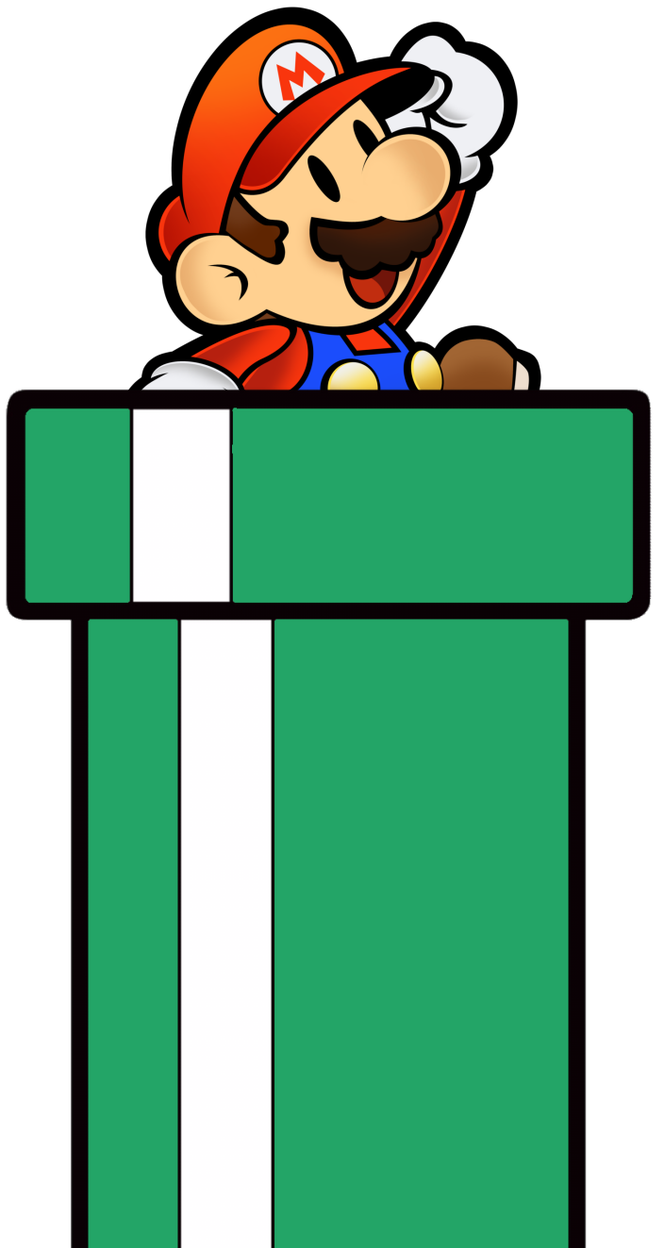

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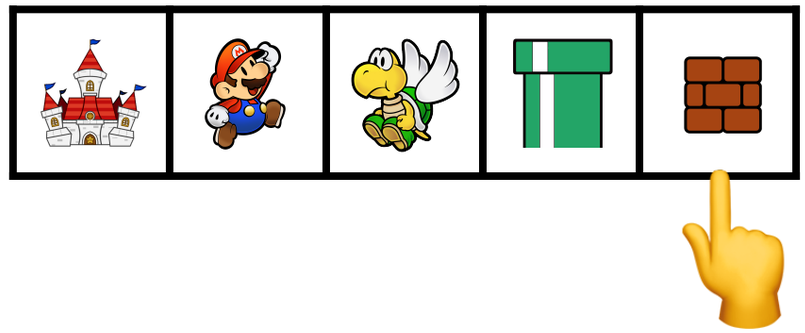

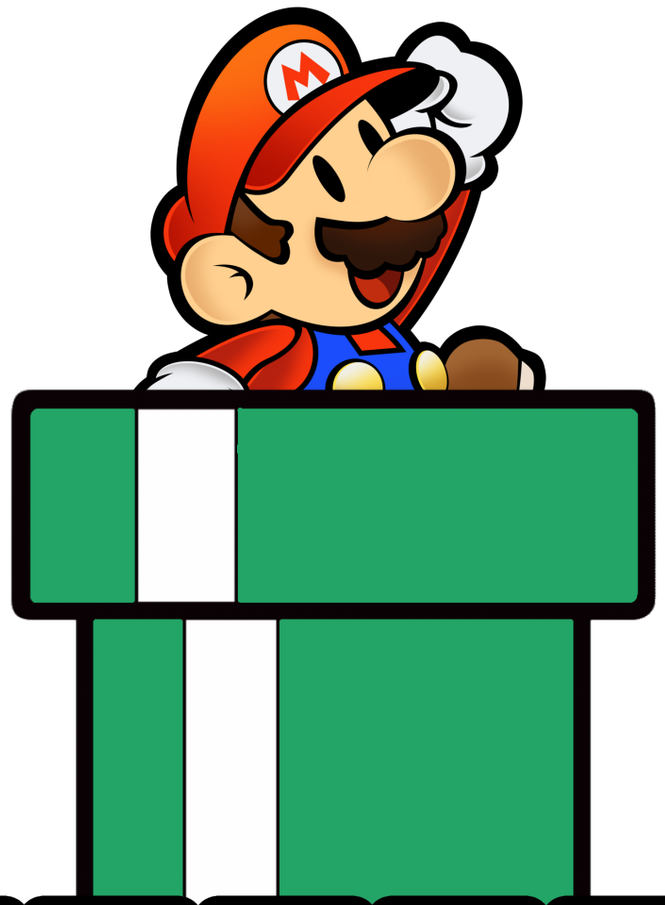

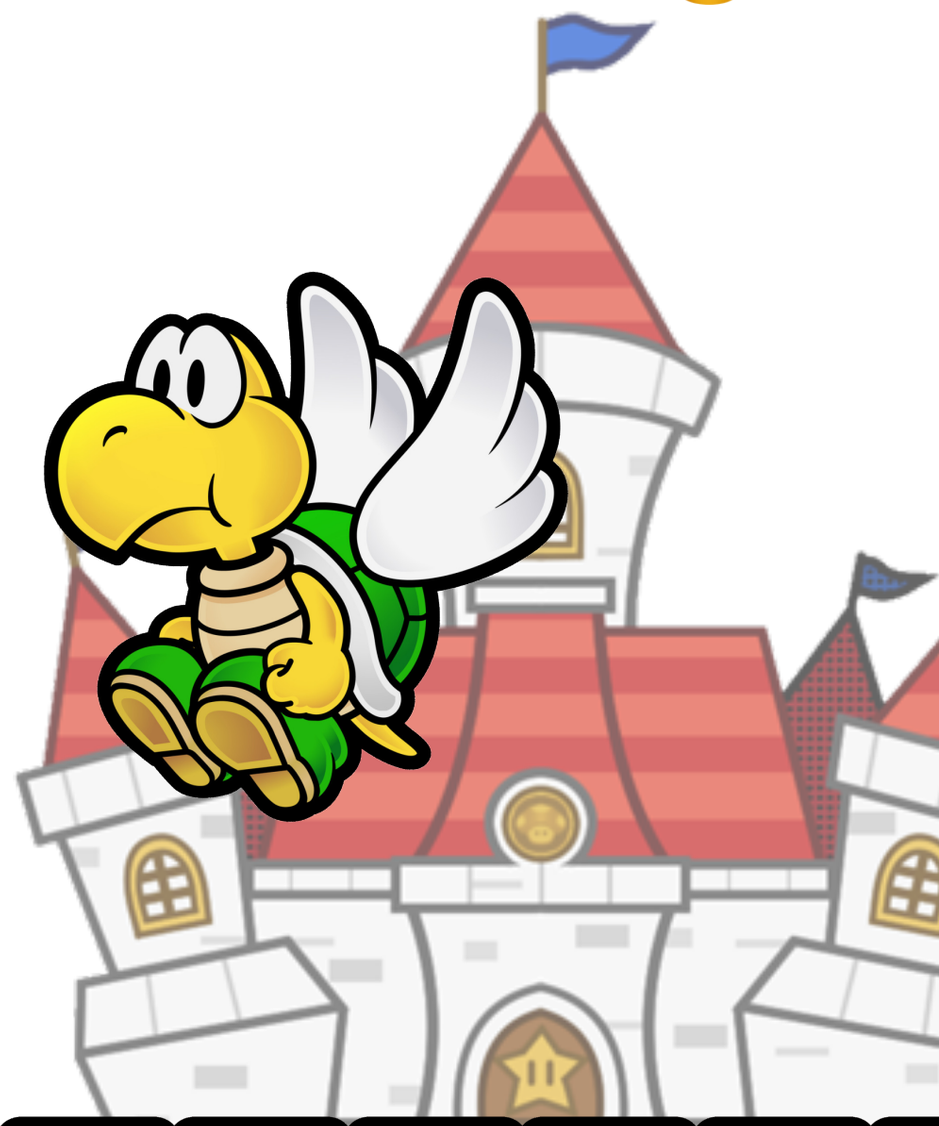

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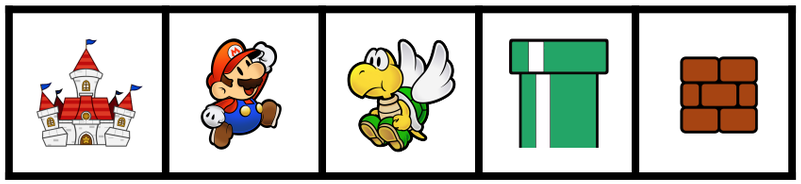
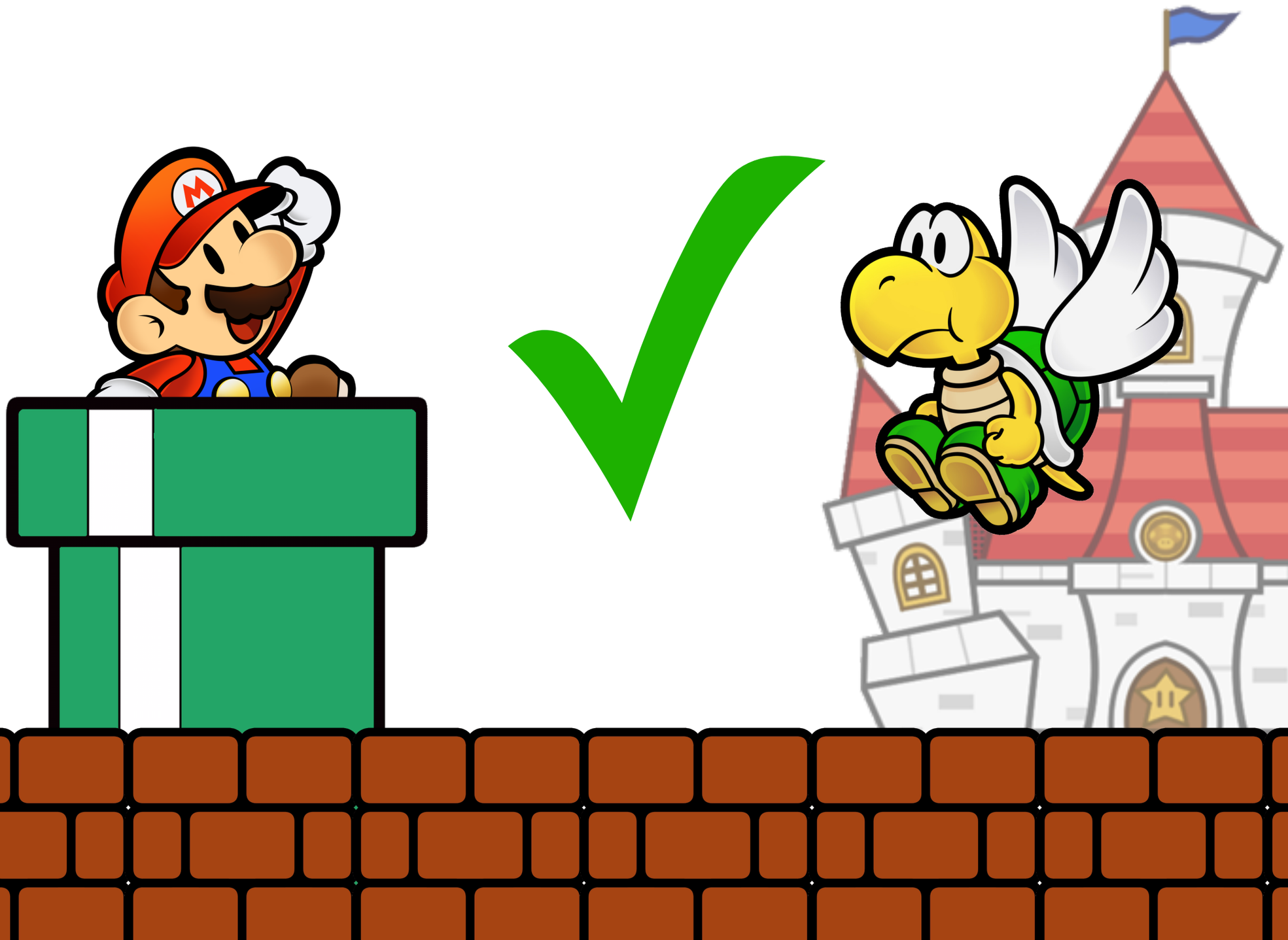
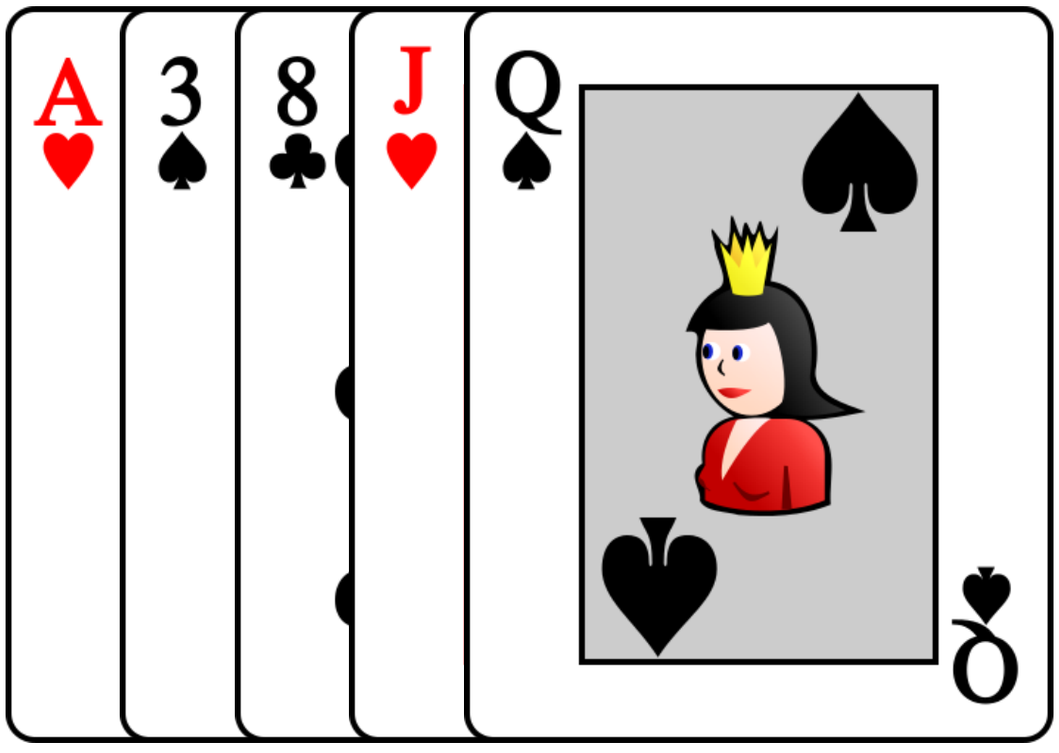


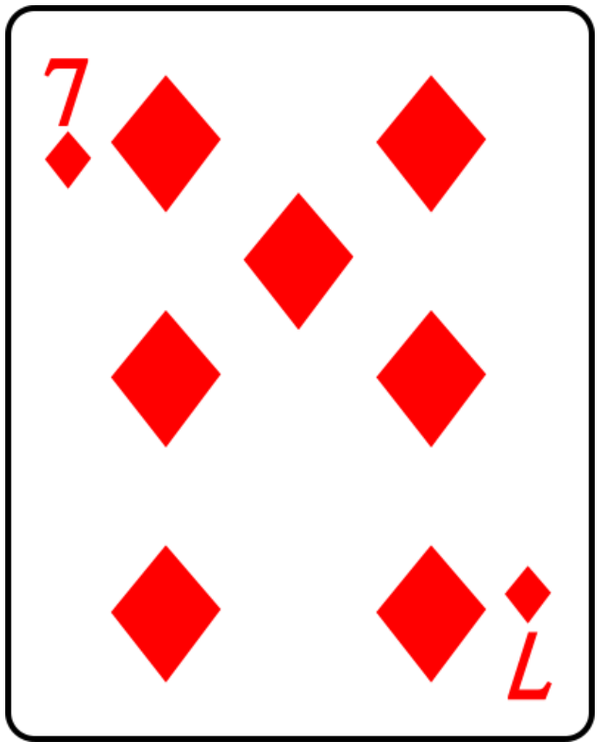


Comment trier un jeu de cartes ?



Tri par insertion

Tri par insertion



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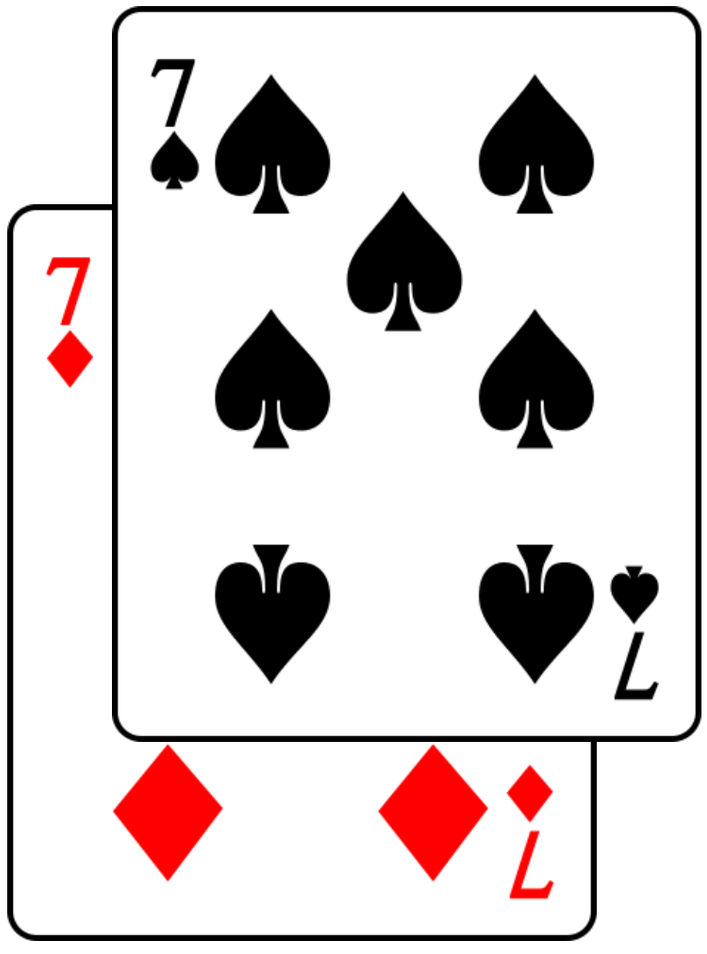

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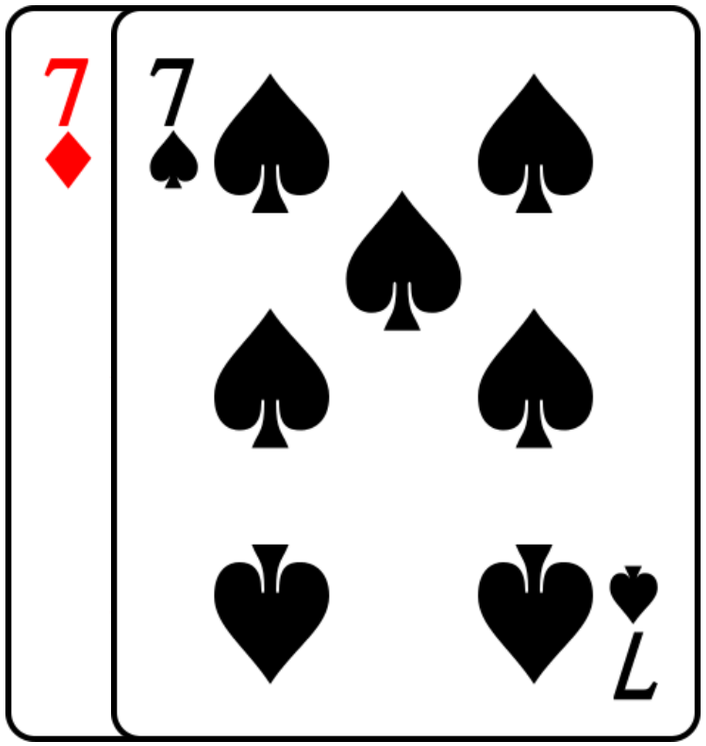

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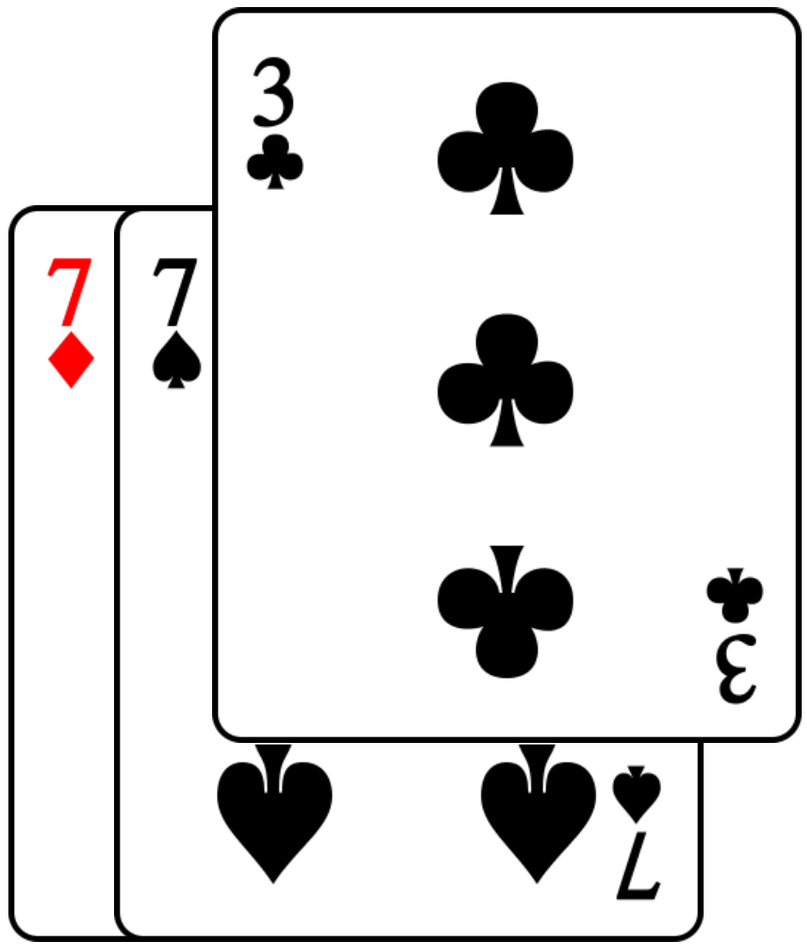

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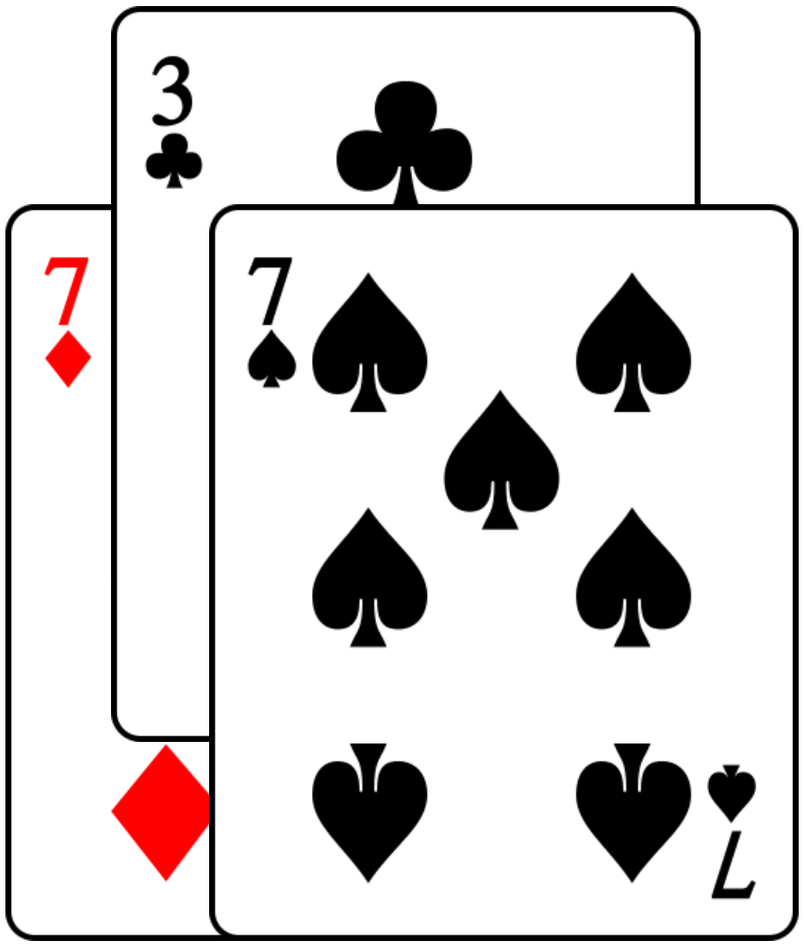

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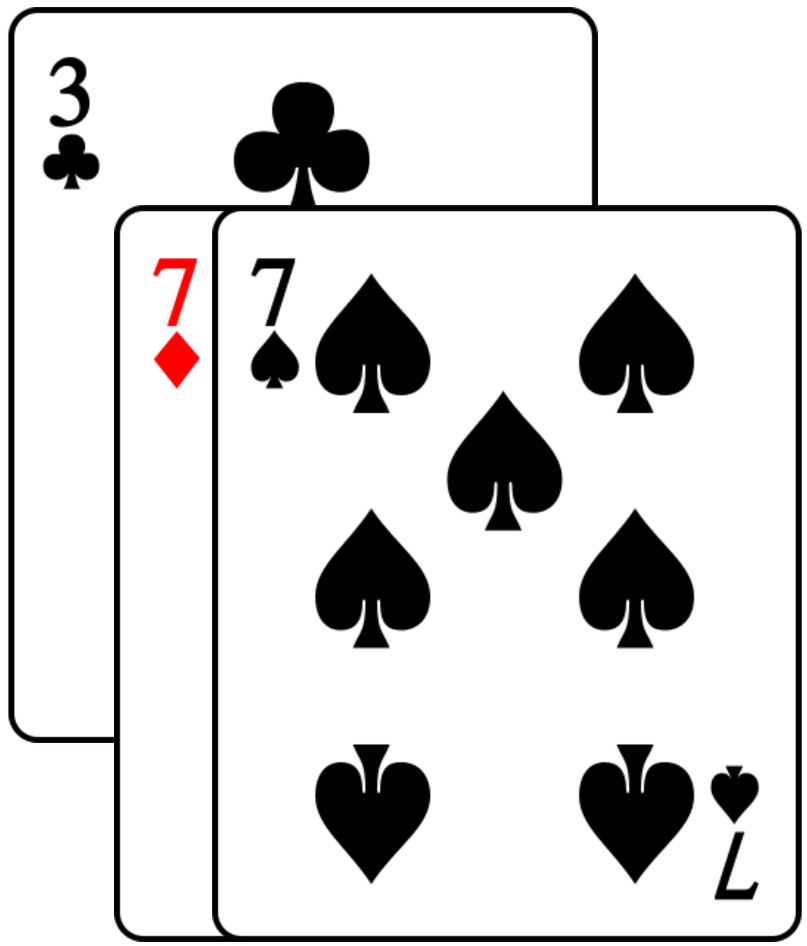

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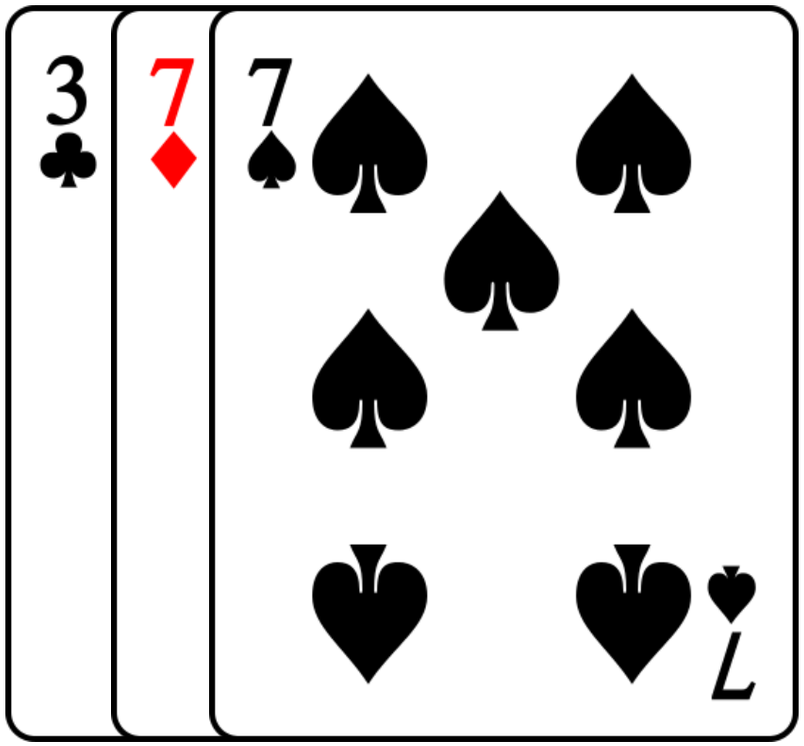

