Introduction à l’informatique CM7

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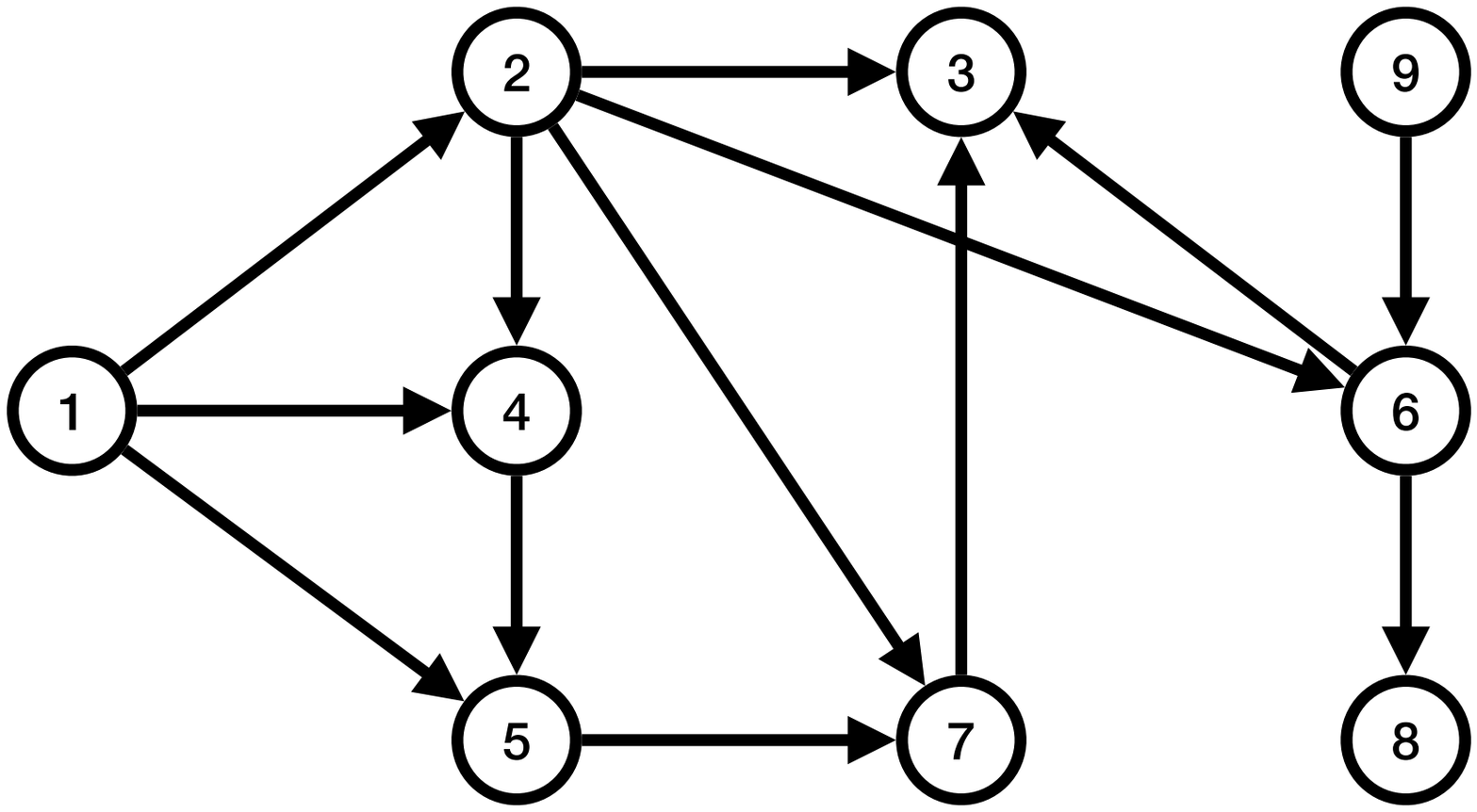
Parcours en largeur

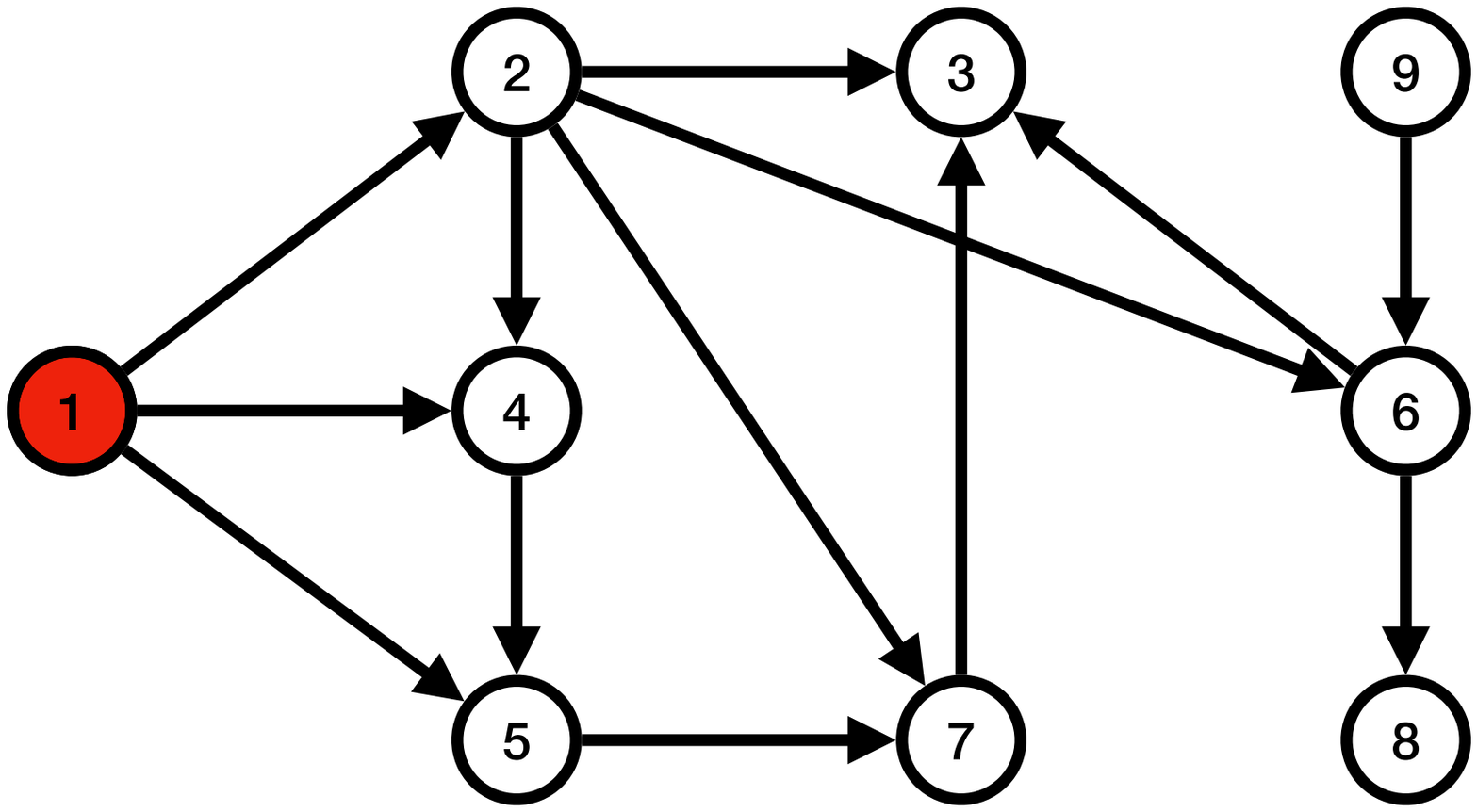
```
def parcours_en_largeur(G, s):
    n = len(G)
    H = nouveau_graphe(n)          # graphe vide
    F = nouvelle_file()            # file vide
    couleur = ['blanc'] * n        # n cases blanches
    couleur[s] = 'rouge'
    enfiler(F, s)
    while not est_vide(F):
        u = défiler(F)
        for v in range(n):
            if G[u][v] == 1 and couleur[v] == 'blanc':
                couleur[v] = 'rouge'
                enfiler(F, v)
                H[u][v] = 1
        couleur[u] = 'vert'
    return H                       # graphe des chemins min
```

Un autre exemple



File →

Un autre exemple



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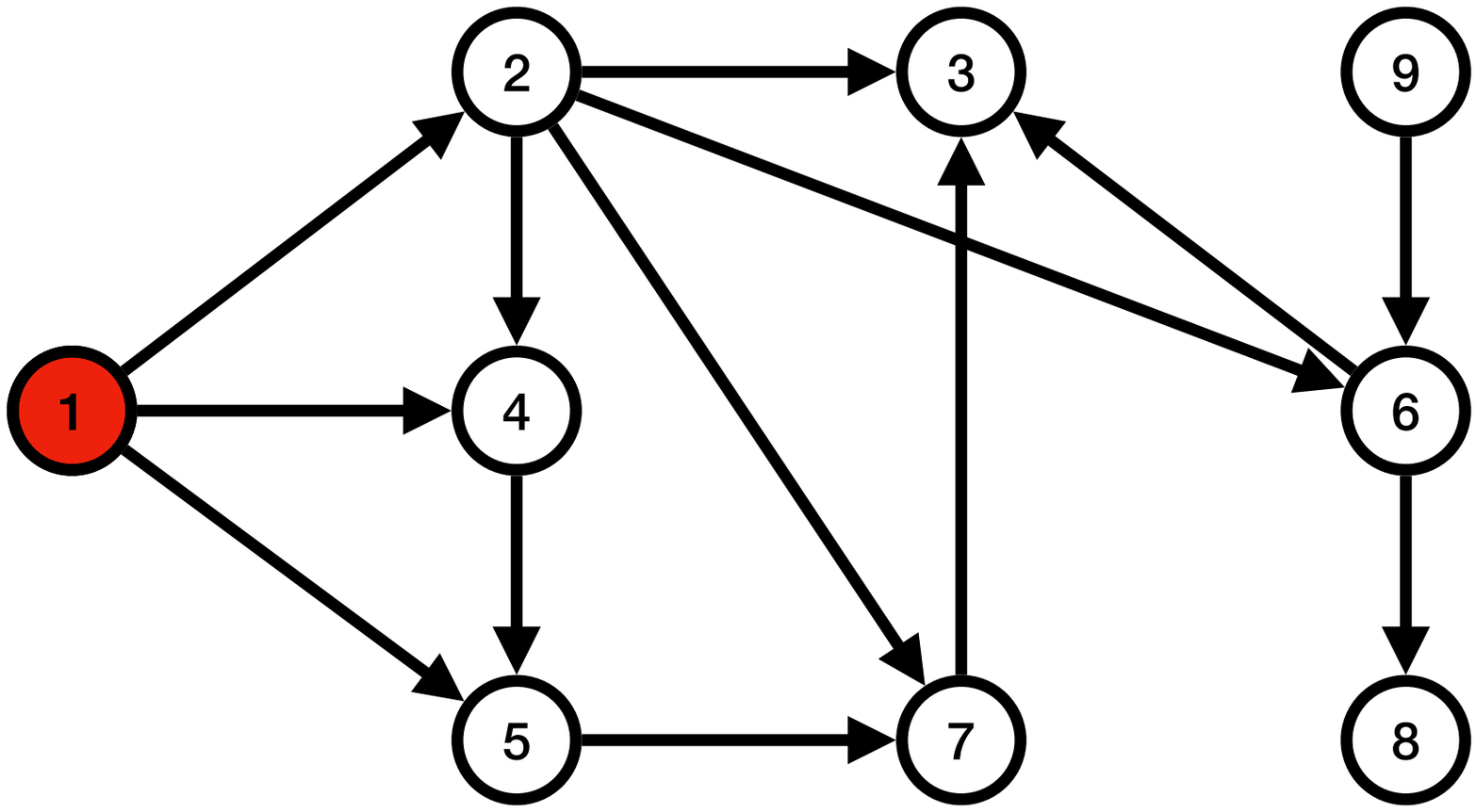

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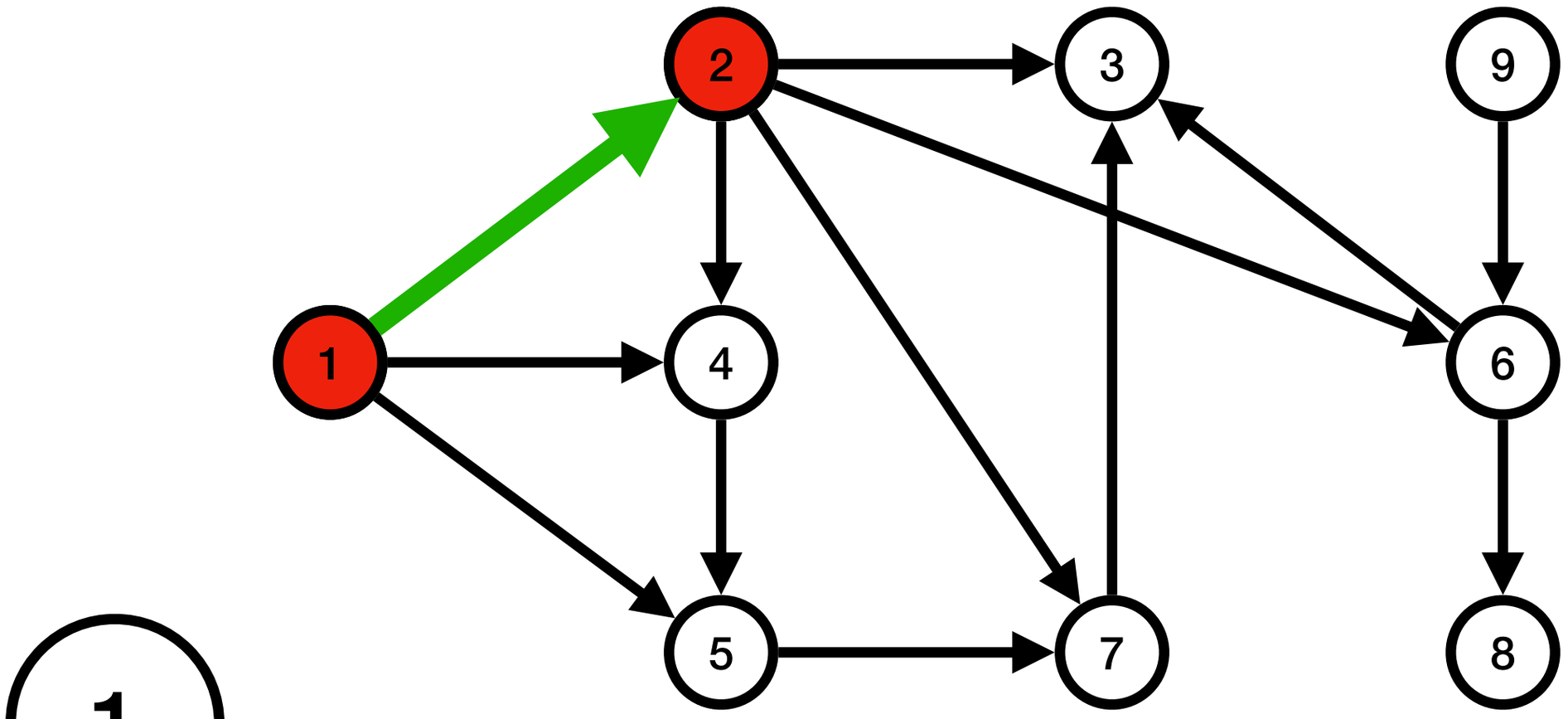

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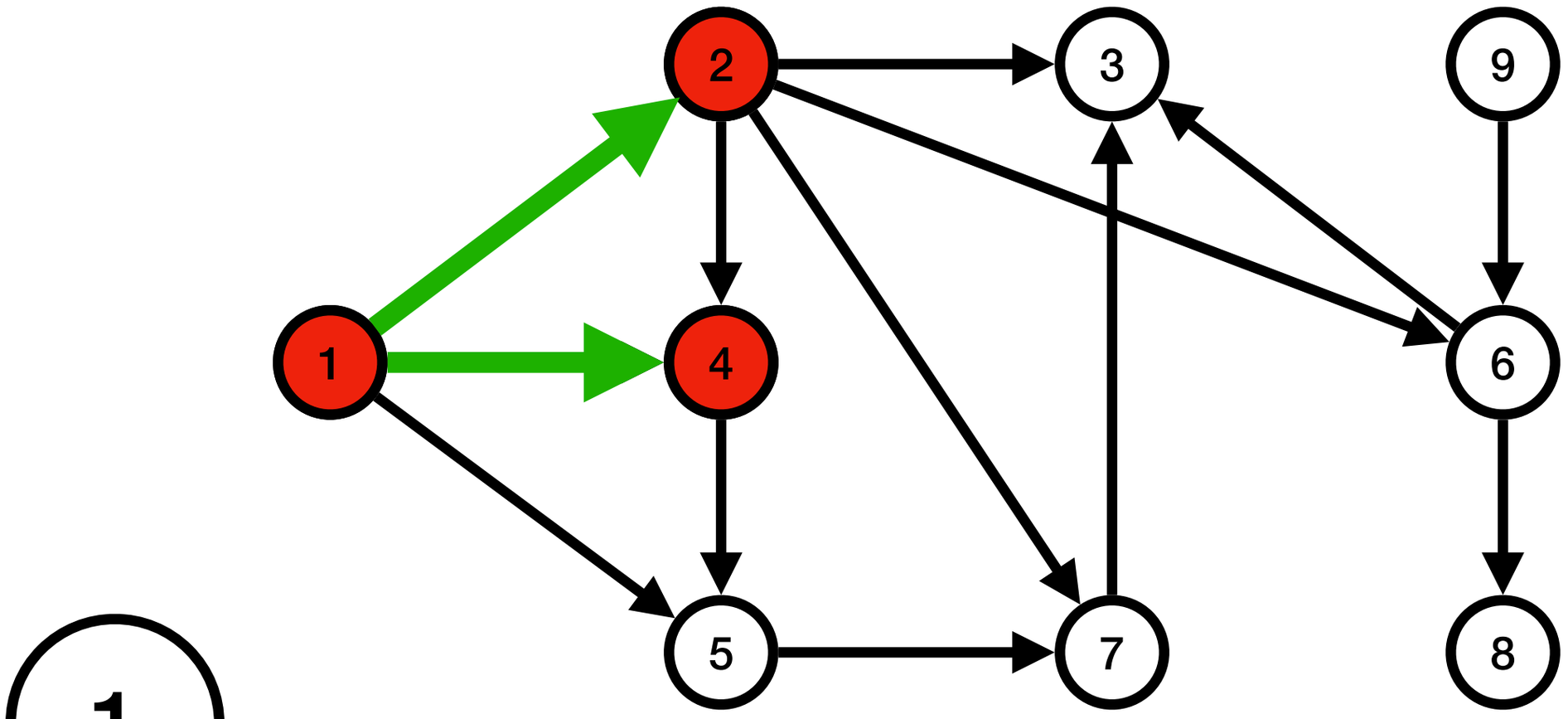

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File → 2 4

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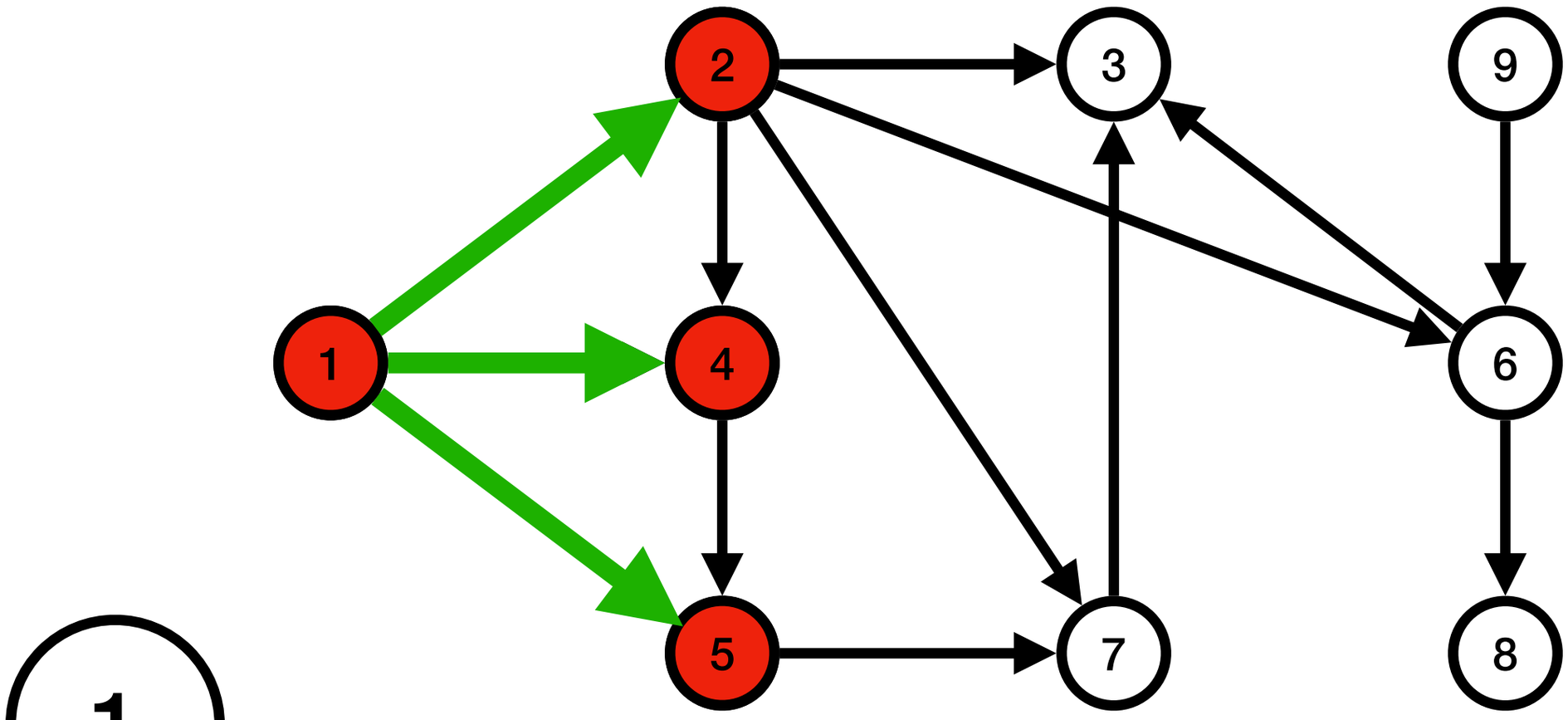

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File → 2 4 5

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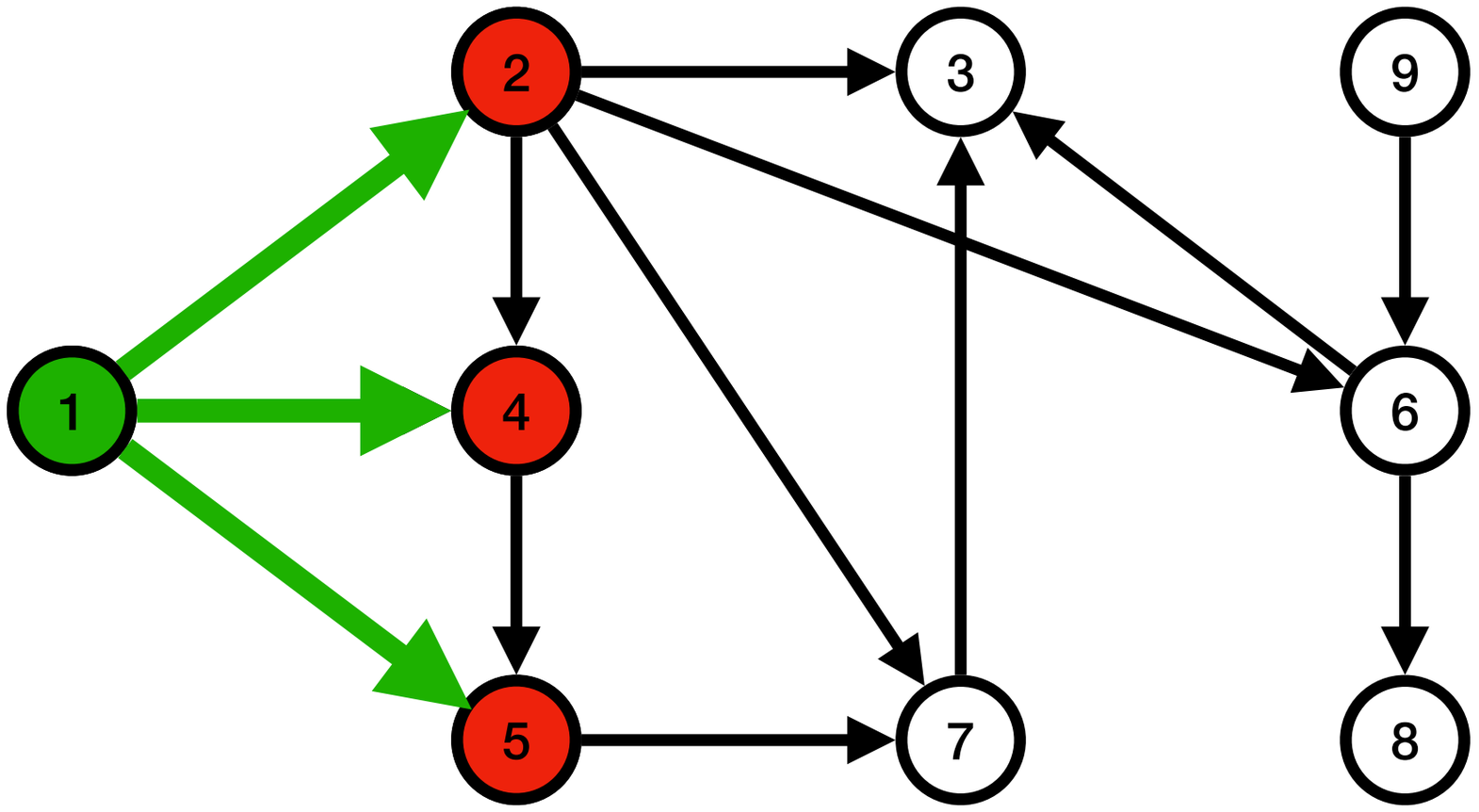