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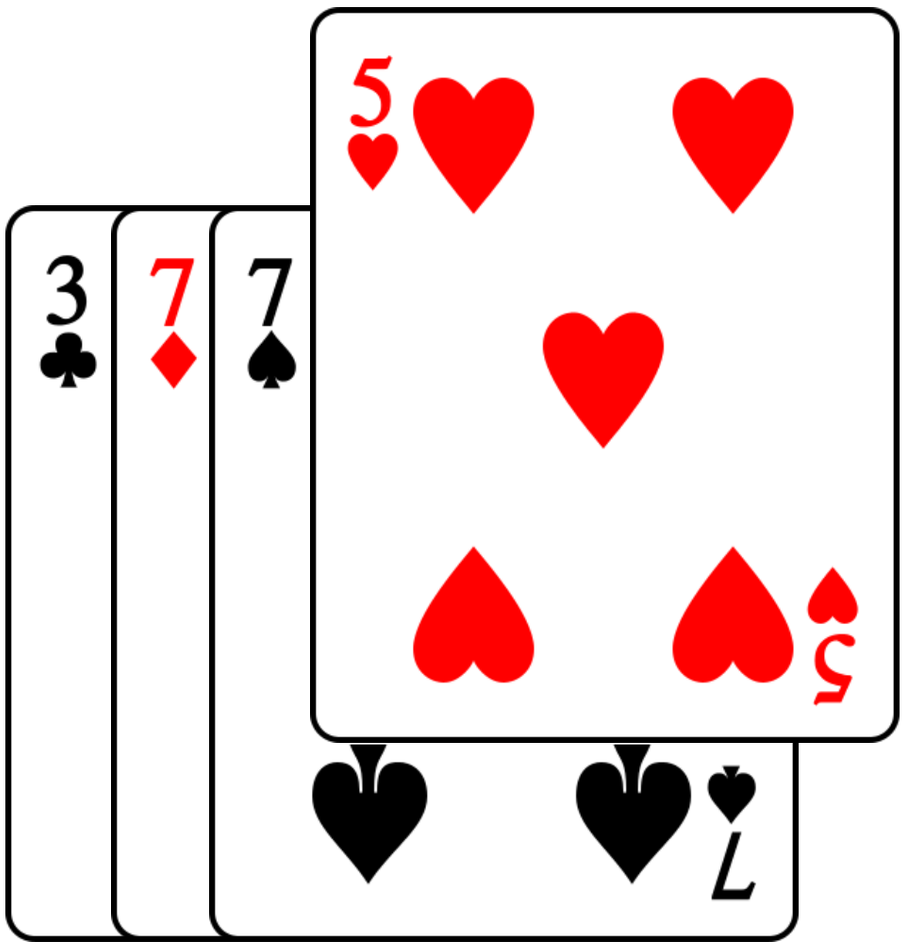

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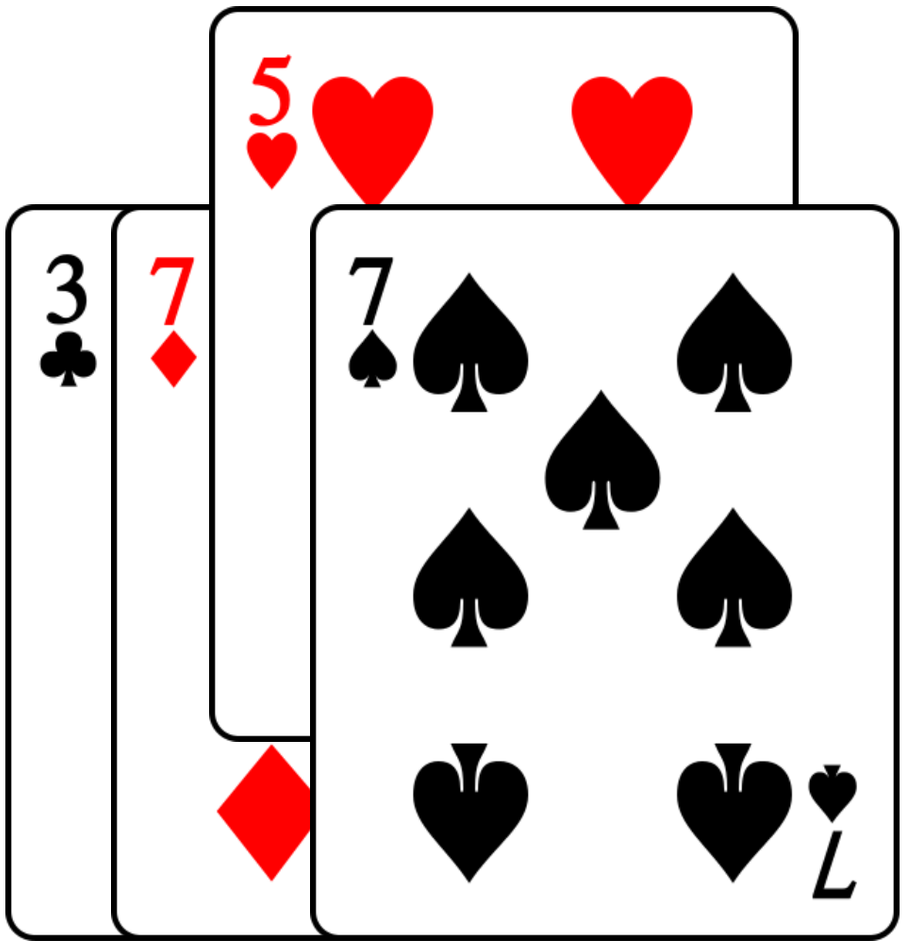

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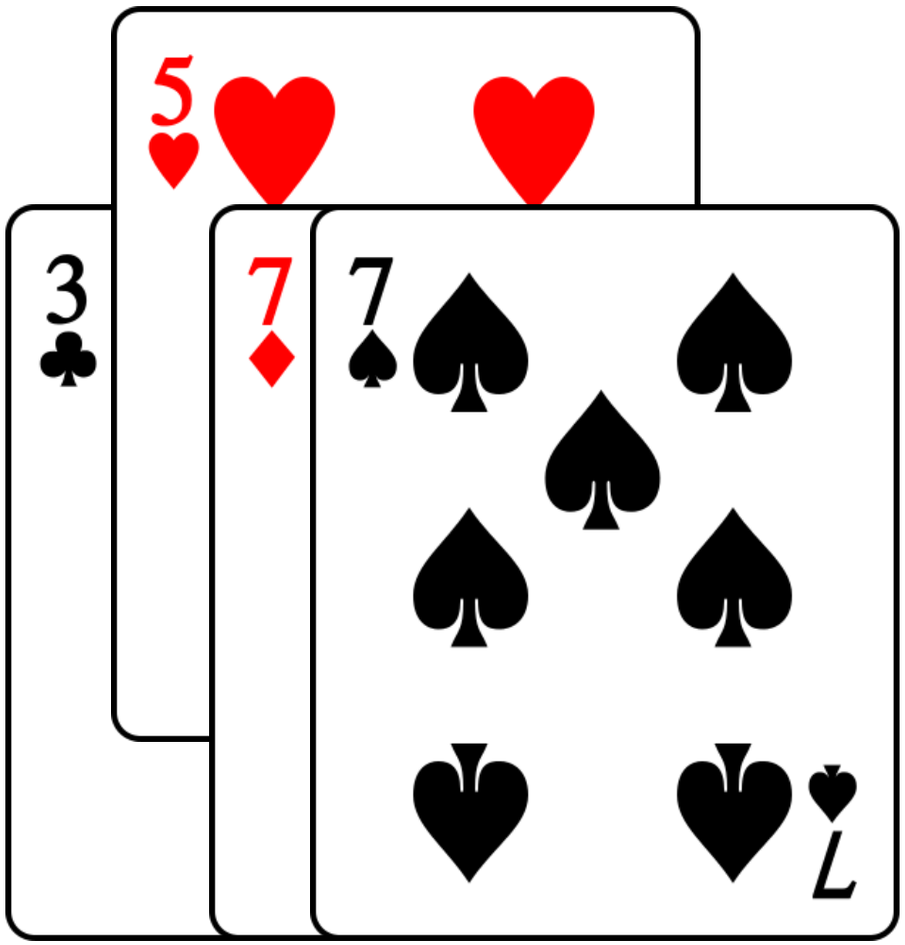

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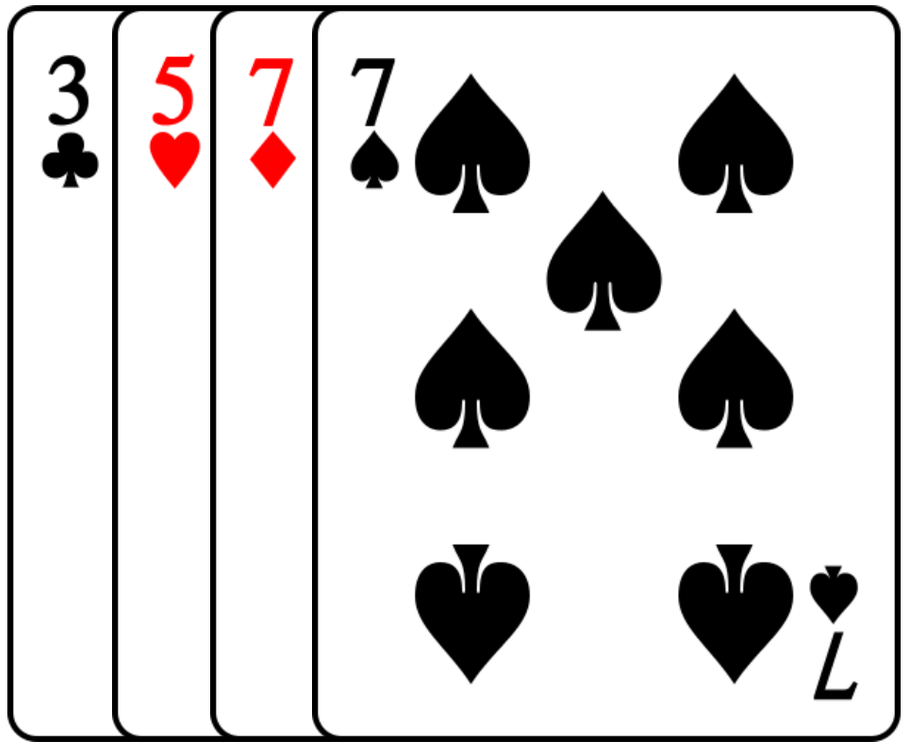

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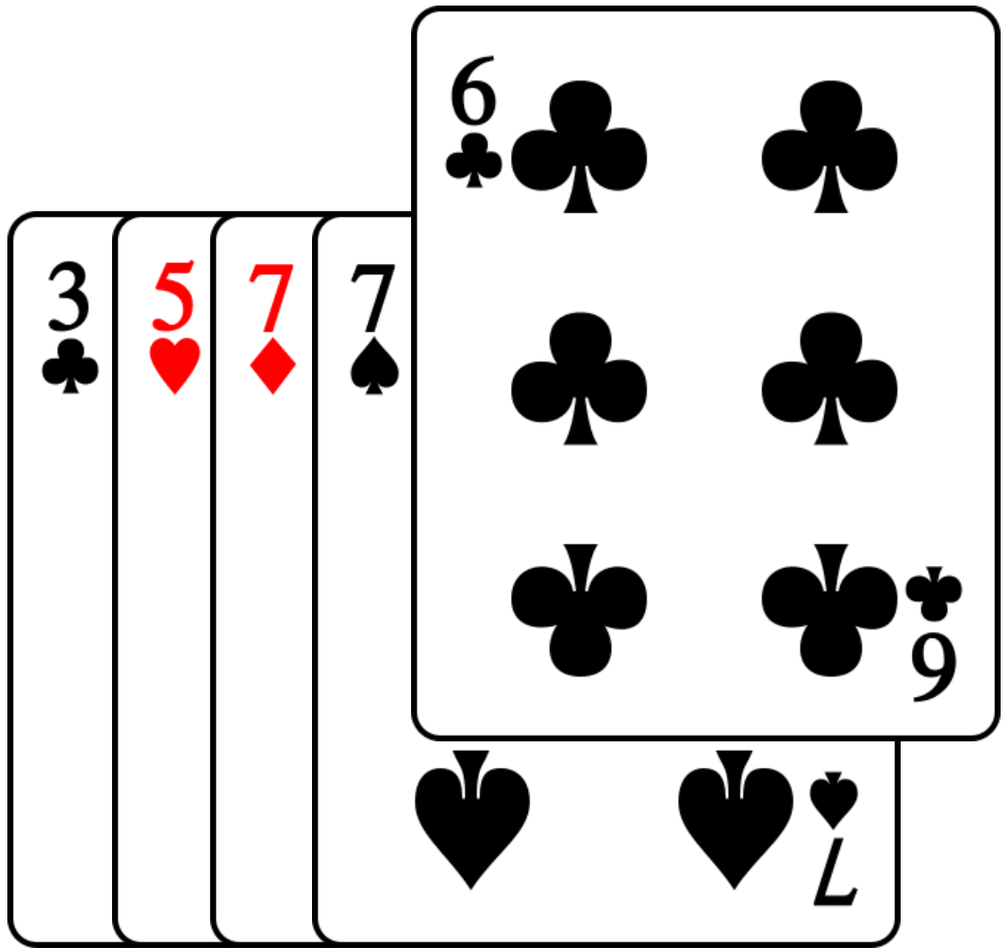
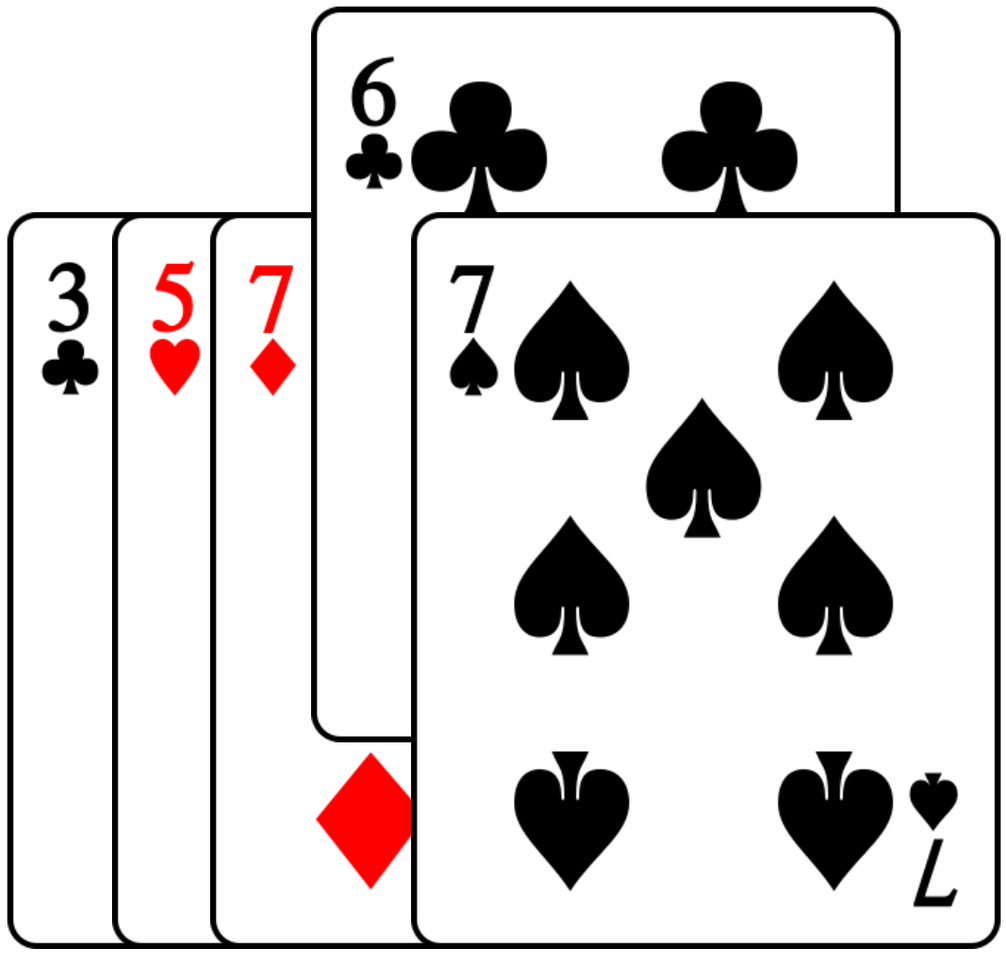
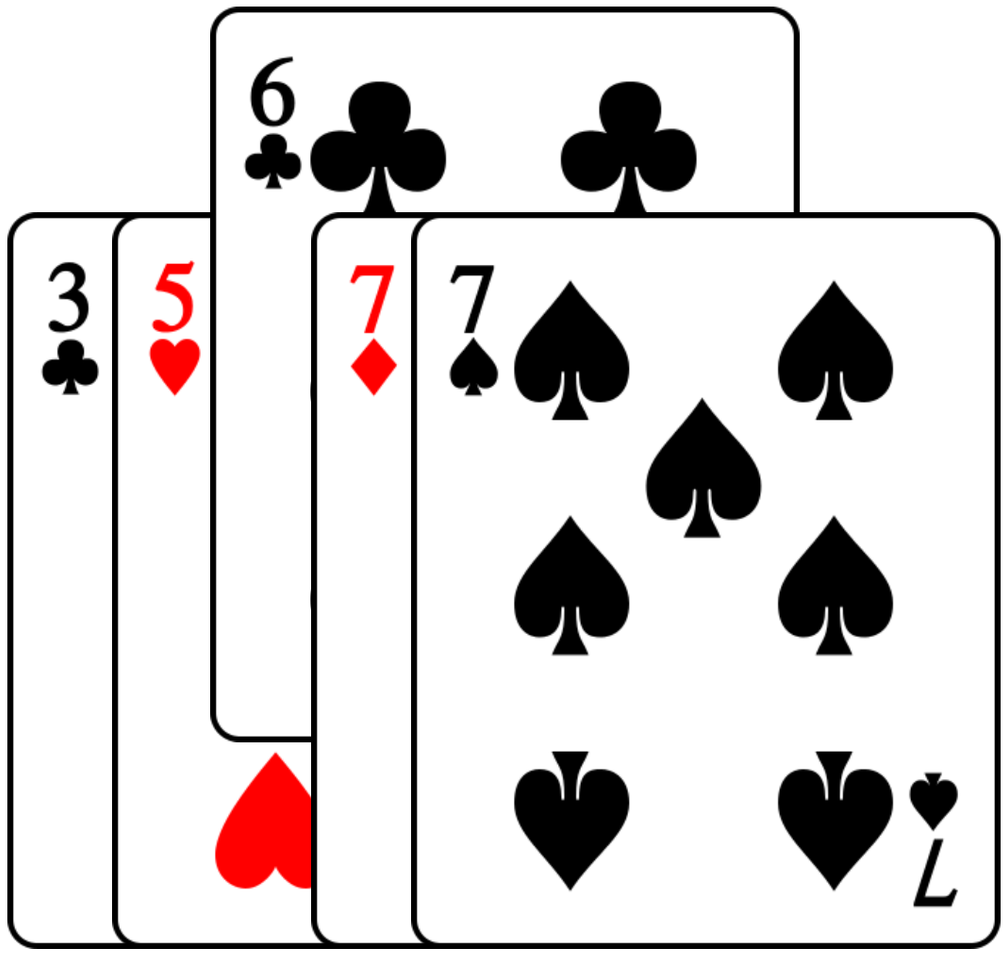


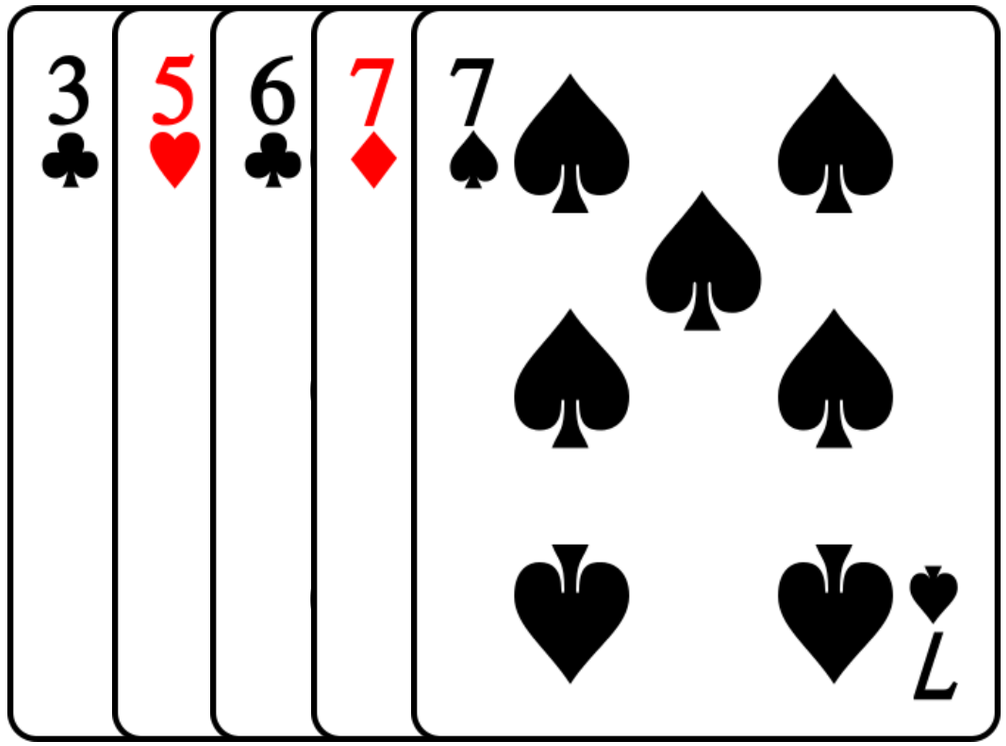
Tri par insertion



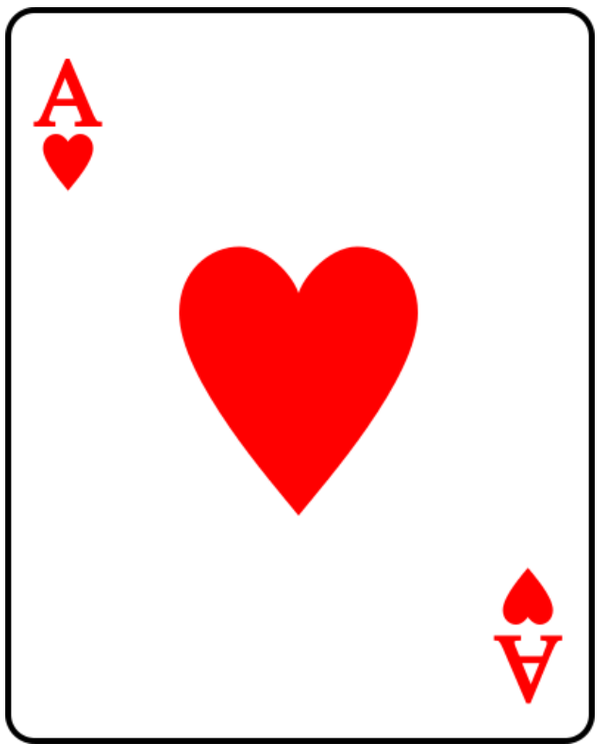
Tri par insertion



Tri par insertion

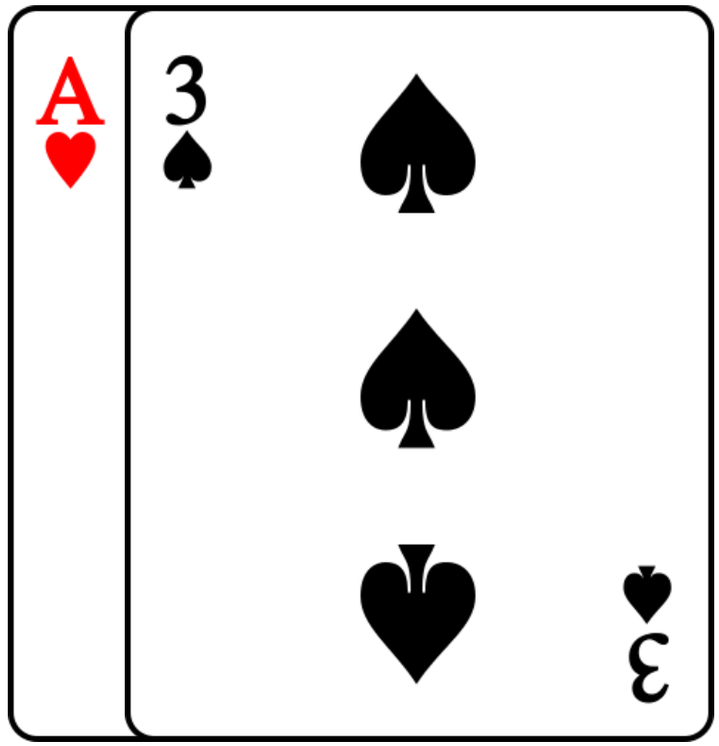


Le meilleur des cas



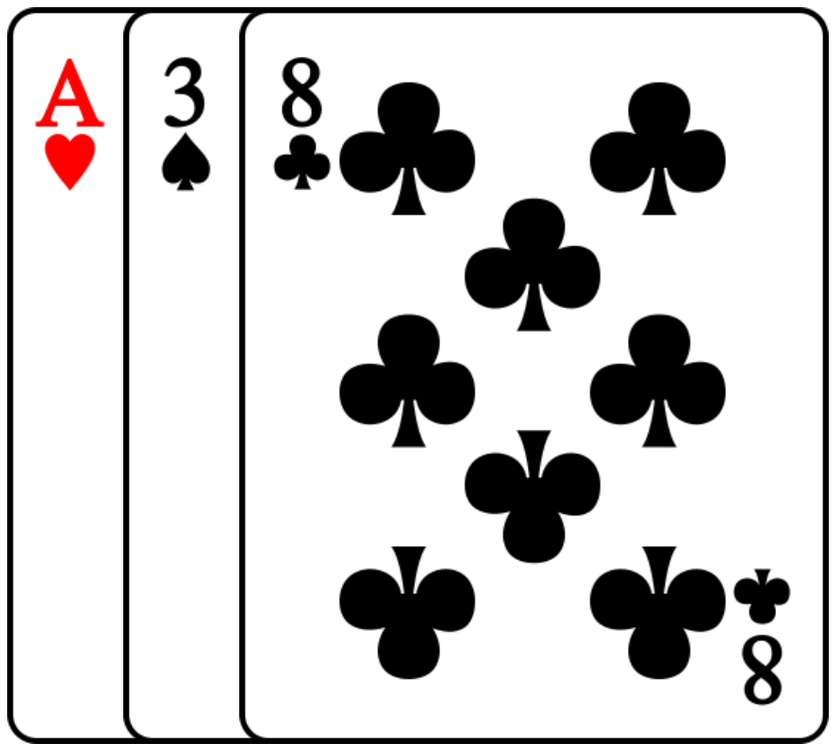
№ operations = 1

Le meilleur des cas



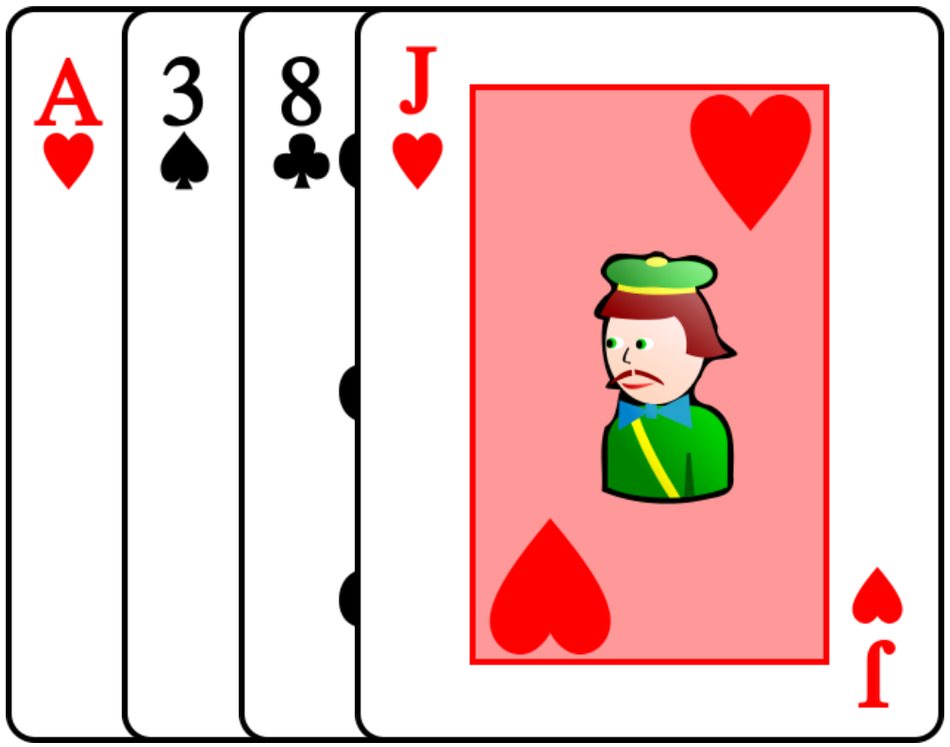
№ operations = 2

Le meilleur des cas



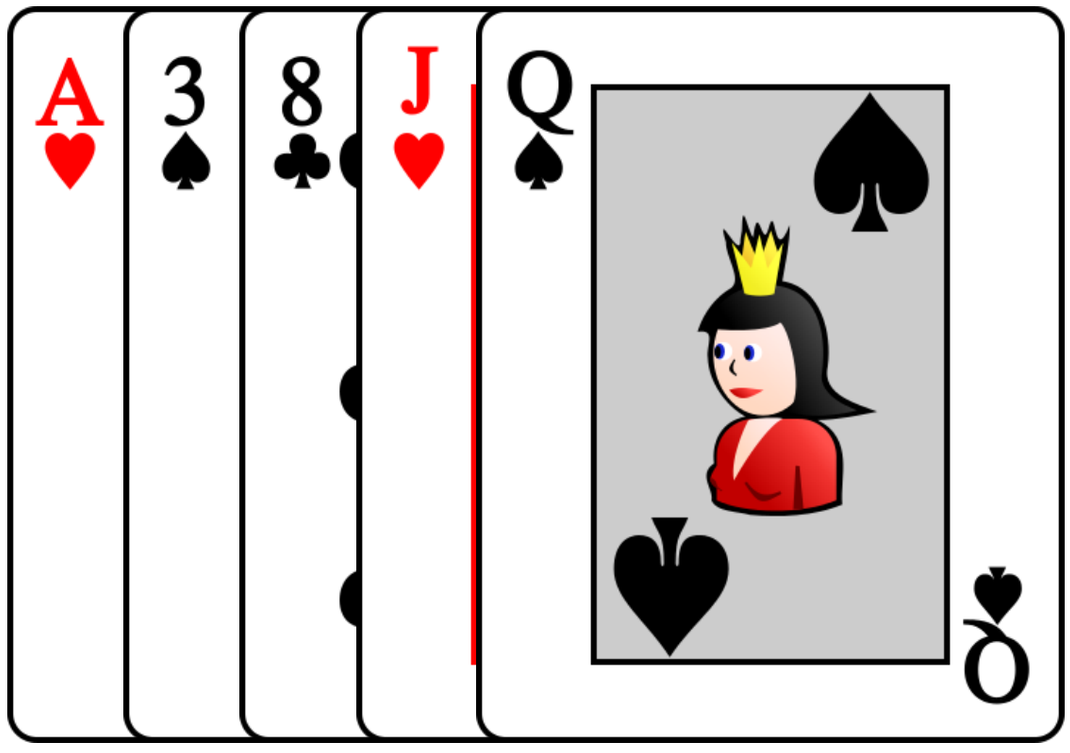
№ operations = 3

Le meilleur des cas



№ operations = 4

Le meilleur des cas



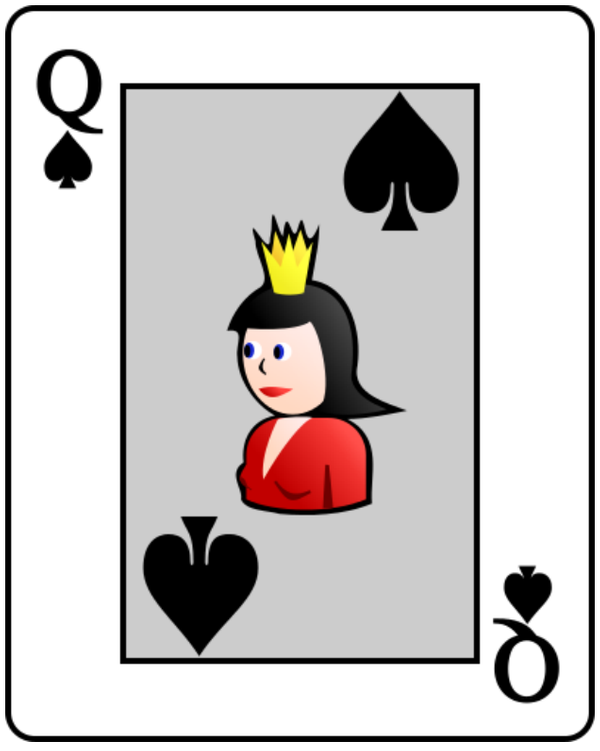
№ operations = 5

Le meilleur des cas

- Les cartes arrivent déjà triées

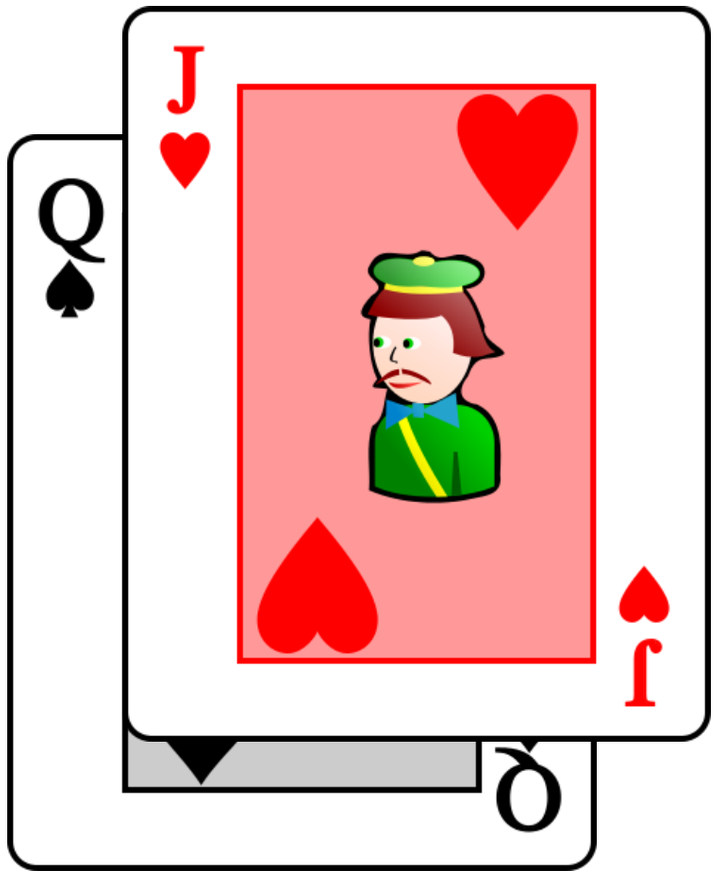
- On fait n opérations (déplacements de cartes)