File → 2 4 5

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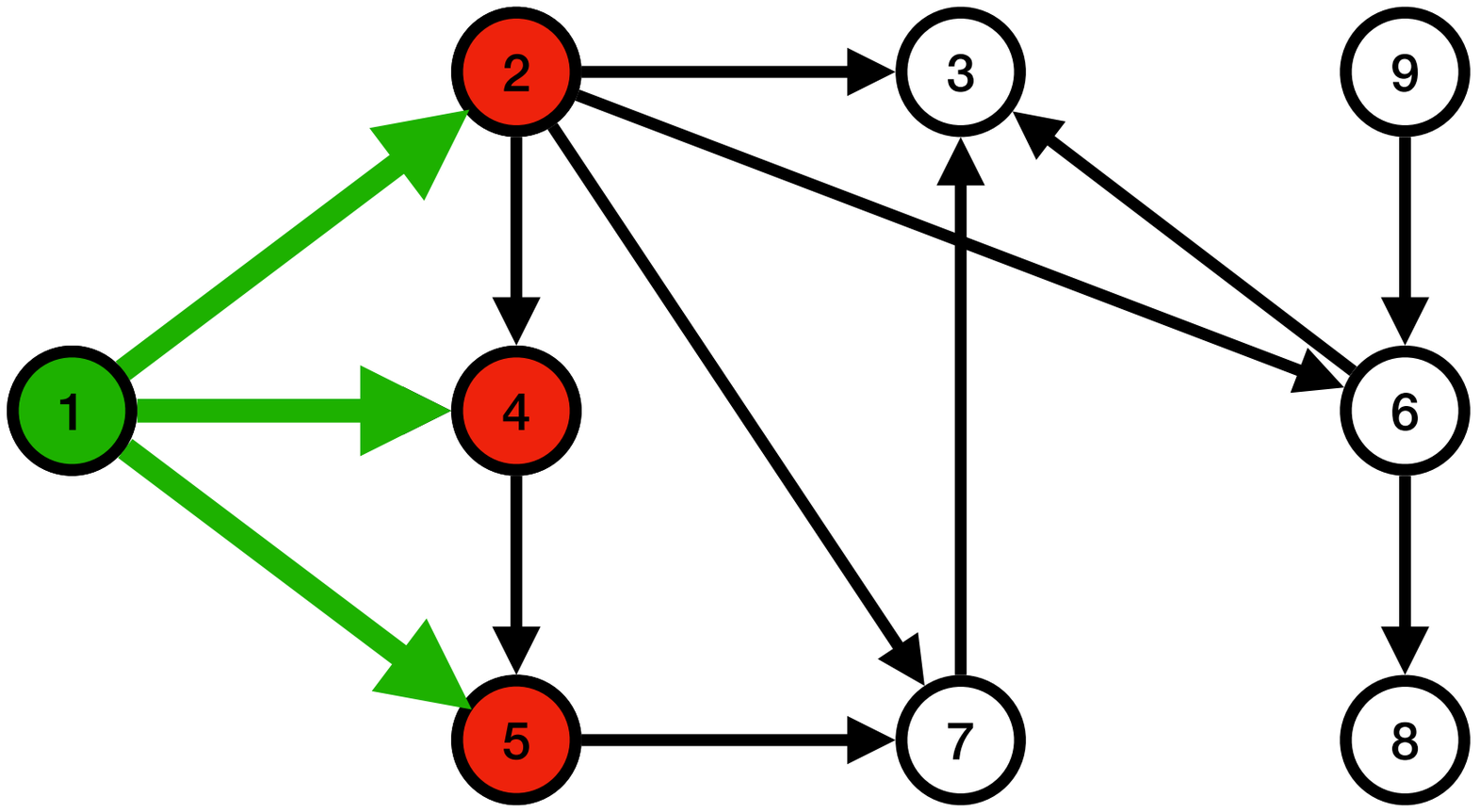
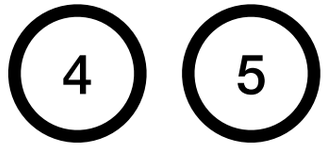


File → 

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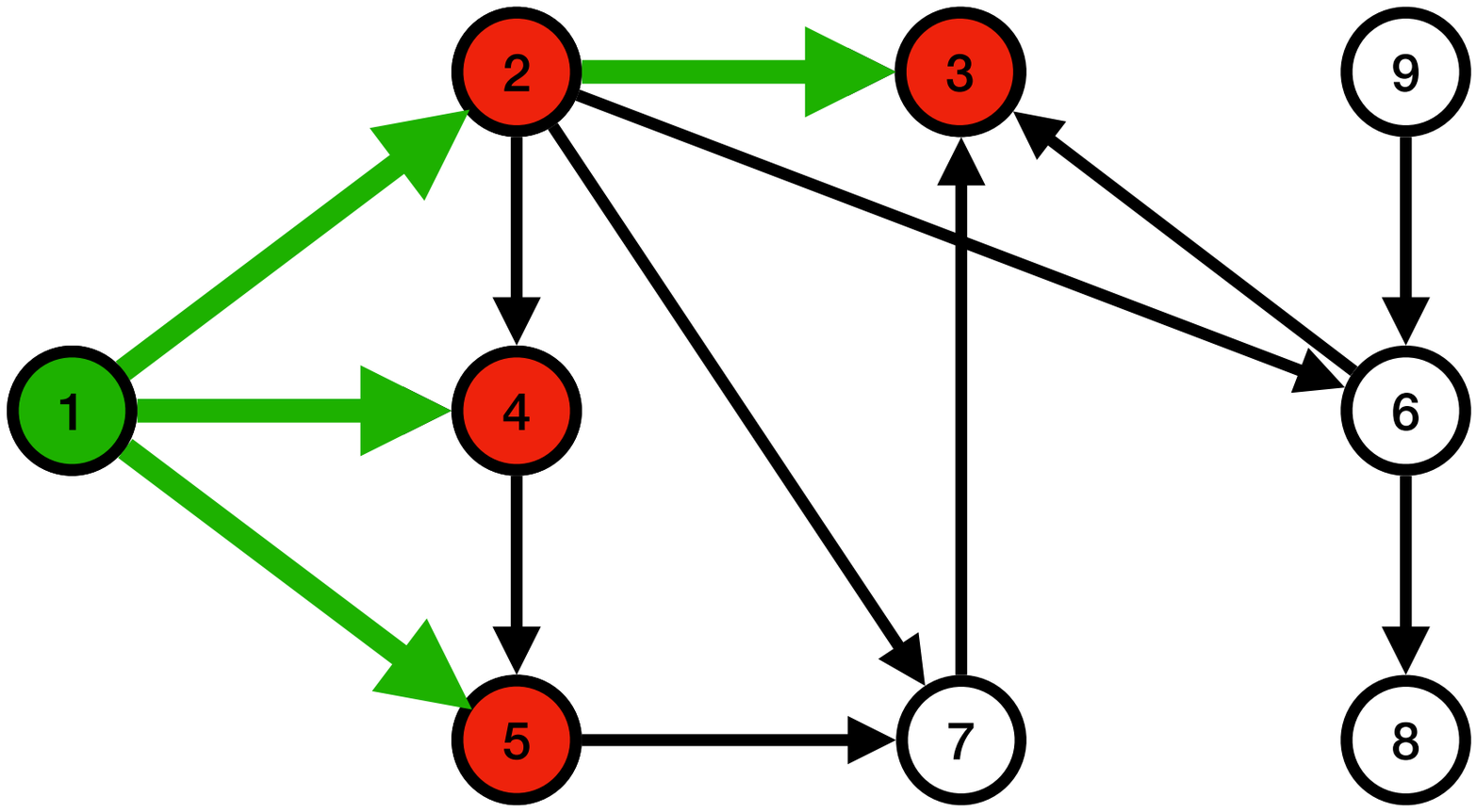

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File → 4 5 3

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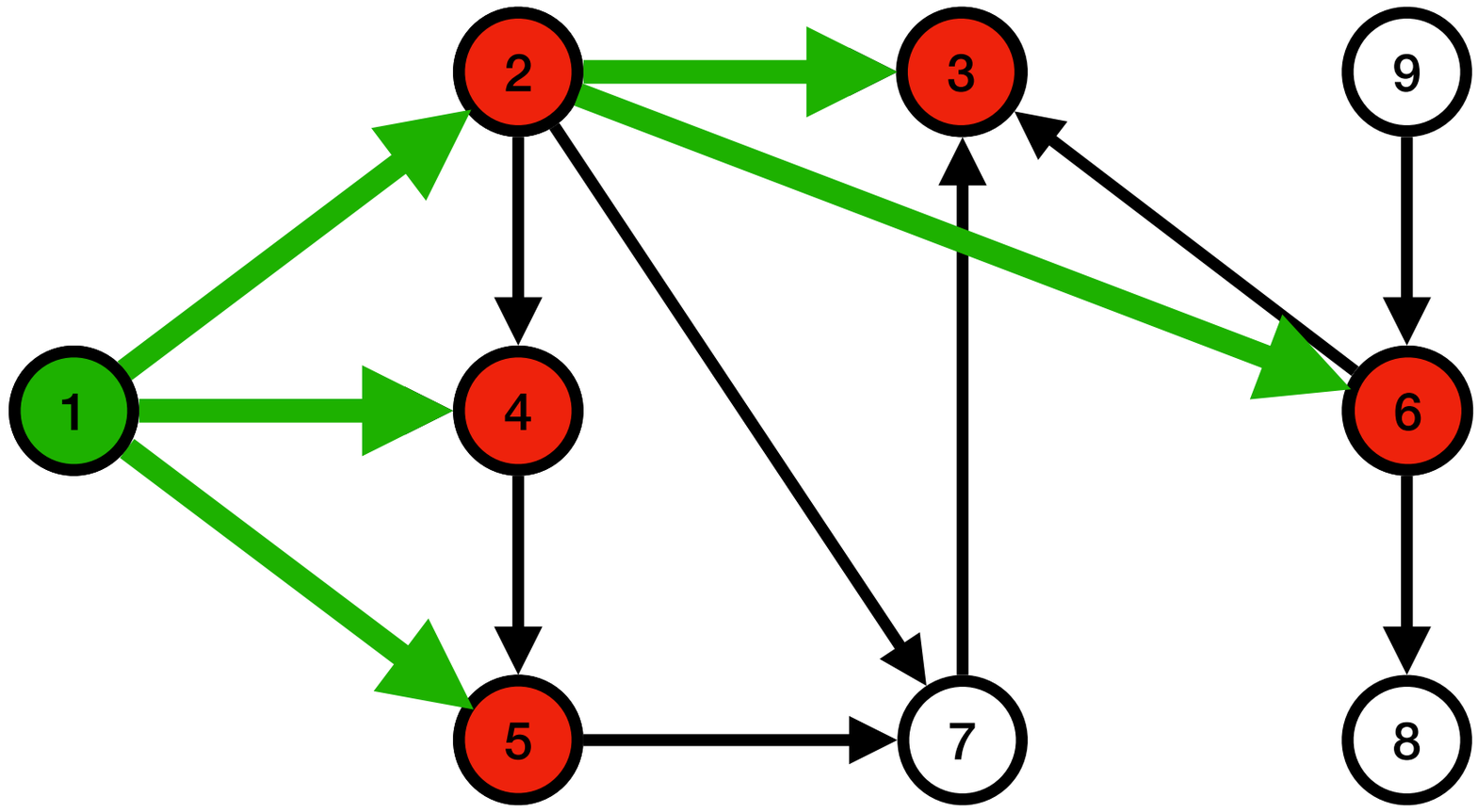

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File → 4 5 3 6

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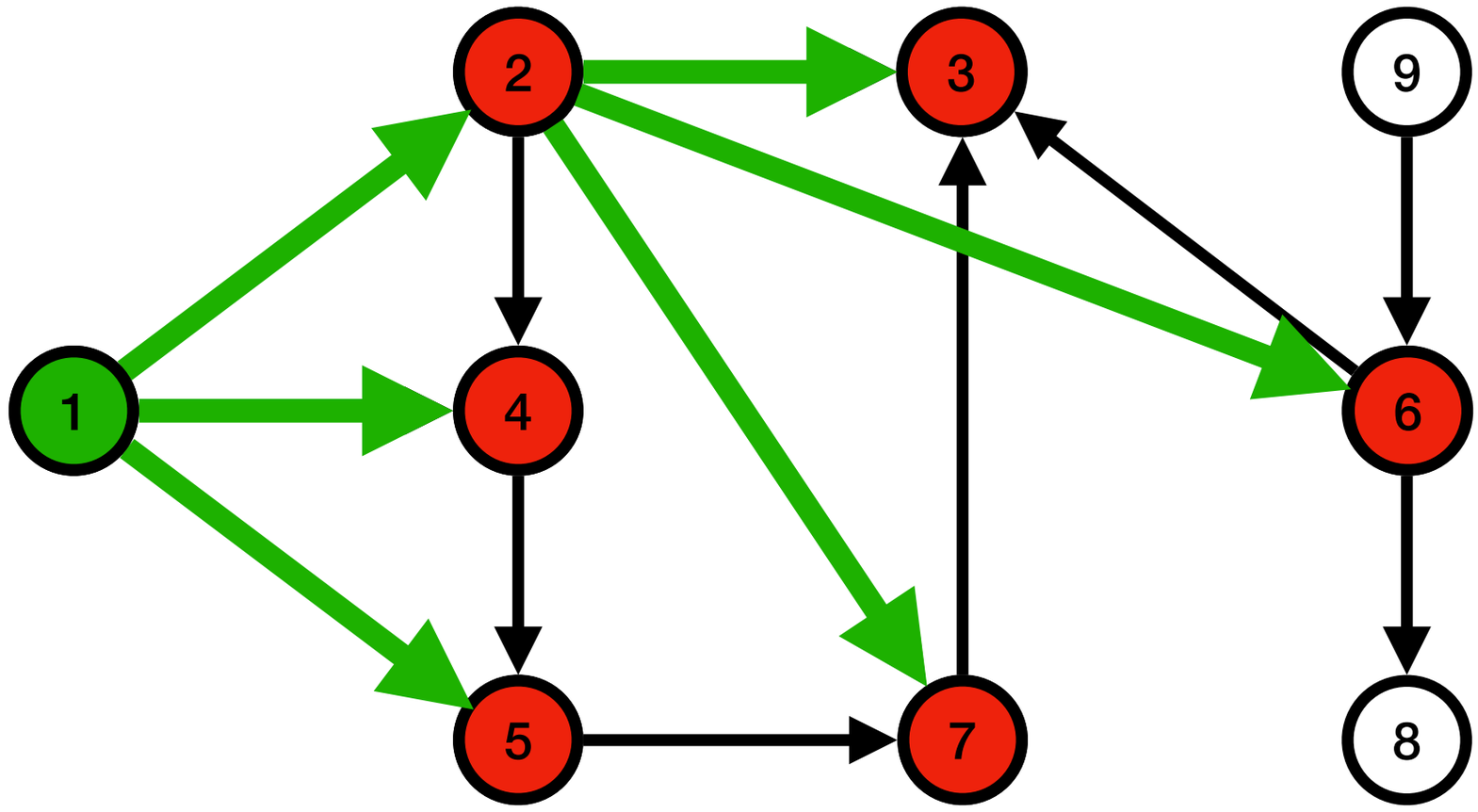

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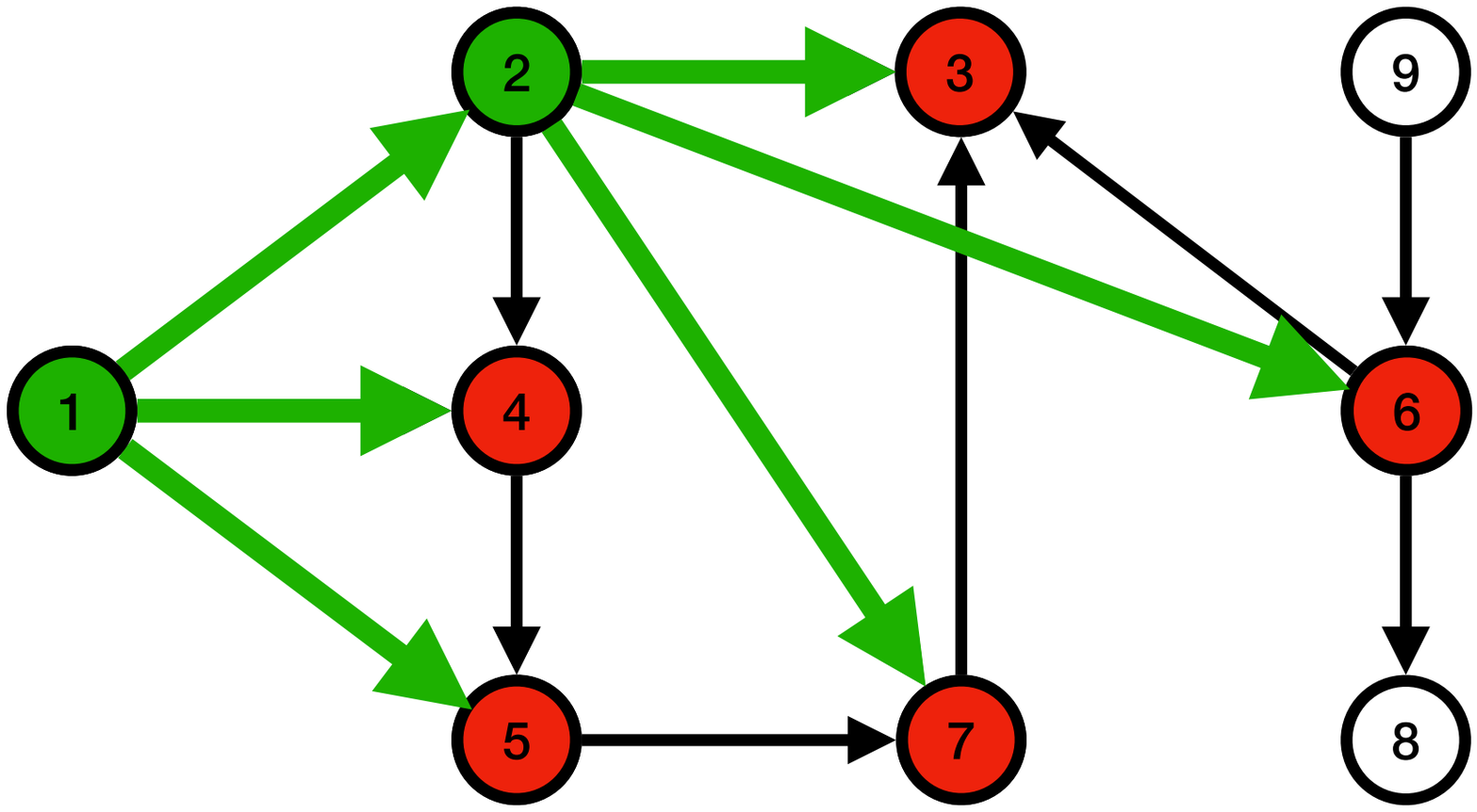

File → 4 5 3 6 7

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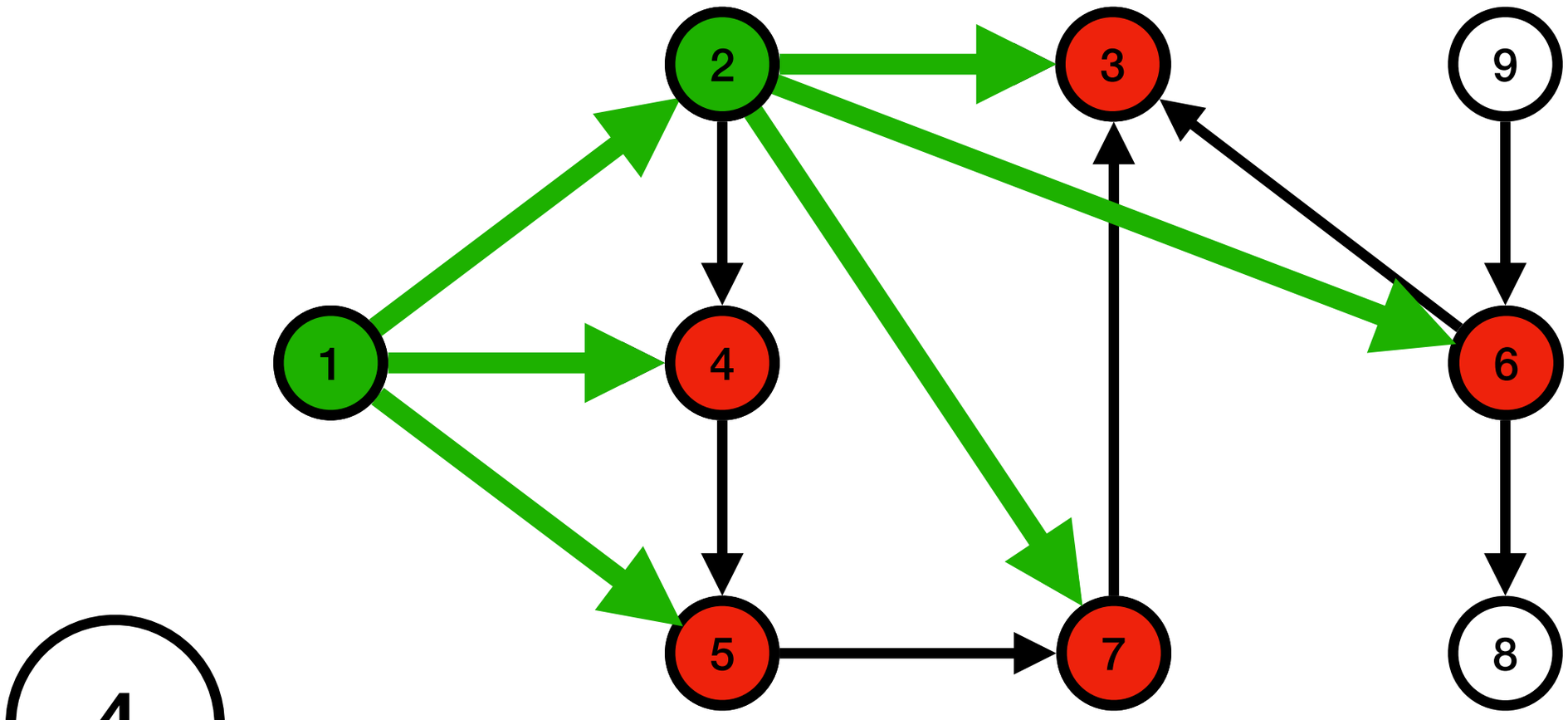