Le pire des cas



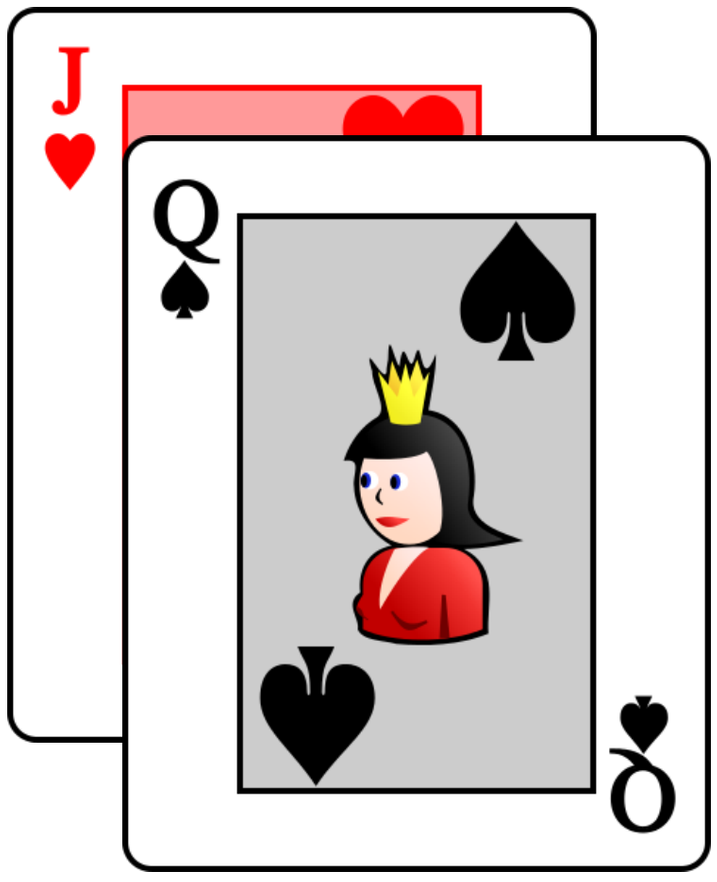
№ operations = 1

Le pire des cas



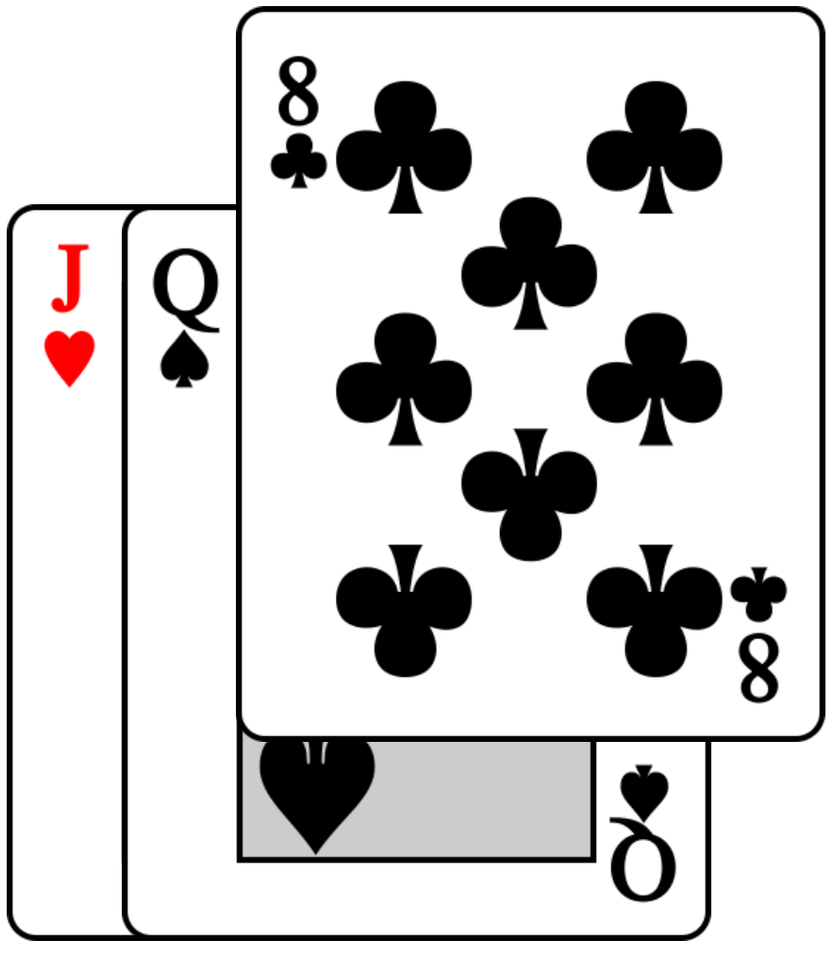
№ operations = 1 + 1

Le pire des cas



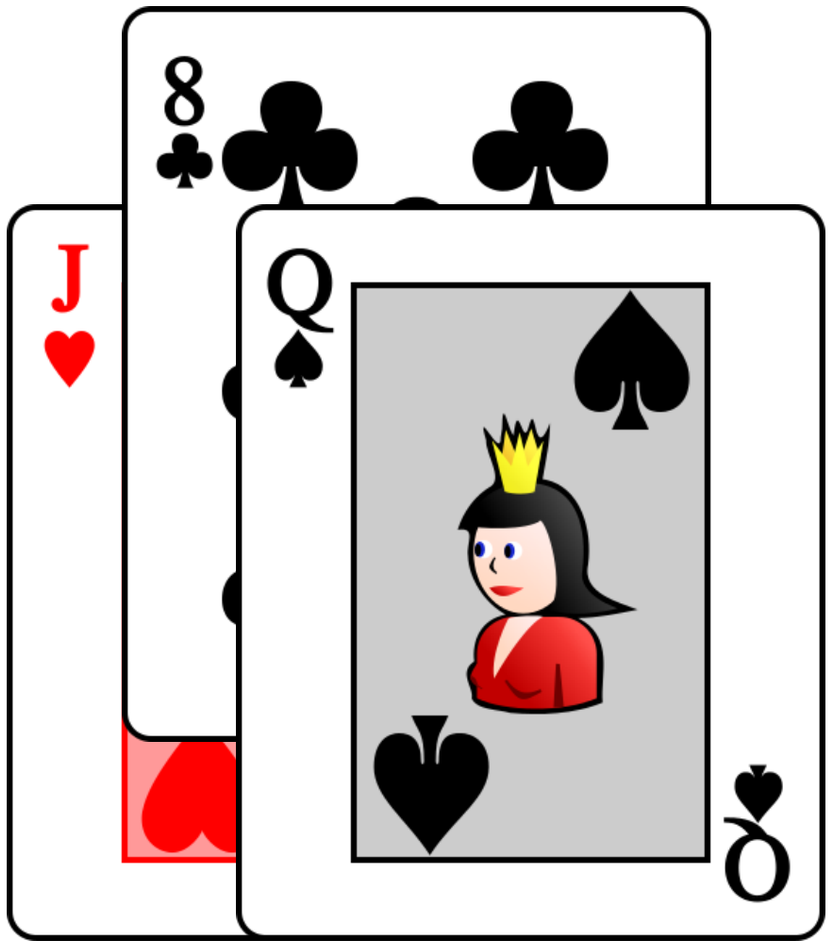
№ operations = 1 + 2

Le pire des cas



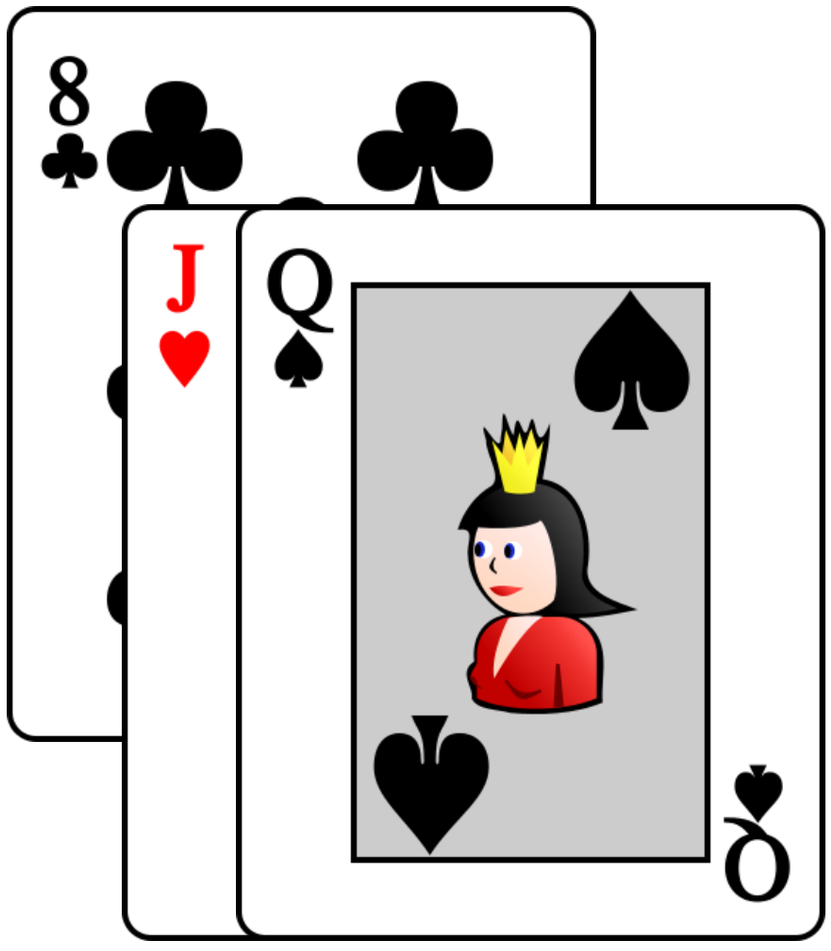
№ operations = 1 + 2 + 1

Le pire des cas



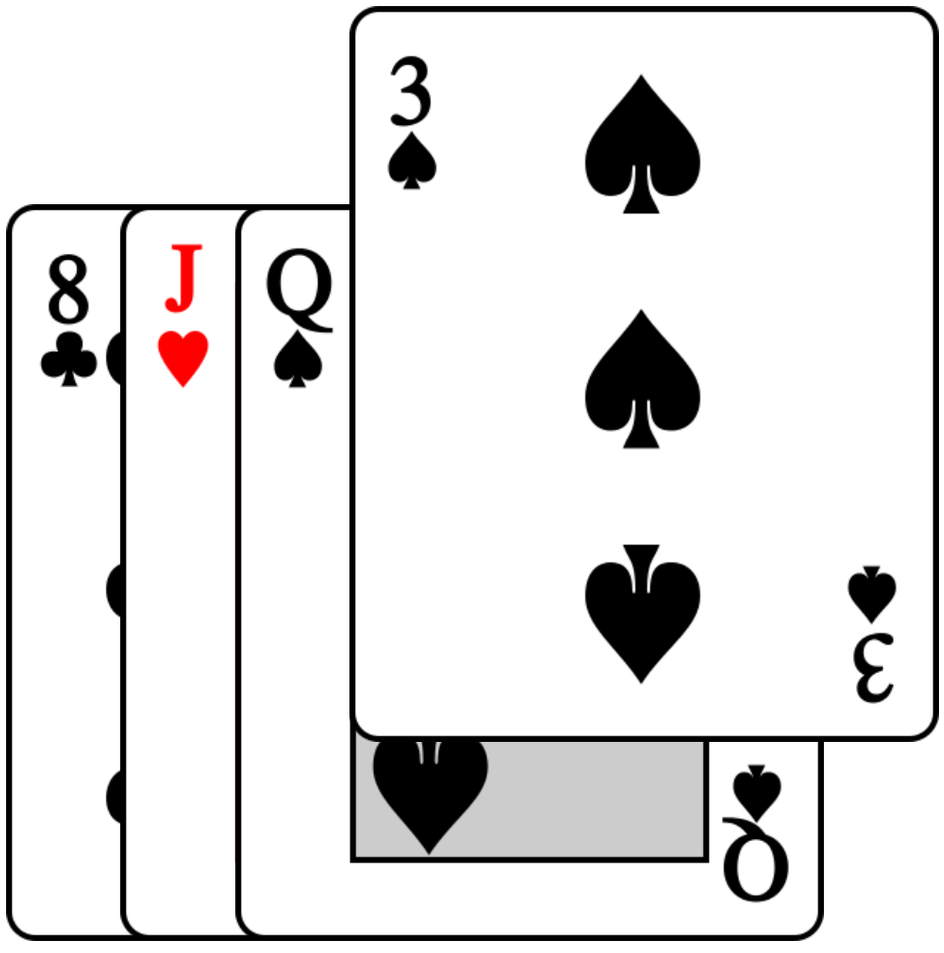
№ operations = 1 + 2 + 2

Le pire des cas



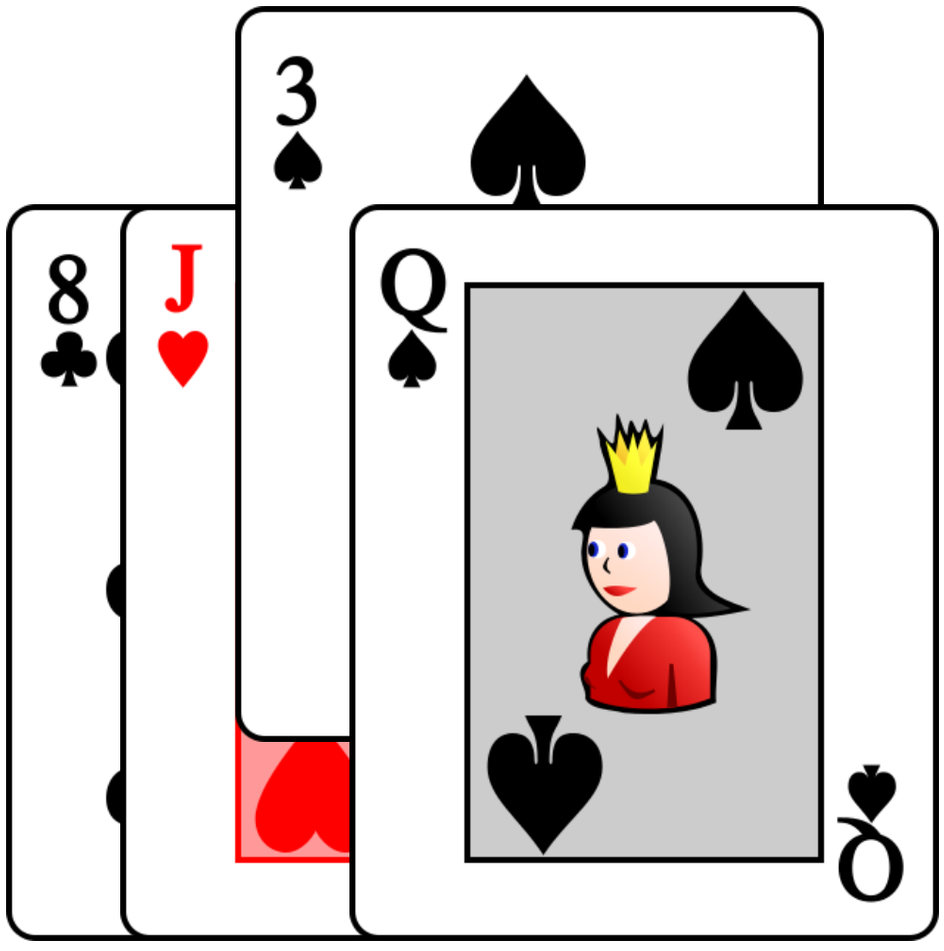
№ operations = 1 + 2 + 3

Le pire des cas



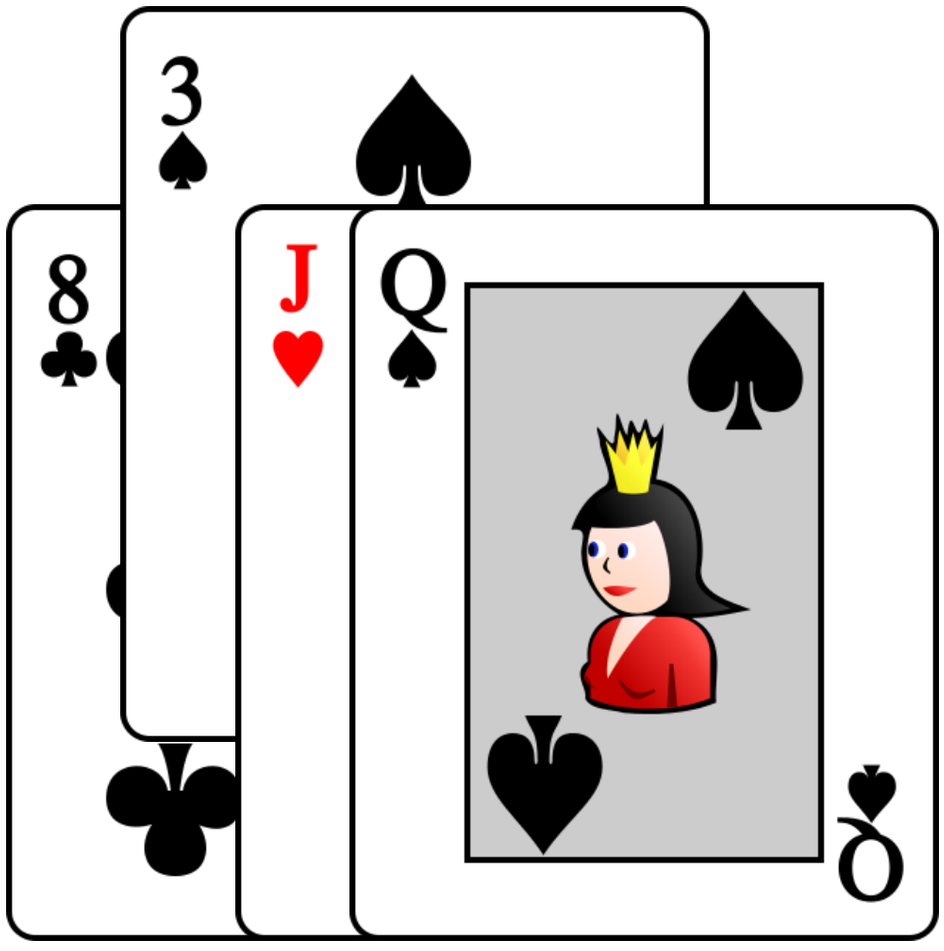
№ operations = 1 + 2 + 3 + 1

Le pire des cas



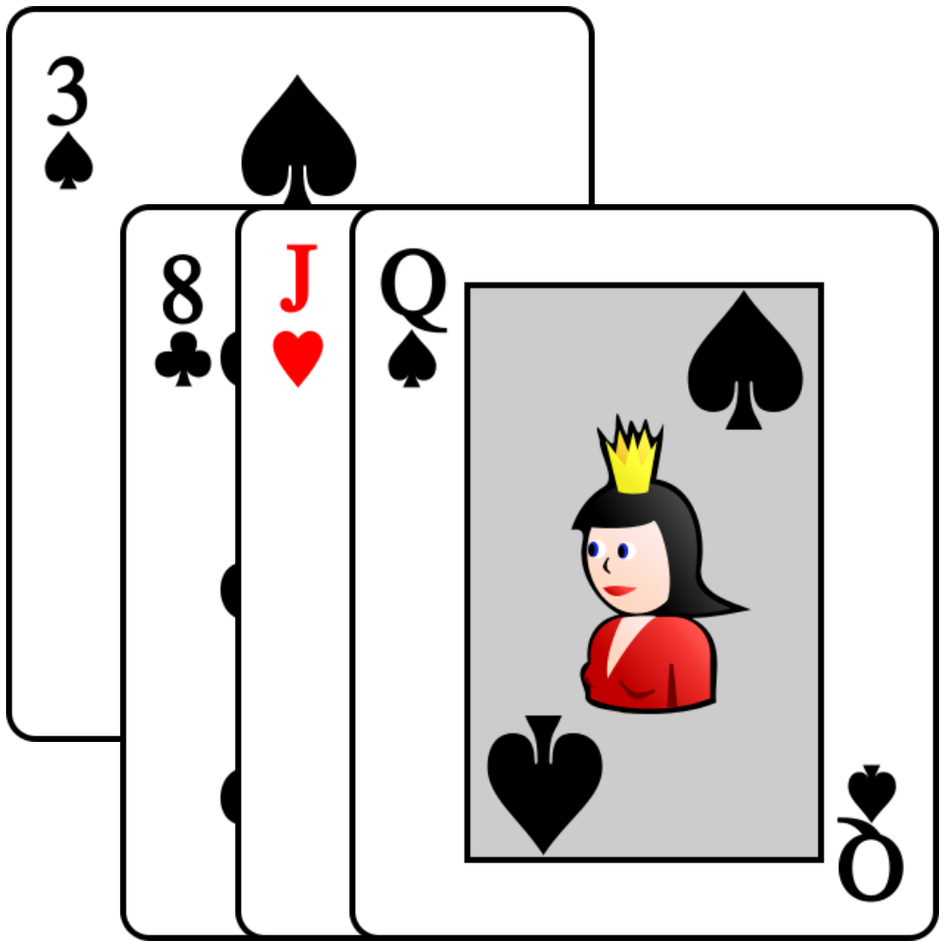
№ operations = 1 + 2 + 3 + 2

Le pire des cas



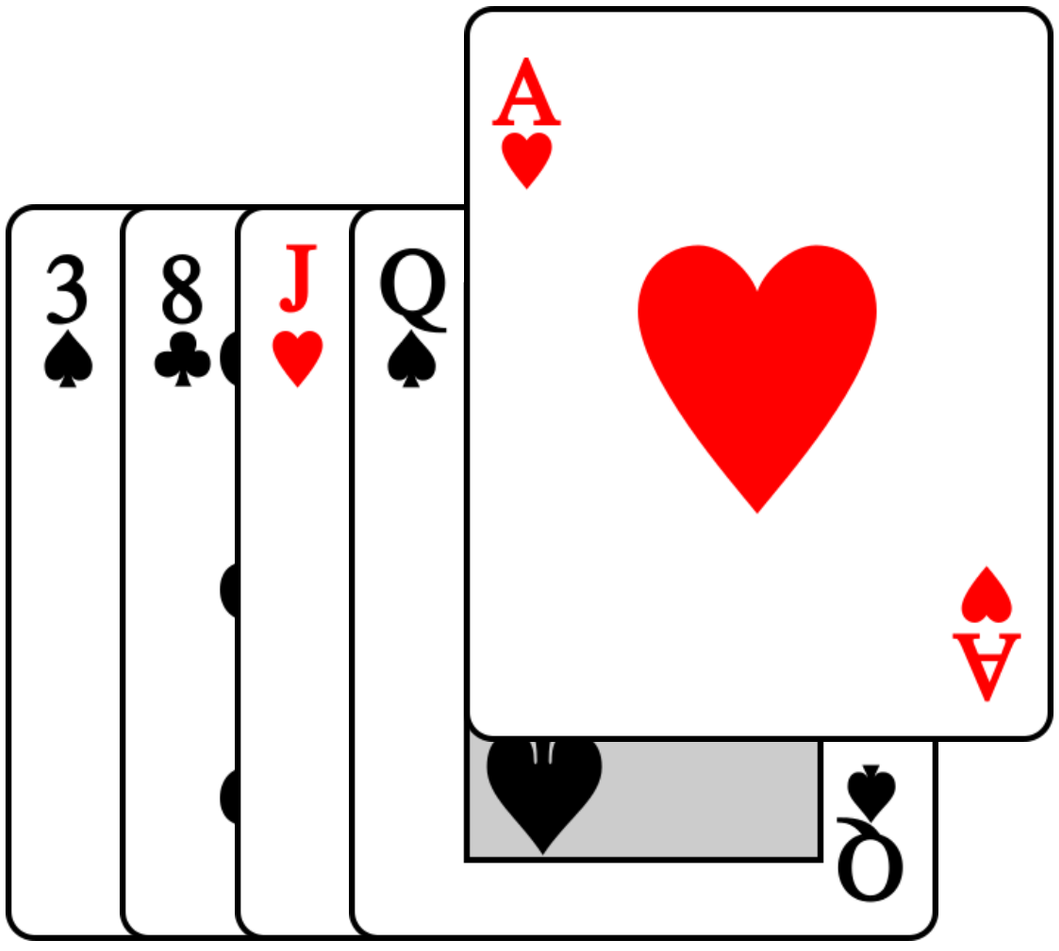
№ operations = 1 + 2 + 3 + 3

Le pire des cas



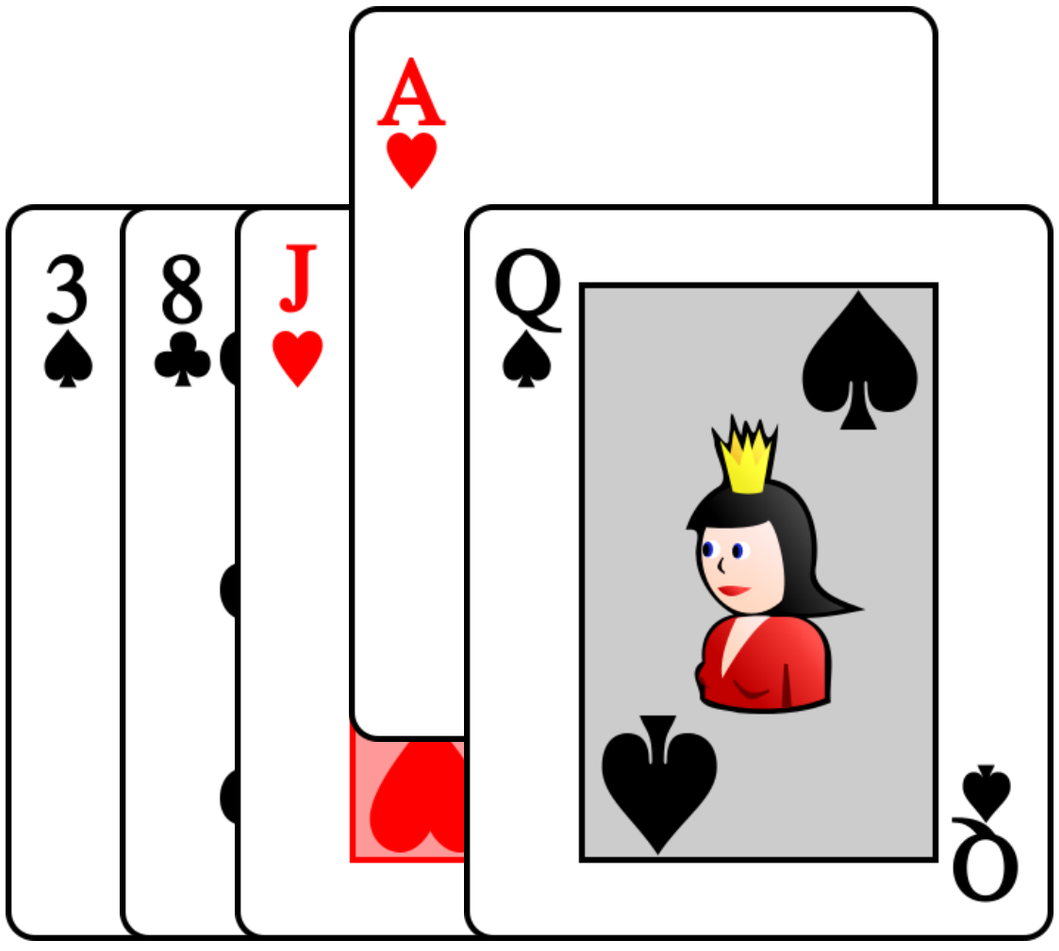
№ operations = 1 + 2 + 3 + 4

Le pire des cas



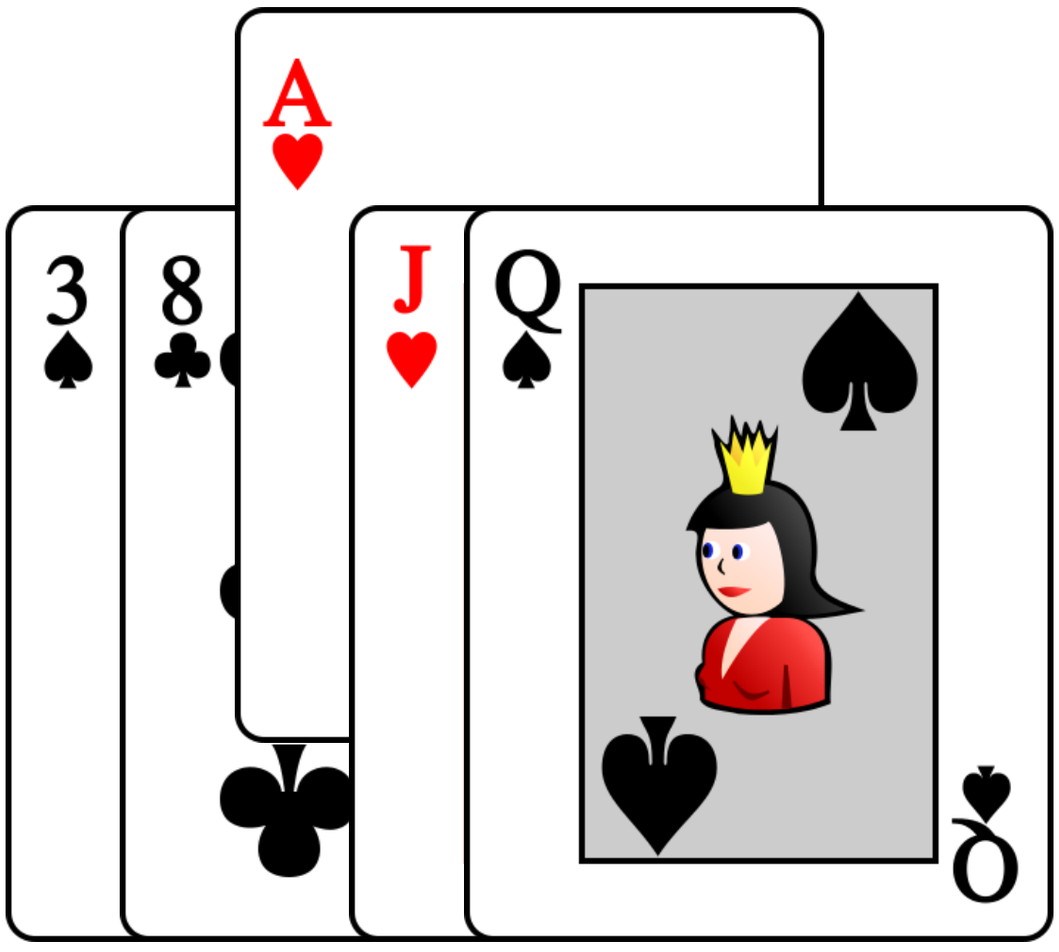
№ operations = 1 + 2 + 3 + 4 + 1

Le pire des cas



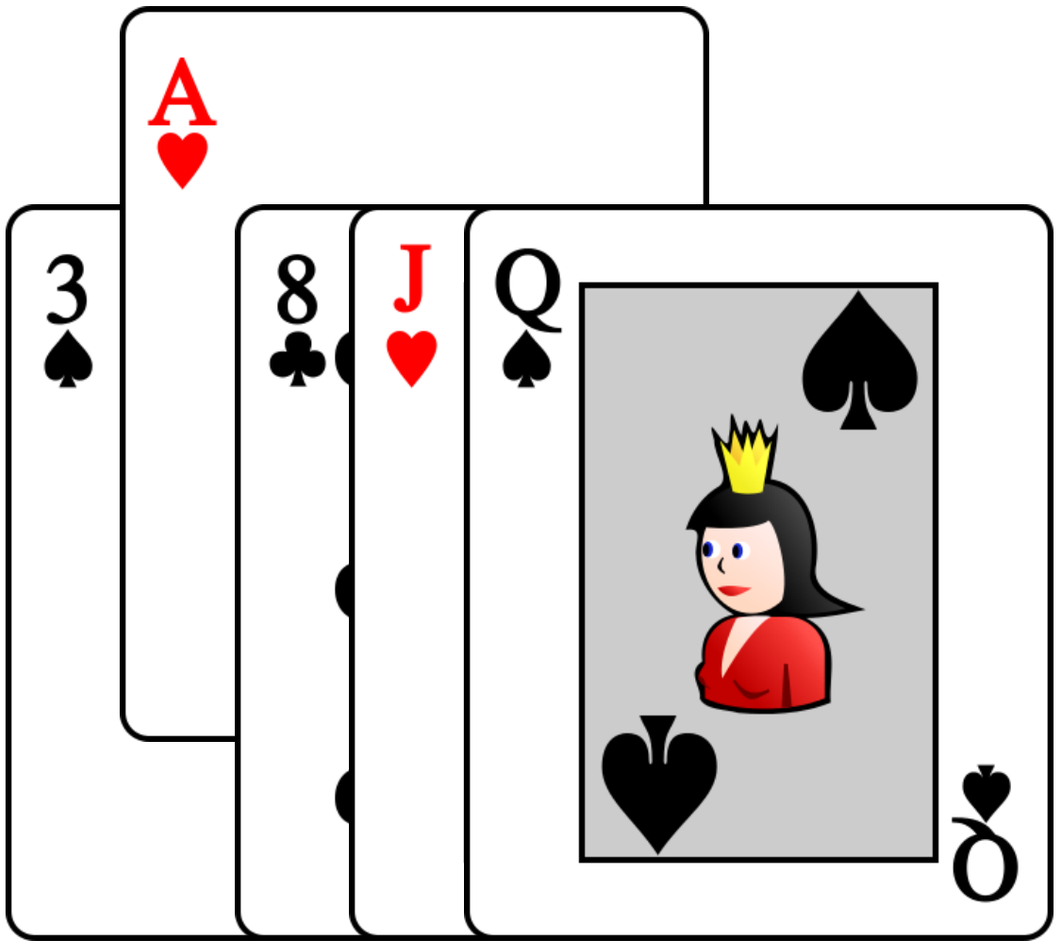
№ operations = 1 + 2 + 3 + 4 + 2

Le pire des cas



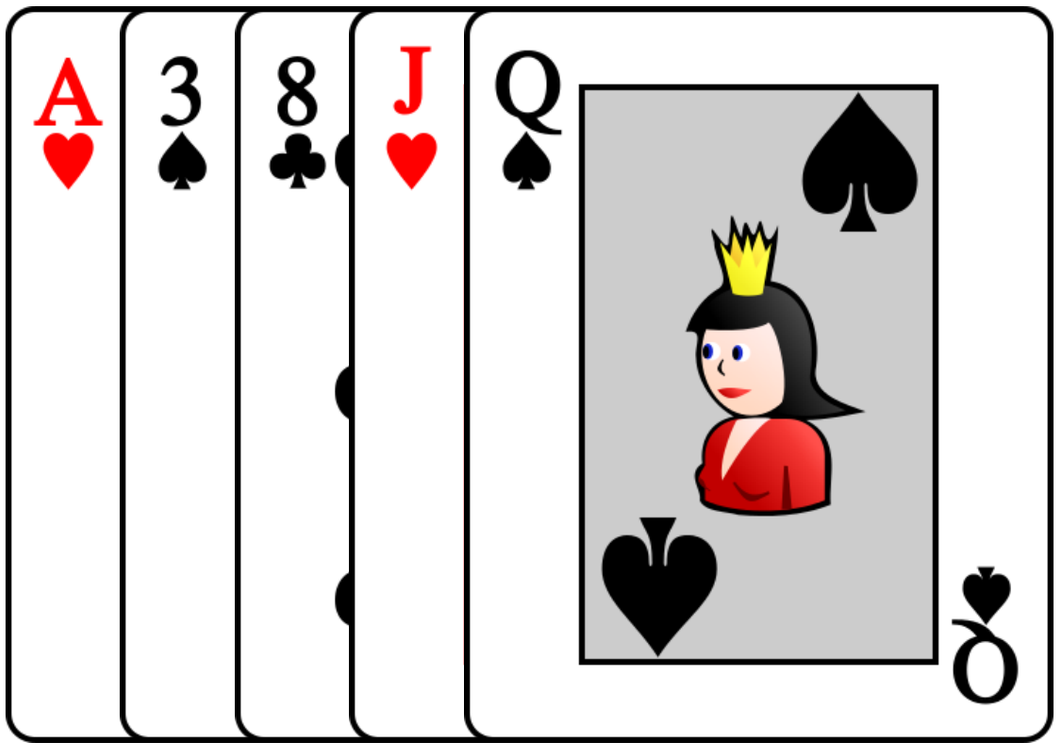
№ operations = 1 + 2 + 3 + 4 + 3

Le pire des cas



№ operations = 1 + 2 + 3 + 4 + 4

Le pire des cas



№ operations = 1 + 2 + 3 + 4 + 5

Le pire des cas

- Les cartes arrivent en ordre décroissant
- On fait i opérations pour la i-ème carte
- Le nombre totale est $1 + 2 + 3 + \cdots + n$

$$\sum_{i=1}^{n} i = \frac{1}{2}n(n+1) = \frac{1}{2}(n^2 + n) = \frac{1}{2}n^2 + \frac{1}{2}n \in O(n^2)$$

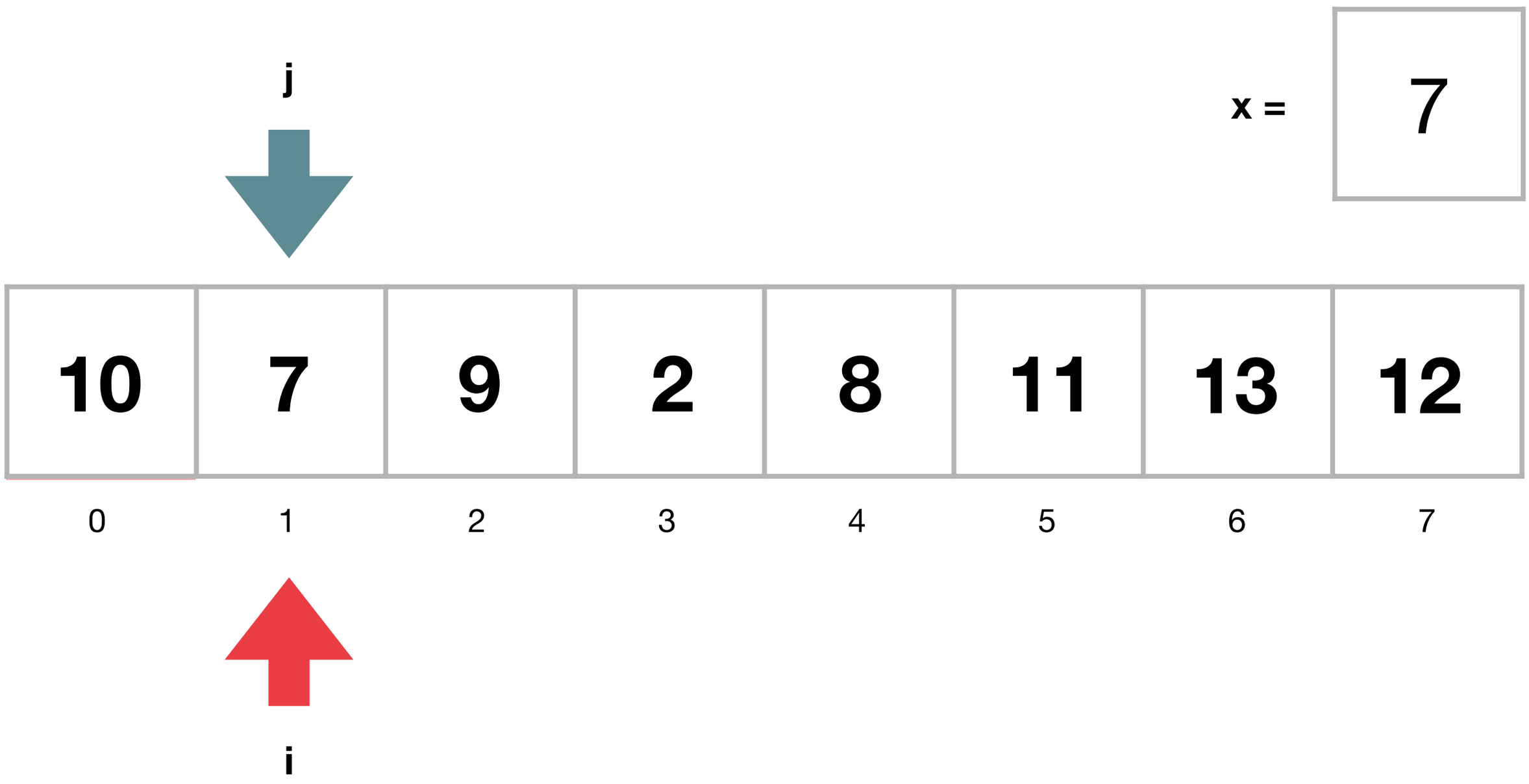
Tri par insertion

```
def trier_par_insertion(A) :
    n = len(A)
    for i in range(1, n):
        x = A[i]
        # insérer x parmi les i premiers éléments
        j = i
        while j > 0 and x < A[j - 1]:
            # décaler d'un élément
            A[j] = A[j - 1]
            j = j - 1
            # ici, x >= A[j - 1] ou bien j == 0
        A[j] = x
        # le tableau est trié !
```