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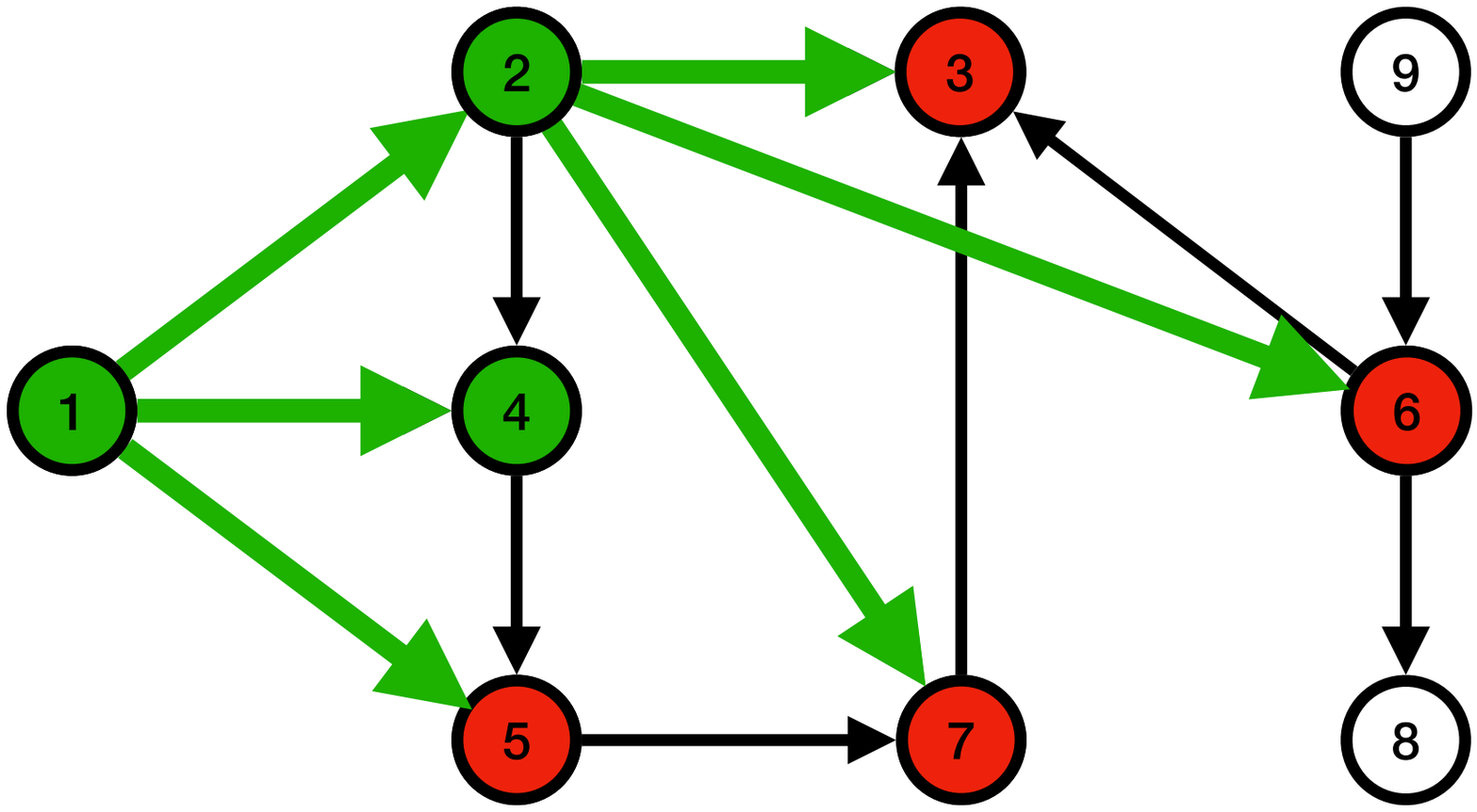

File → 5 3 6 7

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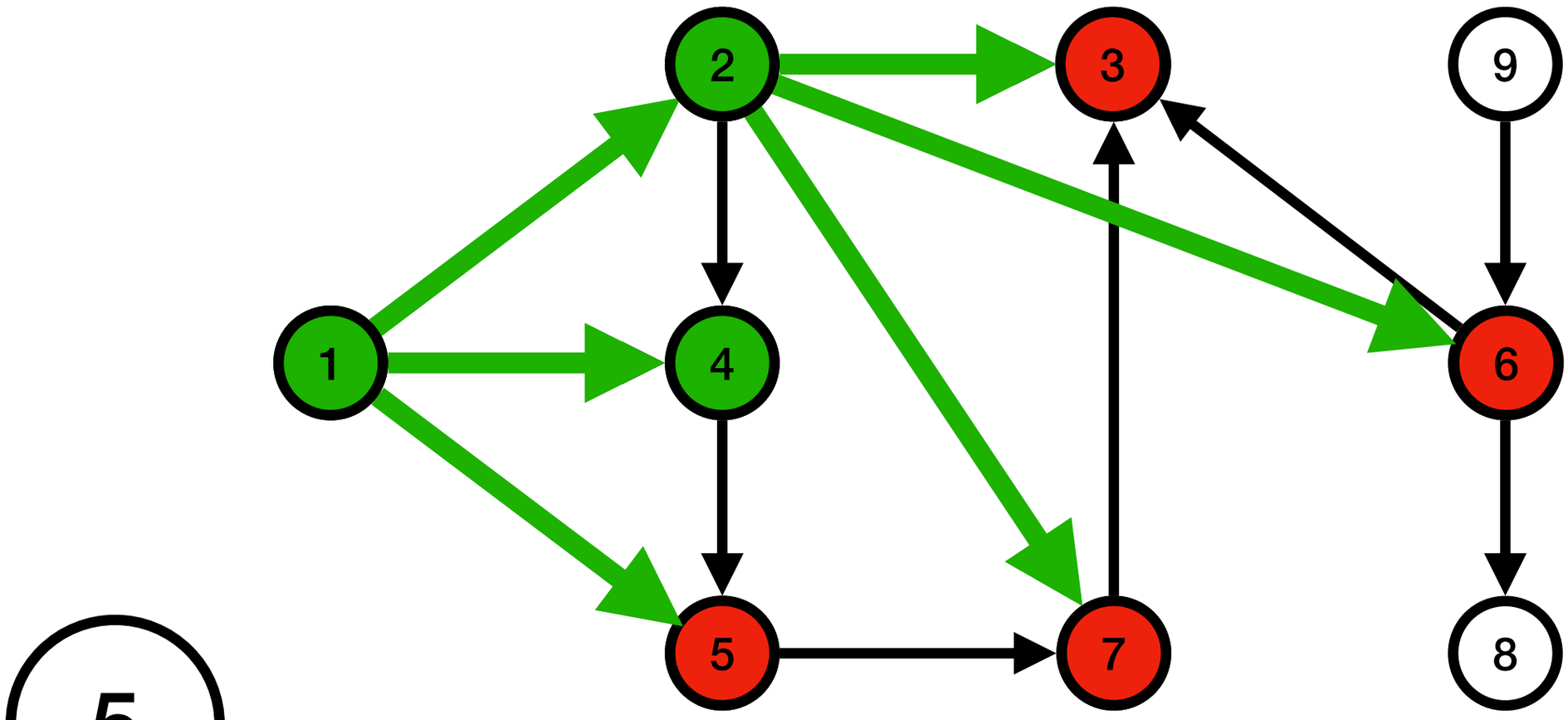

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File → 3 6 7

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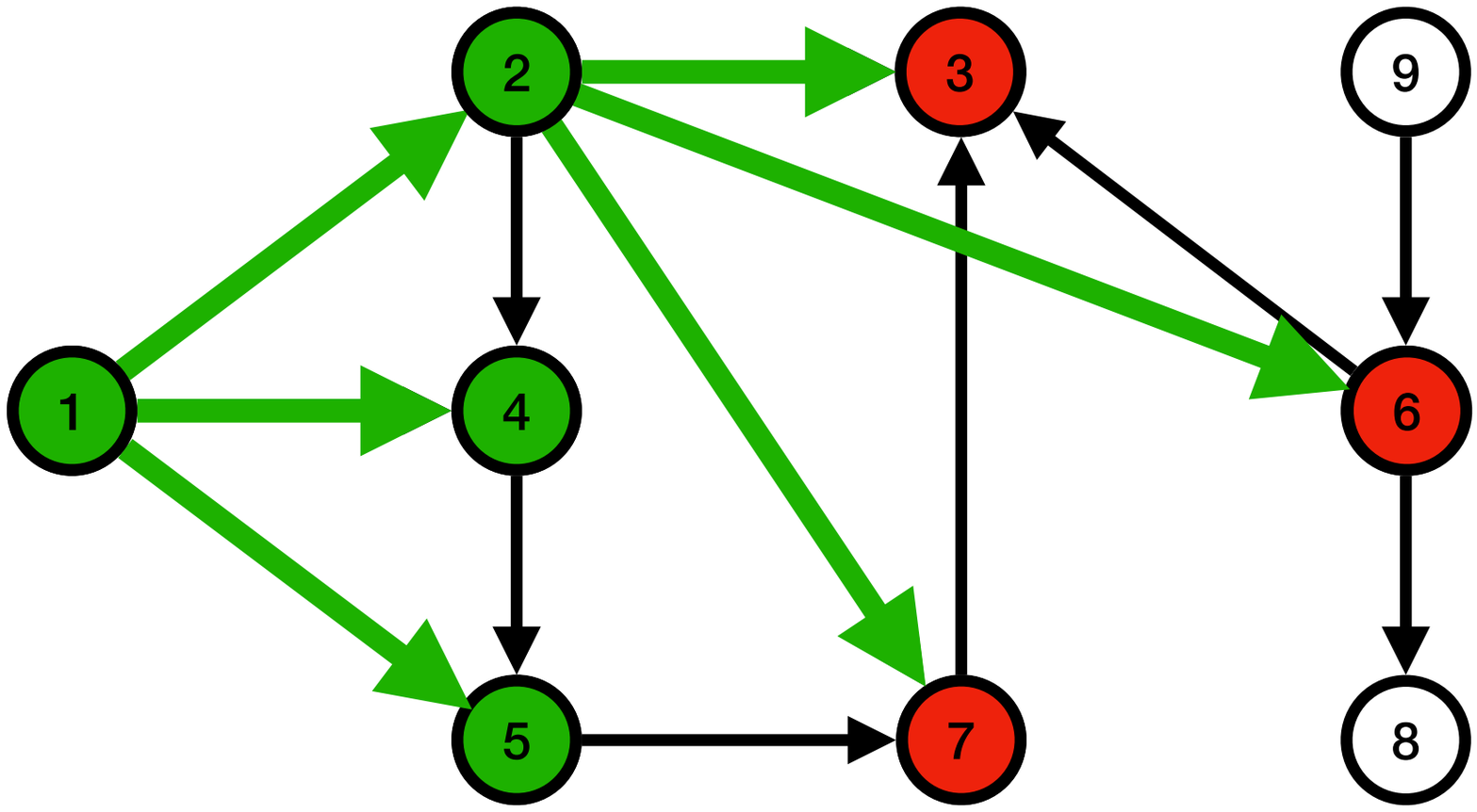

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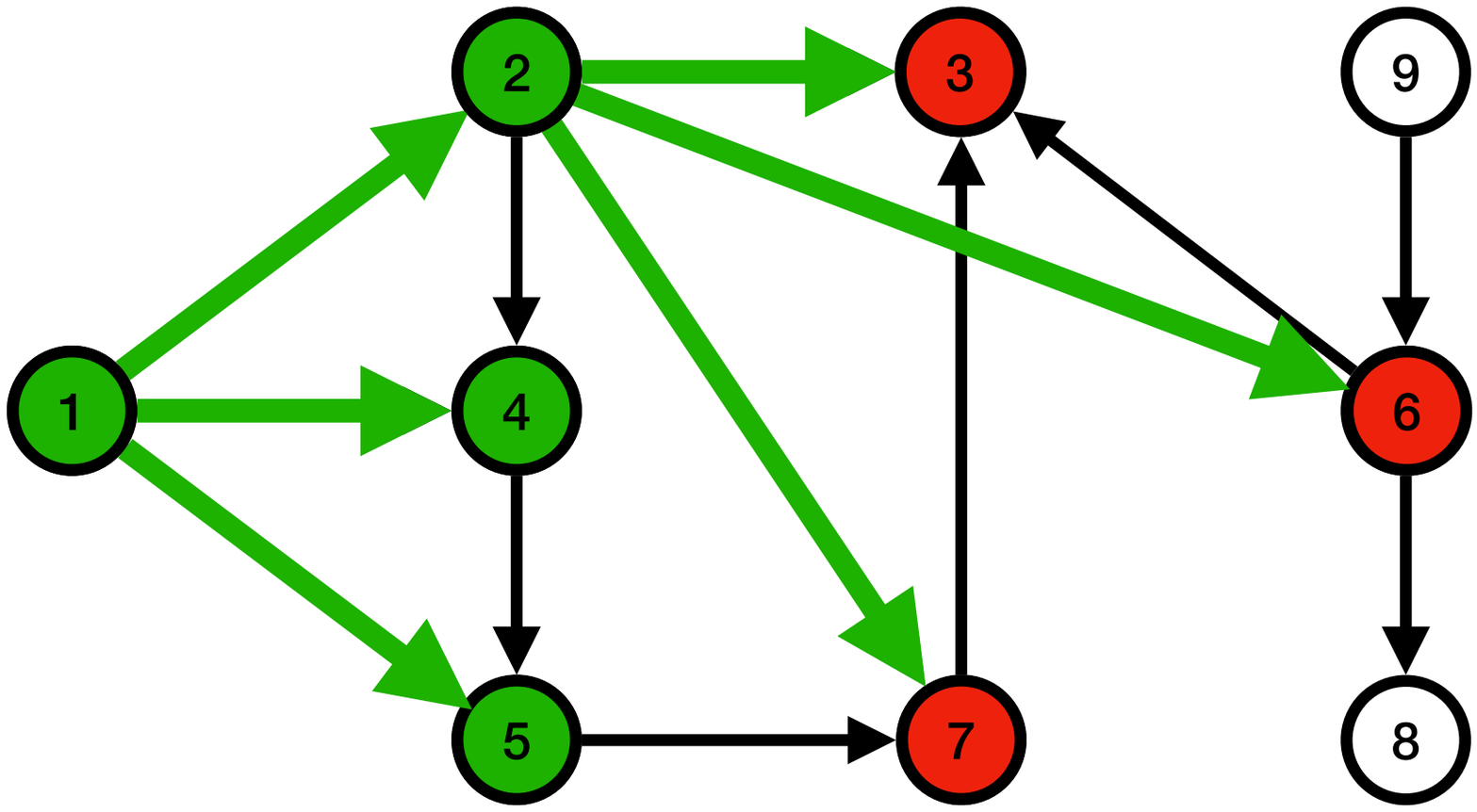

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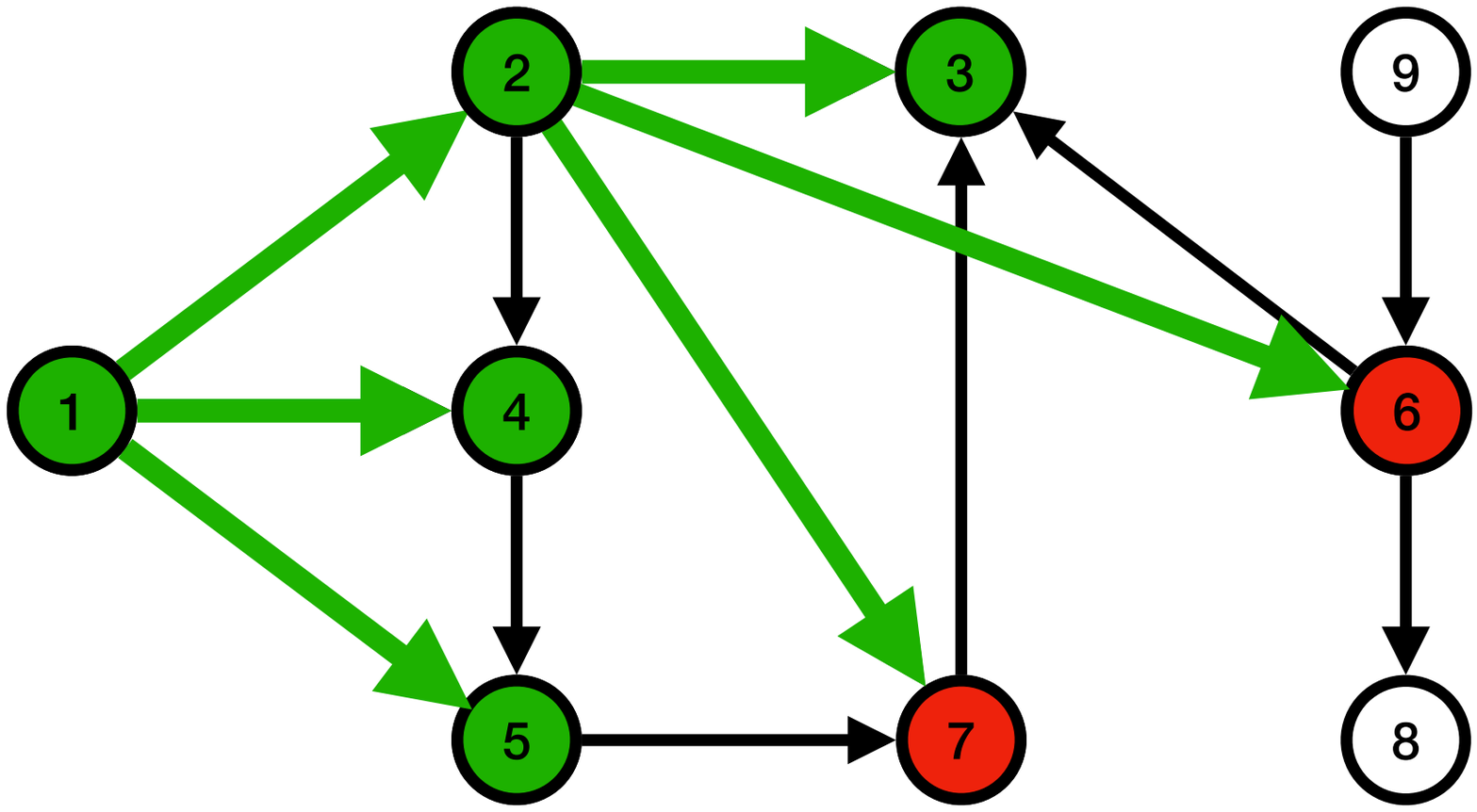

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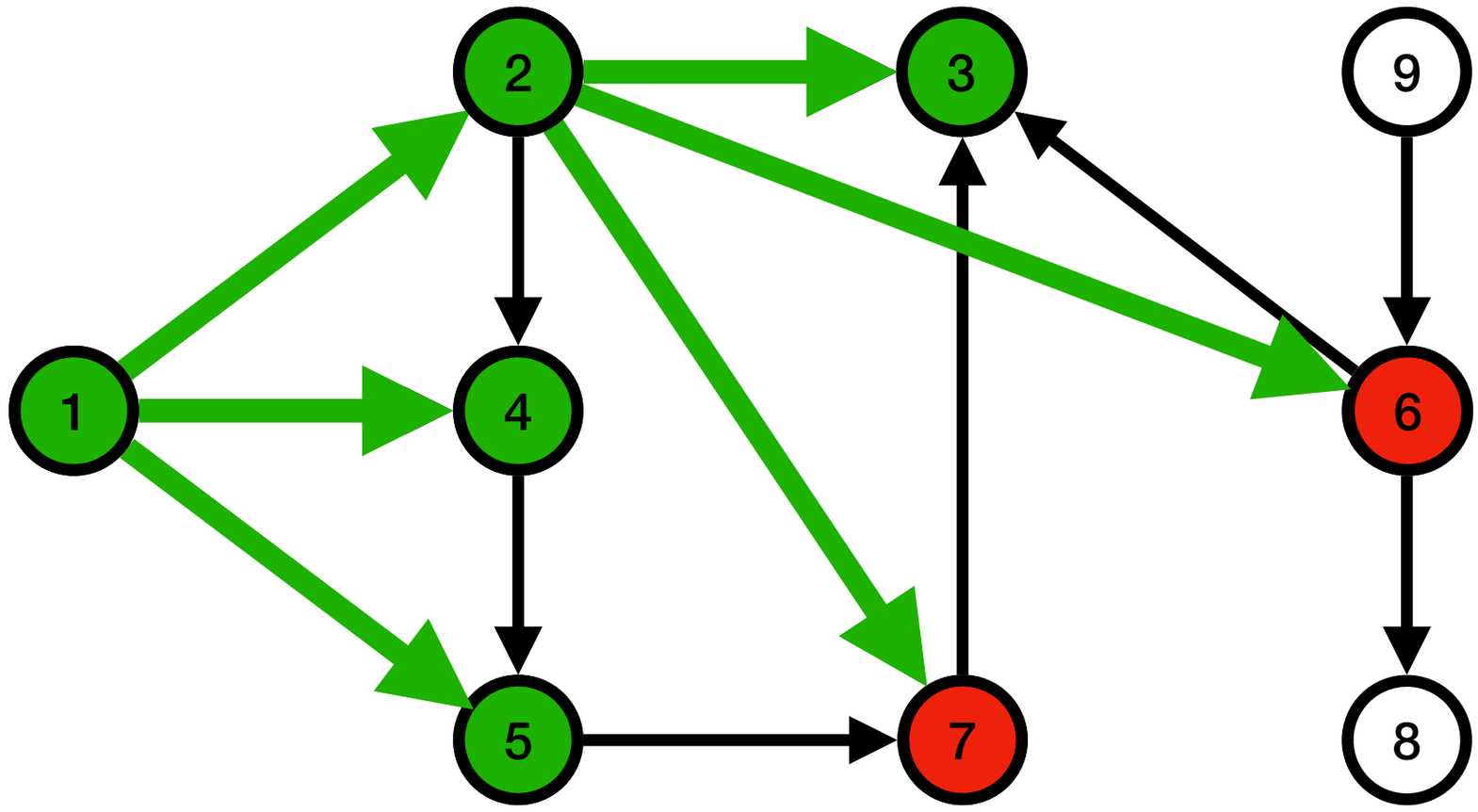

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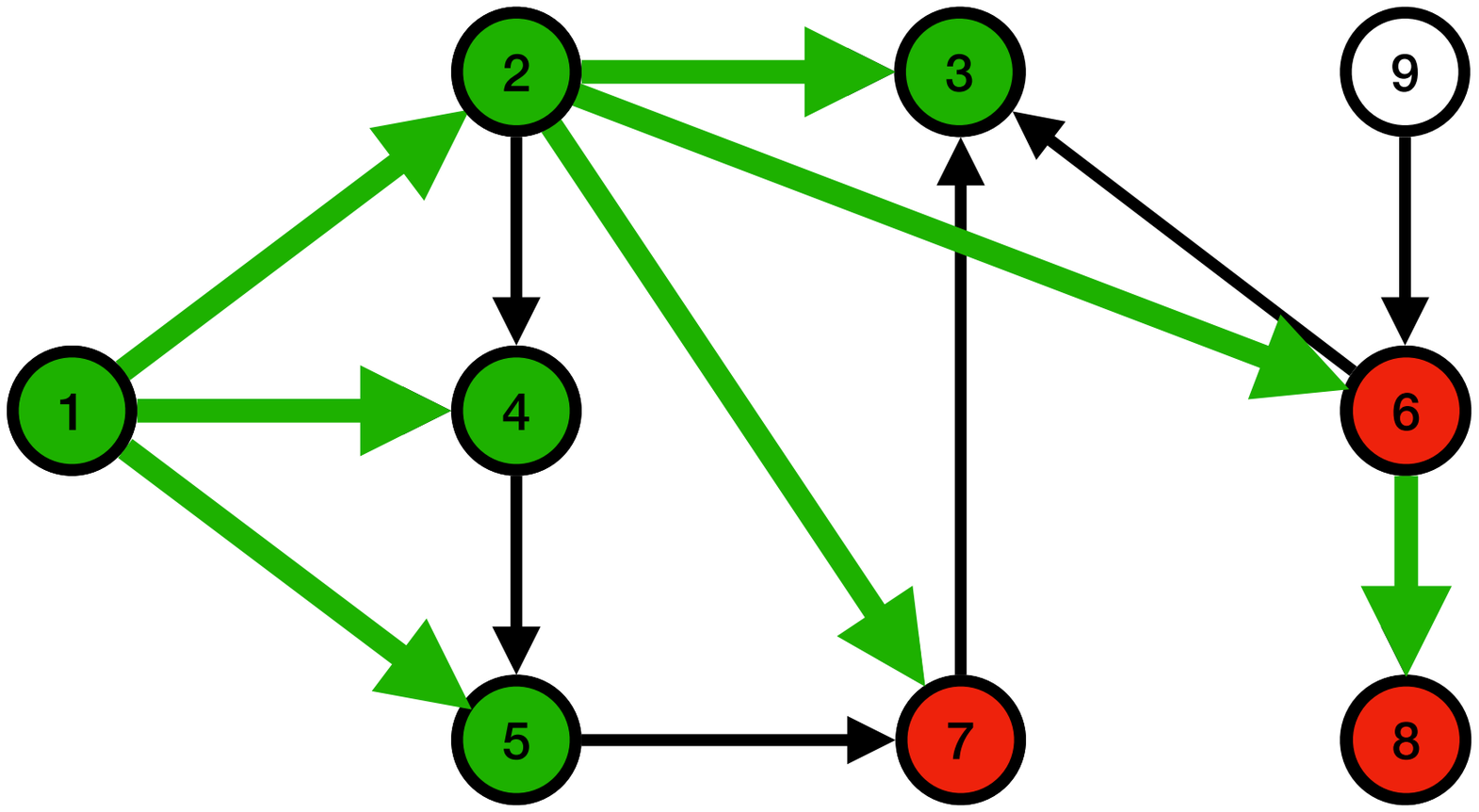

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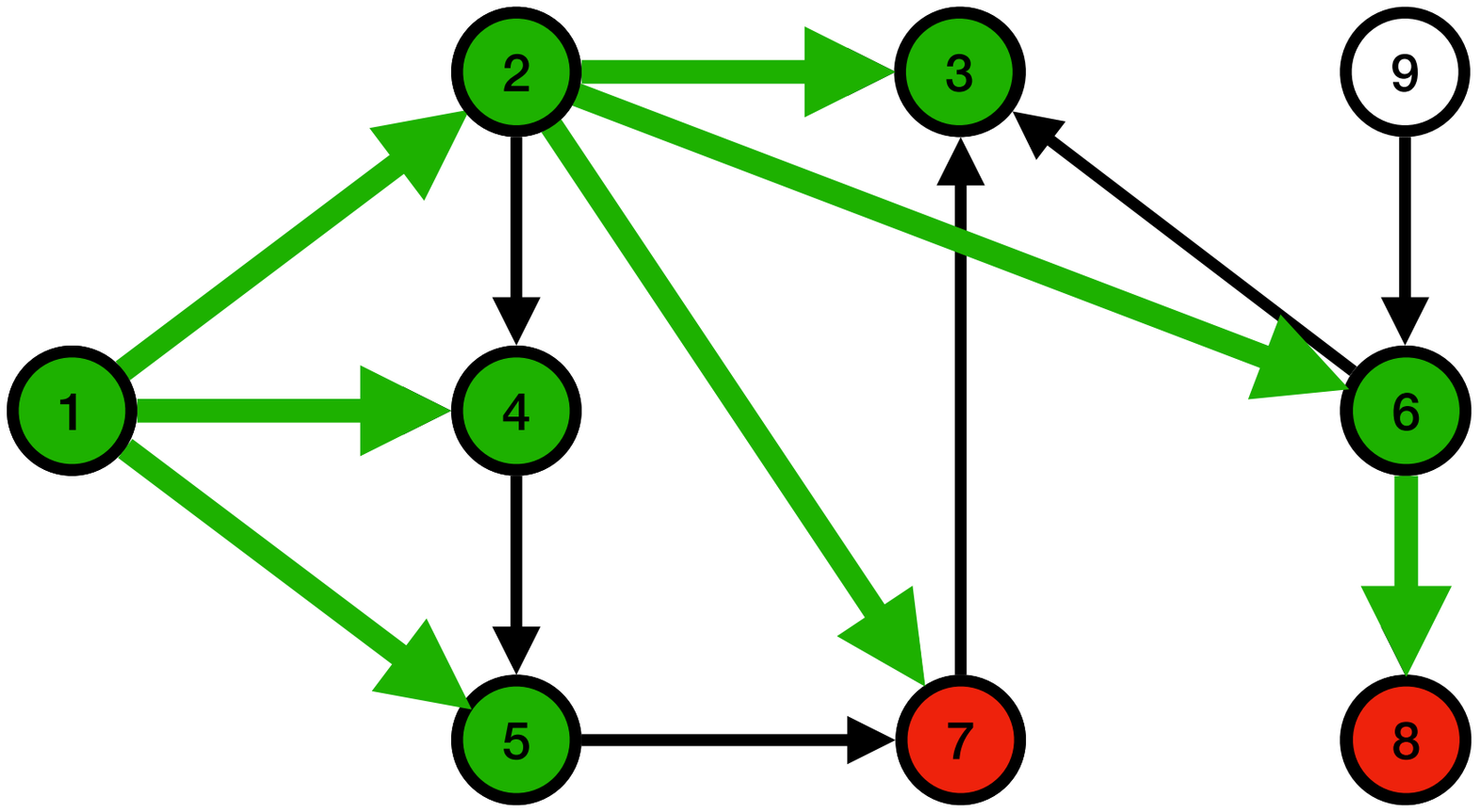

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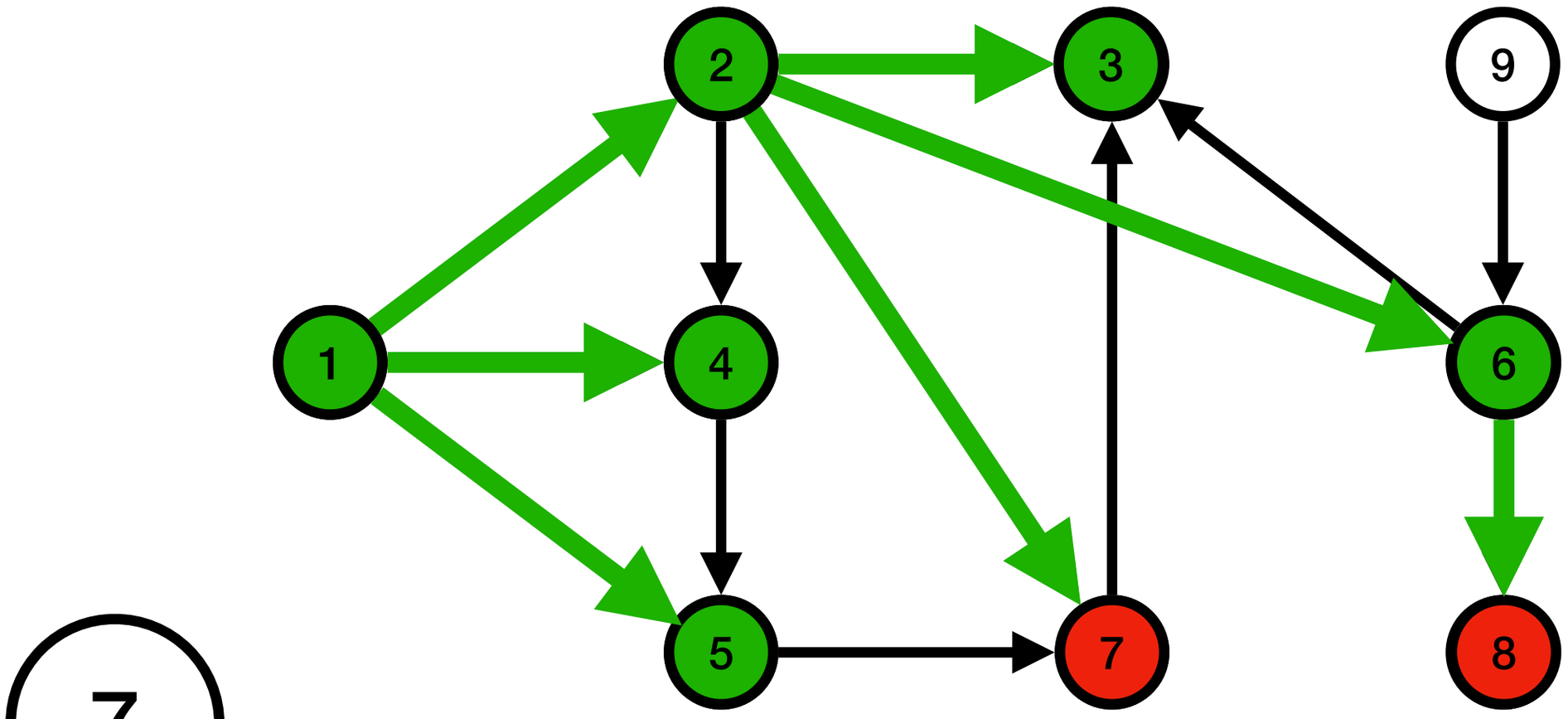

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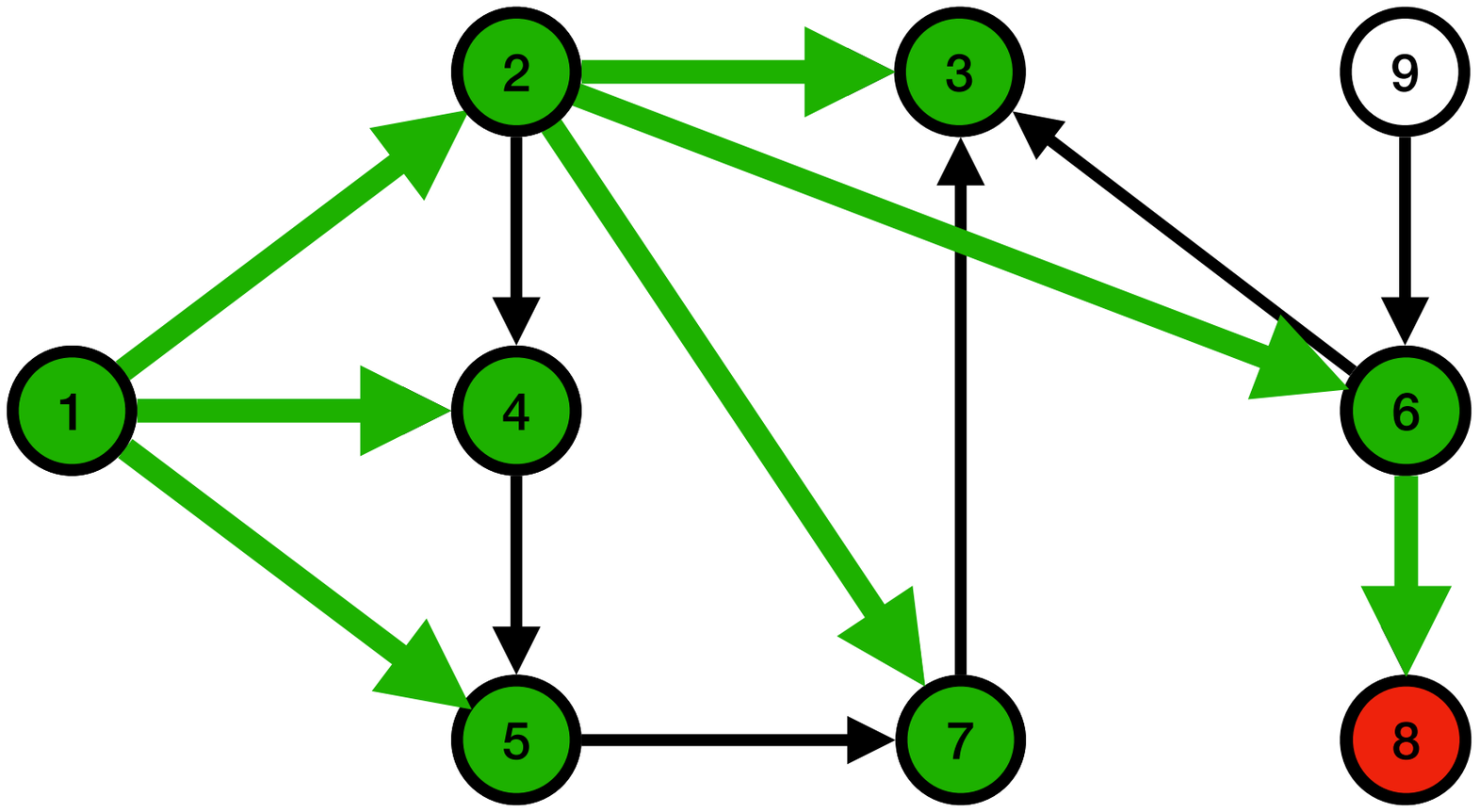

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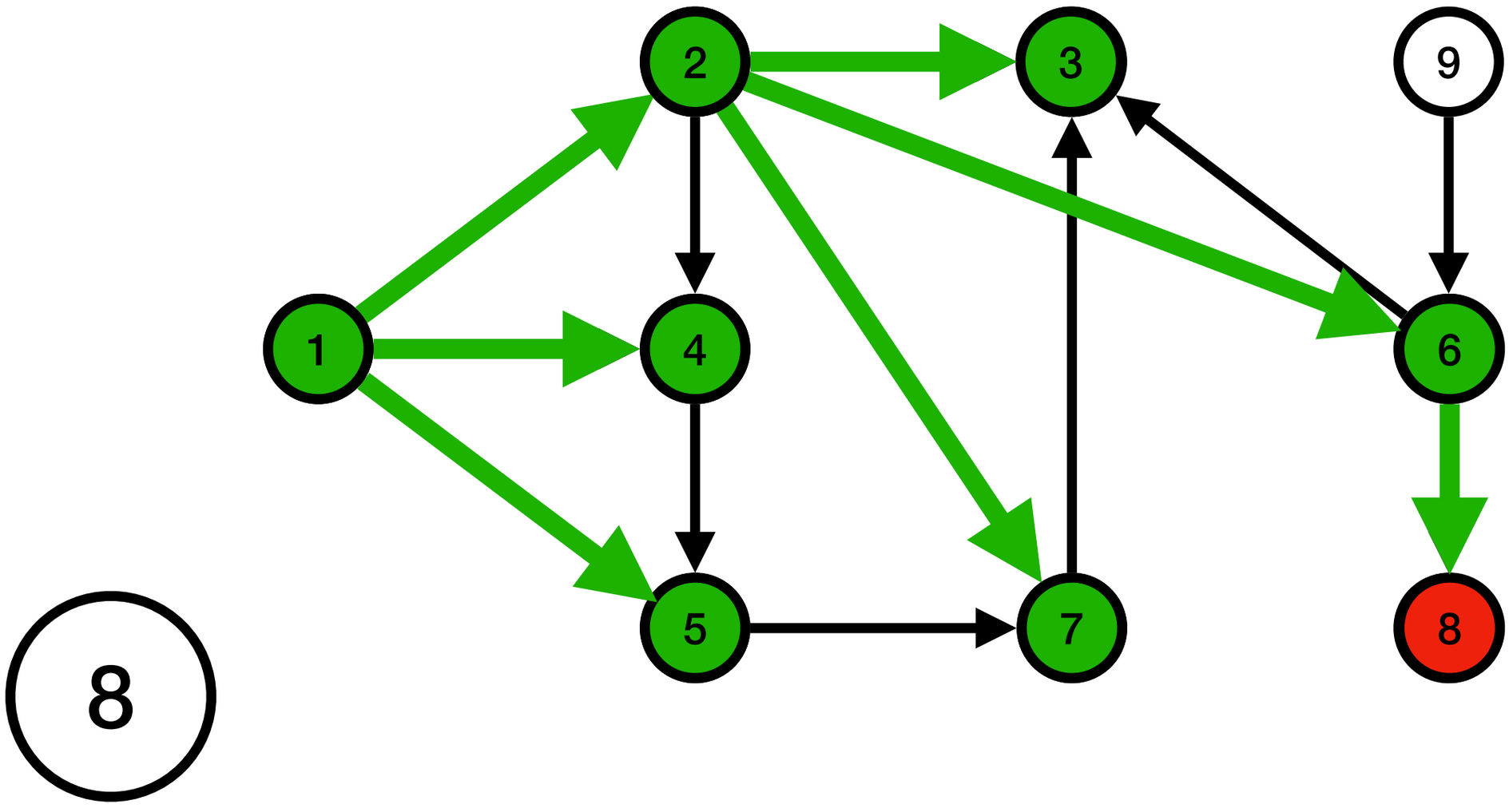

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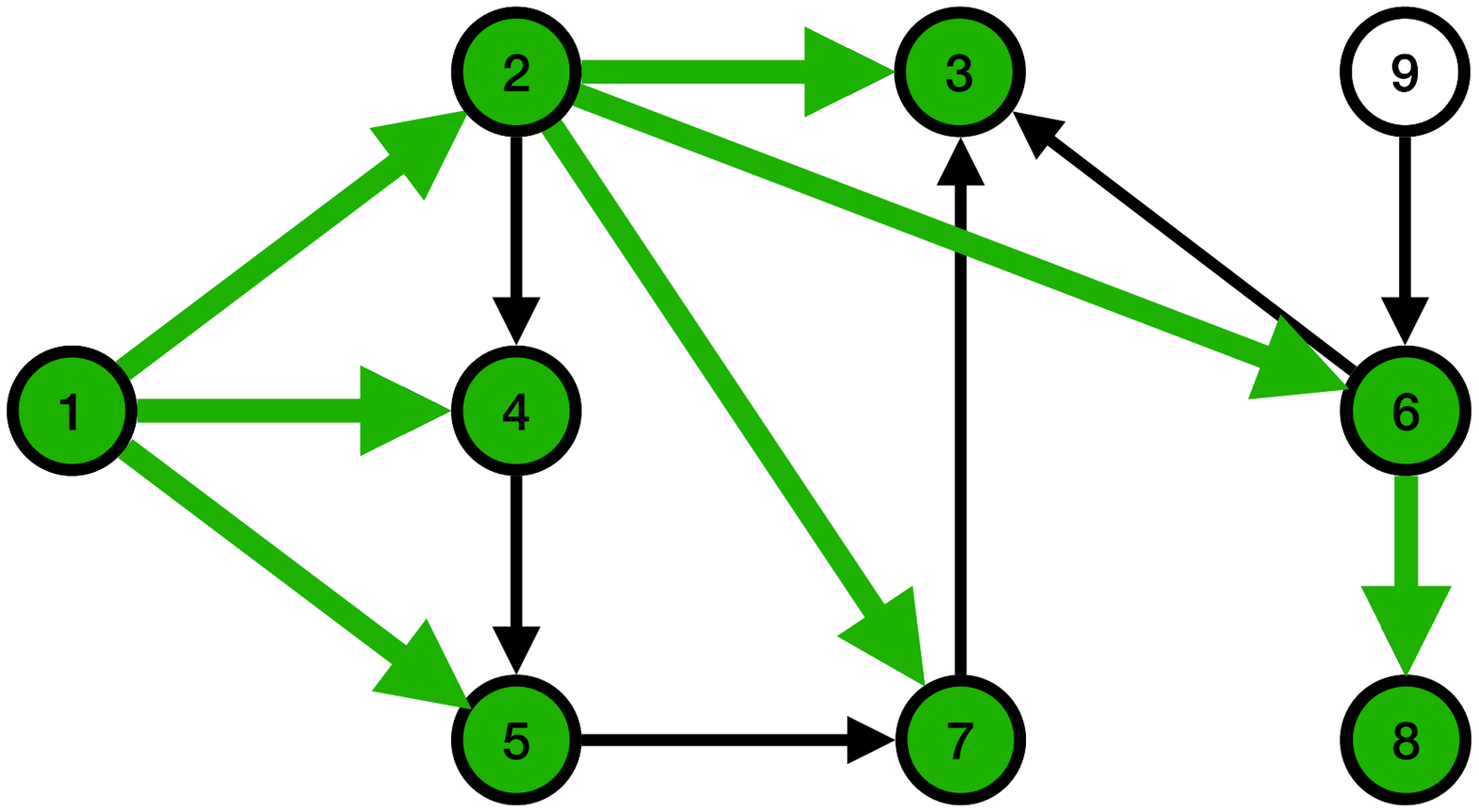

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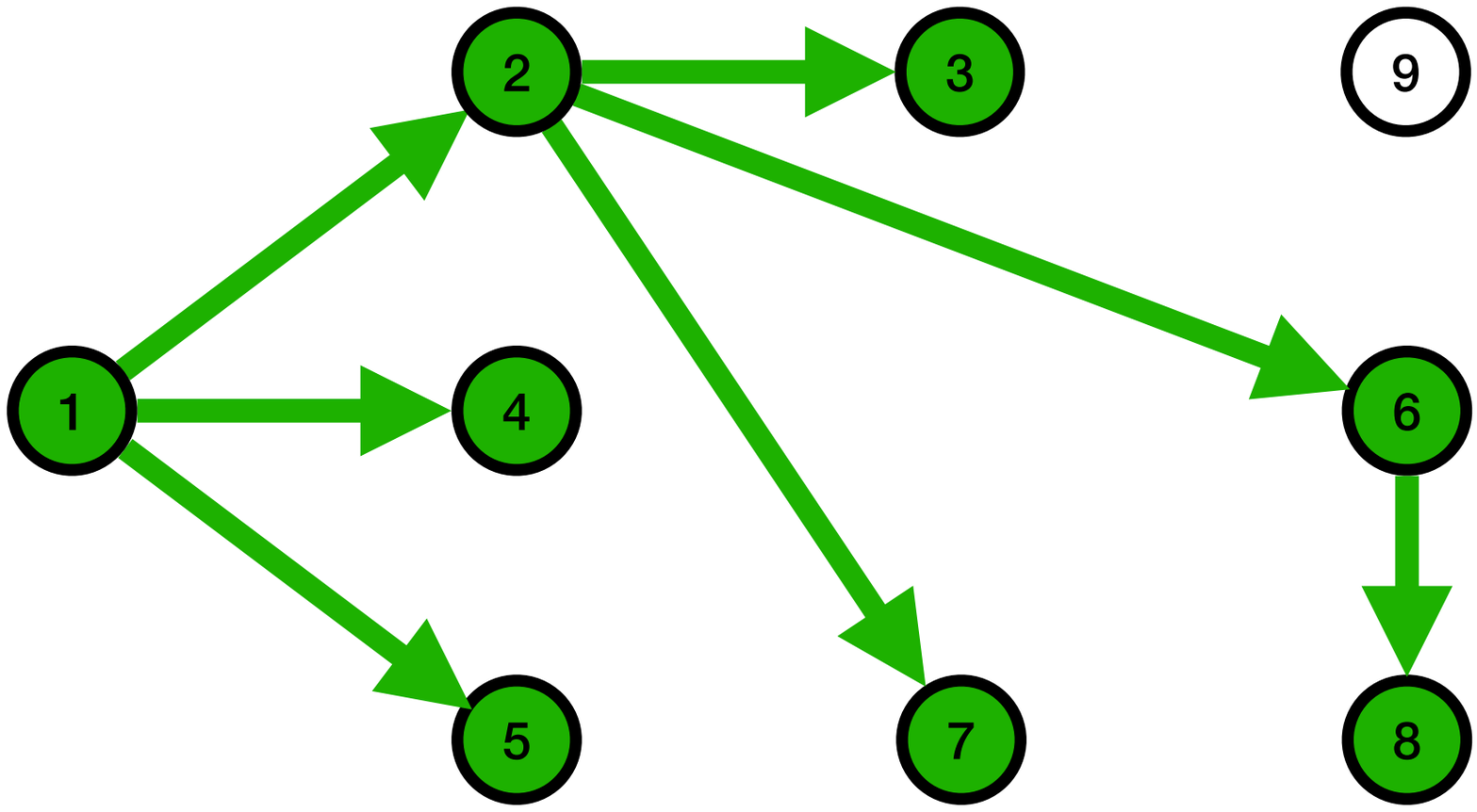