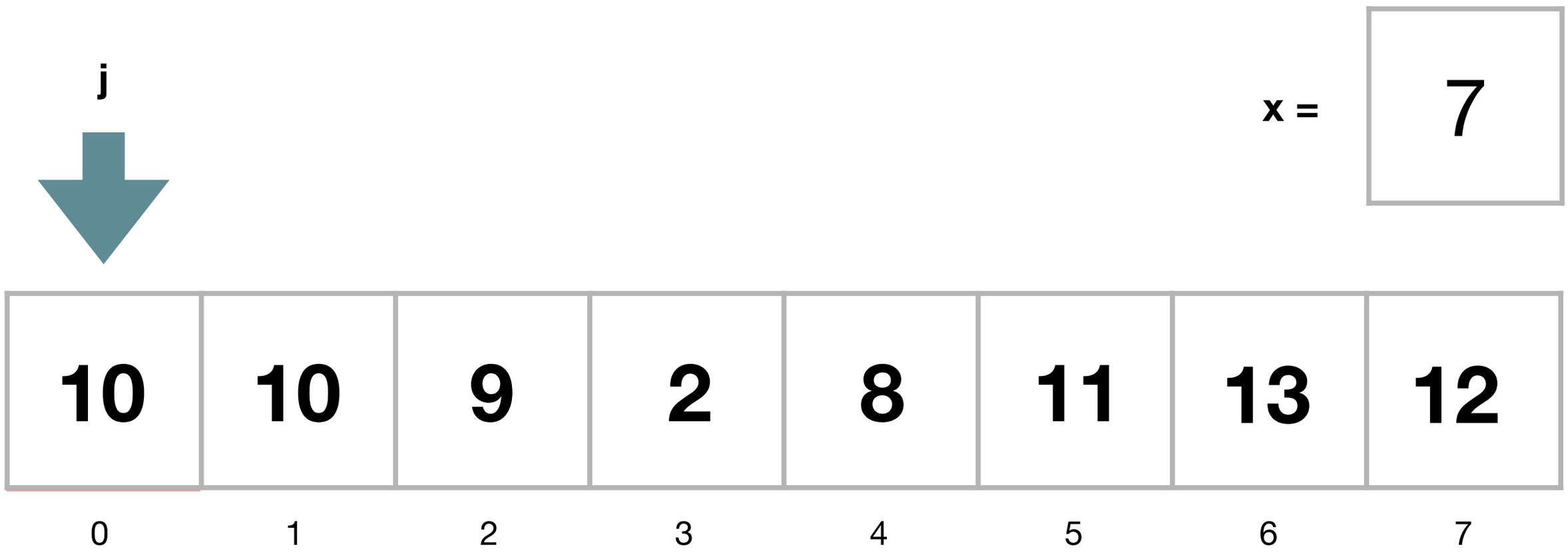
Tri par insertion

10	7	9	2	8	11	13	12
0	1	2	3	4	5	6	7

Tri par insertion



Tri par insertion



Tri par insertion

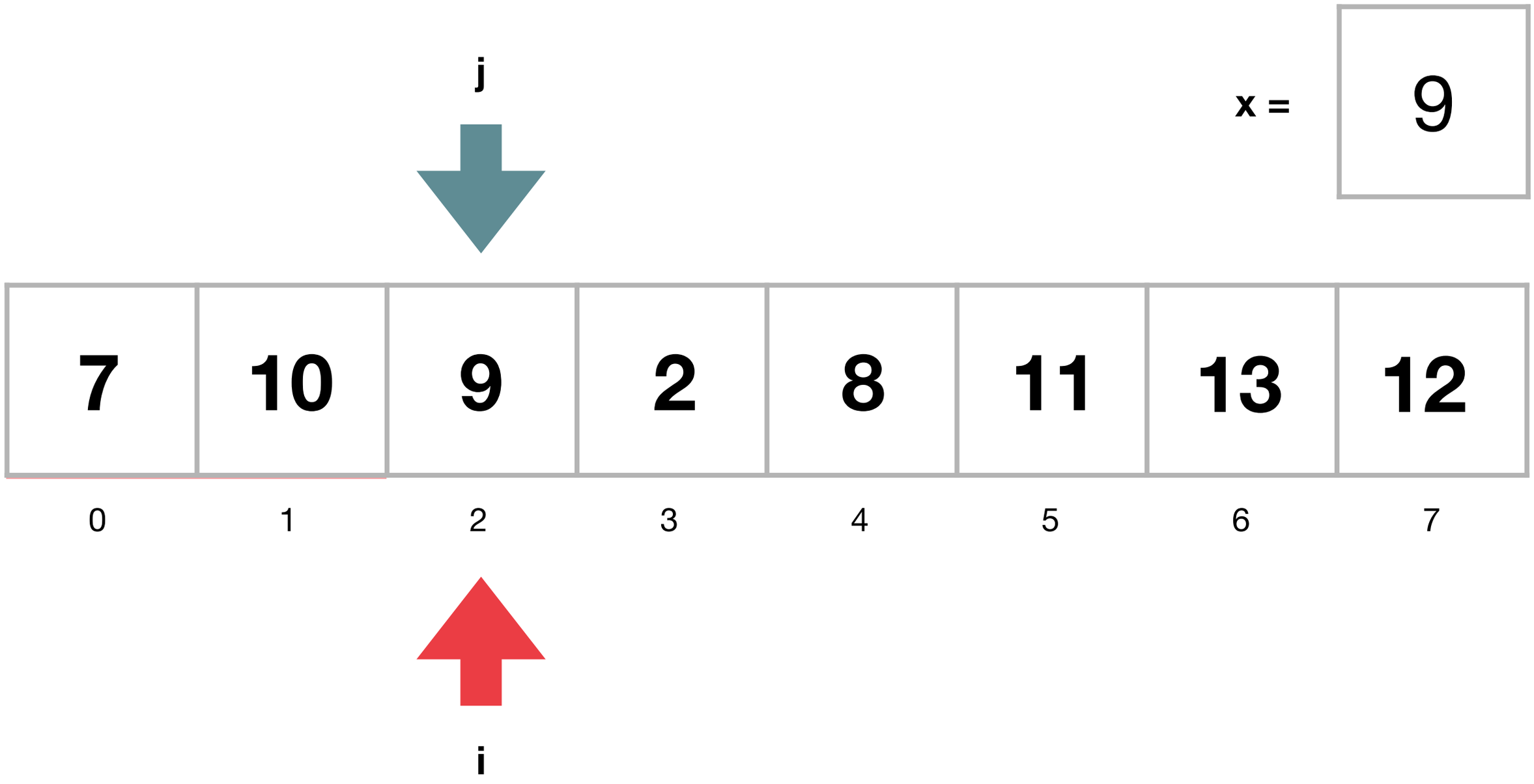
j

x = 7

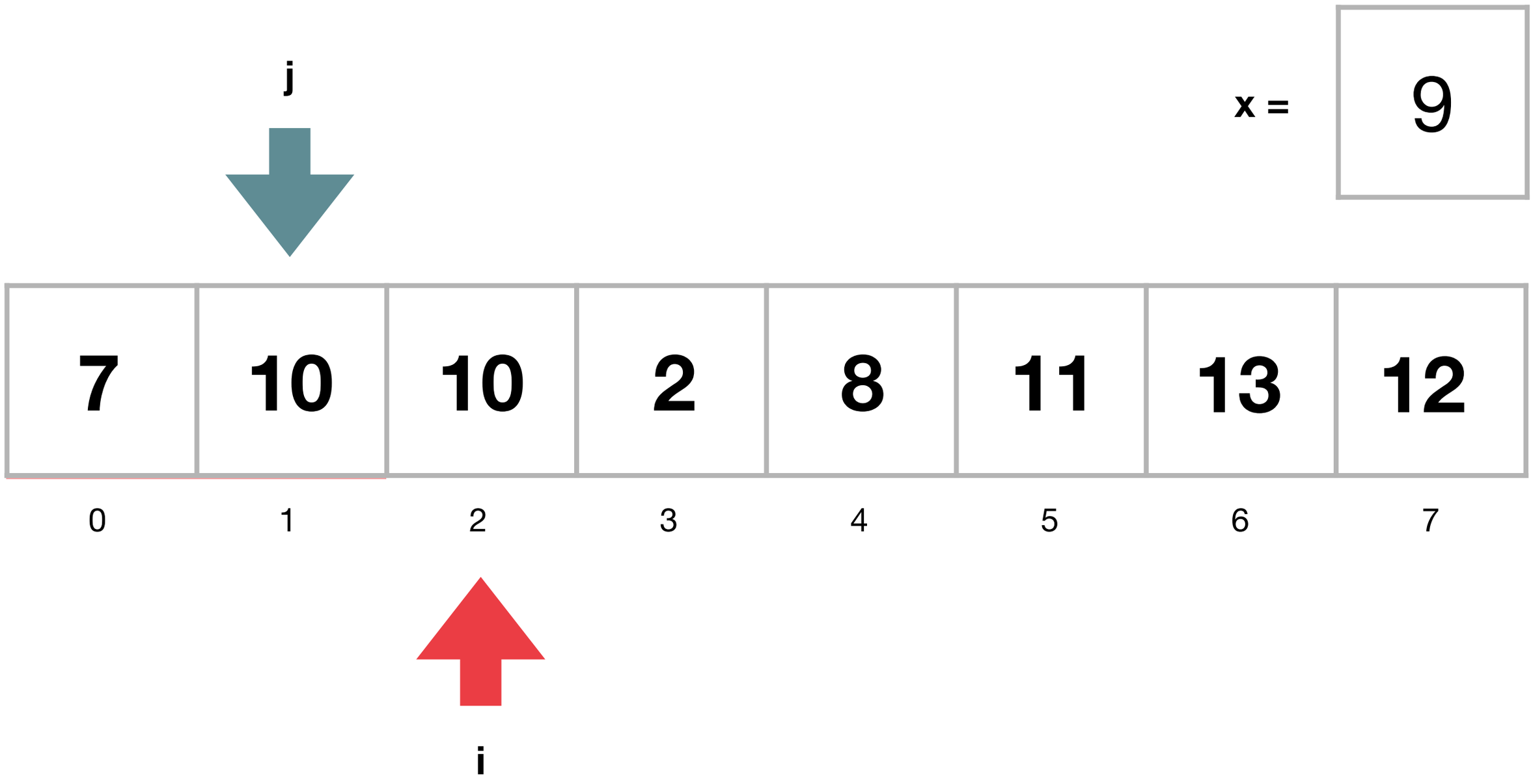
7	10	9	2	8	11	13	12
0	1	2	3	4	5	6	7

i

Tri par insertion



Tri par insertion



Tri par insertion

j

x = 9

7	9	10	2	8	11	13	12
0	1	2	3	4	5	6	7

i

Tri par insertion

j

x = 2

7	9	10	2	8	11	13	12
0	1	2	3	4	5	6	7

i

Tri par insertion

j

x = 2

7	9	10	10	8	11	13	12
0	1	2	3	4	5	6	7

i

Tri par insertion

j

x = 2

7	9	9	10	8	11	13	12
0	1	2	3	4	5	6	7

i

Tri par insertion

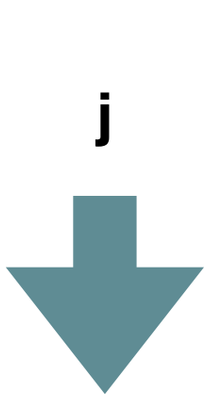
j

x = 2

7	7	9	10	8	11	13	12
0	1	2	3	4	5	6	7

i

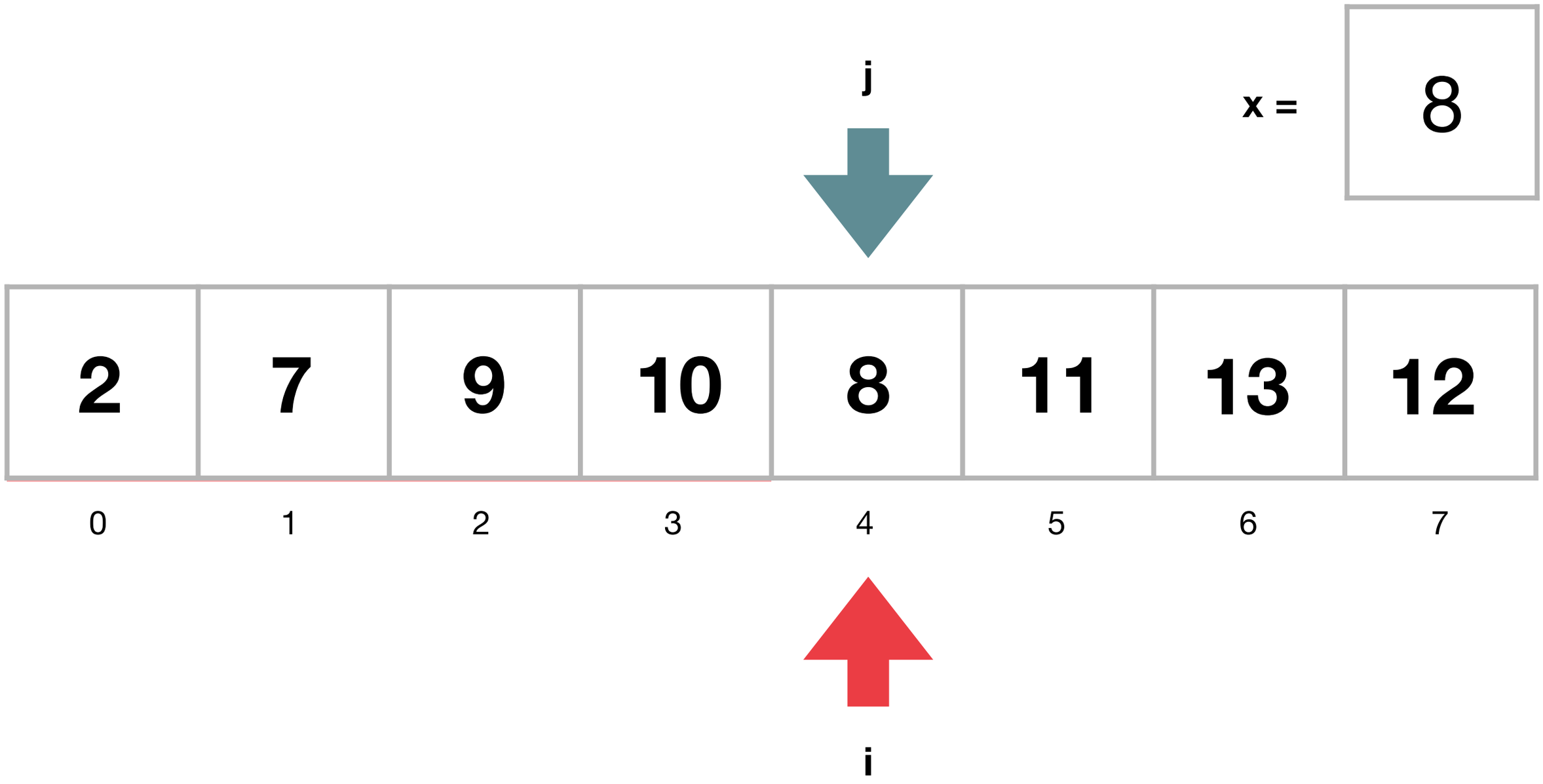
Tri par insertion



x = 2

2	7	9	10	8	11	13	12
0	1	2	3	4	5	6	7

Tri par insertion



Tri par insertion

j

x = 8

2	7	9	10	10	11	13	12
0	1	2	3	4	5	6	7

i

Tri par insertion

j

x = 8

2	7	9	9	10	11	13	12
0	1	2	3	4	5	6	7

i

Tri par insertion

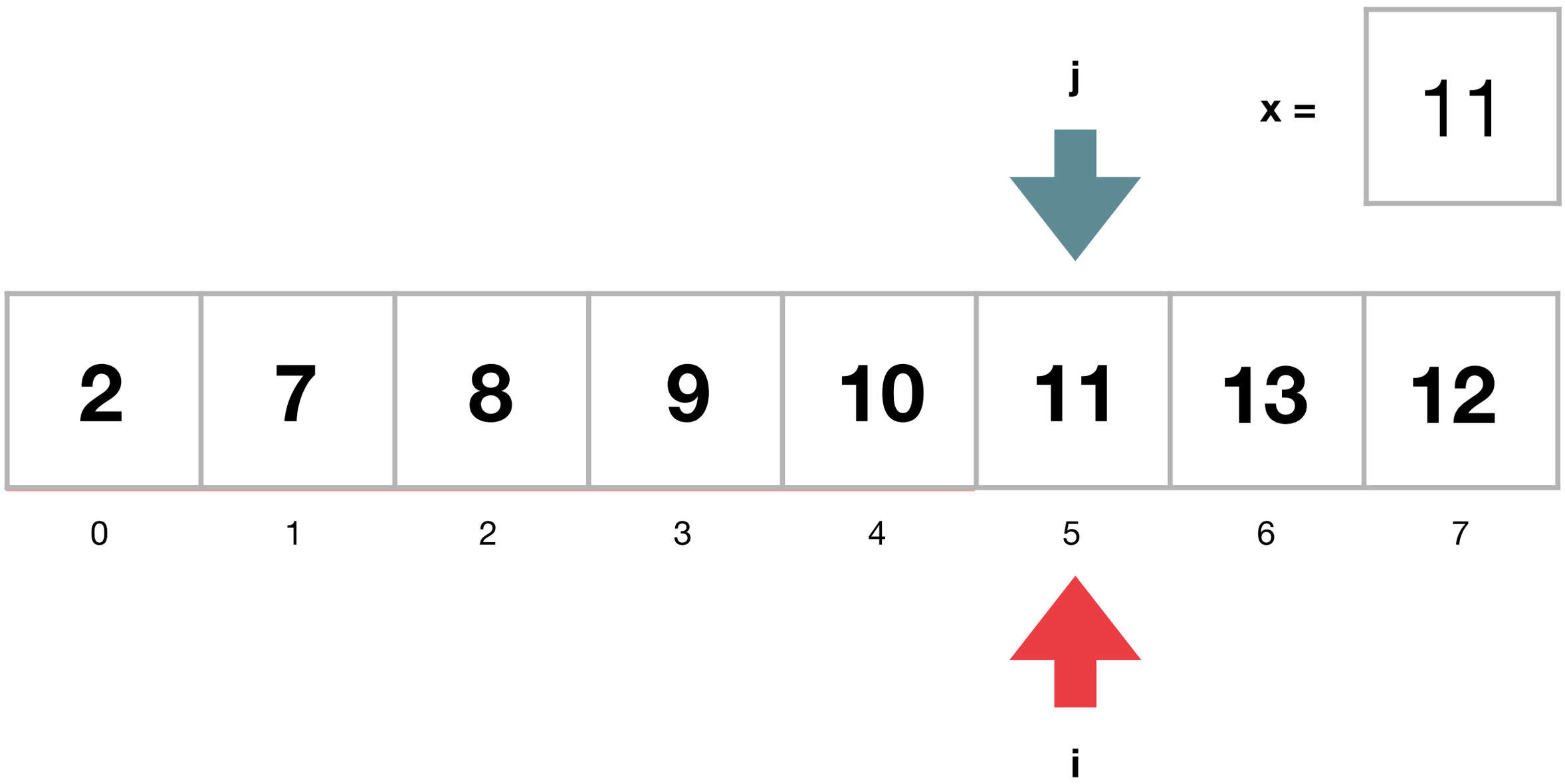
j

x = 8

2	7	8	9	10	11	13	12
0	1	2	3	4	5	6	7

i

Tri par insertion



Tri par insertion

x = 13

j

2	7	8	9	10	11	13	12
0	1	2	3	4	5	6	7

i

Tri par insertion

x = 12

j

2	7	8	9	10	11	13	12
0	1	2	3	4	5	6	7

i

Tri par insertion

x = 12

j

2	7	8	9	10	11	13	13
0	1	2	3	4	5	6	7

i

Tri par insertion

x = 12

j

2	7	8	9	10	11	12	13
0	1	2	3	4	5	6	7

i

Terminaison

```
def trier_par_insertion(A) :
    n = len(A)
    for i in range(1, n):
        x = A[i]
        j = i
        while j > 0 and x < A[j - 1]:
            A[j] = A[j - 1]
            j = j - 1
        A[j] = x
```

Terminaison

```
def trier_par_insertion(A) :
    n = len(A)
    for i in range(1, n):
        x = A[i]
        j = i
        while j > 0 and x < A[j - 1]:
            A[j] = A[j - 1]
            j = j - 1
        A[j] = x
```

- La boucle **for** termine toujours

Terminaison

```
def trier_par_insertion(A) :
    n = len(A)
    for i in range(1, n):
        x = A[i]
        j = i
        while j > 0 and x < A[j - 1]:
            A[j] = A[j - 1]
            j = j - 1
        A[j] = x
```

- La boucle **`for`** termine toujours
- La boucle **`while`** termine (au pire) quand $j = 0$

Correction

```
def trier_par_insertion(A) :
    n = len(A)
    for i in range(1, n):
        x = A[i]
        j = i
        while j > 0 and x < A[j - 1]:
            A[j] = A[j - 1]
            j = j - 1
            A[j] = x
```

Correction

```
def trier_par_insertion(A) :
    n = len(A)
    for i in range(1, n):
        x = A[i]
        j = i
        while j > 0 and x < A[j - 1]:
            A[j] = A[j - 1]
            j = j - 1
            A[j] = x
```

- Le sous-tableau $A[0,\ldots,i-1]$ est trié au début des instructions de la boucle **for**

Correction

```
def trier_par_insertion(A) :
    n = len(A)
    for i in range(1, n):
        x = A[i]
        j = i
        while j > 0 and x < A[j - 1]:
            A[j] = A[j - 1]
            j = j - 1
            A[j] = x
```

- Le sous-tableau $A[0,\ldots,i-1]$ est trié au début des instructions de la boucle **for**

- À la fin on a $i = n - 1$, donc le tableau $A[0,\ldots,n-2]$ est trié, et on déplace l'élément $A[n-1]$ à la bonne position, ce qui donne un tableau entier trié

Efficacité

```
def trier_par_insertion(A) :
    n = len(A)
    for i in range(1, n):
        x = A[i]
        j = i
        while j > 0 and x < A[j - 1]:
            A[j] = A[j - 1]
            j = j - 1
            A[j] = x
```

Efficacité

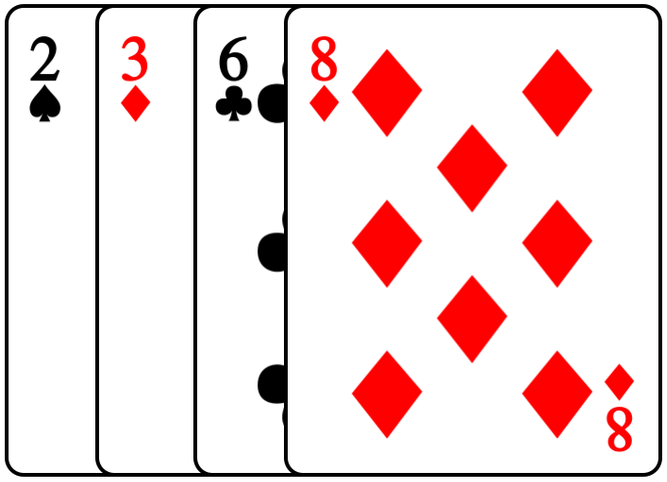
```
def trier_par_insertion(A) :
    n = len(A)
    for i in range(1, n):
        x = A[i]
        j = i
        while j > 0 and x < A[j - 1]:
            A[j] = A[j - 1]
            j = j - 1
            A[j] = x
```

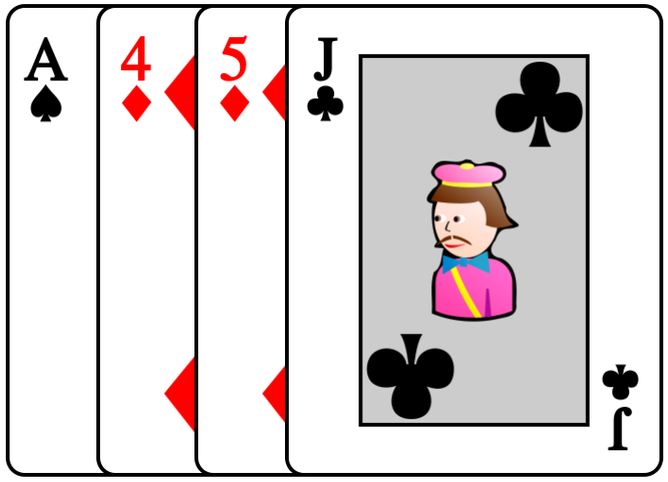
- $O(n)$ opérations dans le meilleur des cas

- $O(n^2)$ opérations dans le pire des cas

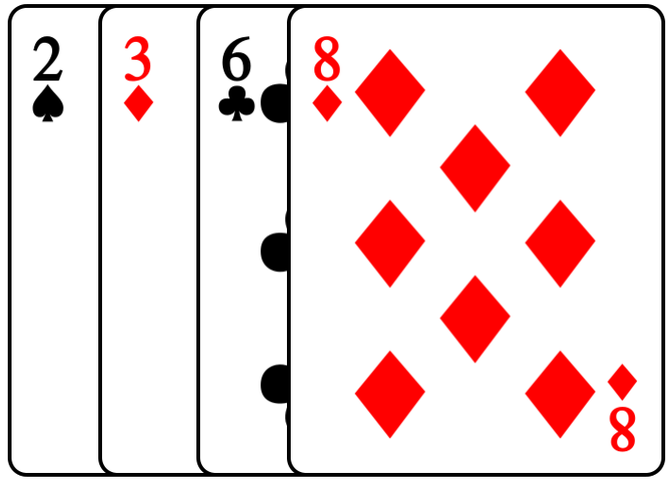
Peut-on faire mieux que $O(n^2)$ pour trier un tableau ?

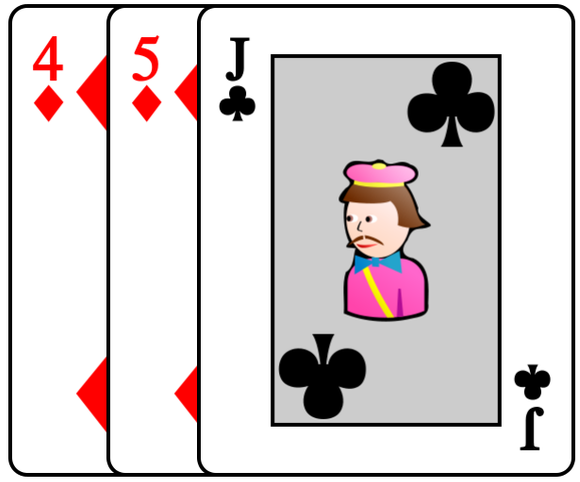
Fusion de tableaux triés

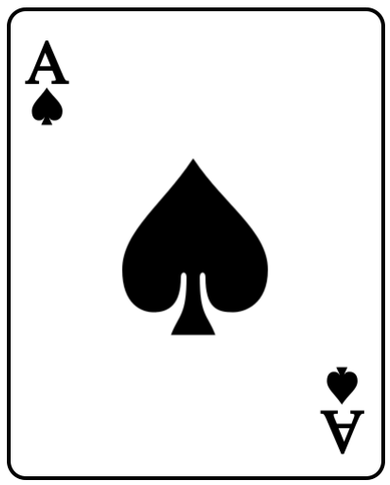




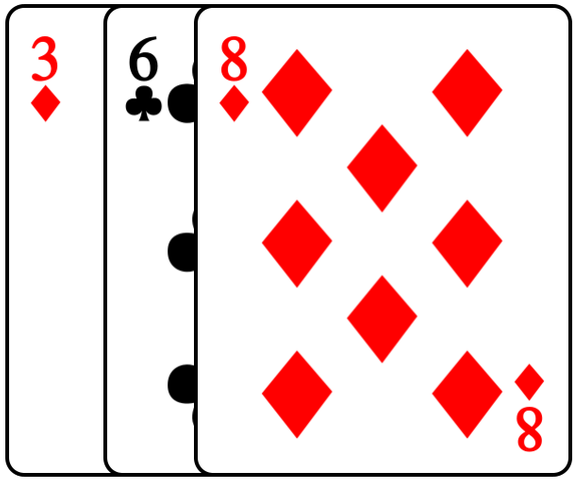
Fusion de tableaux triés

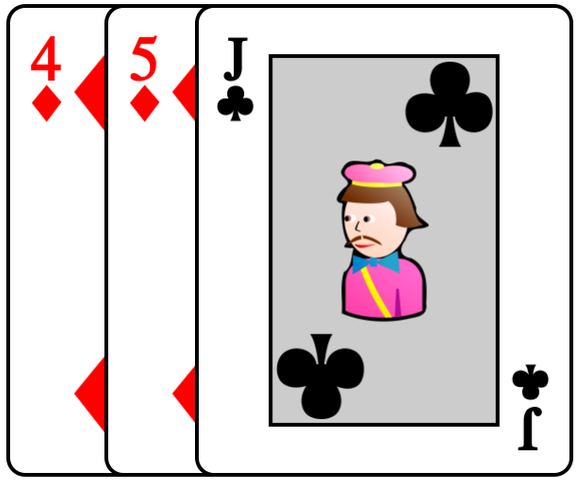


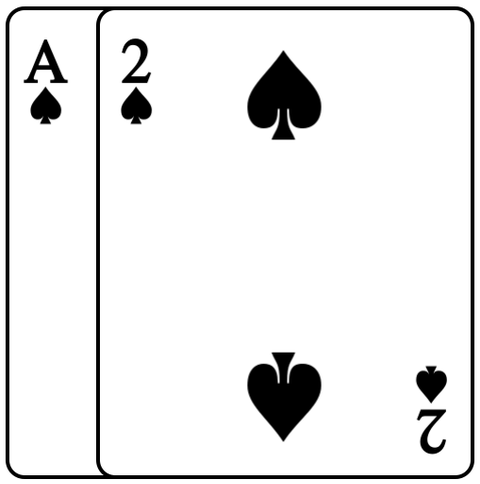




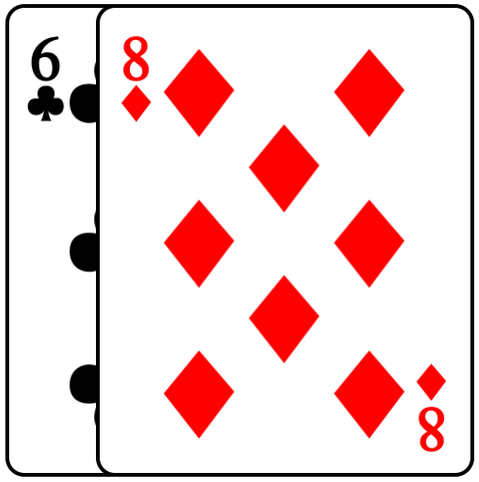
Fusion de tableaux triés

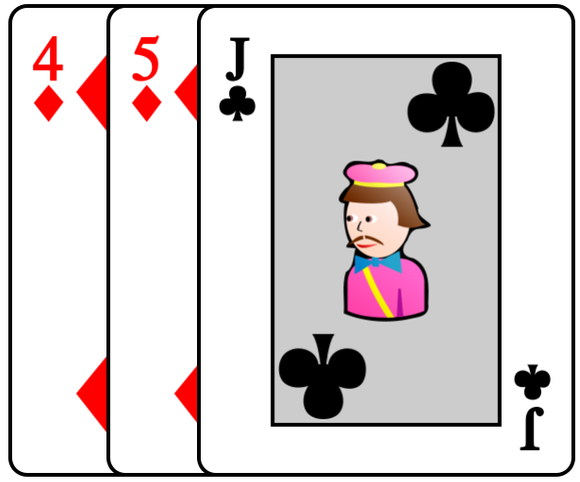


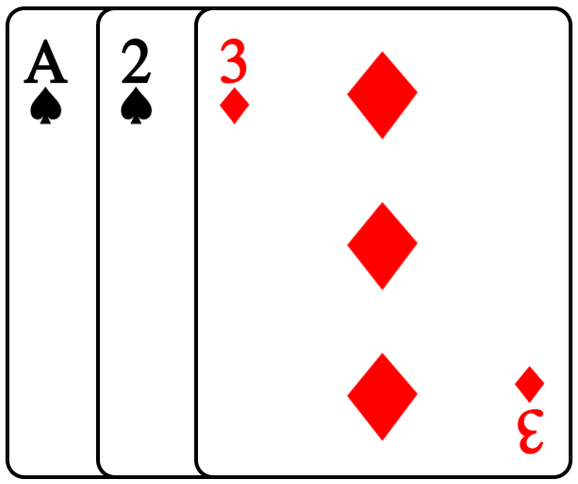




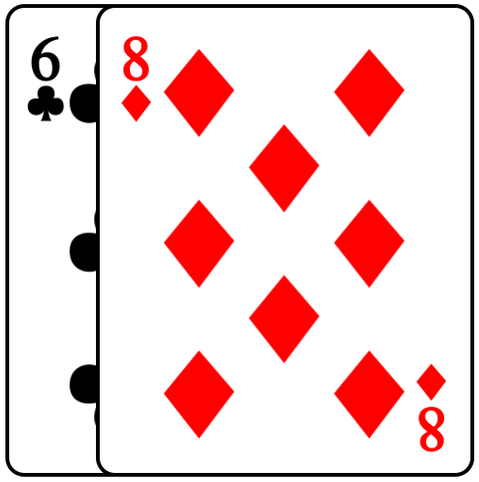
Fusion de tableaux triés

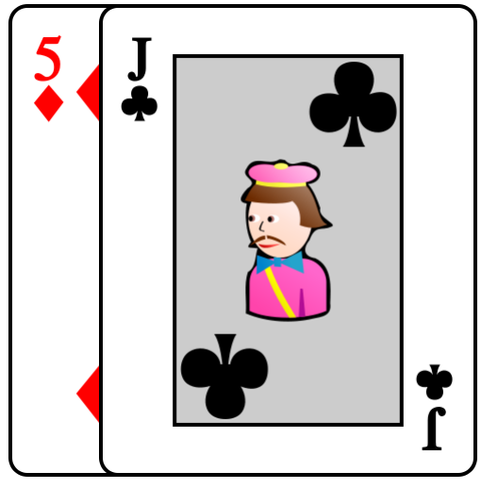


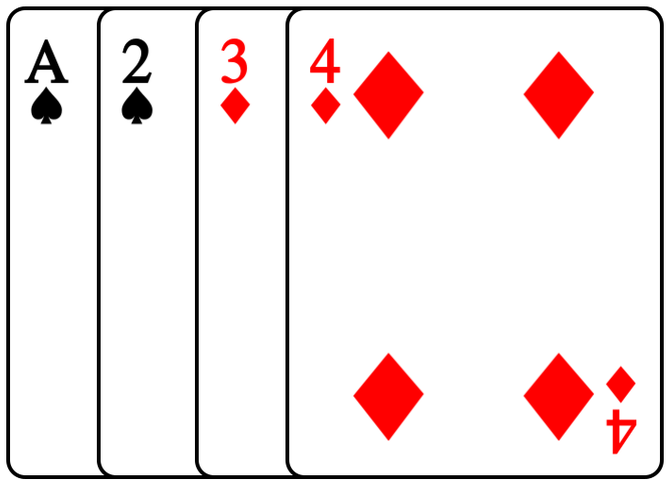




Fusion de tableaux triés







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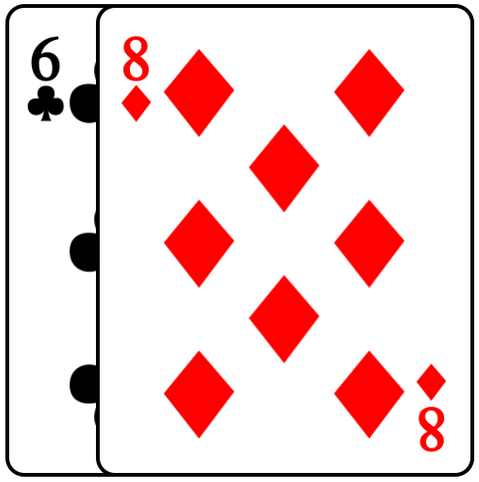

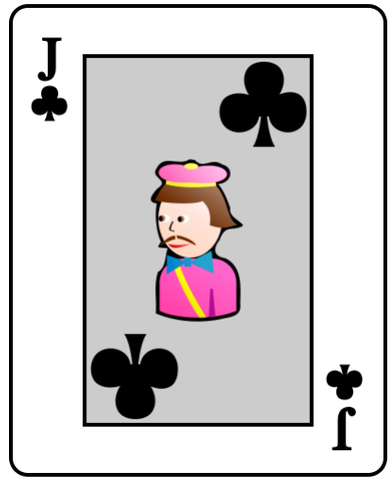

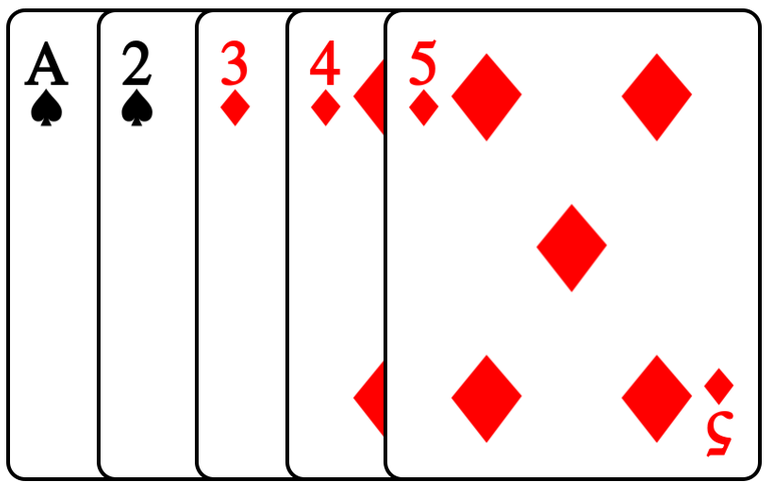

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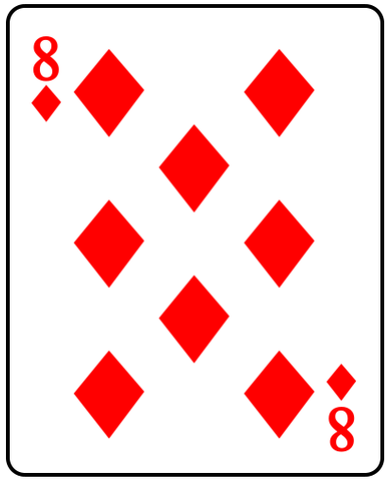

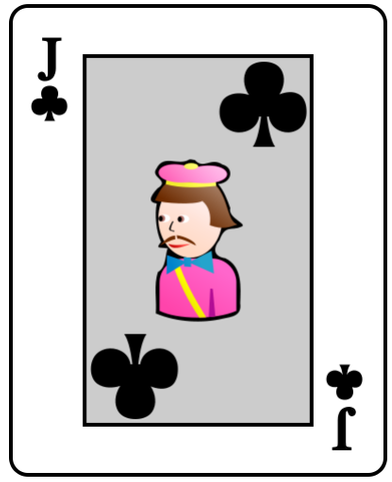

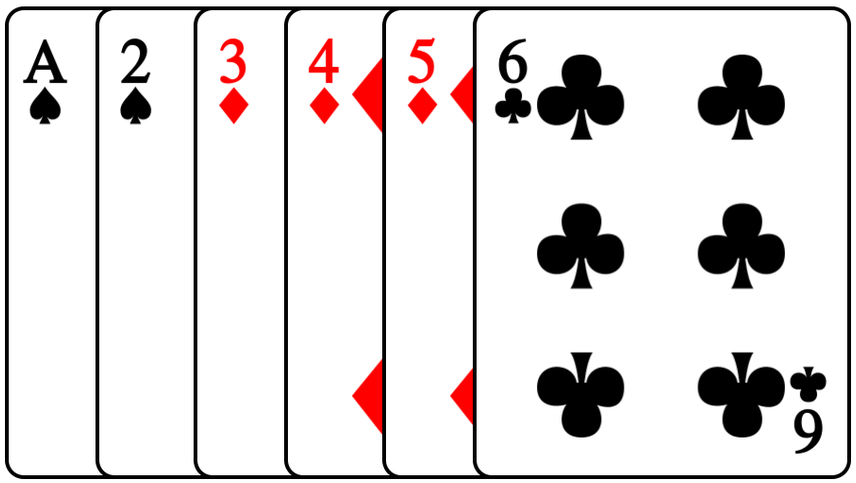

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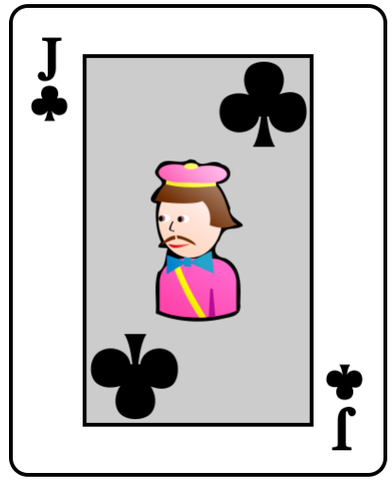

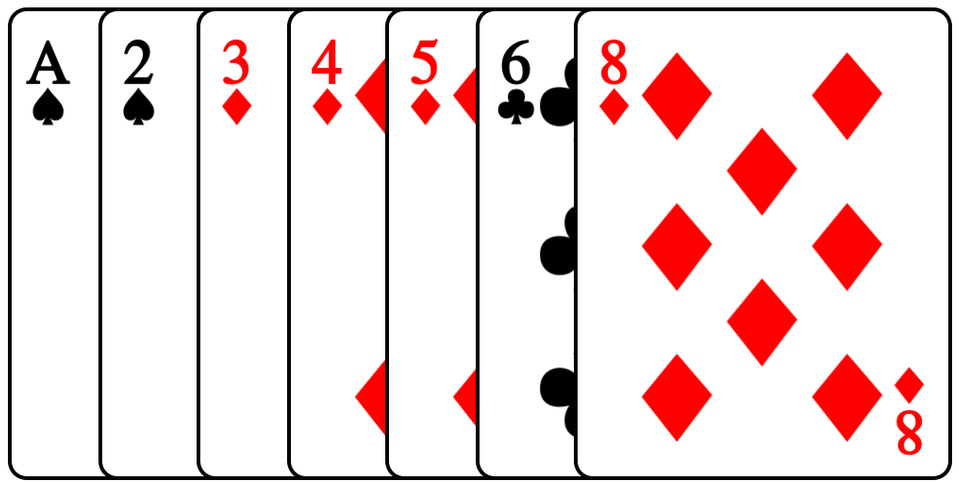

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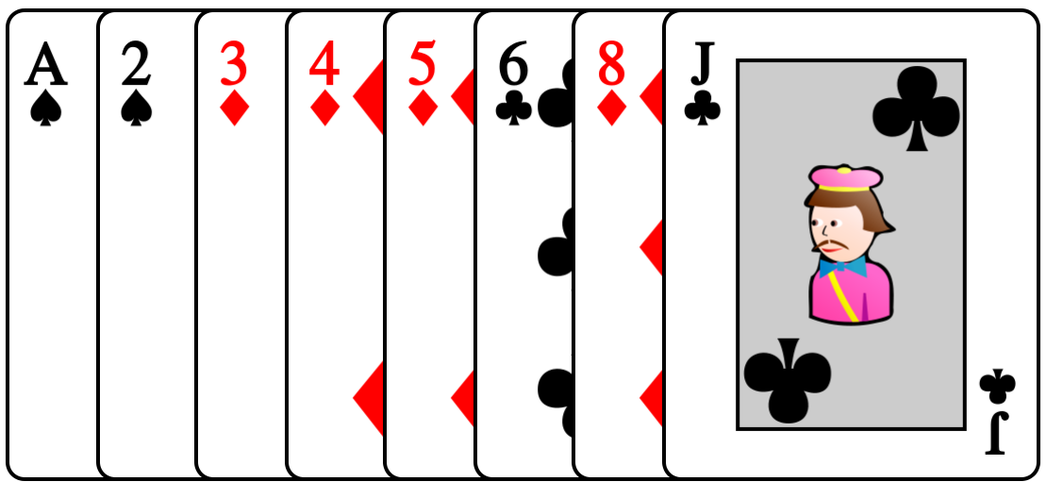
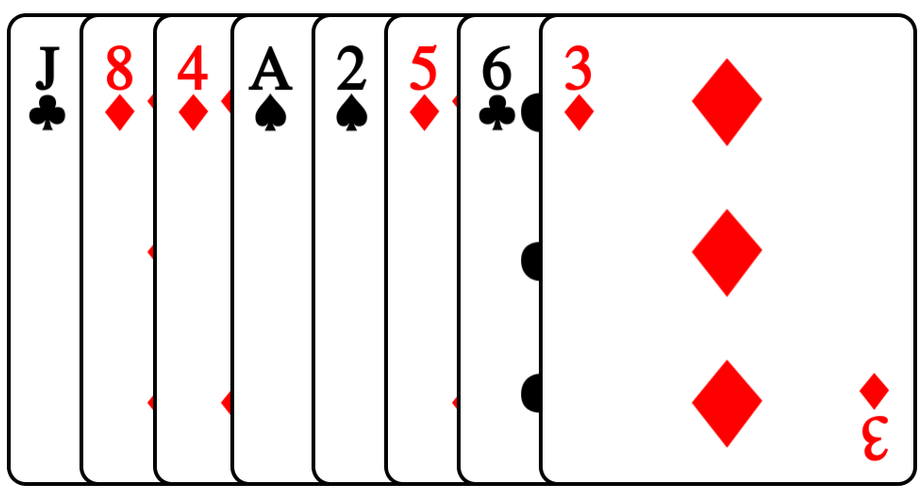


Diviser pour mieux régner !