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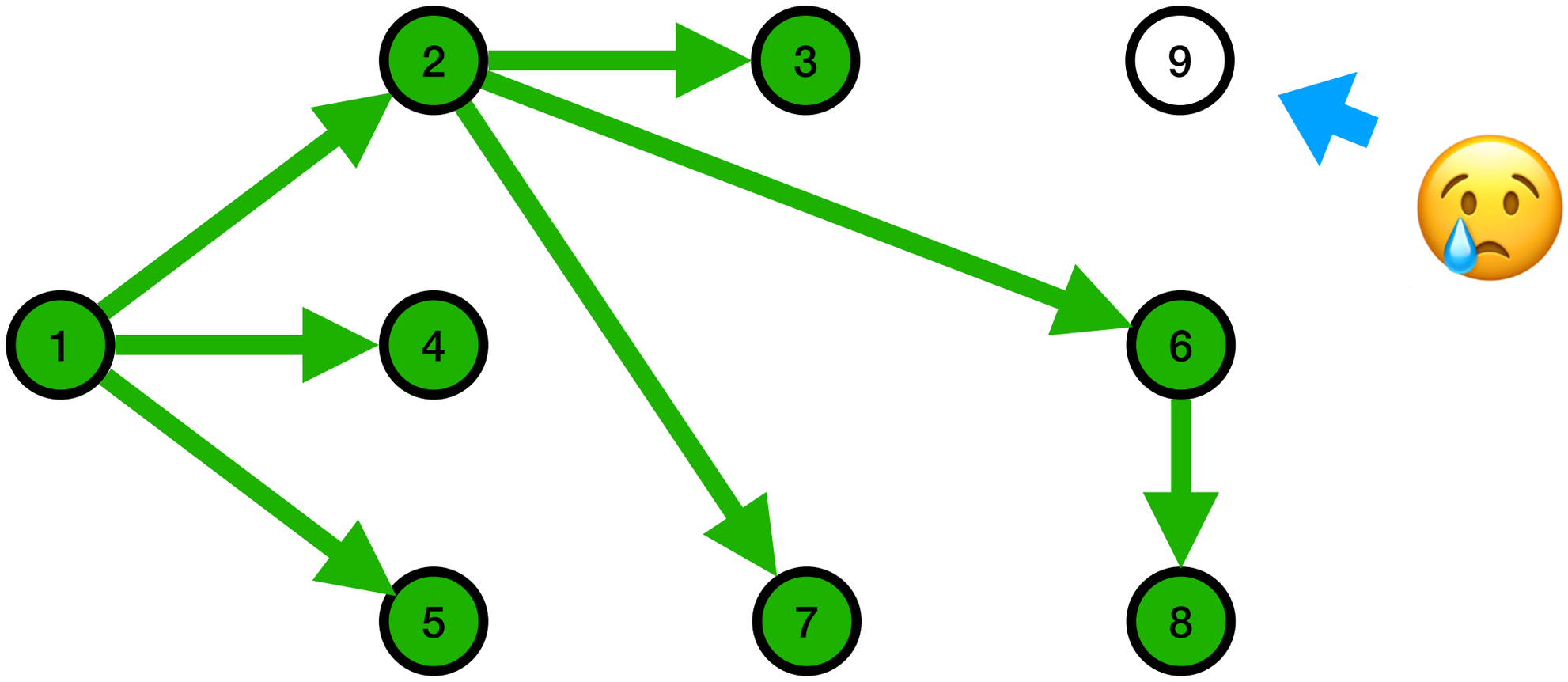
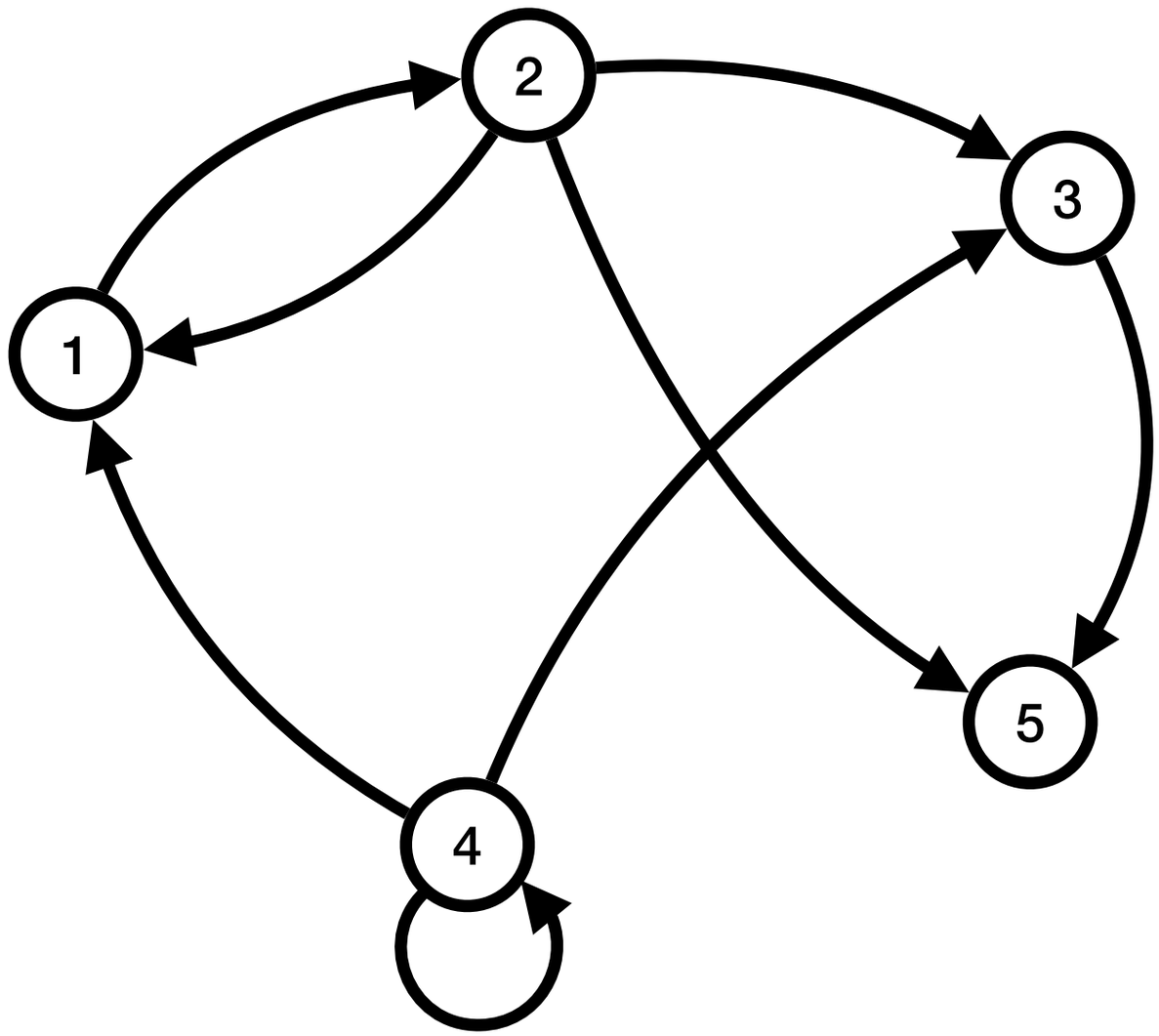
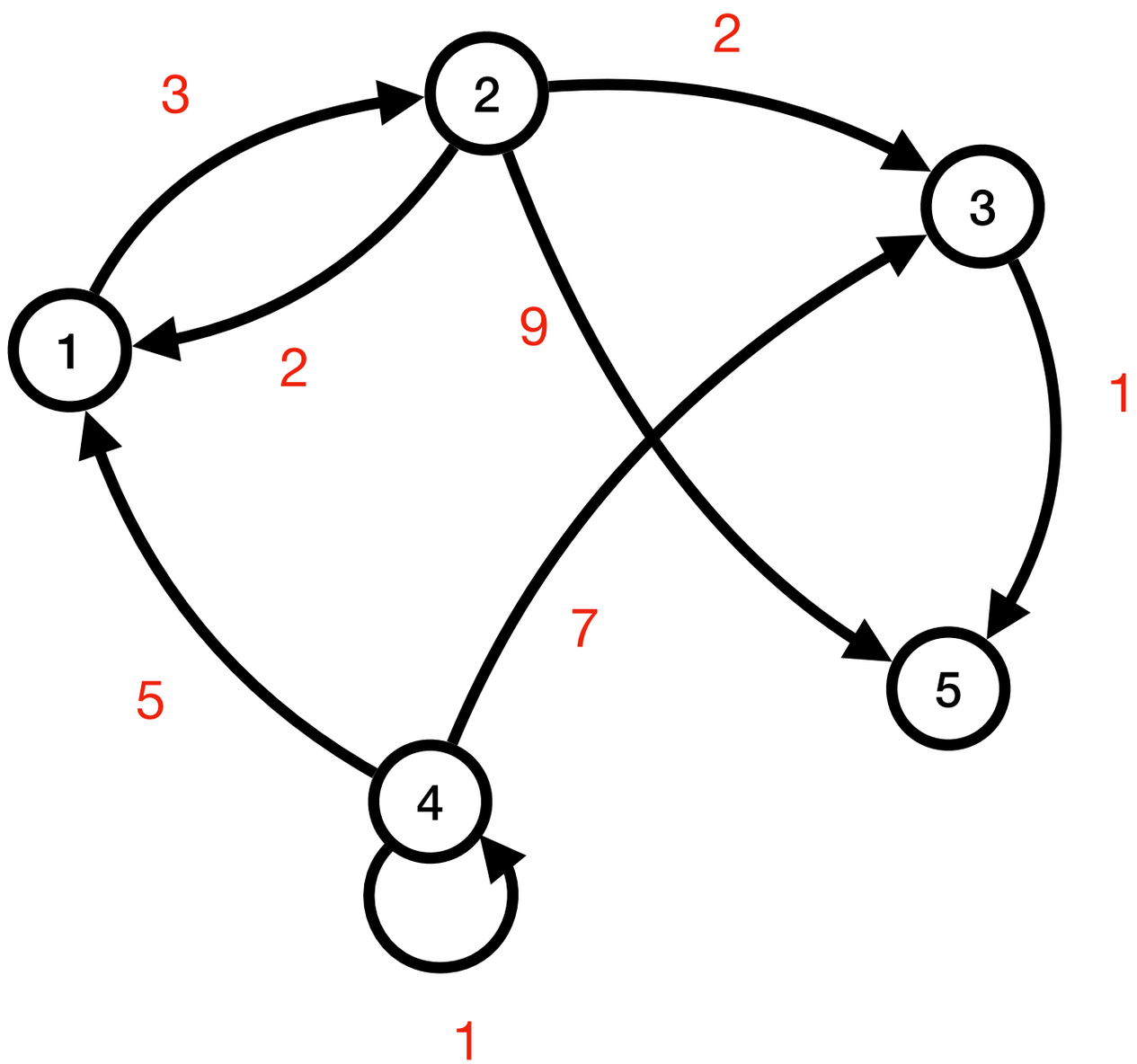


Le plus court chemin

Graphes pondérés



Graphes pondérés



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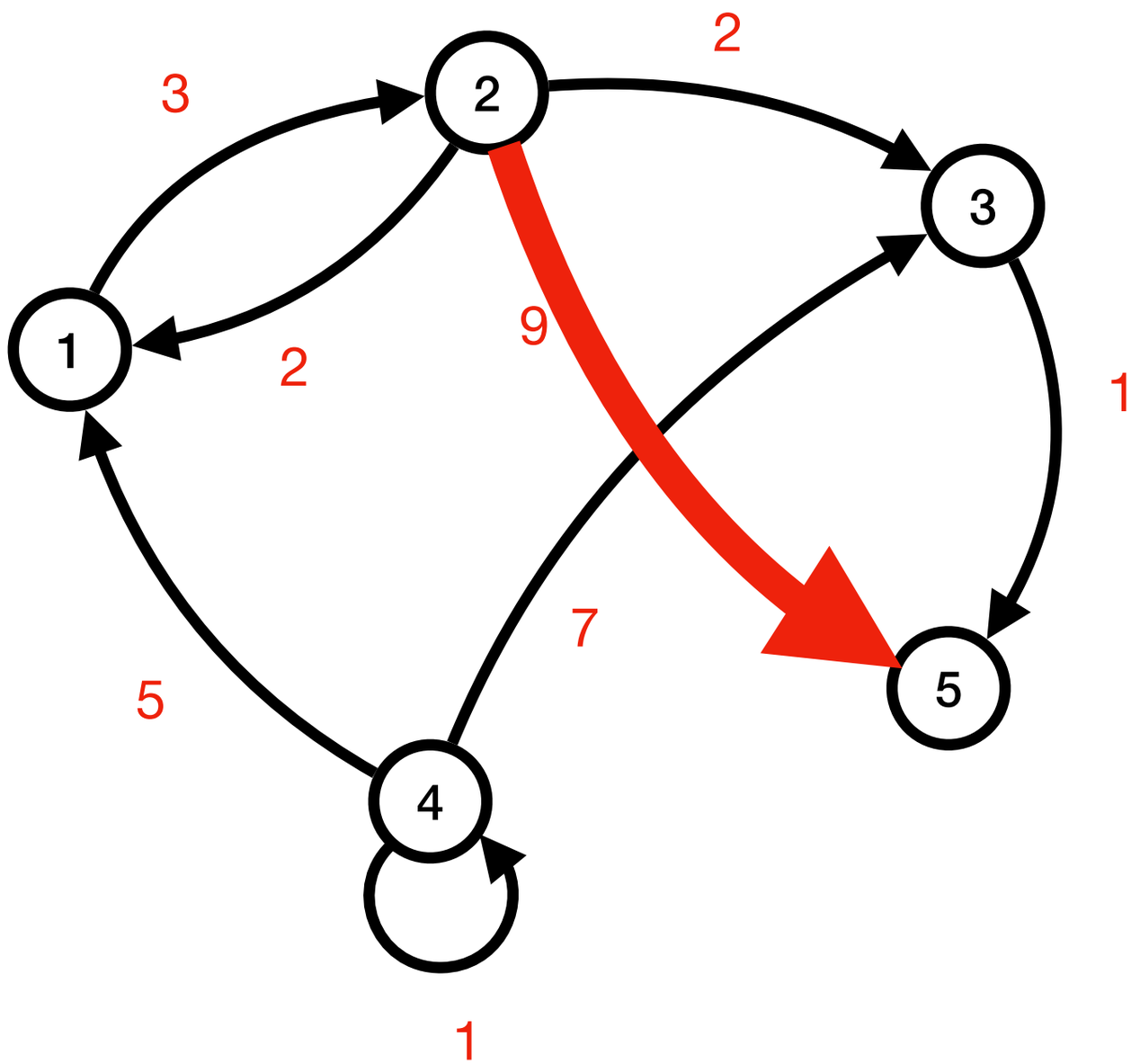
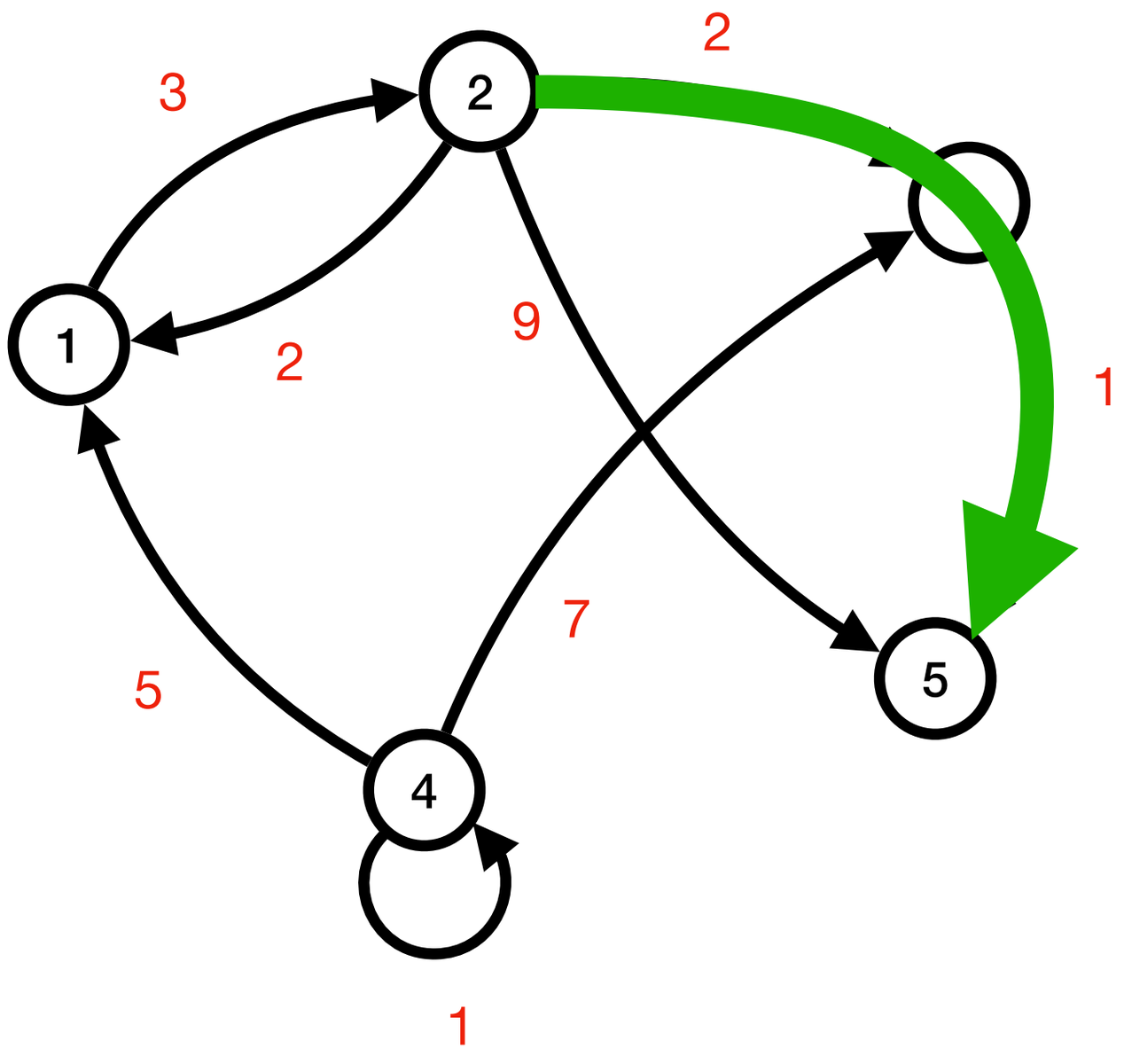
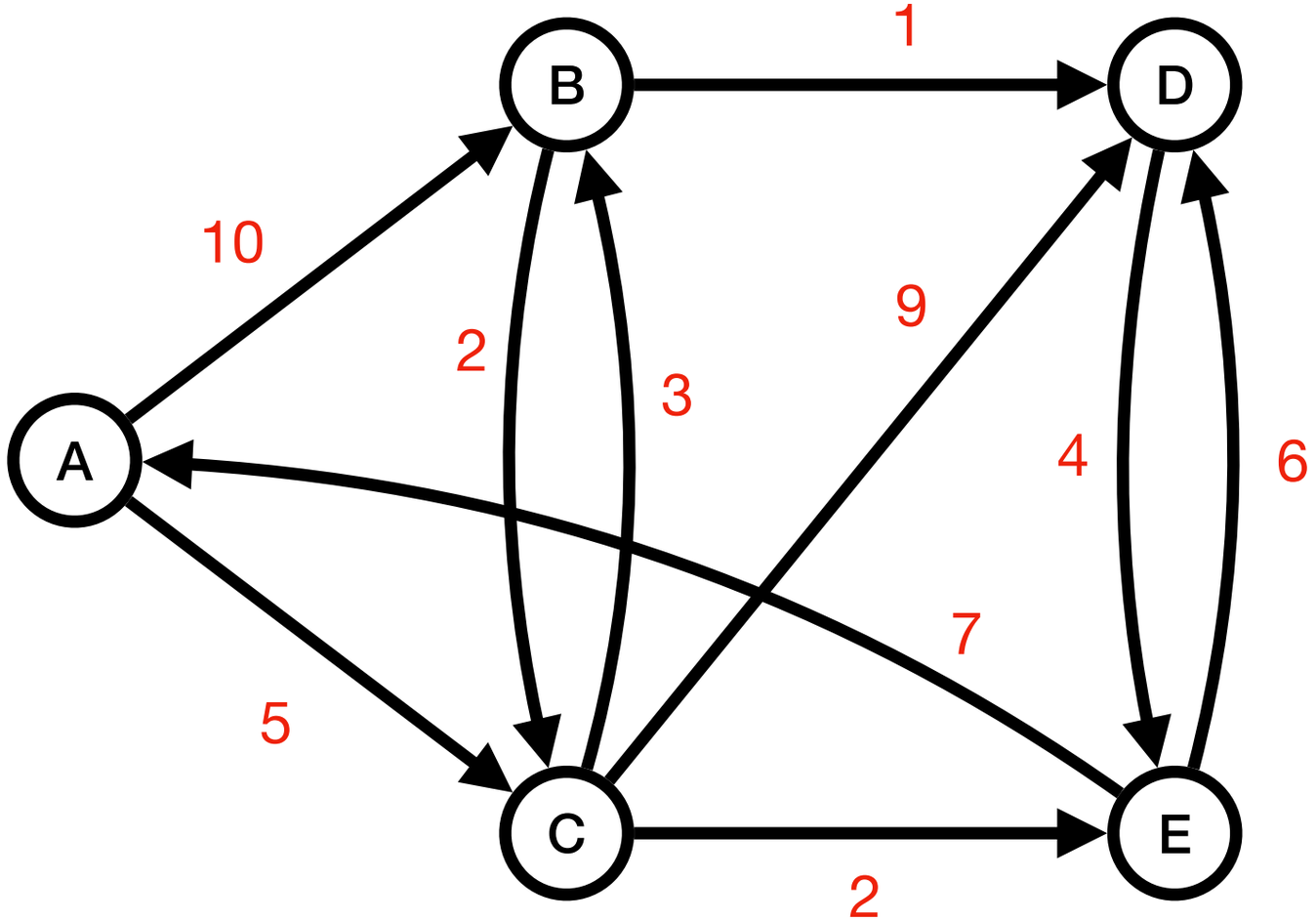


Graphes pondérés

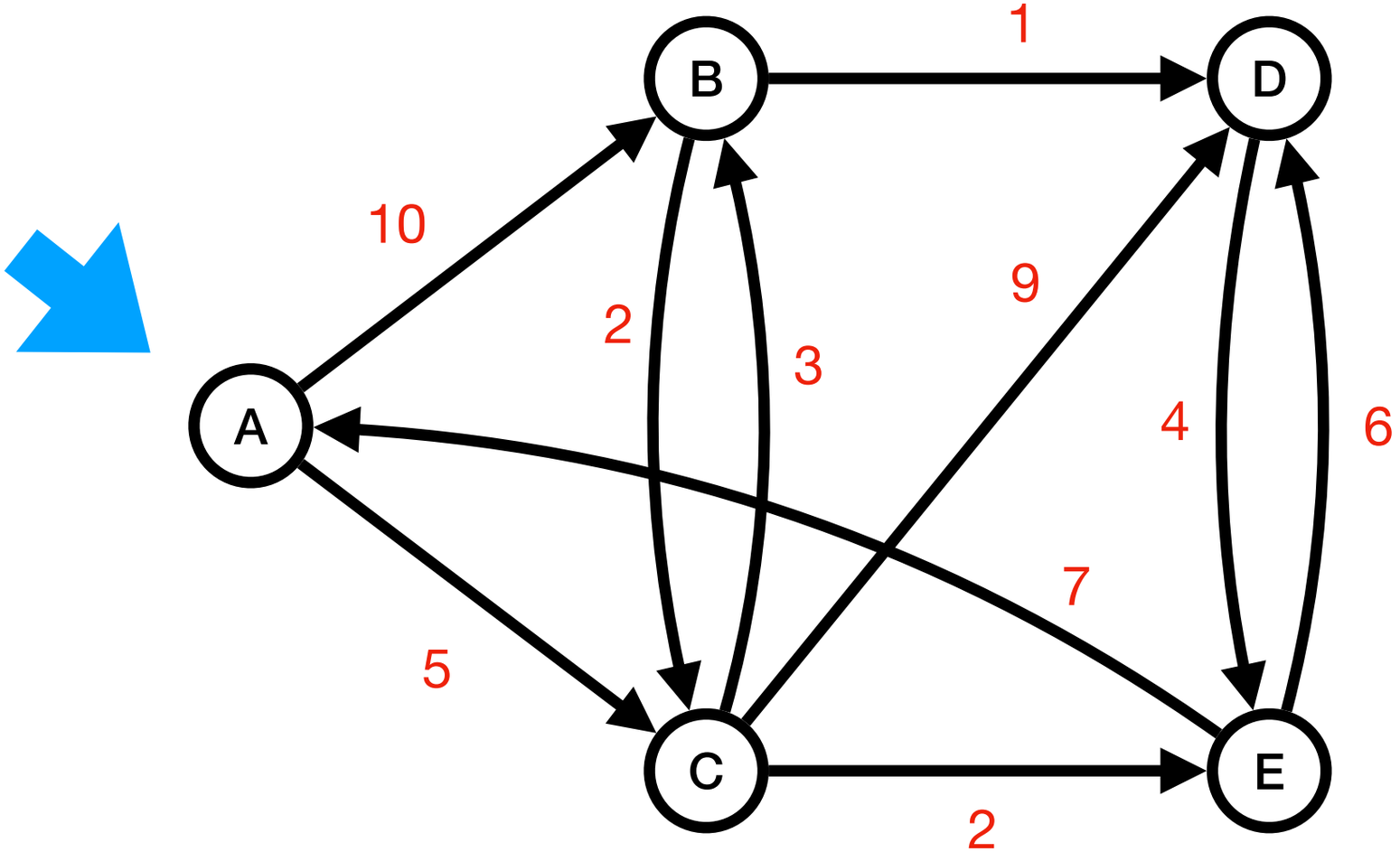


L’algorithme de parcours en largeur ne garantit pas d’obtenir un chemin minimal sur un graphe pondéré