Tri fusion



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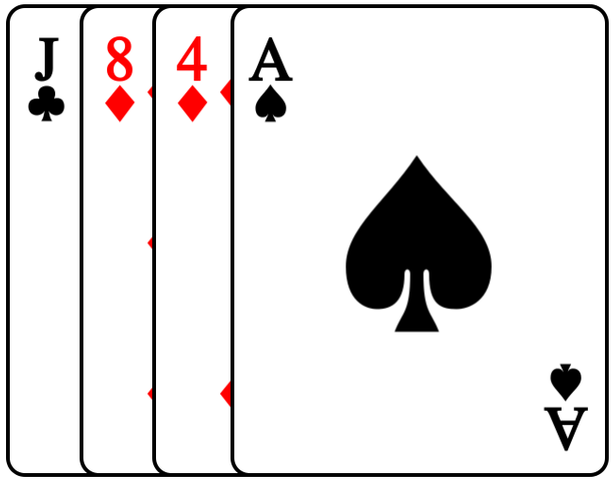

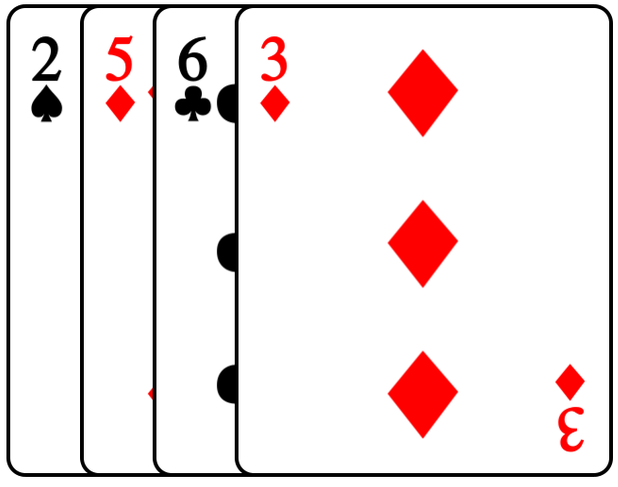

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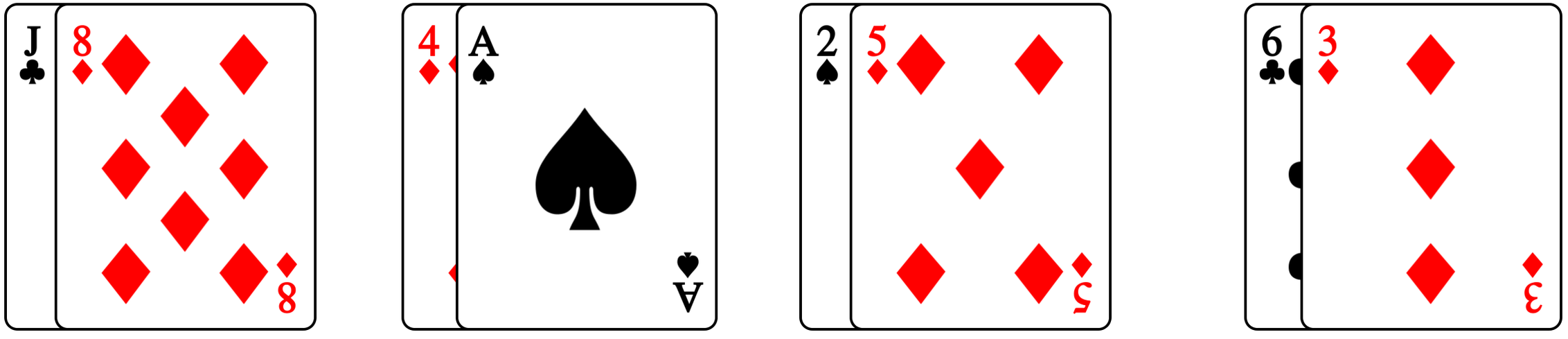

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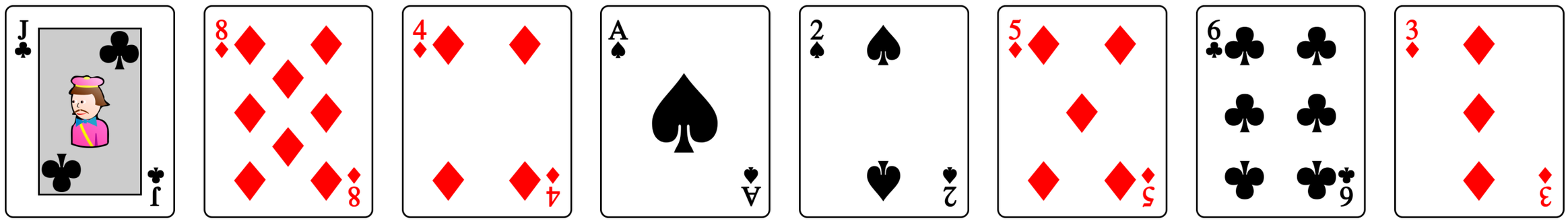
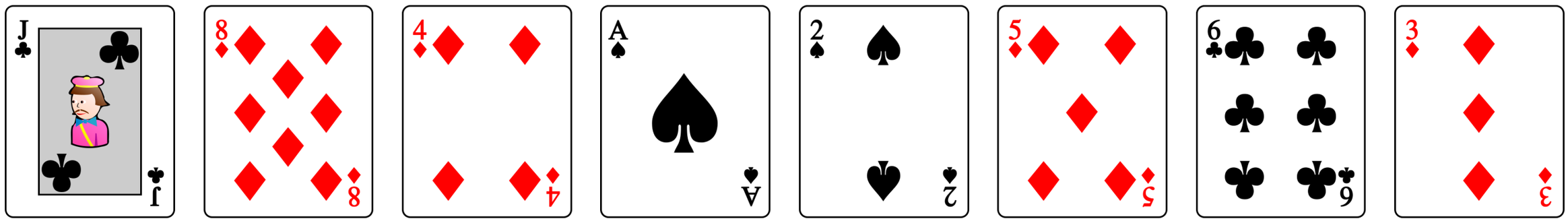


Tous les jeux sont triés !



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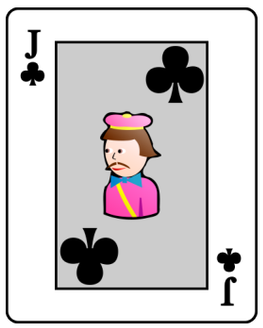
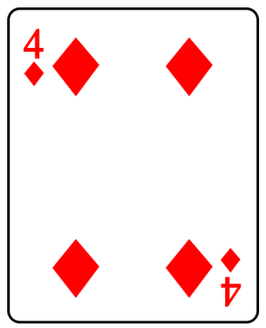
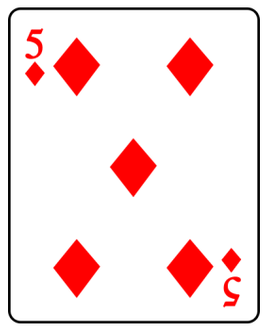
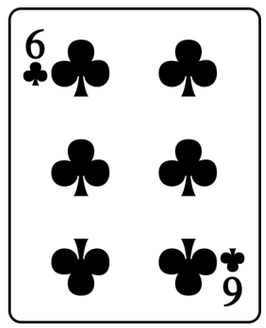
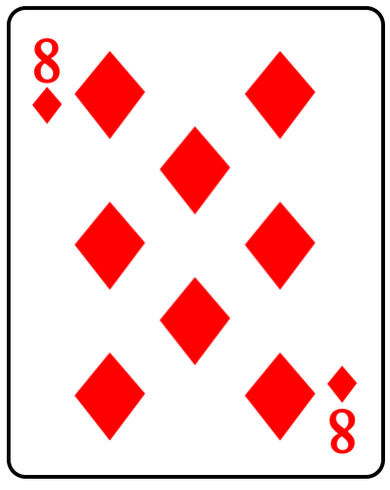
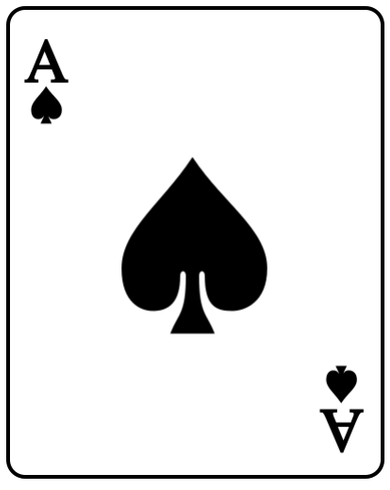
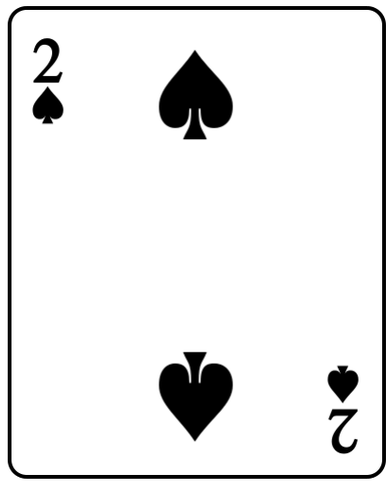
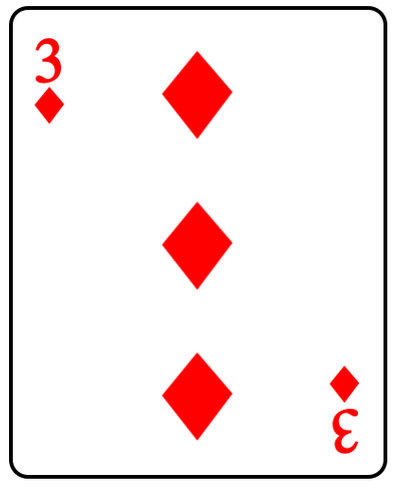

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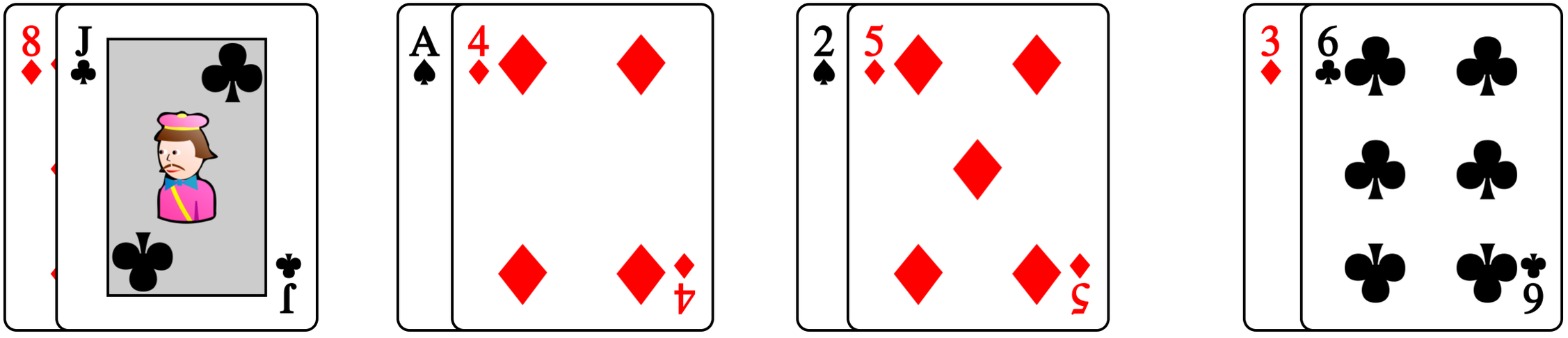
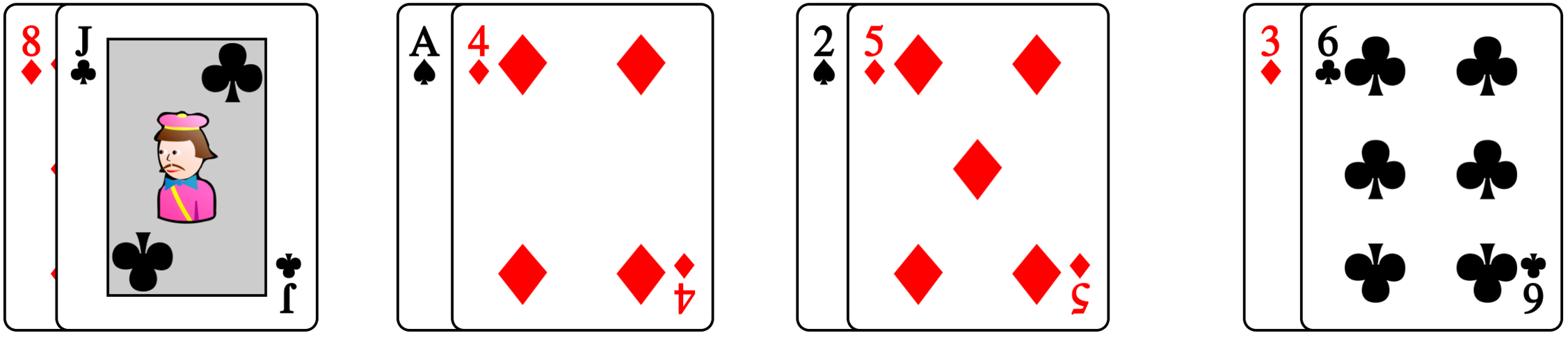


Tous les jeux sont triés !



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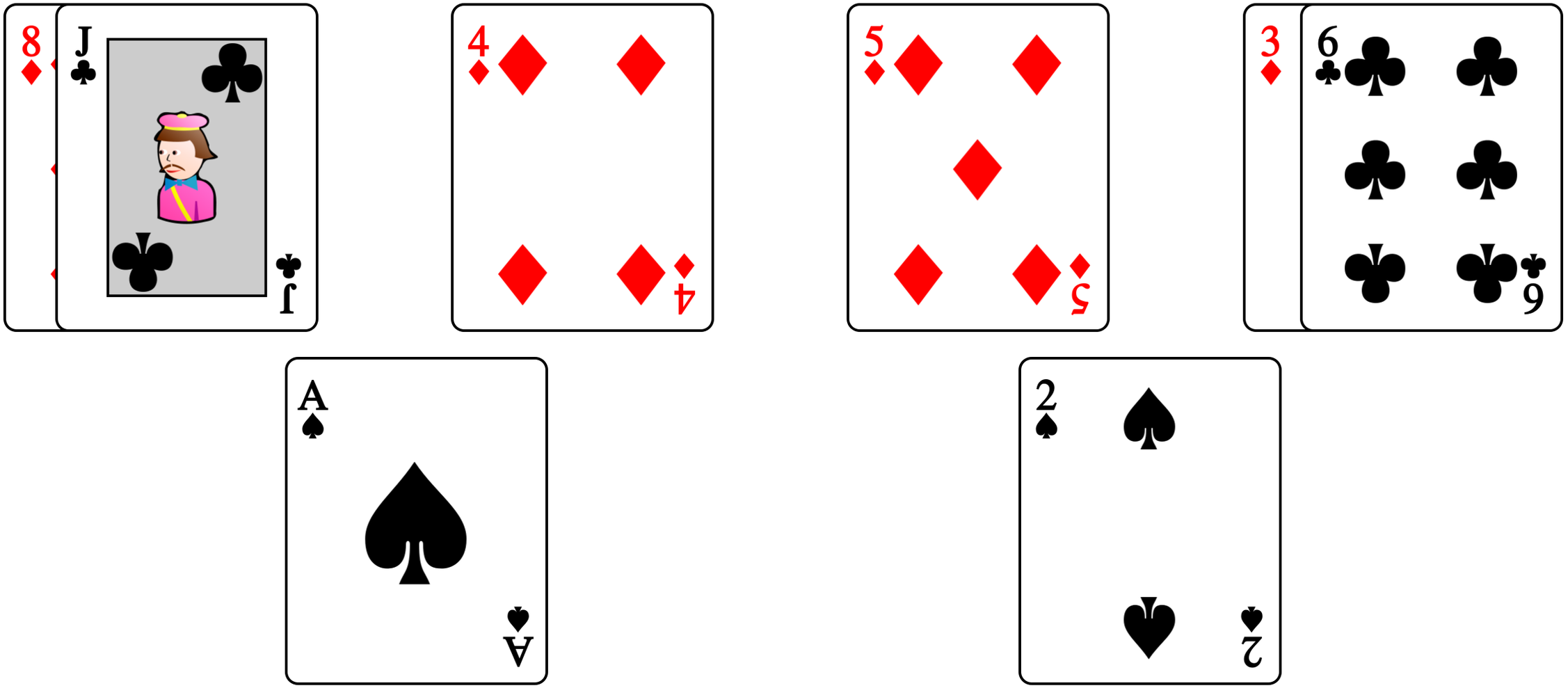

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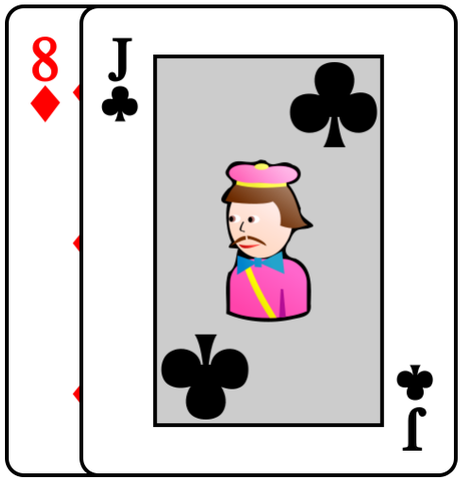

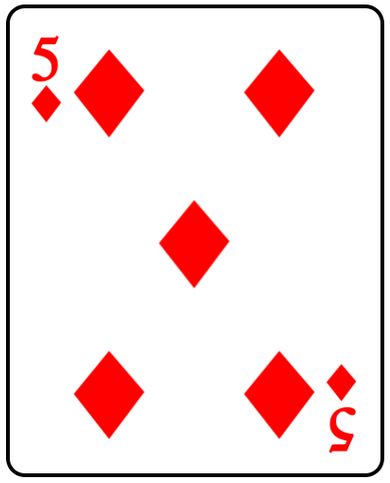

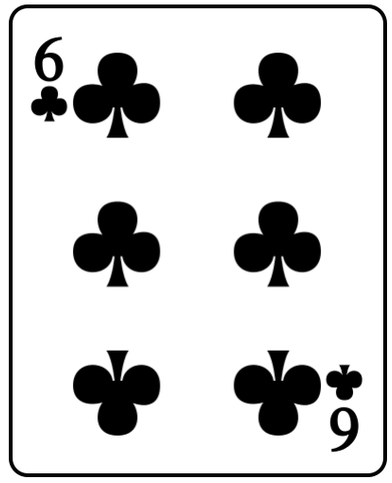

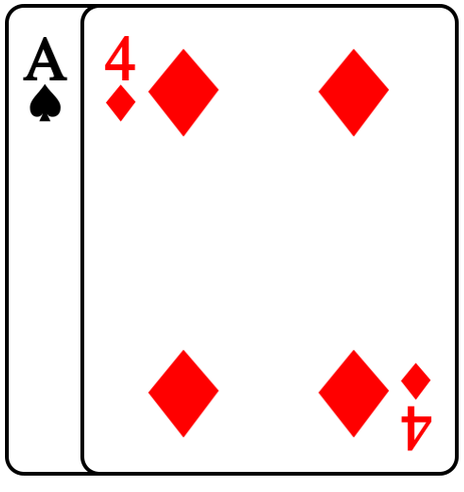

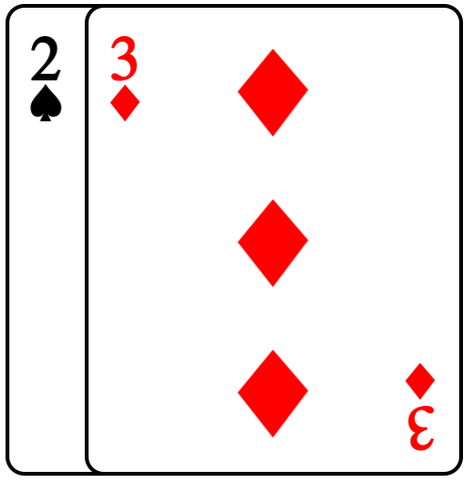

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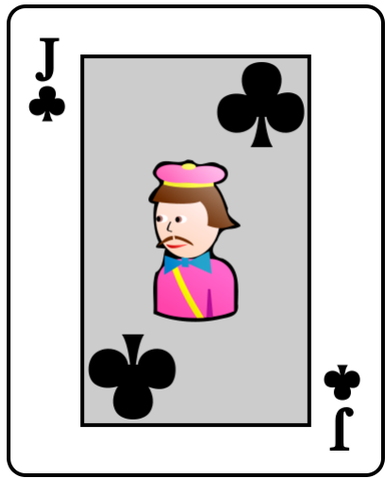

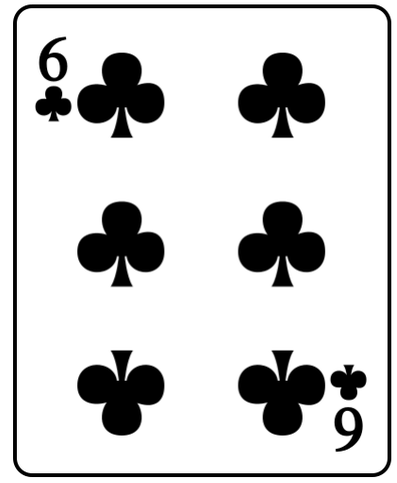

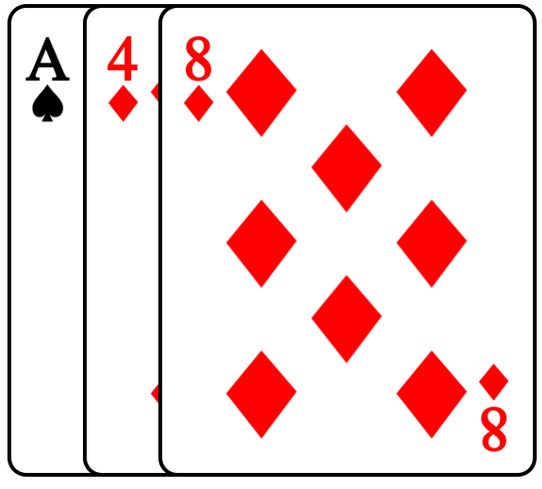

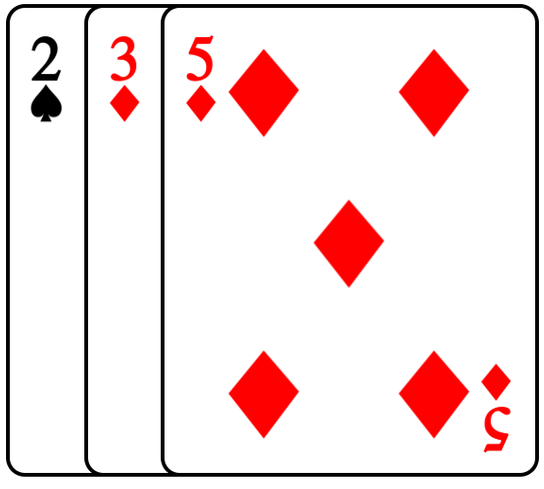

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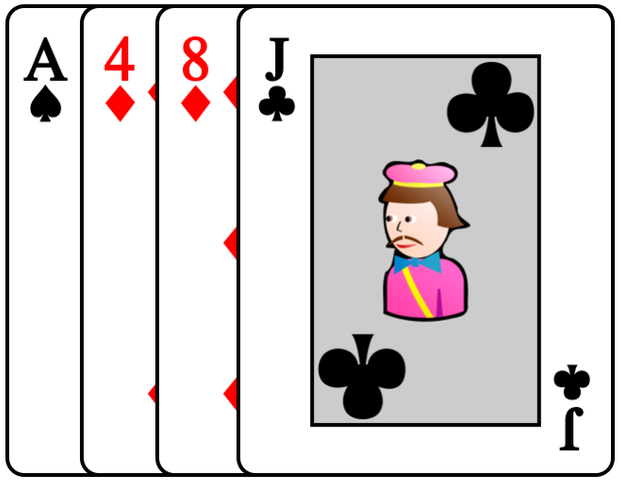
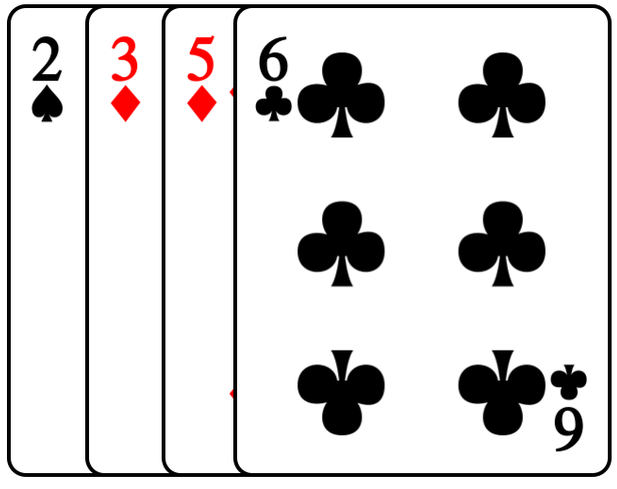
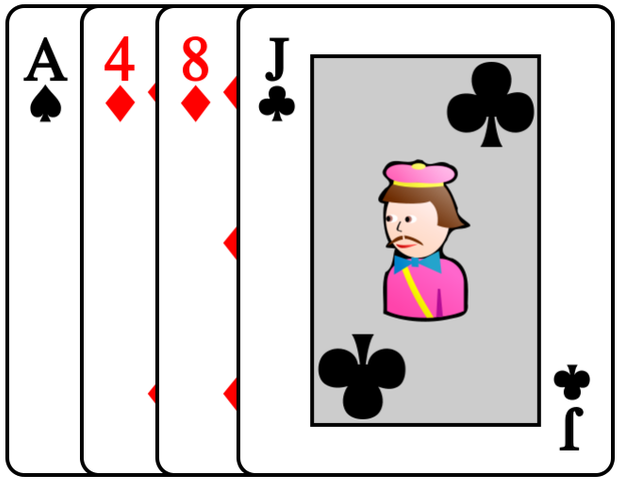
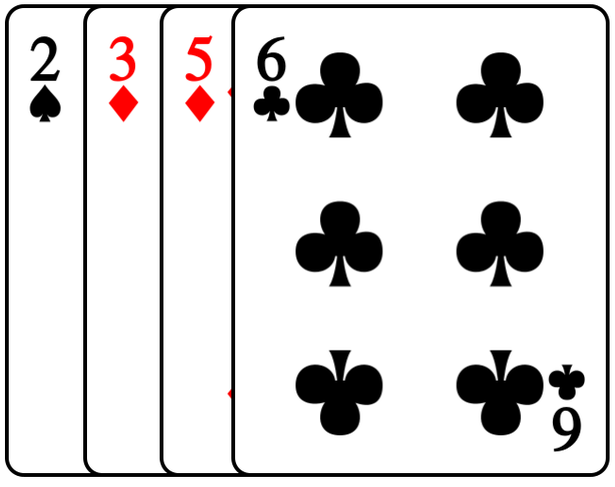




Tous les jeux sont triés !