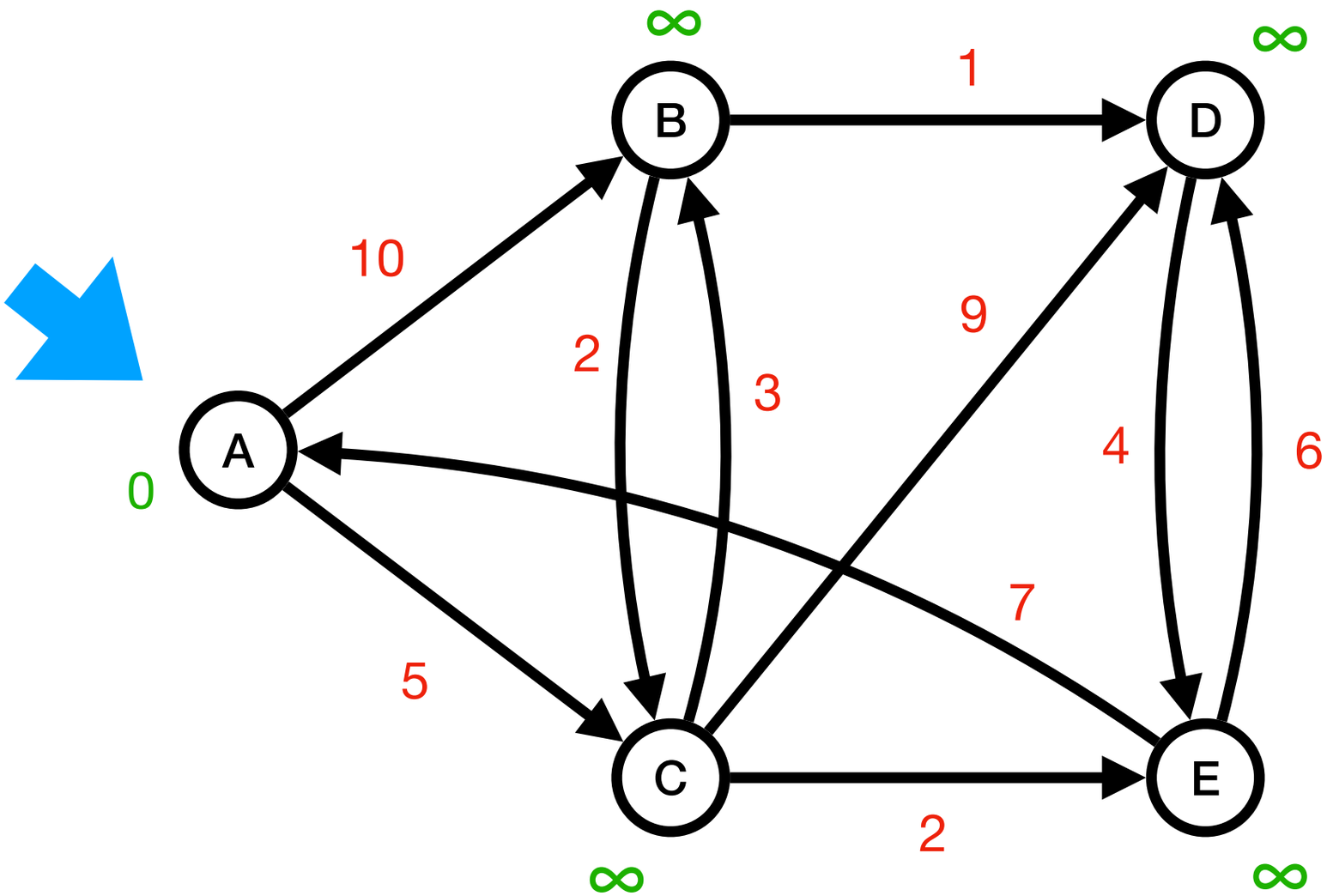
Algorithme de Dijkstra



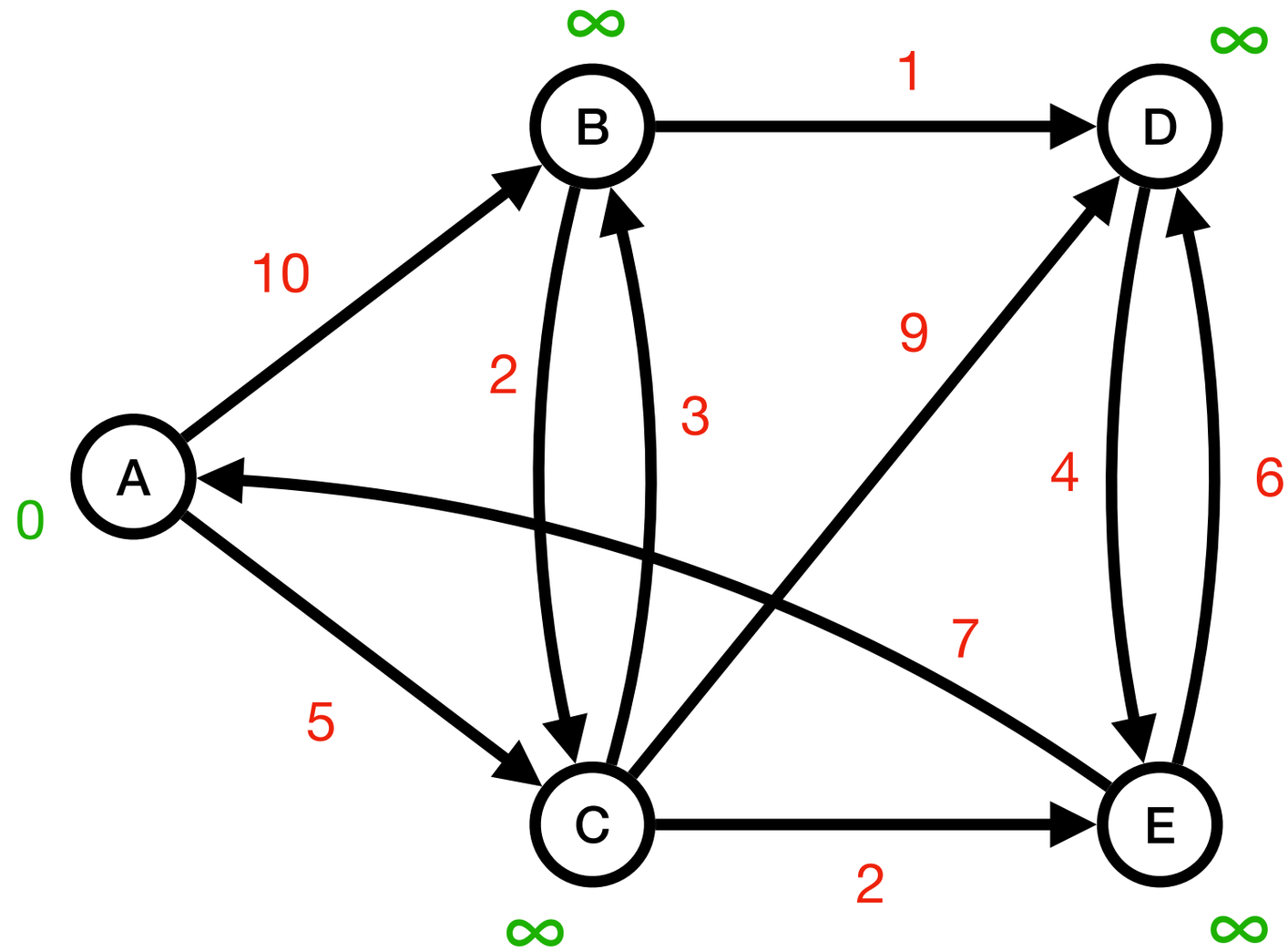
Algorithme de Dijkstra



Algorithme de Dijkstra



Algorithme de Dijkstra



File de priorité → A

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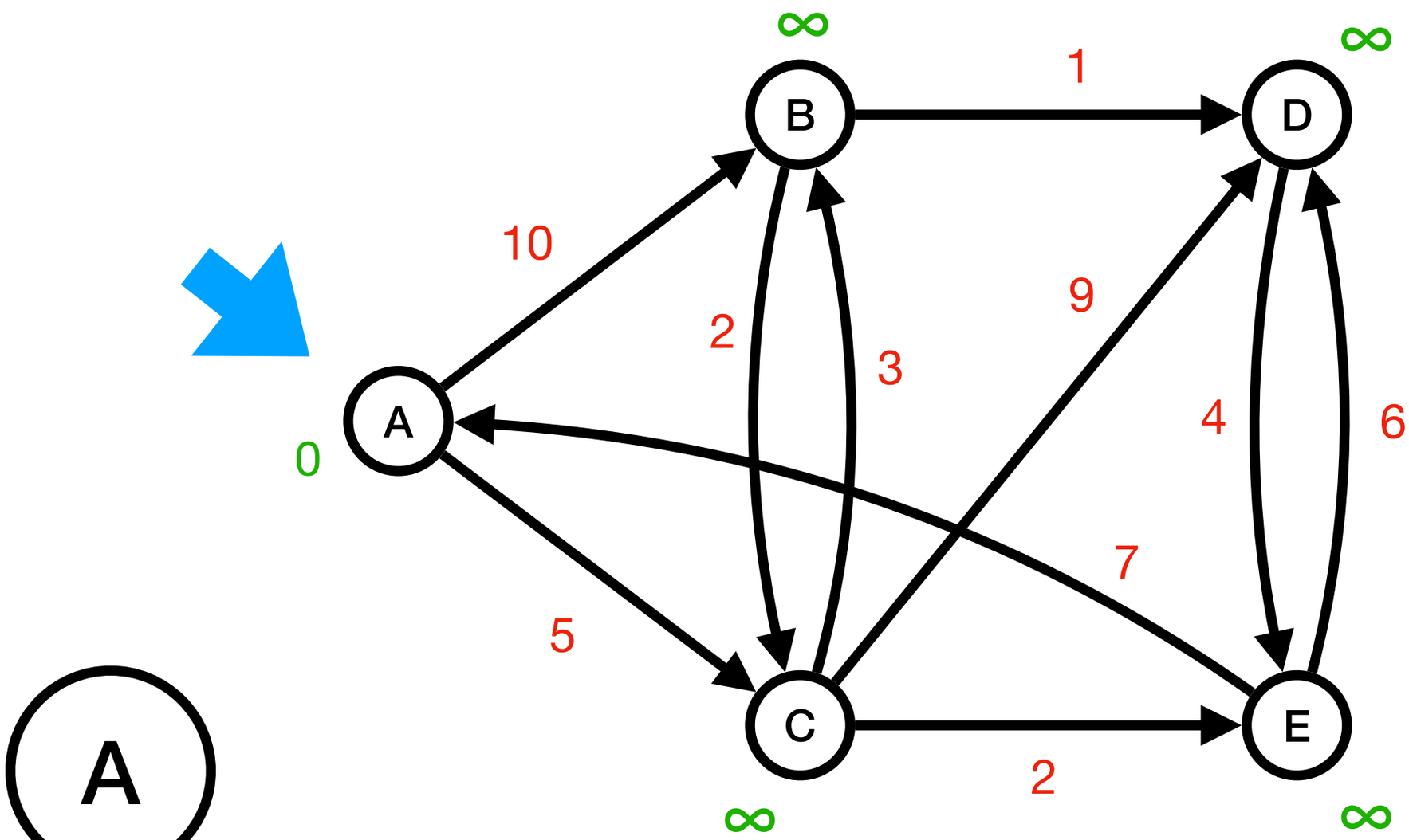
Algorithme de Dijkstra



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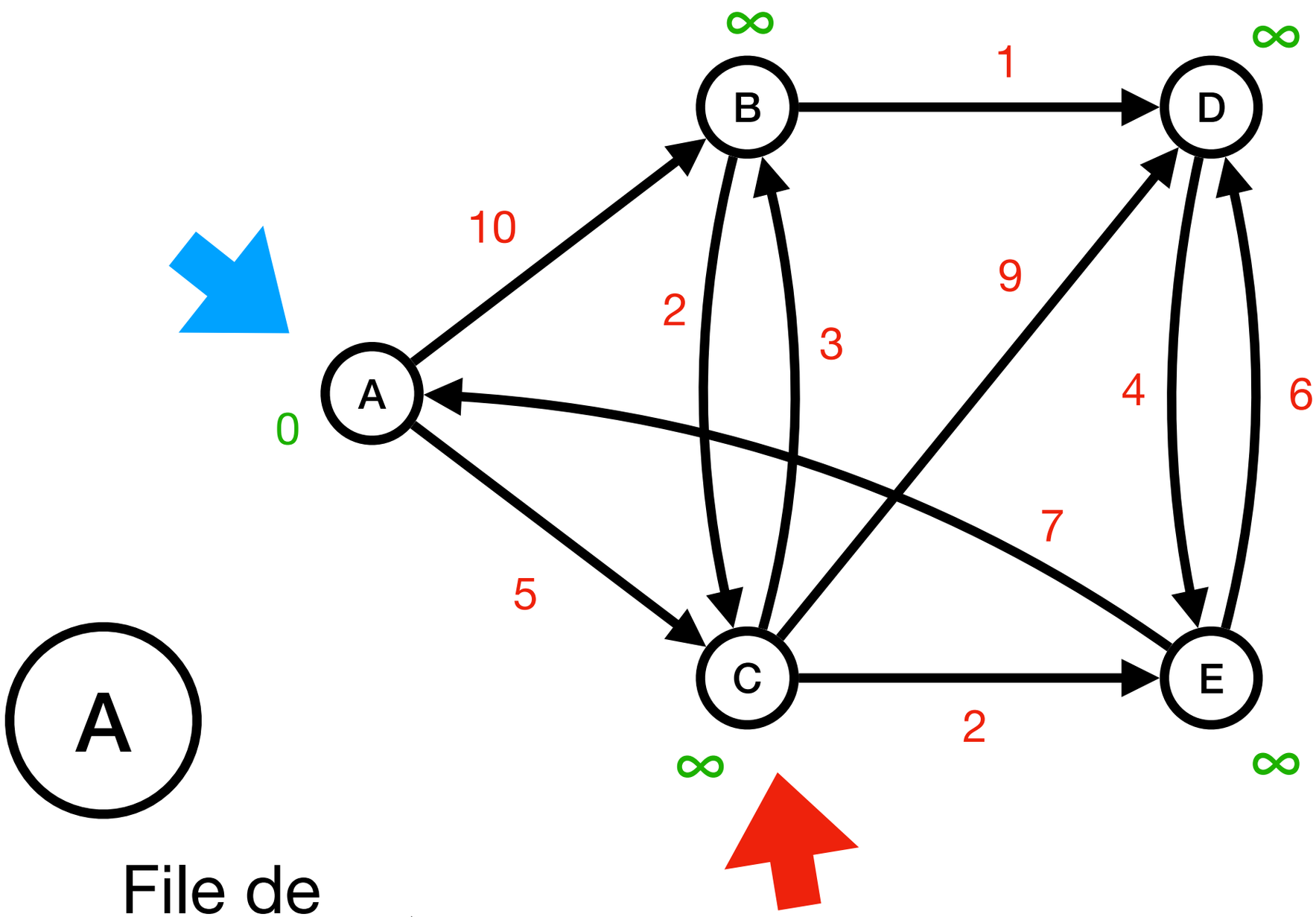
Algorithme de Dijkstra



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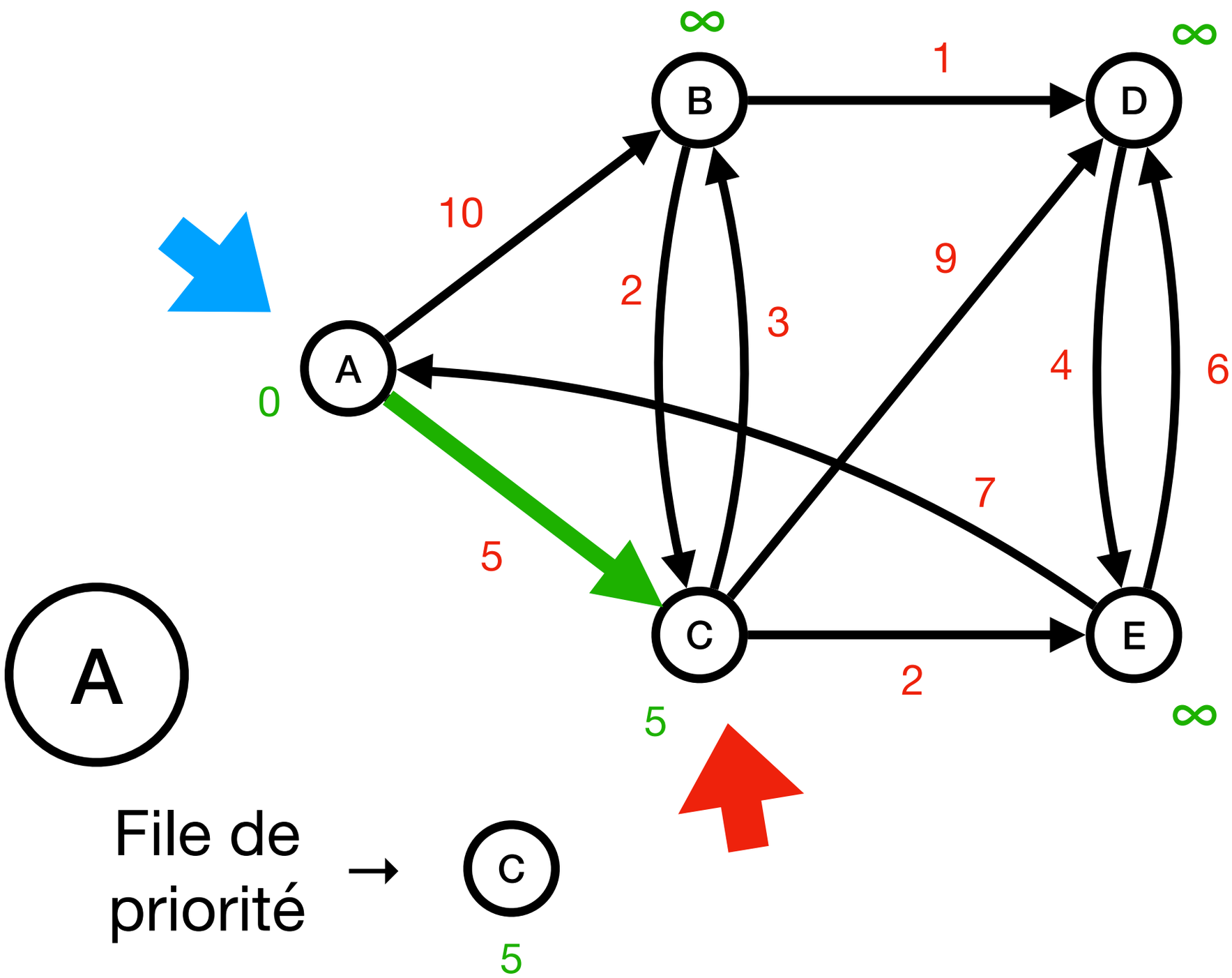

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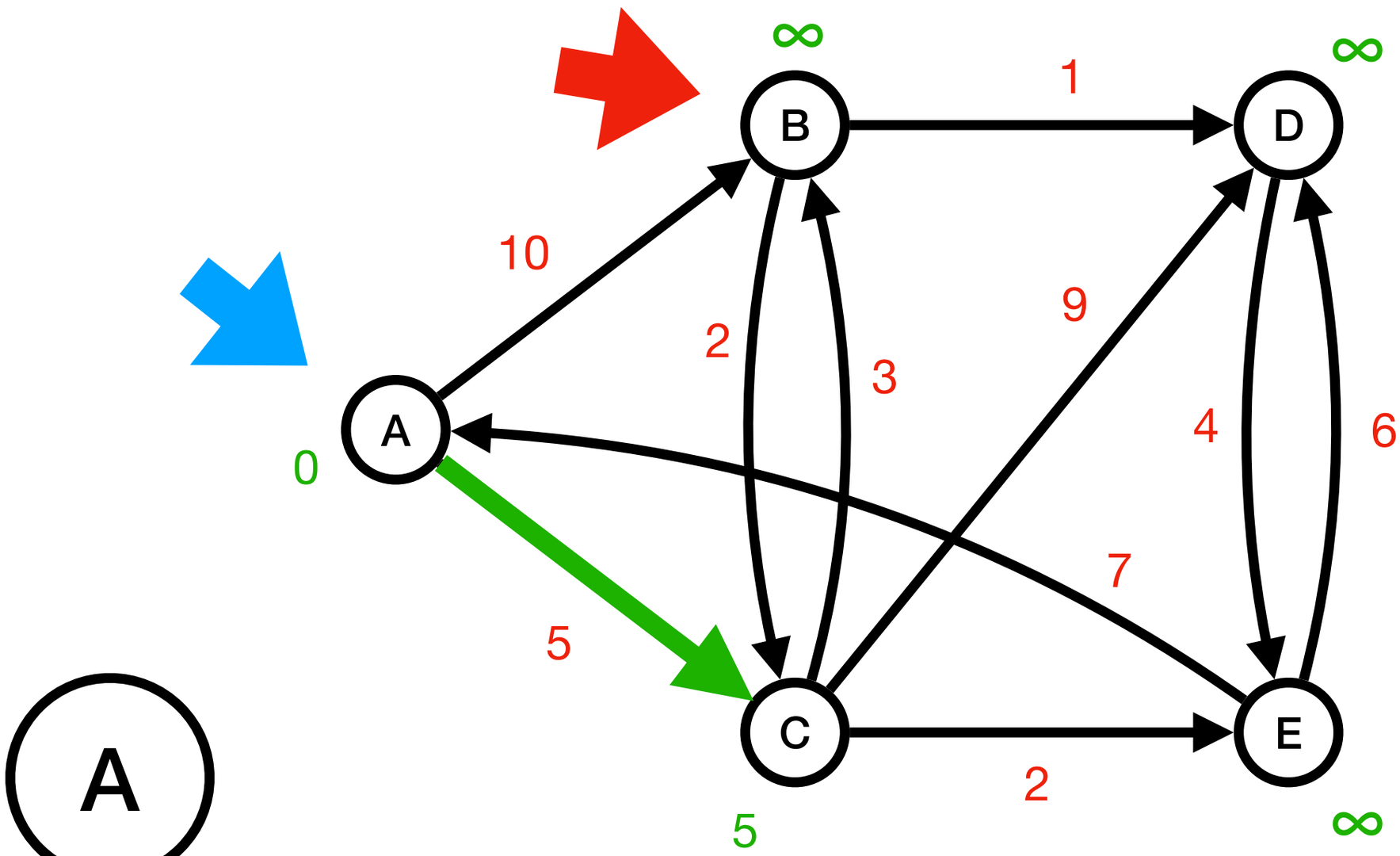
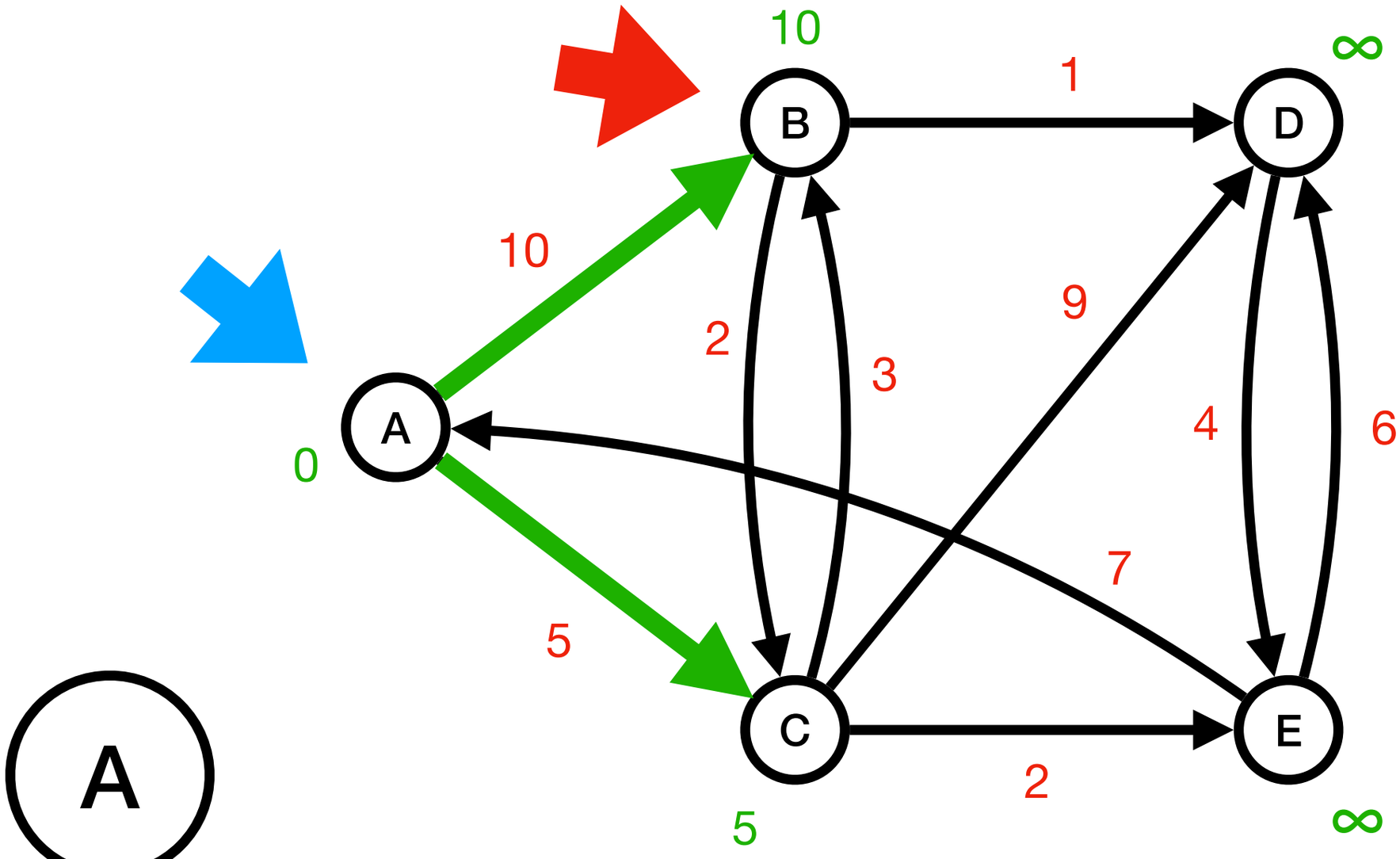


A

File de priorité → C

5

Algorithme de Dijkstra



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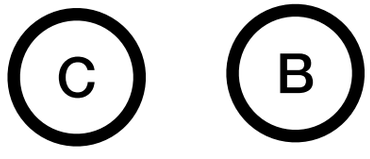
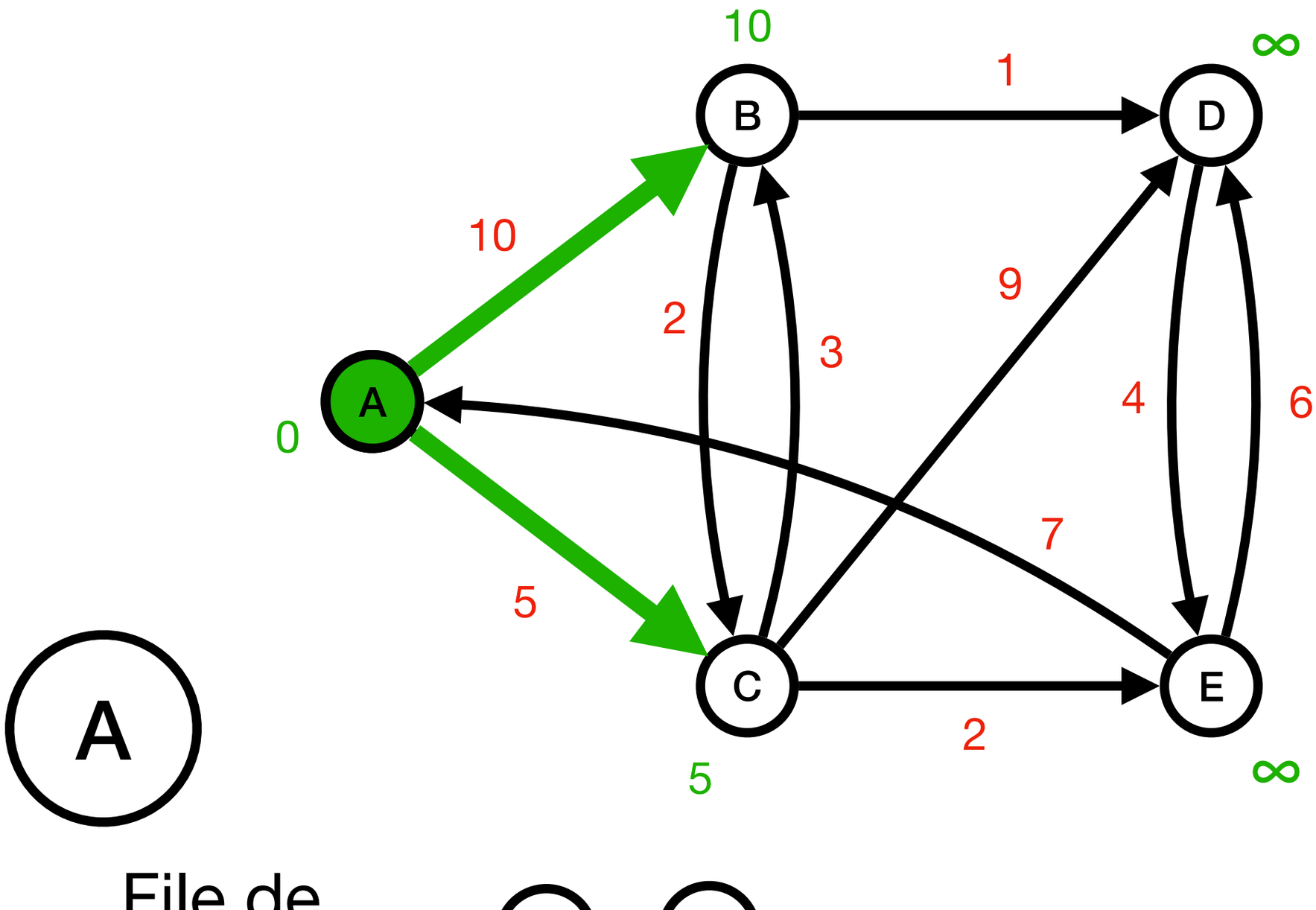


Algorithme de Dijkstra



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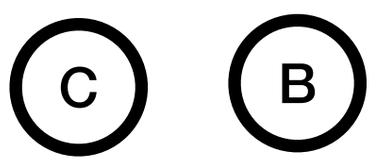

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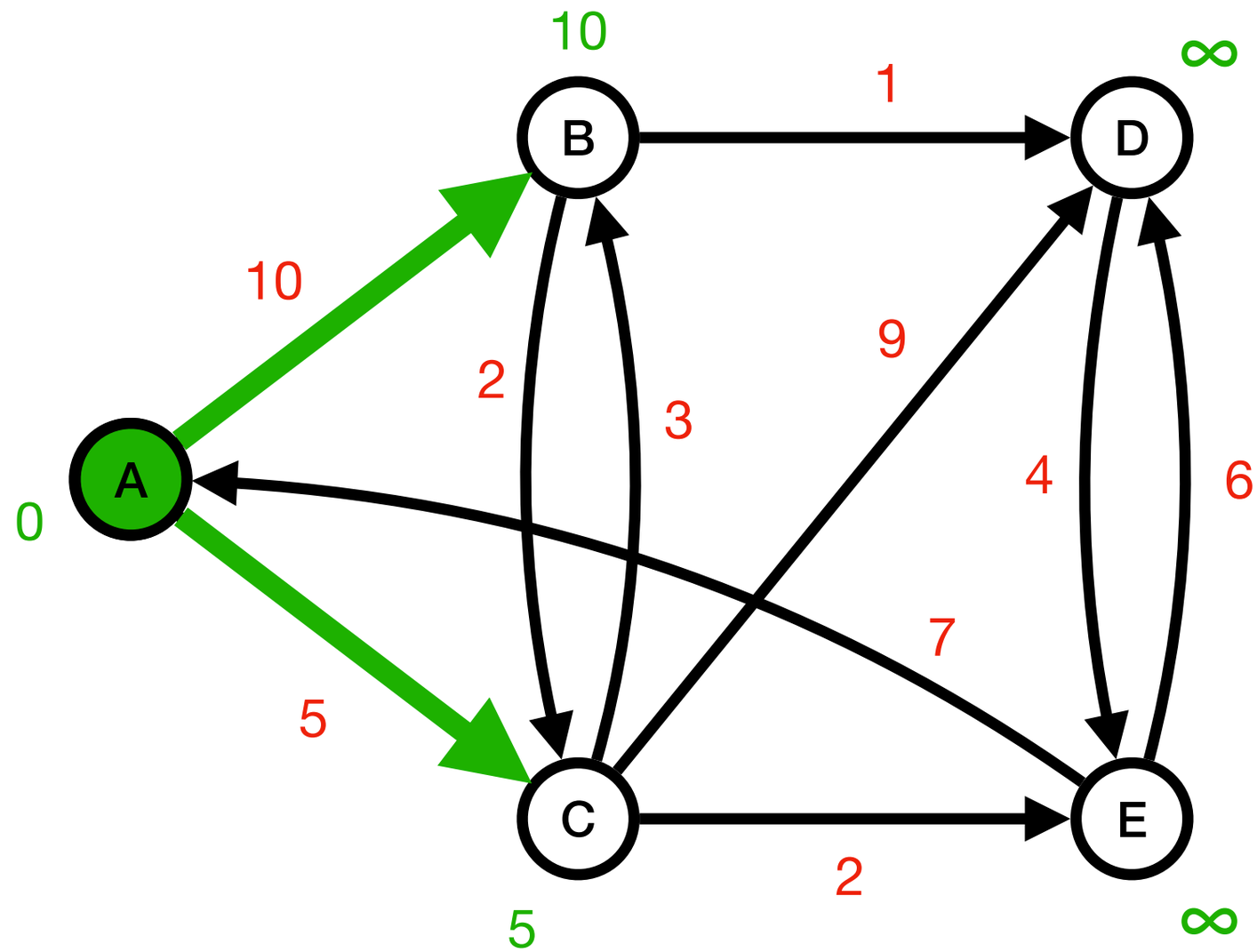

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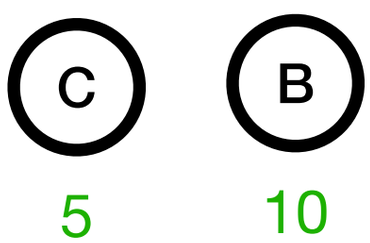

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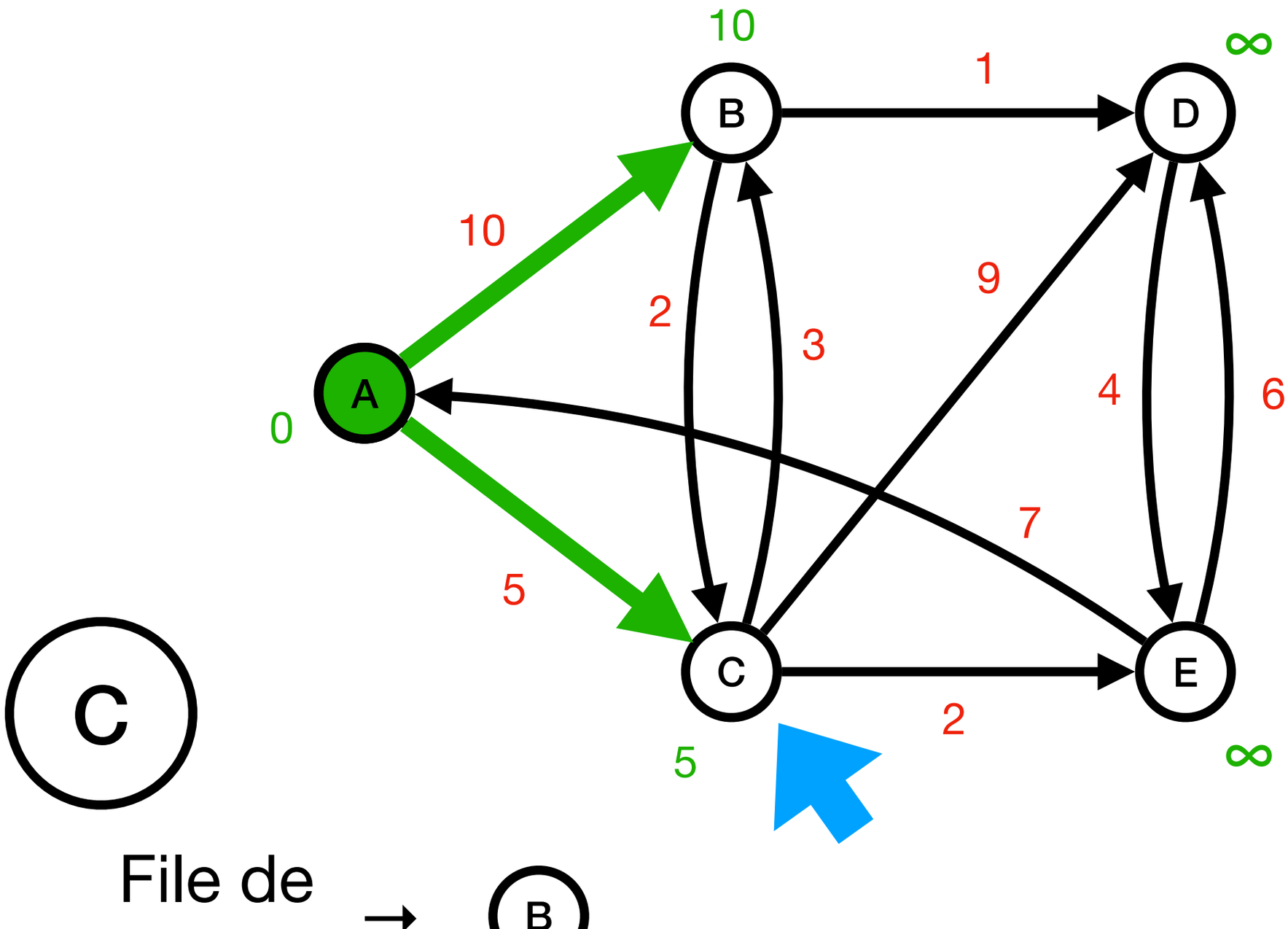
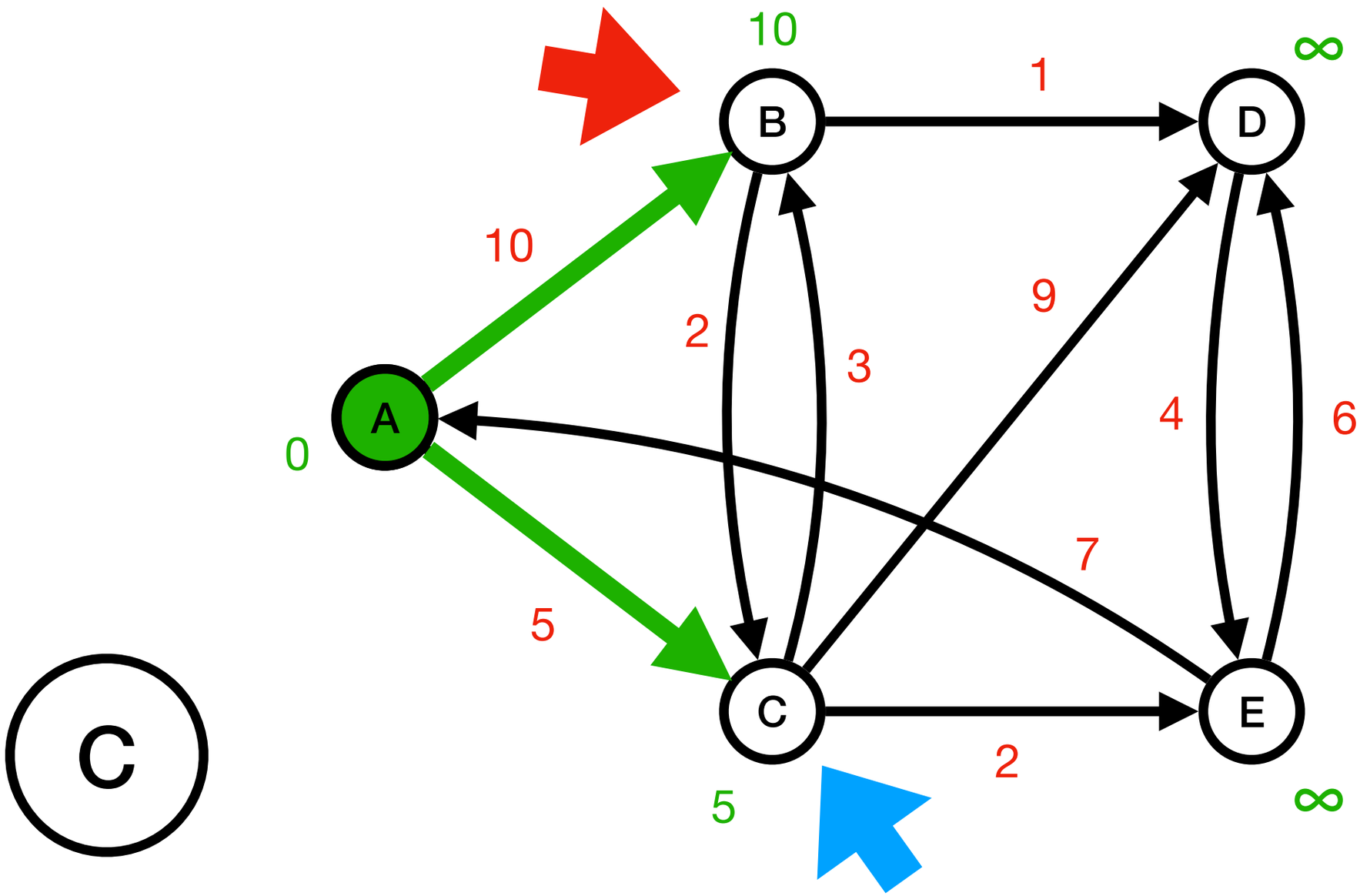


C

File de priorité → B

10

Algorithme de Dijkstra



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File de priorité → B

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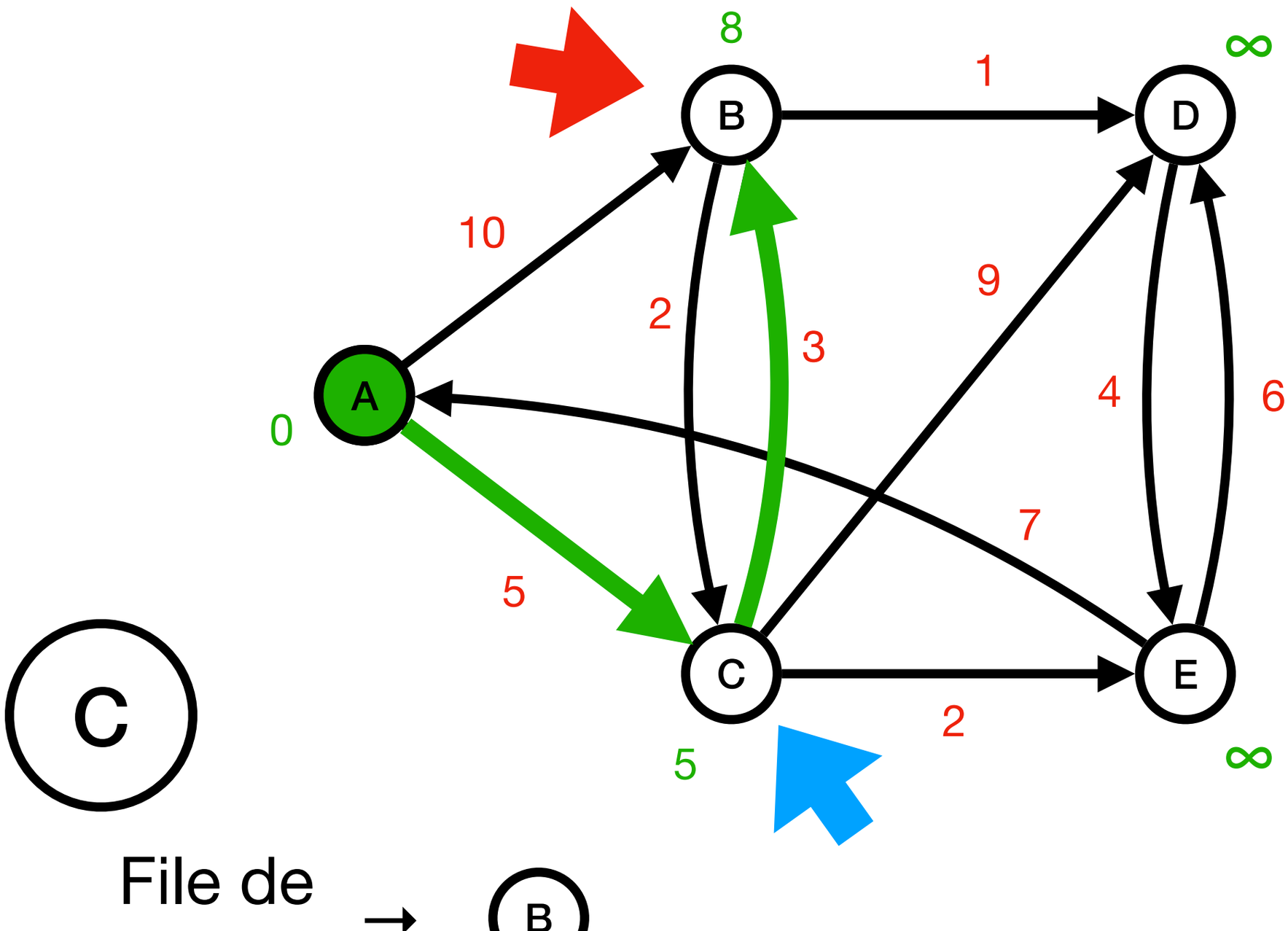
Algorithme de Dijkstra



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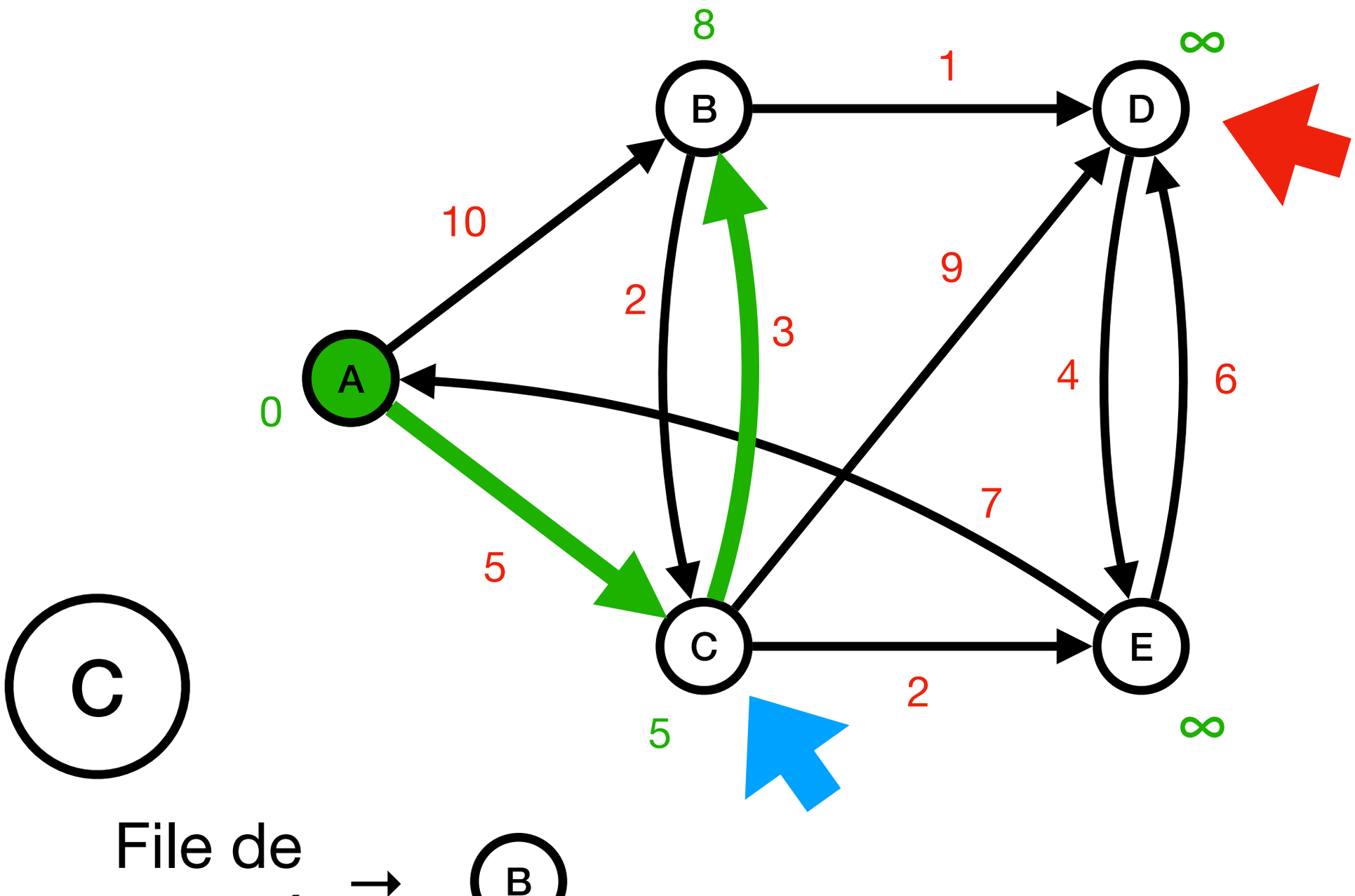
File de priorité → B