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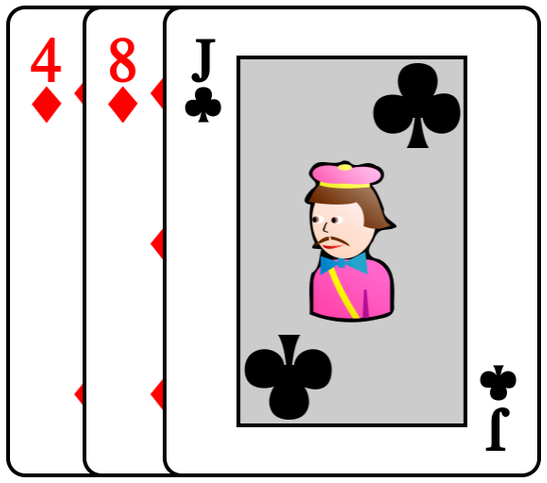

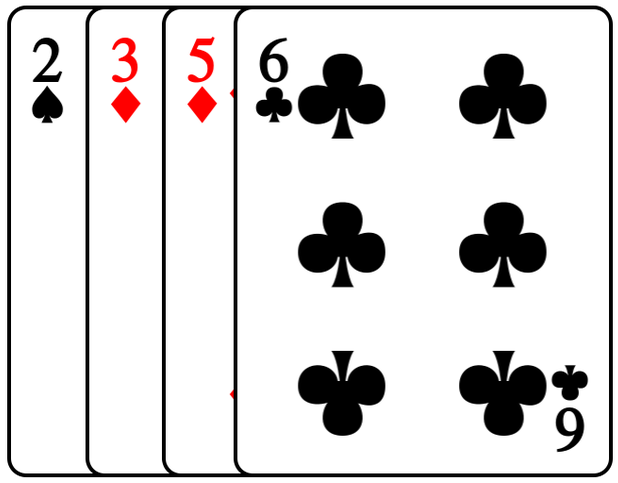

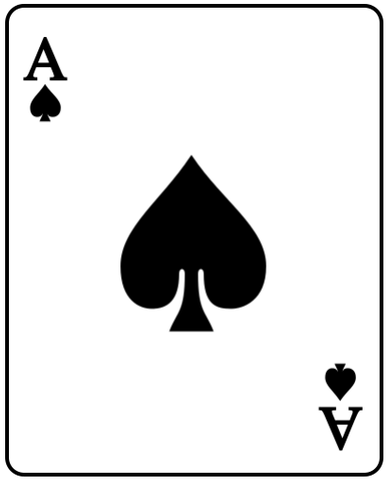

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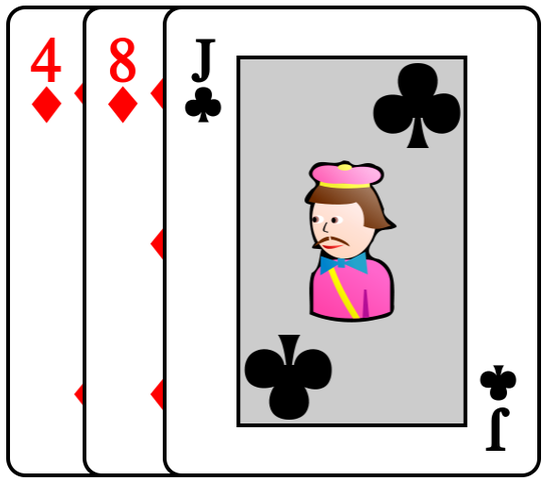

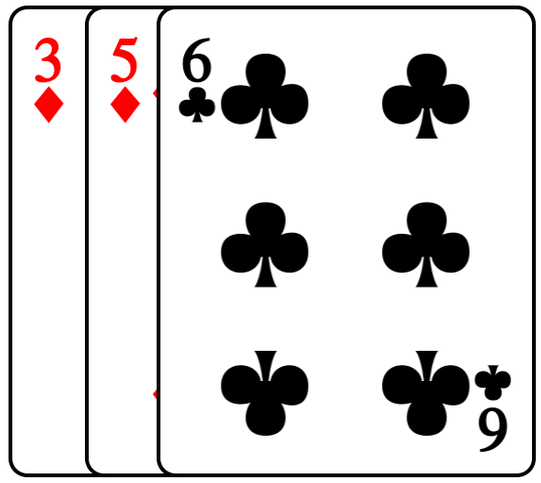

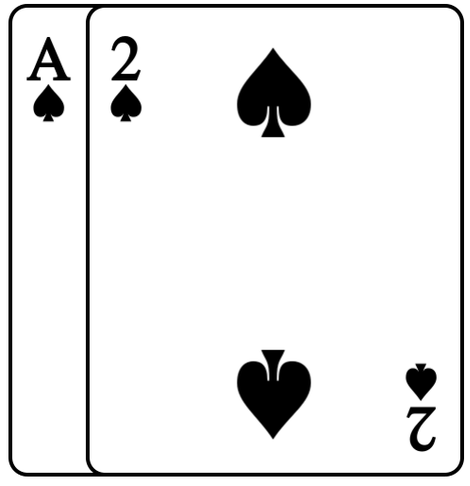

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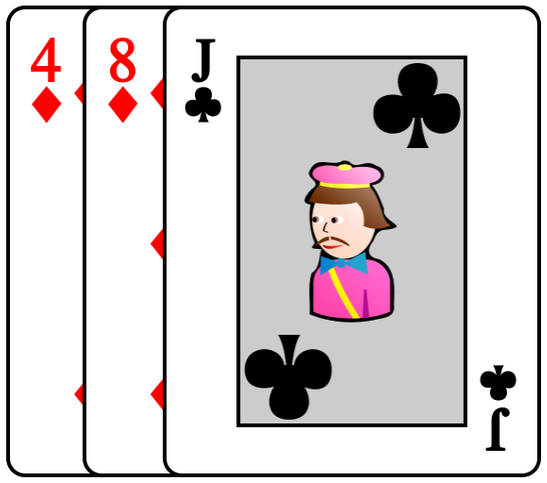

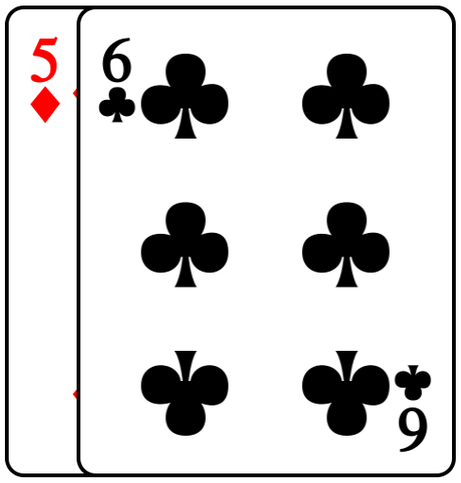

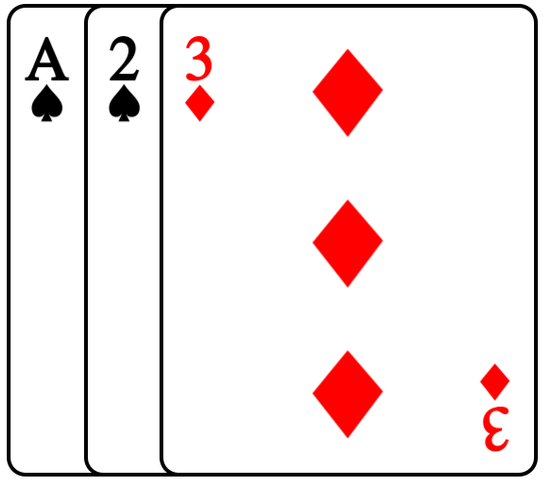

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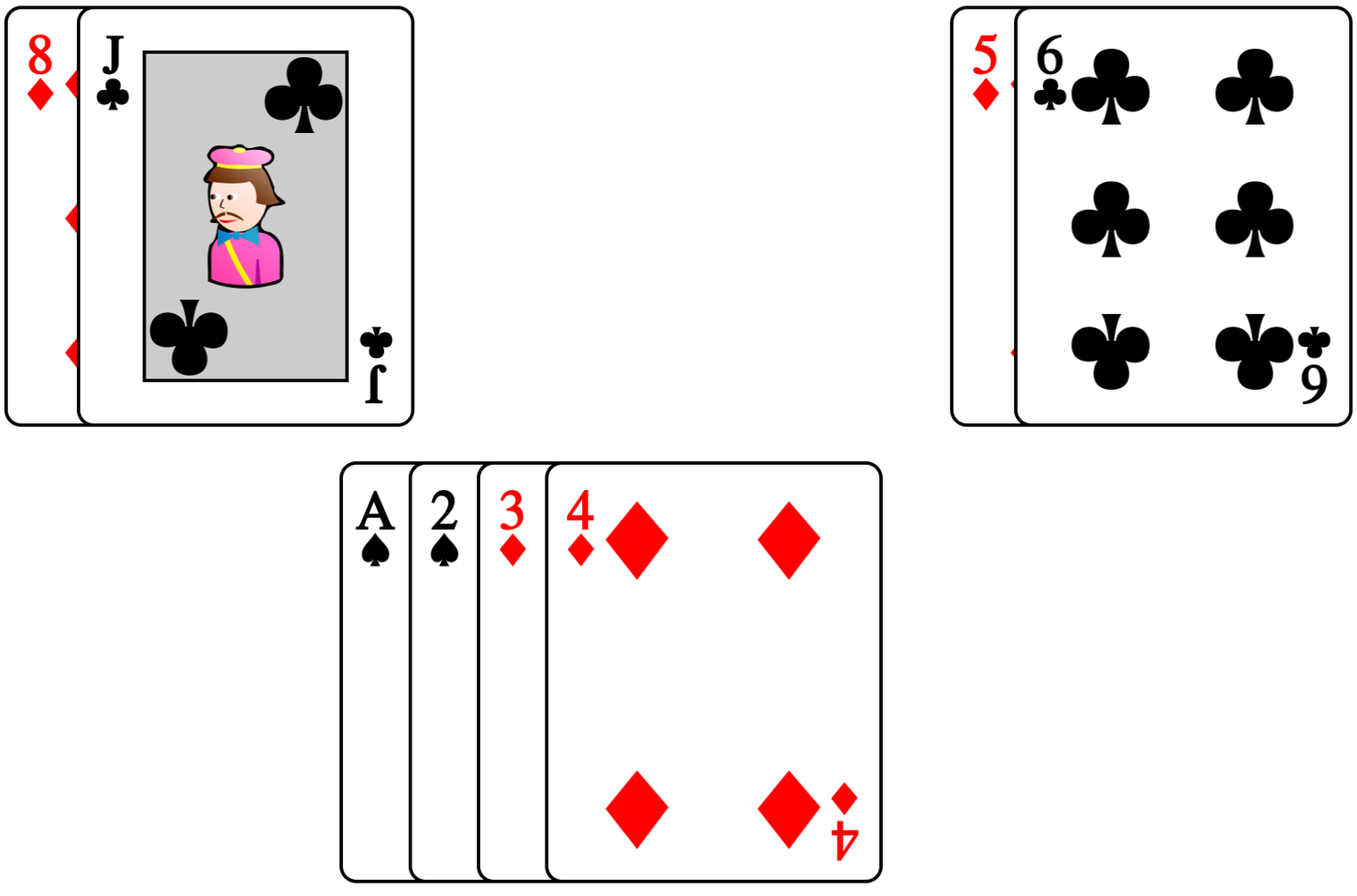

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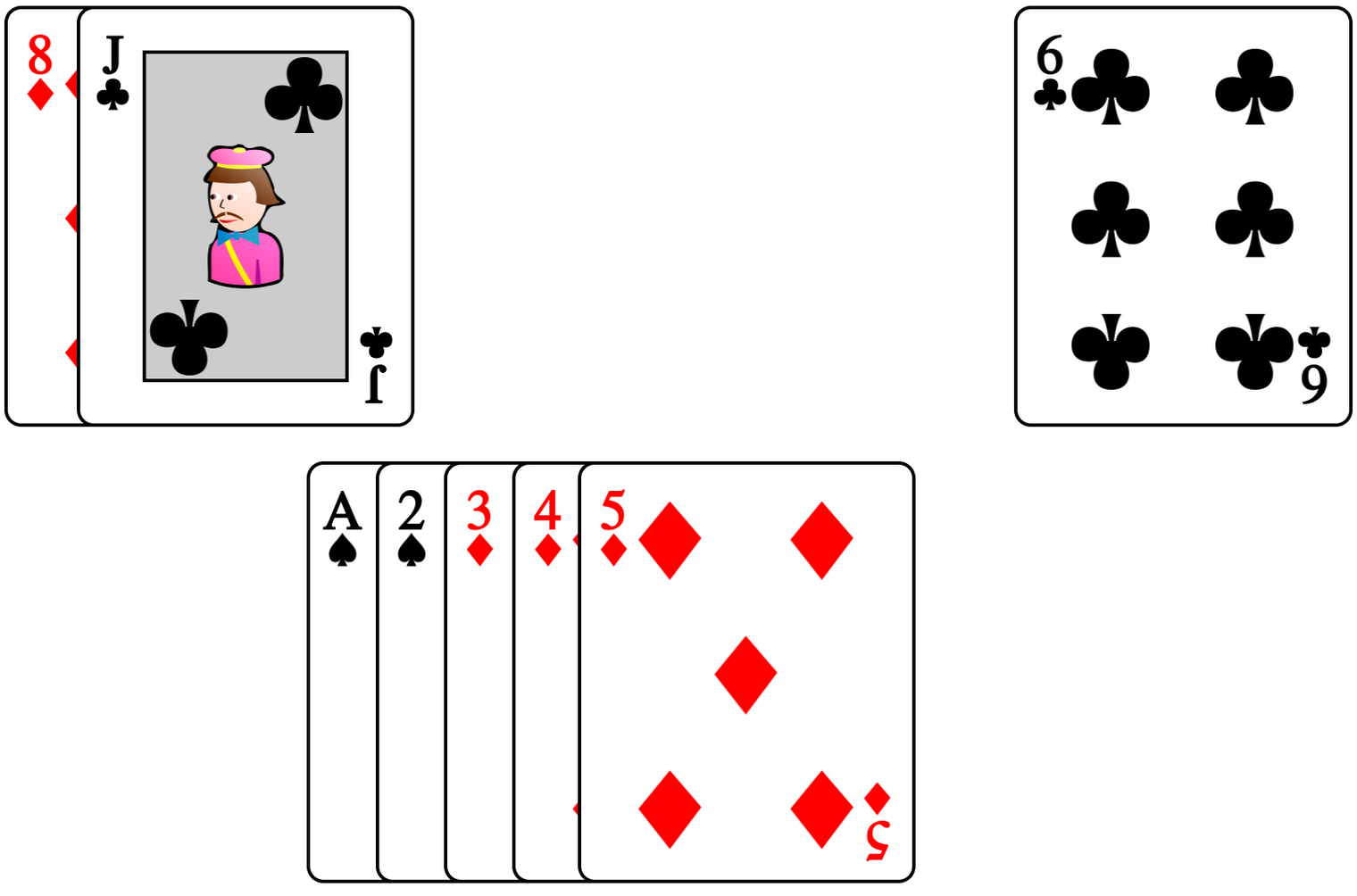

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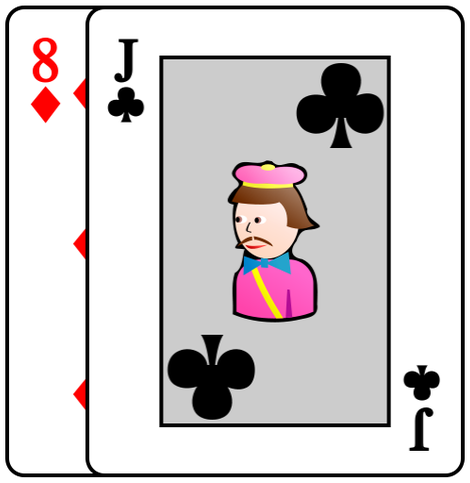

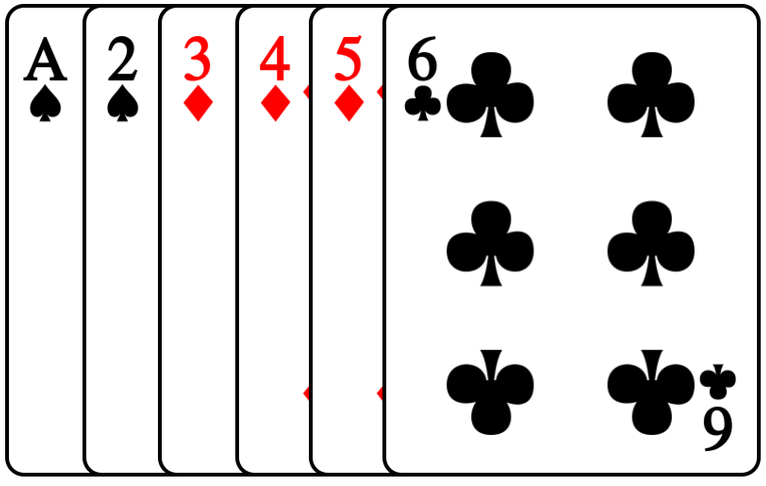

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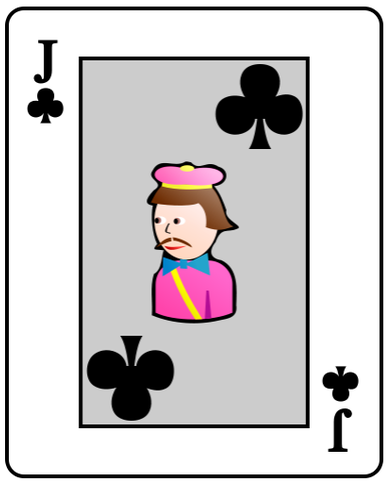
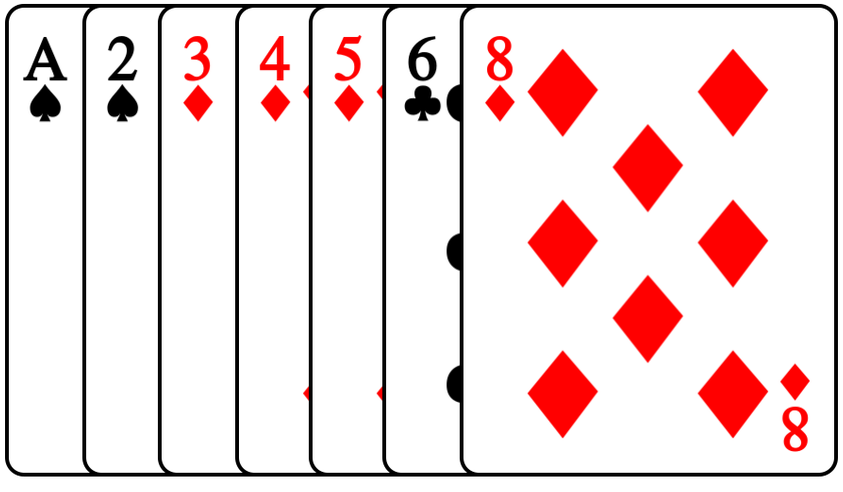
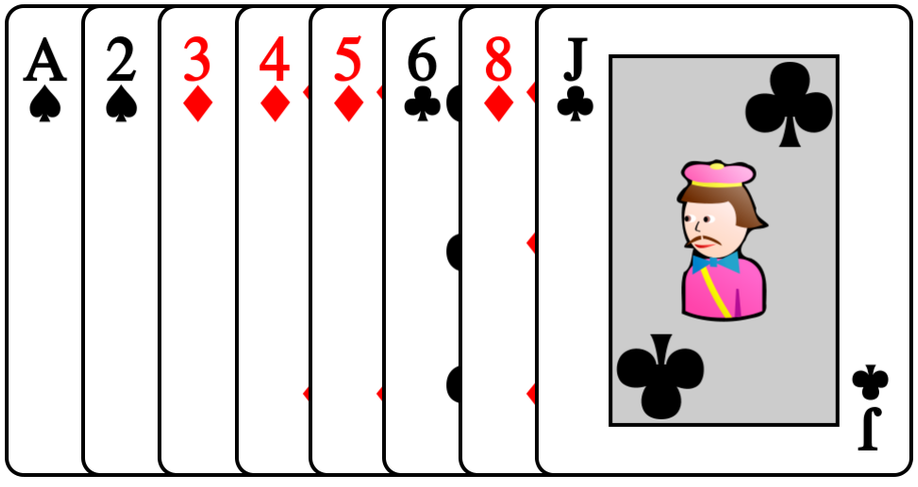
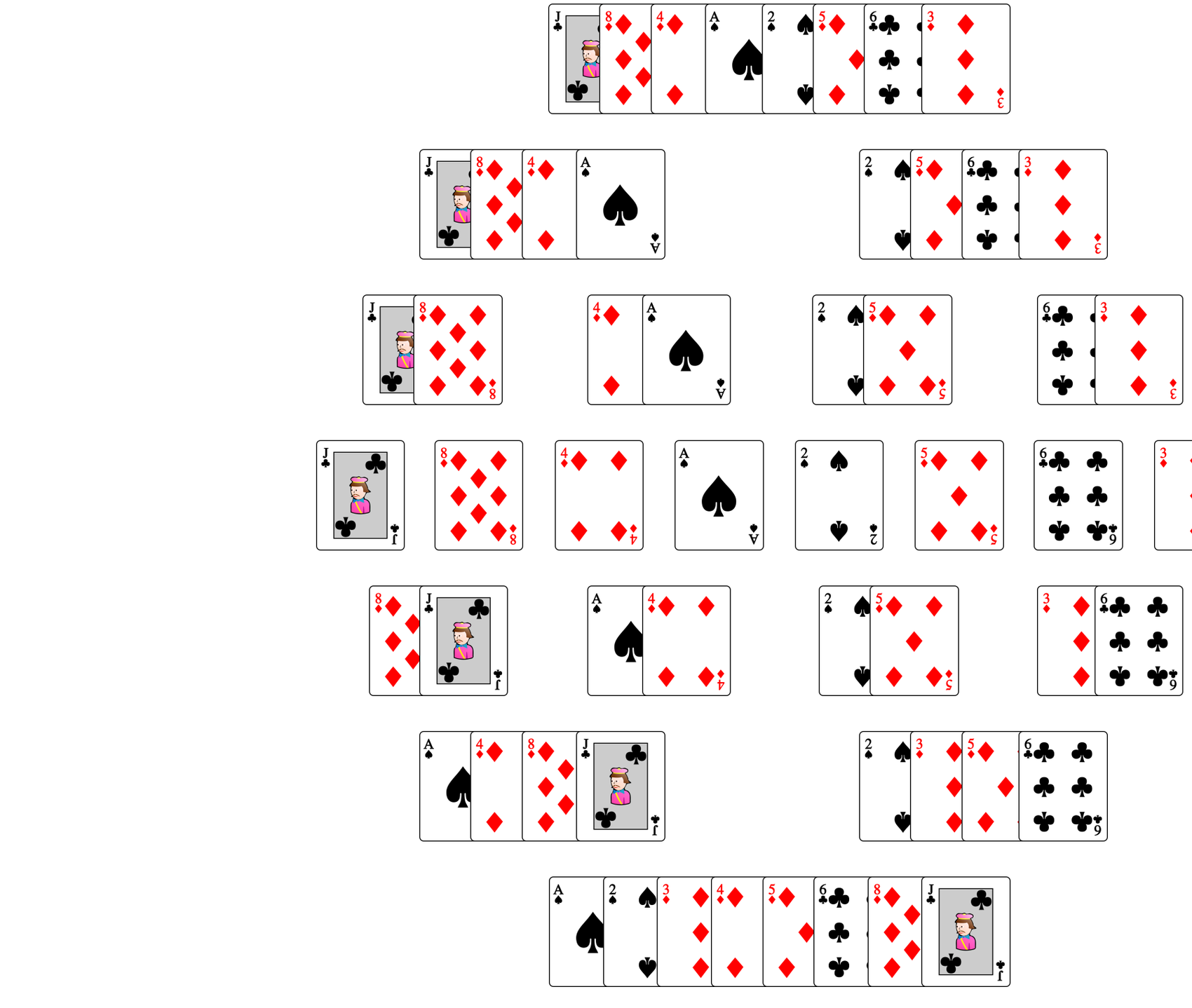




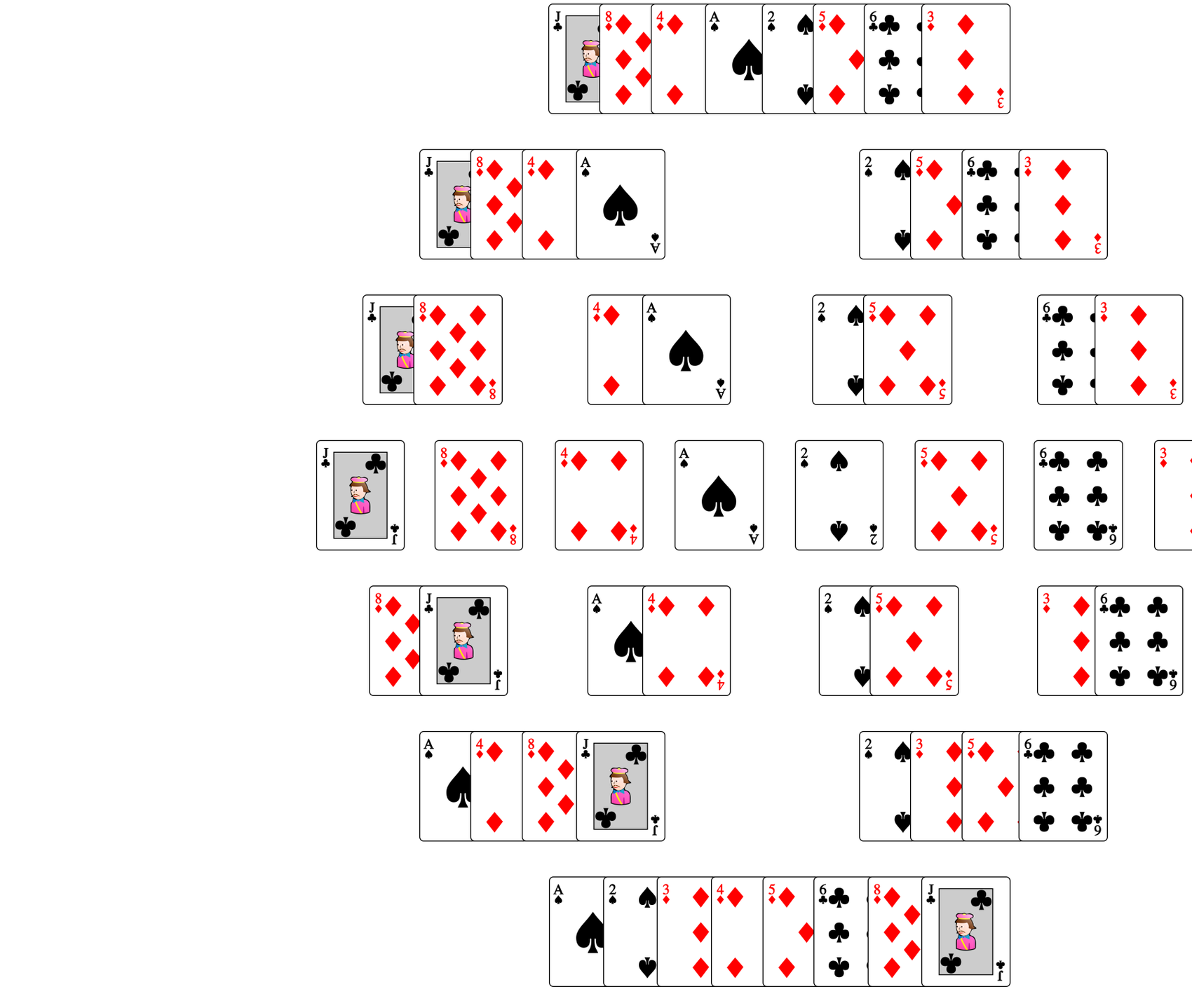
Le jeu est trié !



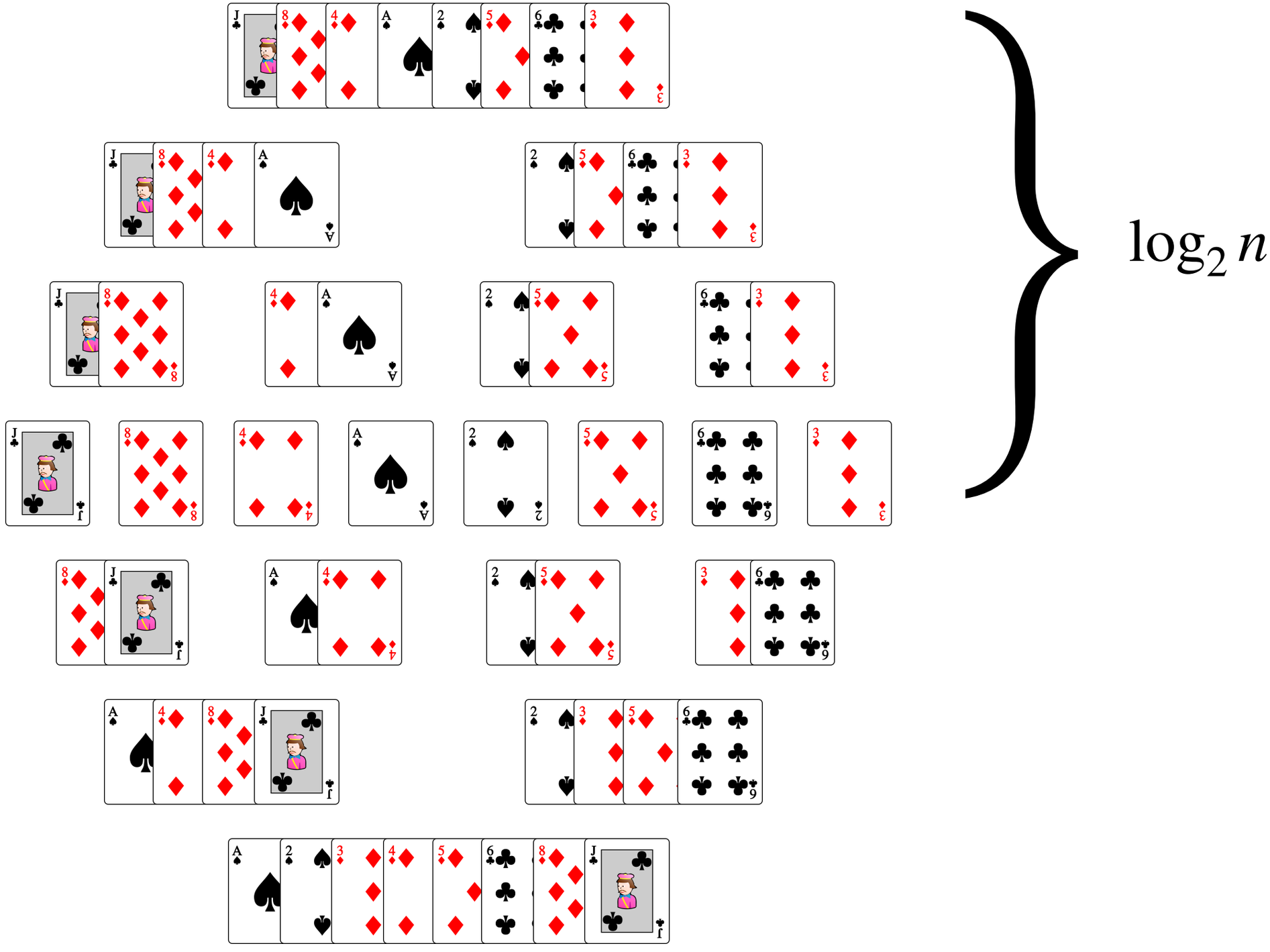
Efficacité du tri fusion



Efficacité du tri fusion

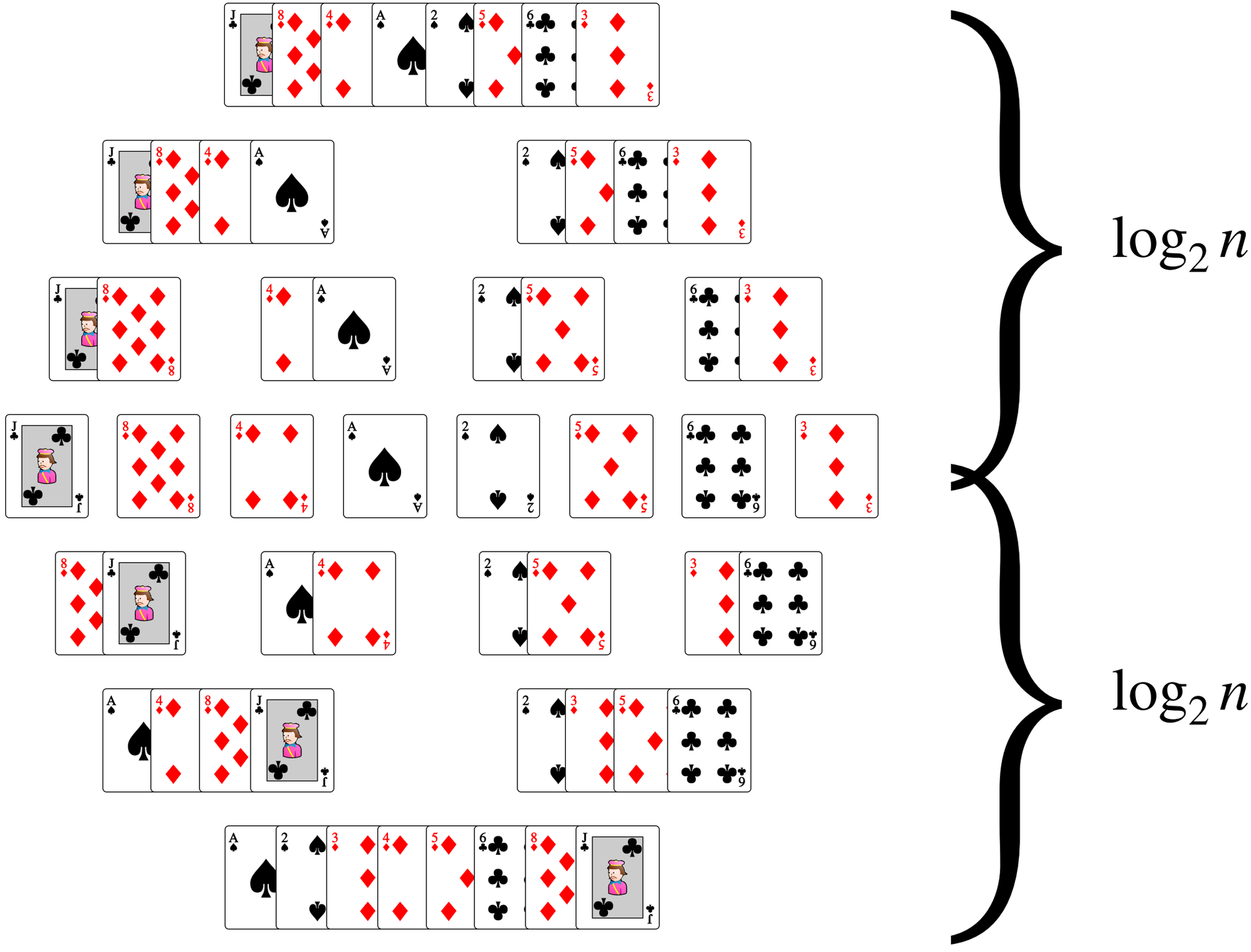


Efficacité du tri fusion



$\log_2 n$

Efficacité du tri fusion

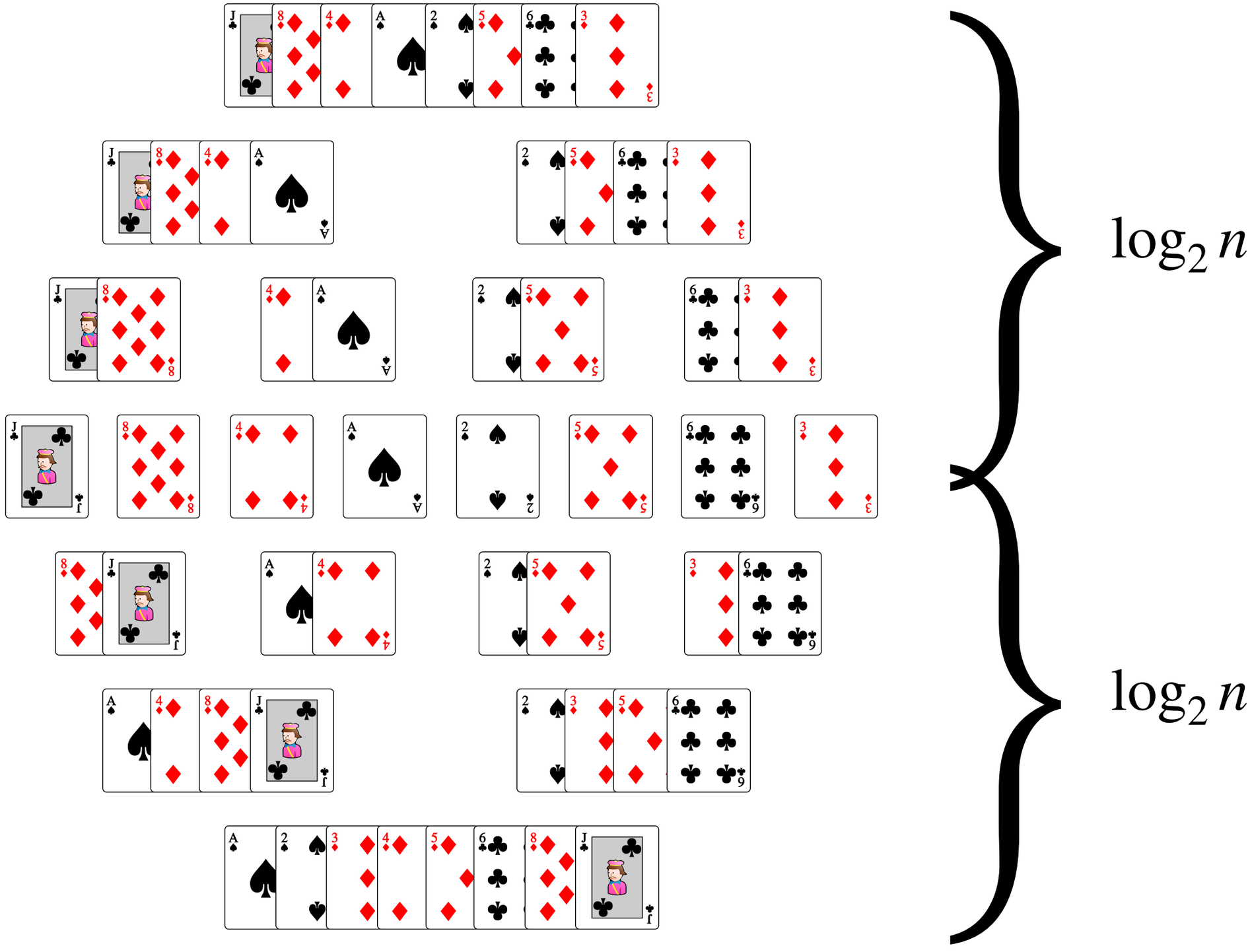


$\log_2 n$

$\log_2 n$

Efficacité du tri fusion

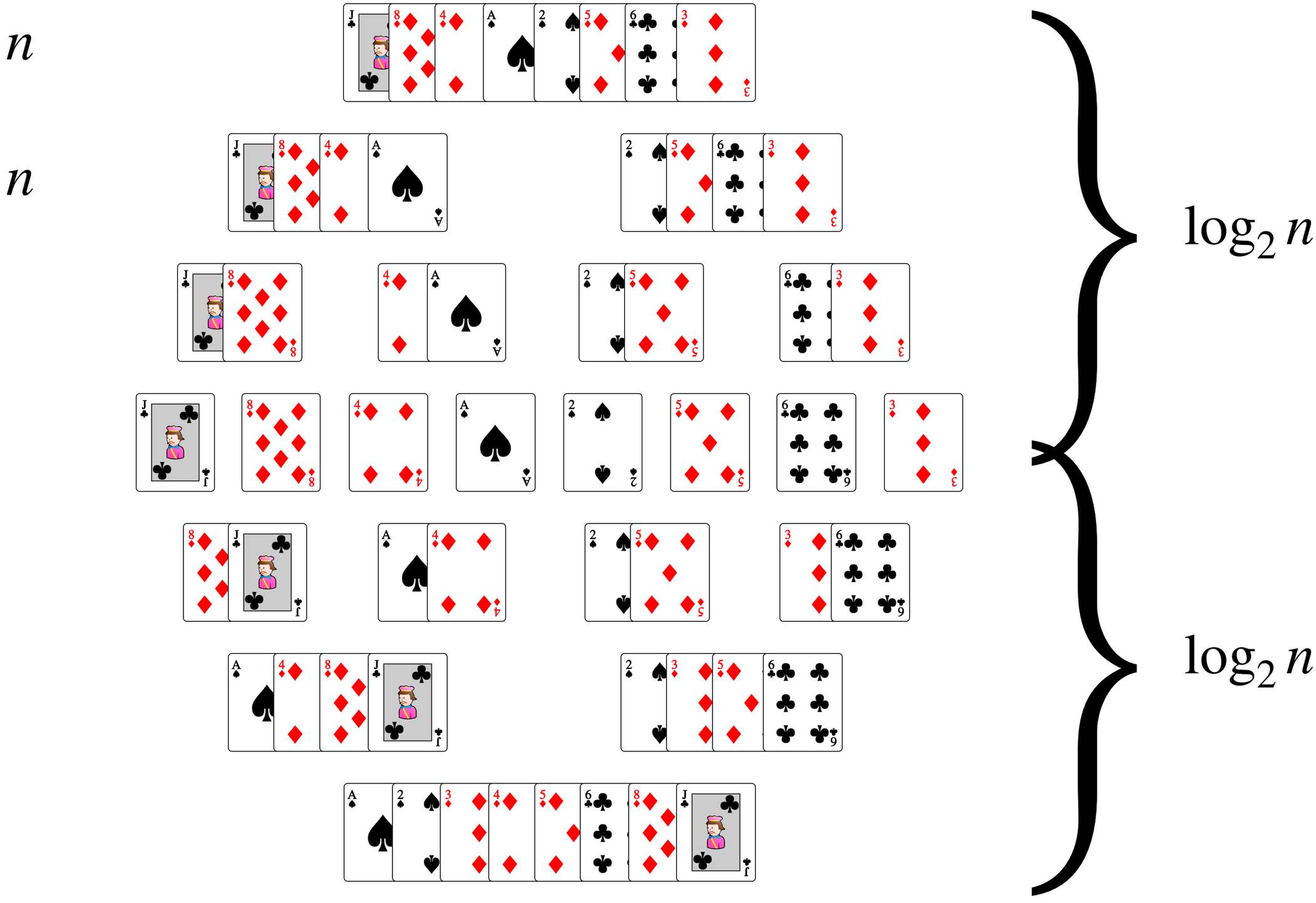
n



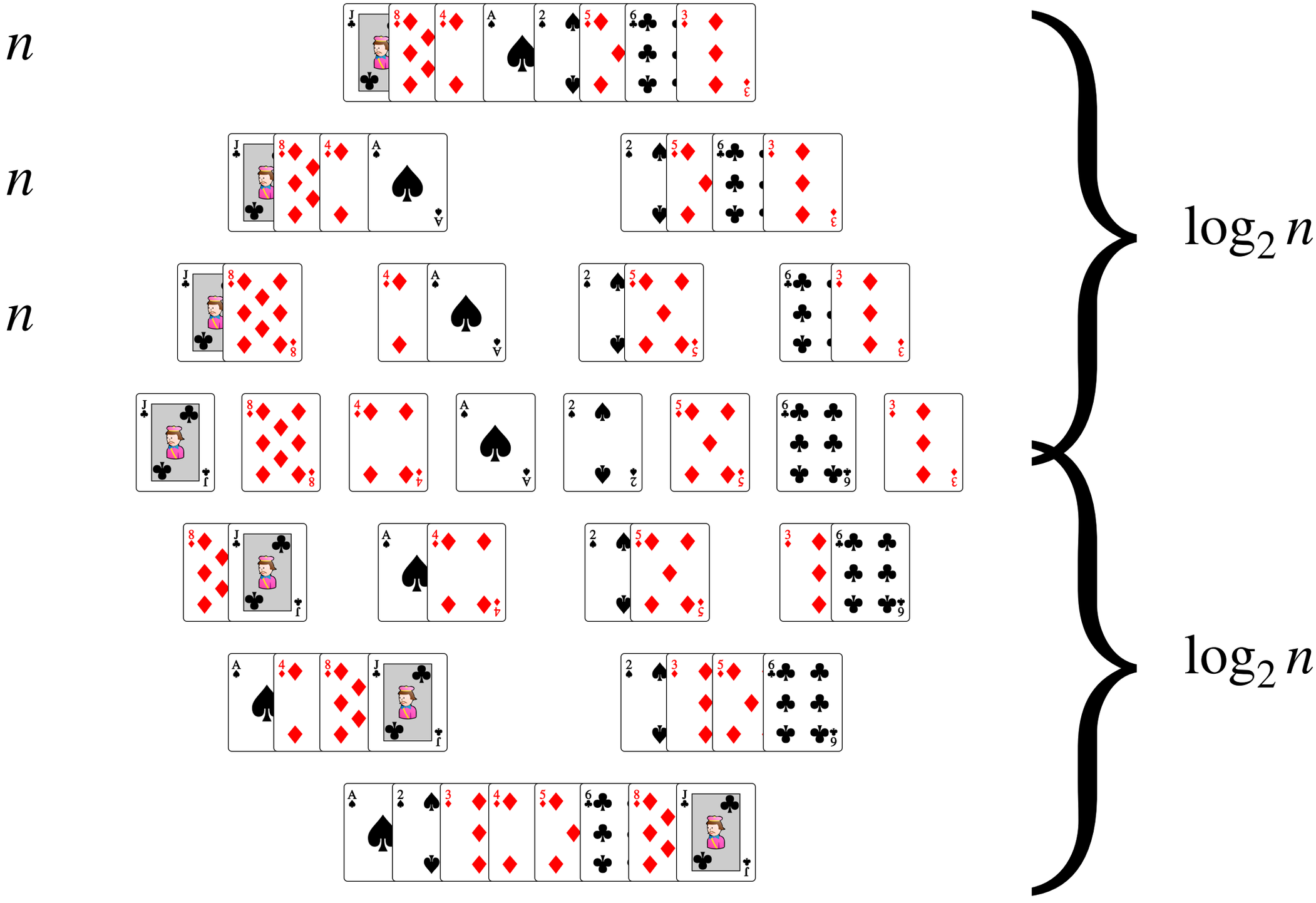
$\log_2 n$

$\log_2 n$

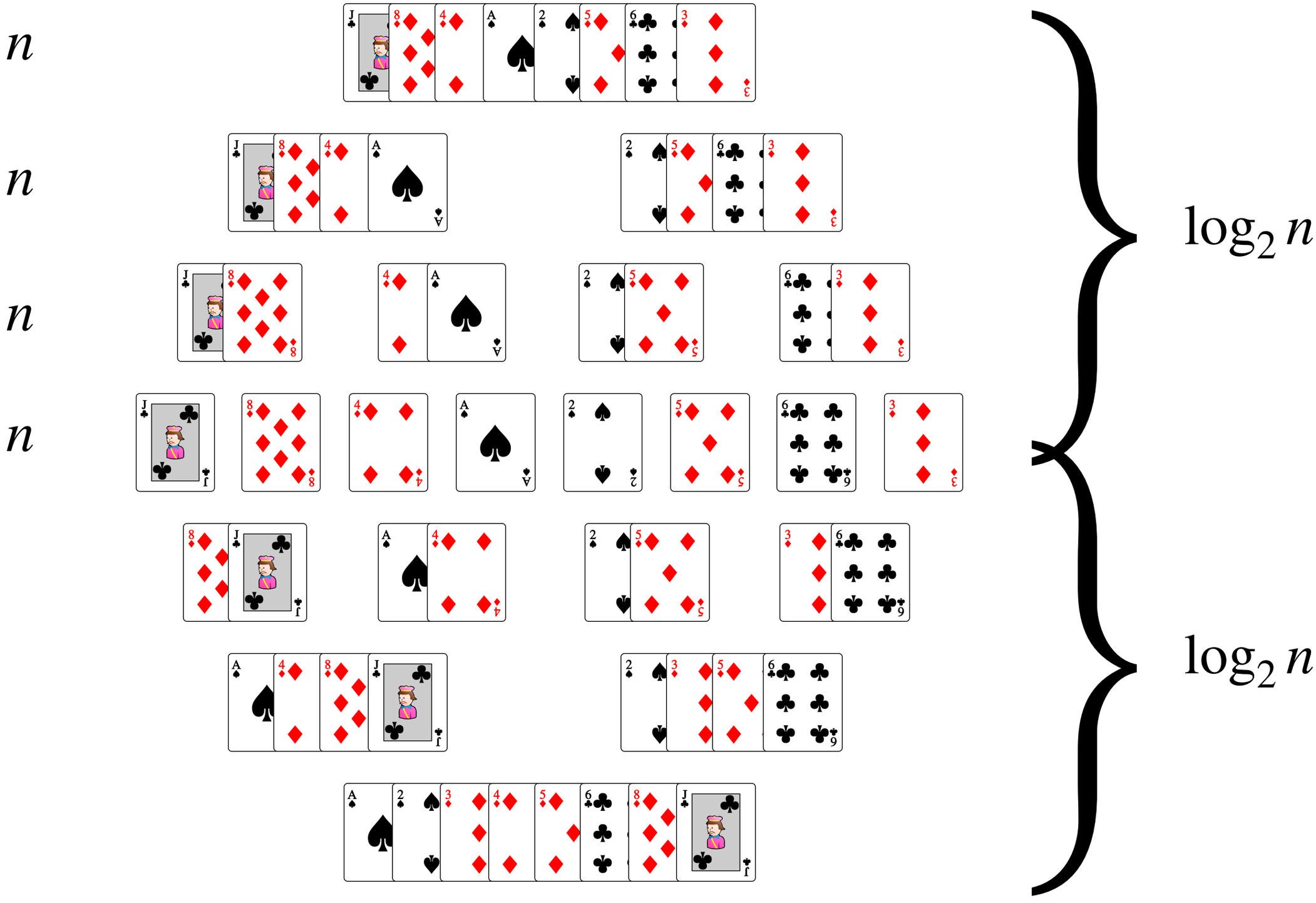
Efficacité du tri fusion



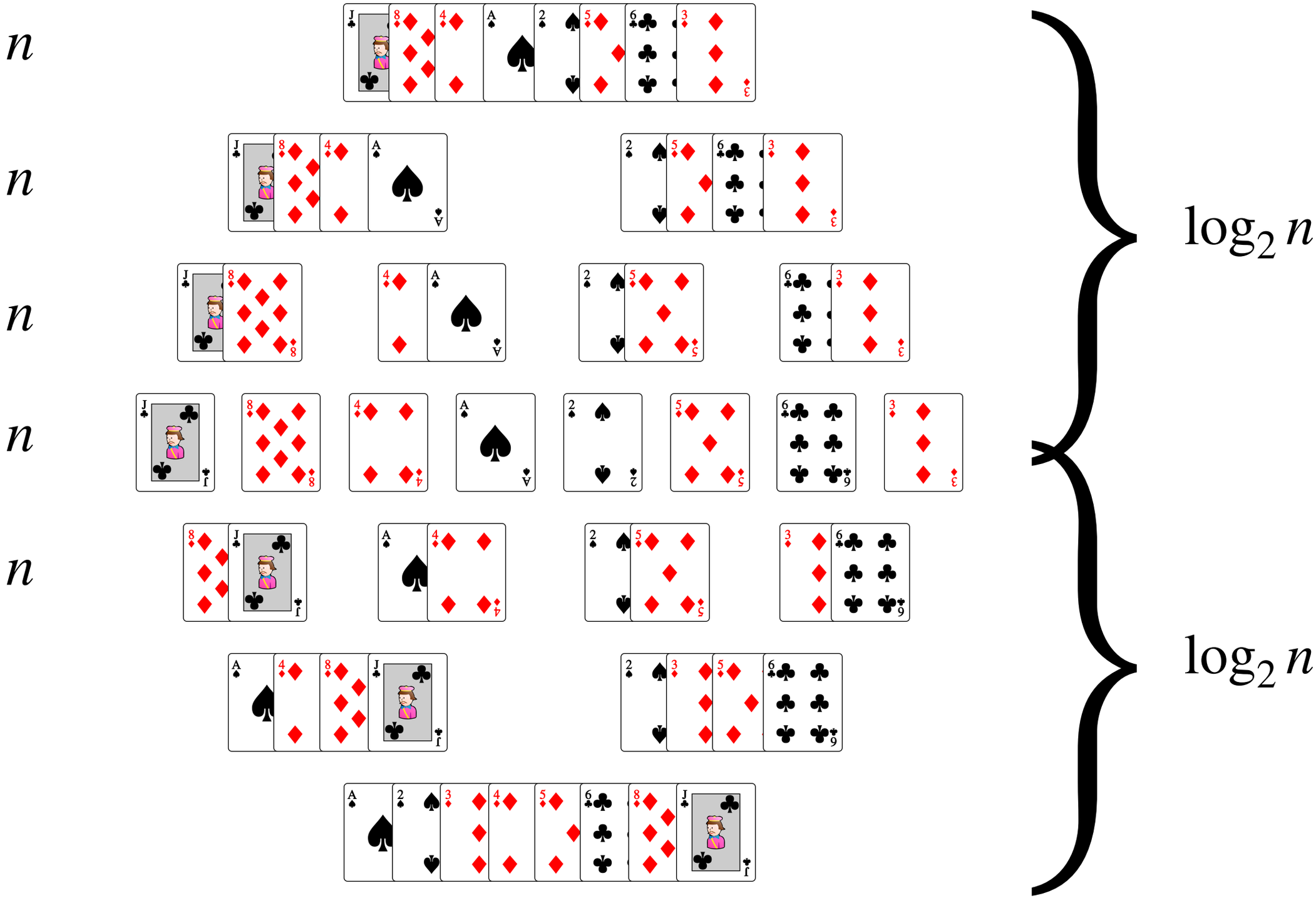
Efficacité du tri fusion



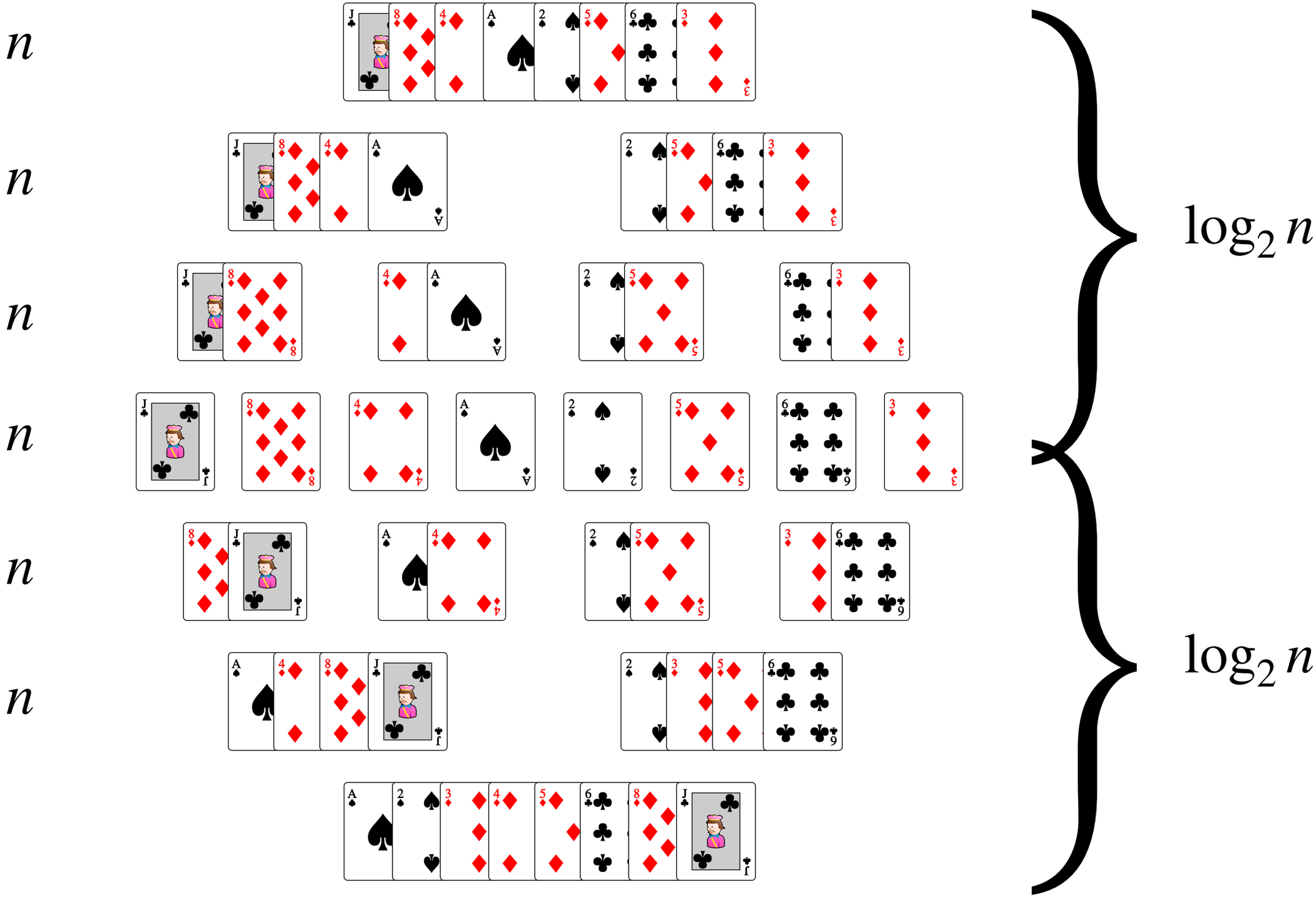
Efficacité du tri fusion



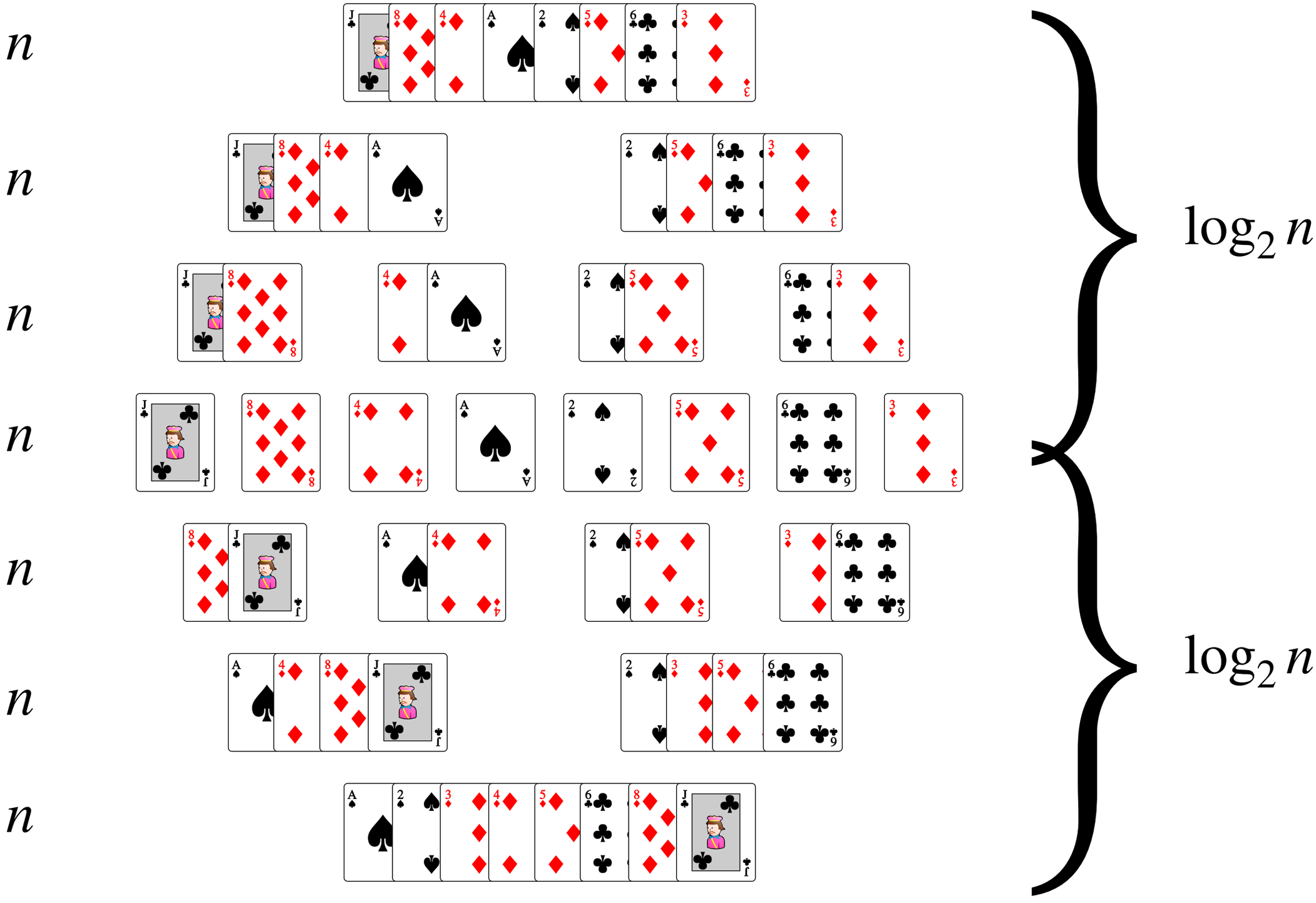
Efficacité du tri fusion



Efficacité du tri fusion



Efficacité du tri fusion



La complexité du tri fusion est

$$O(n \log_2 n)$$