8

Algorithme de Dijkstra



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File de priorité → B

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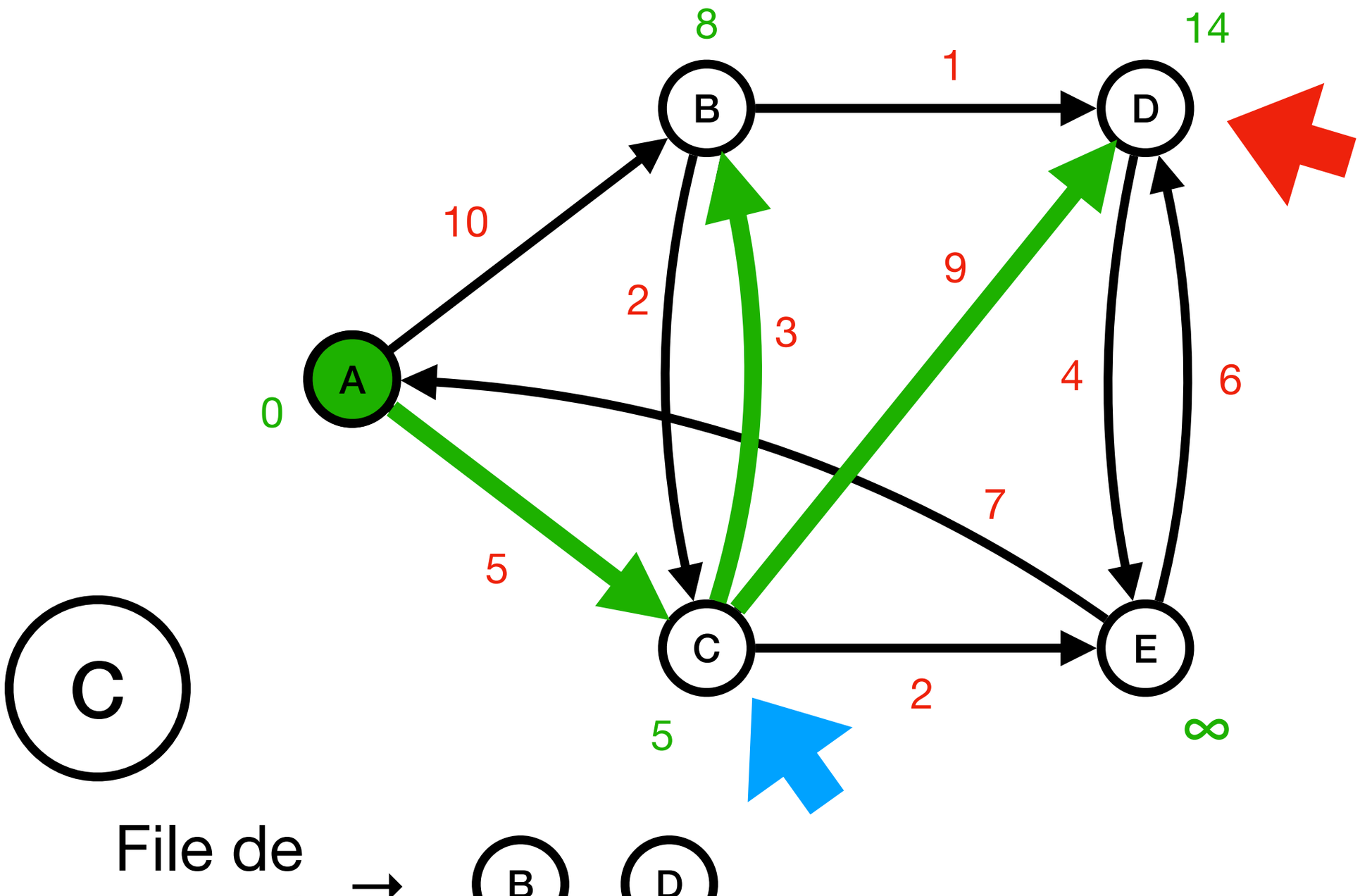
Algorithme de Dijkstra



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File de priorité → B D

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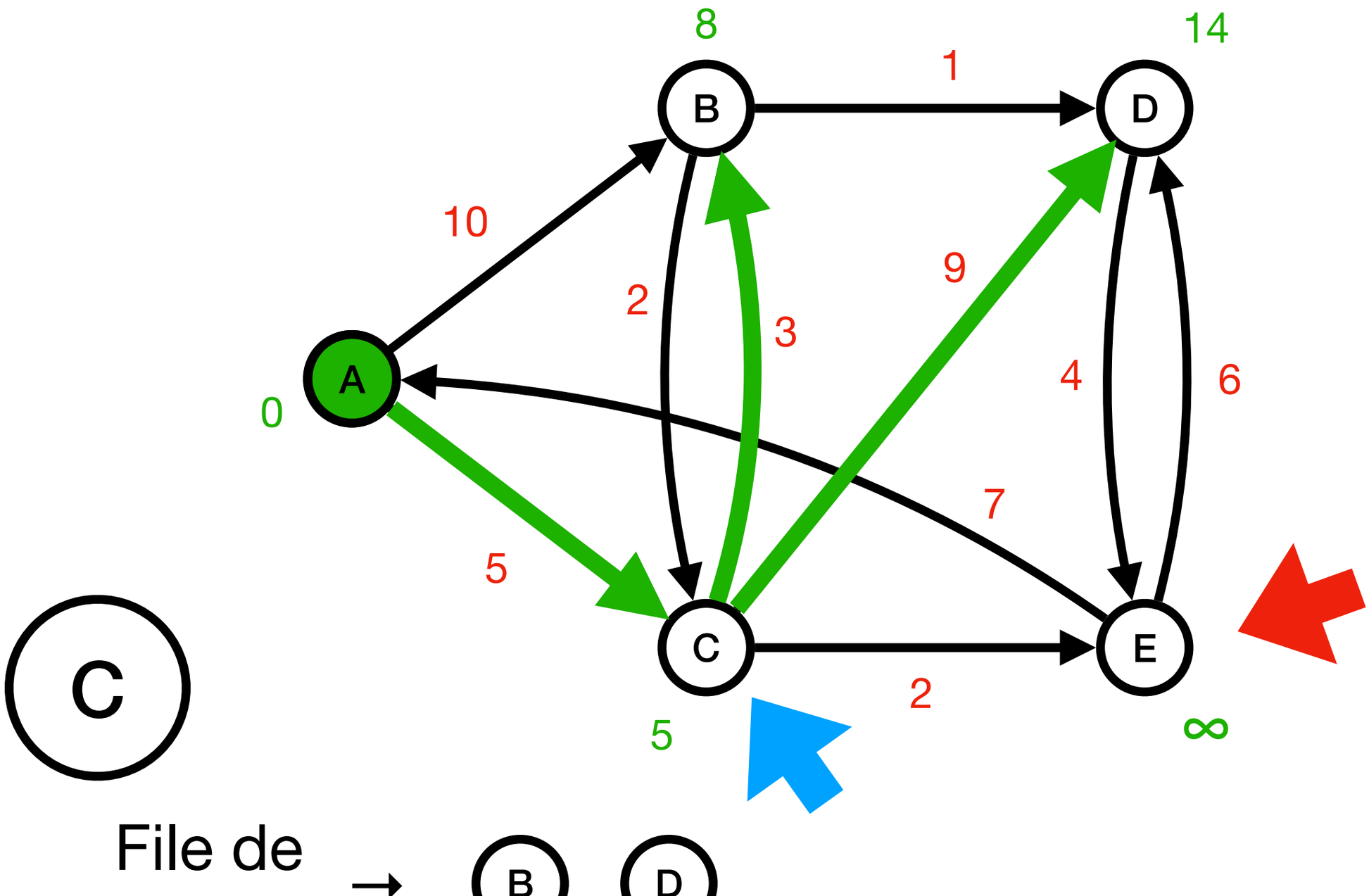
Algorithme de Dijkstra



C

File de priorité → B D

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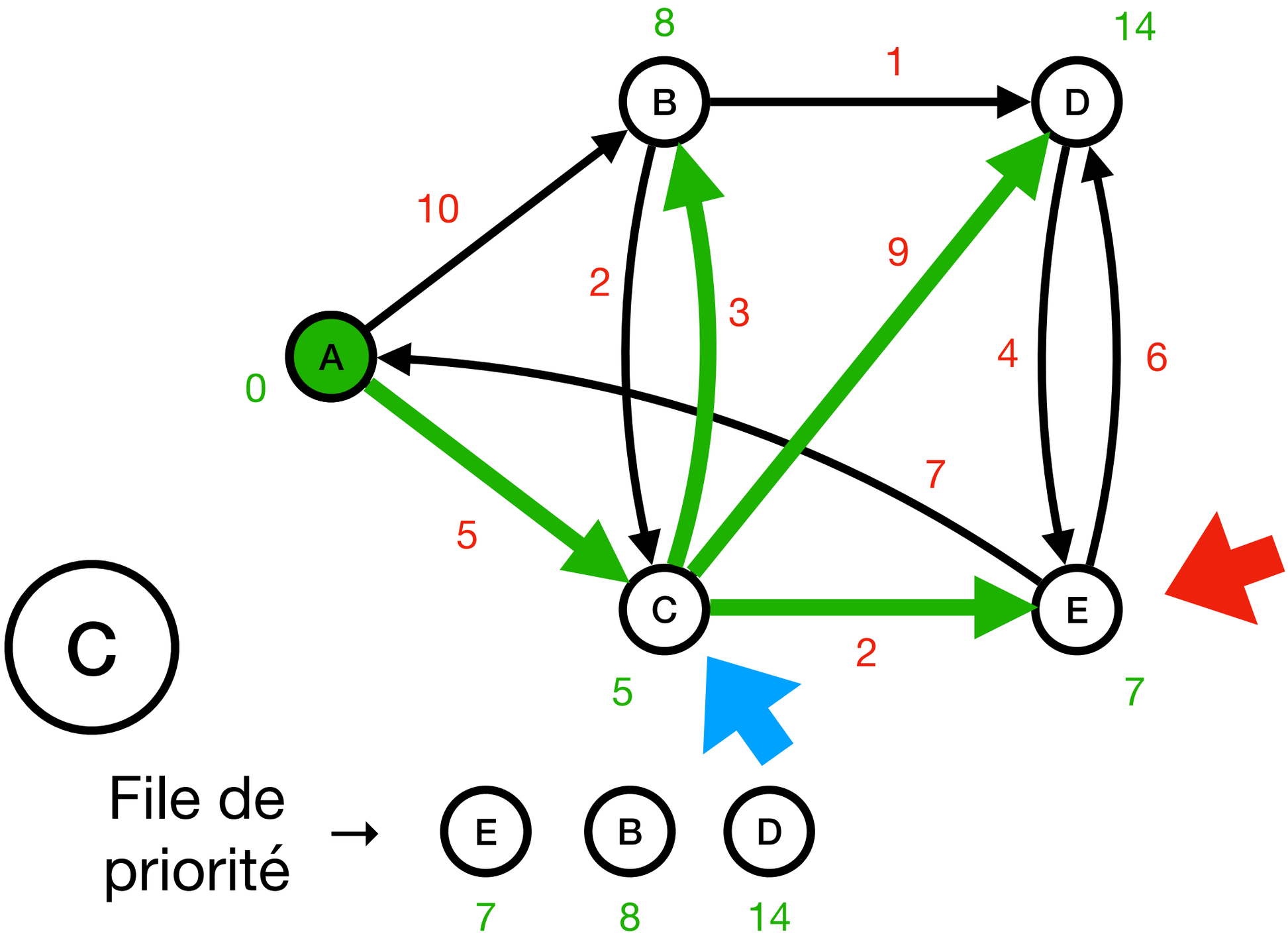
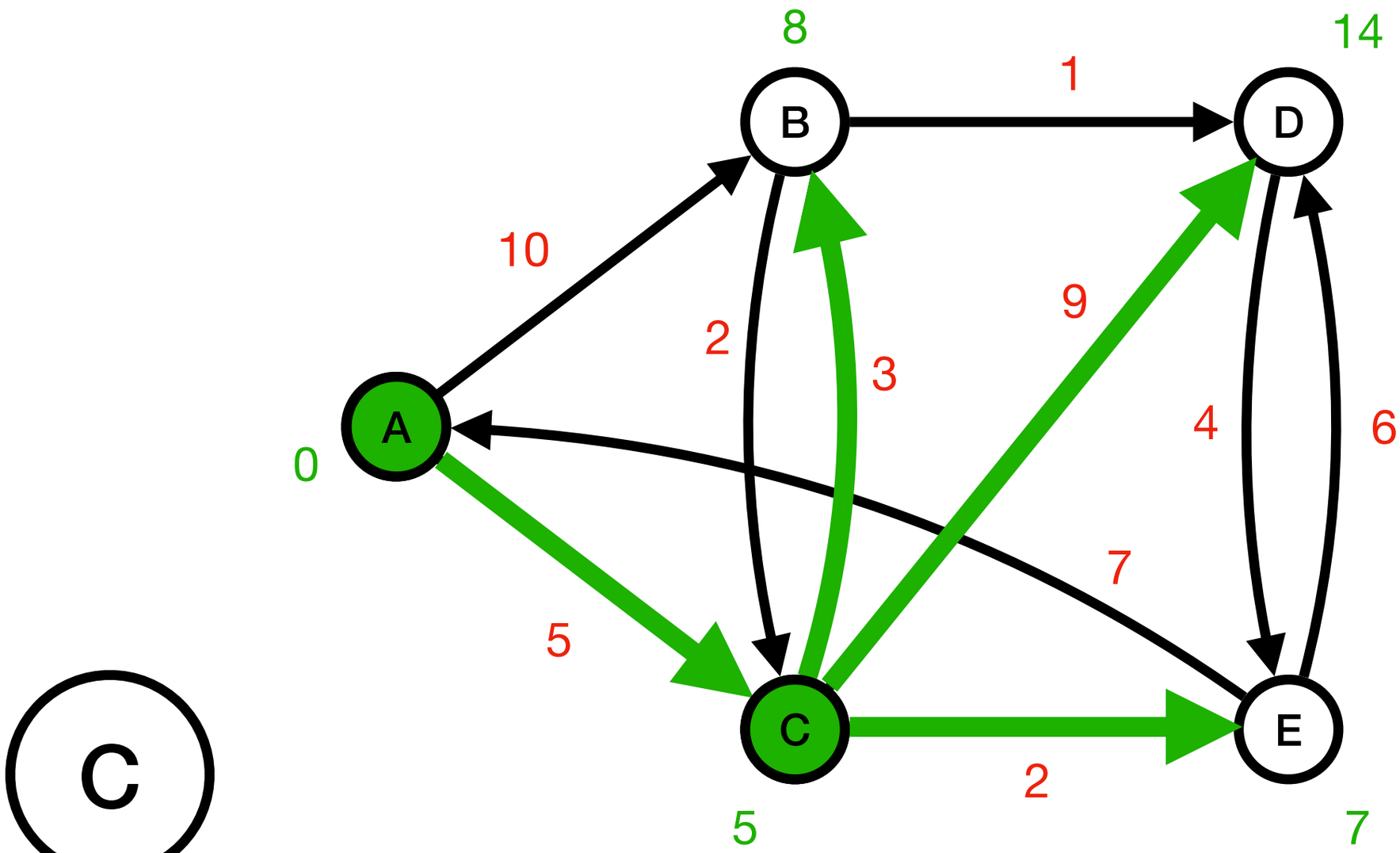


Algorithme de Dijkstra



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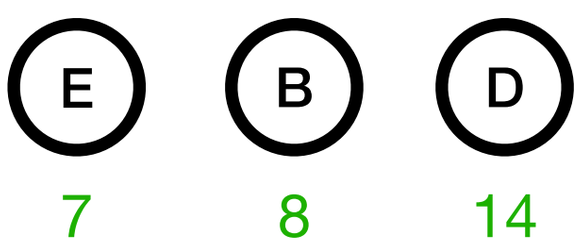
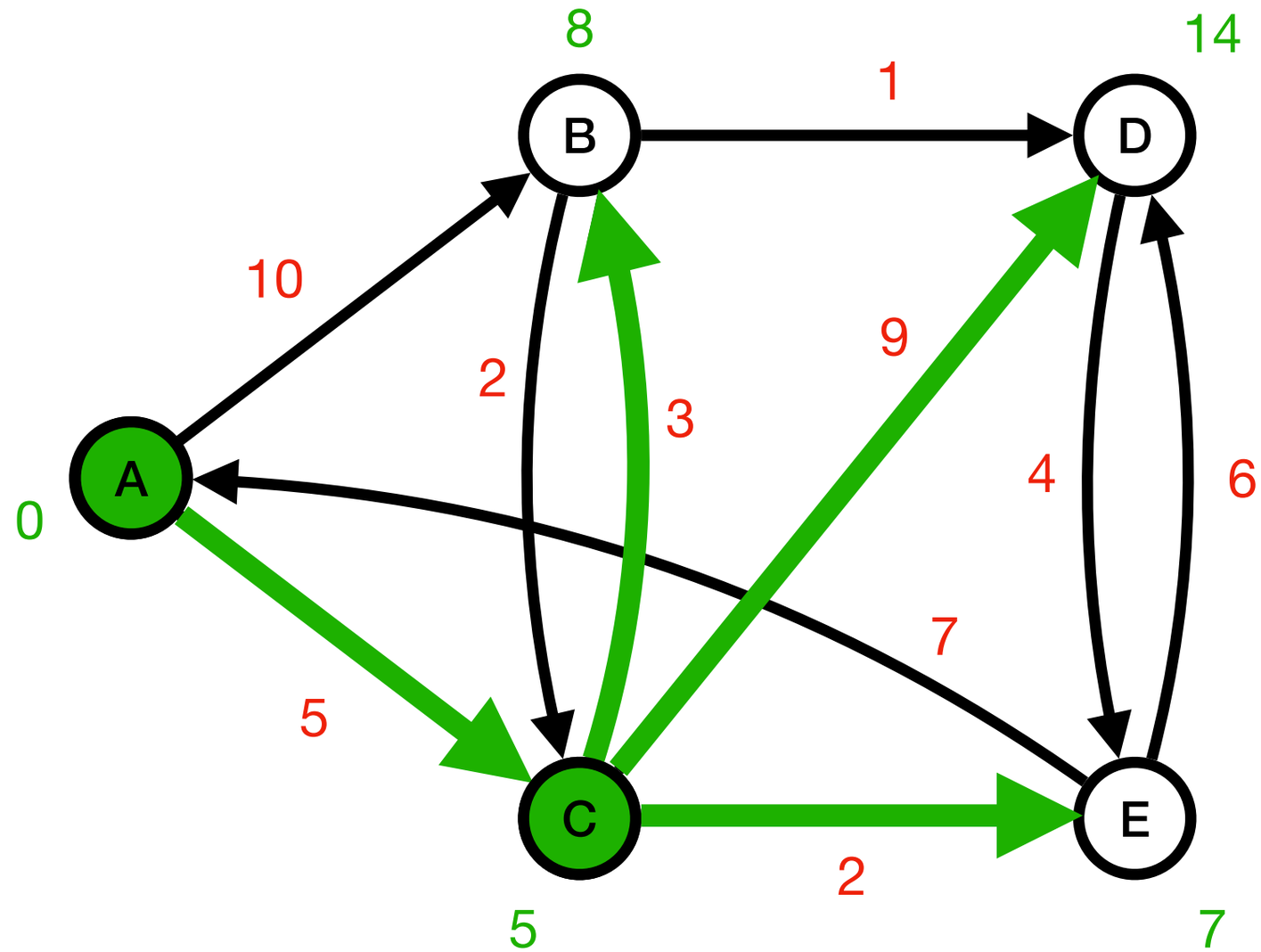


Algorithme de Dijkstra



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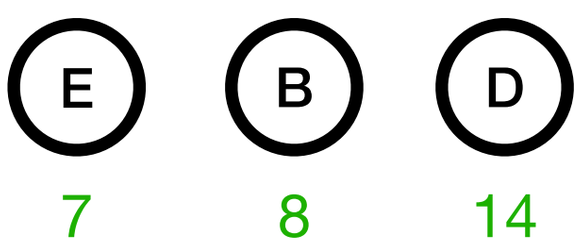

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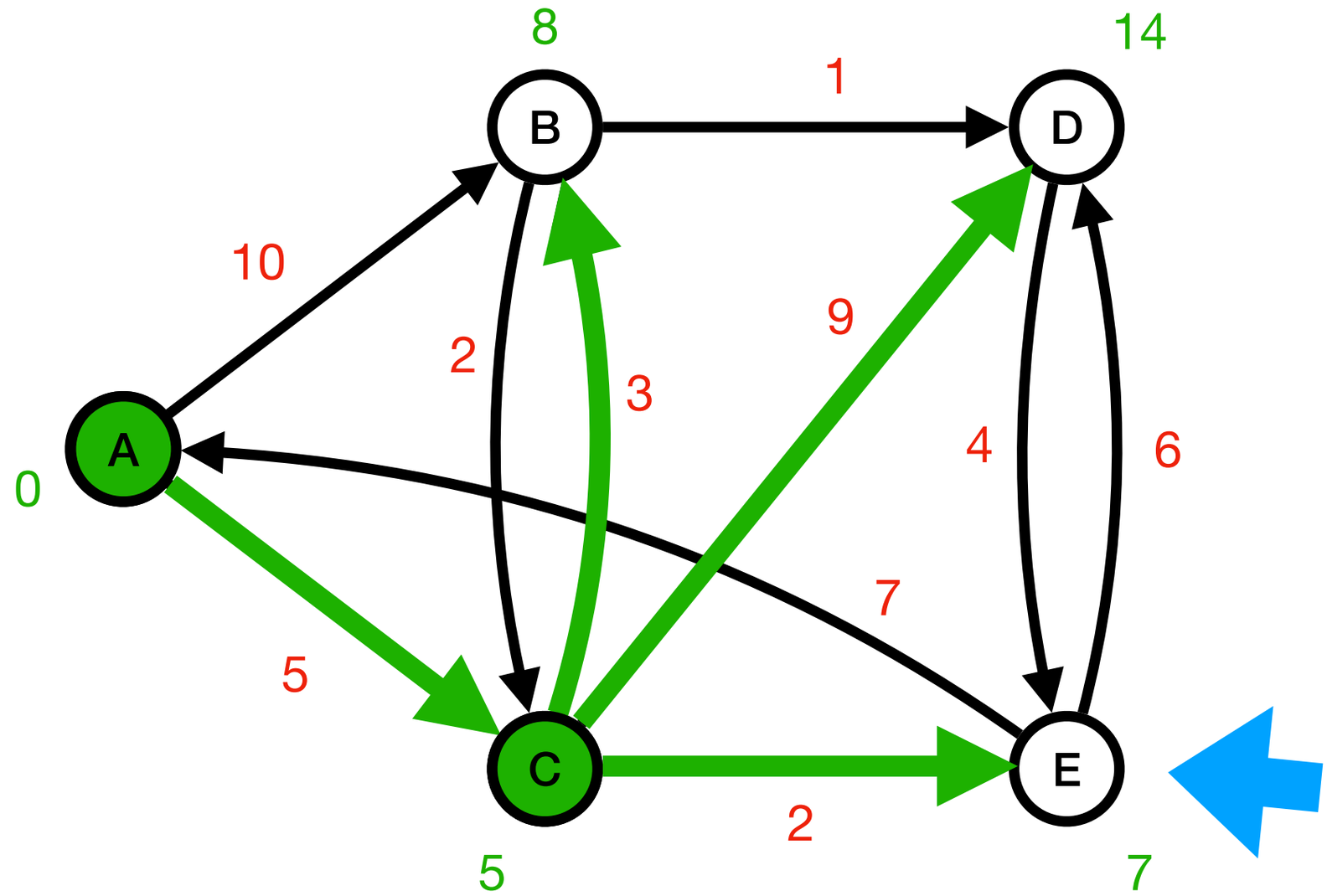

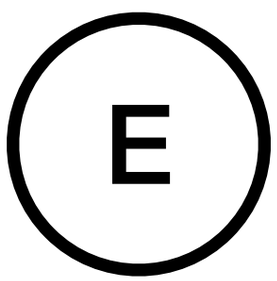

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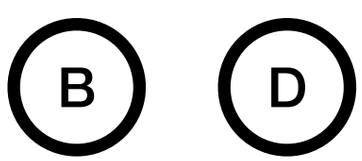

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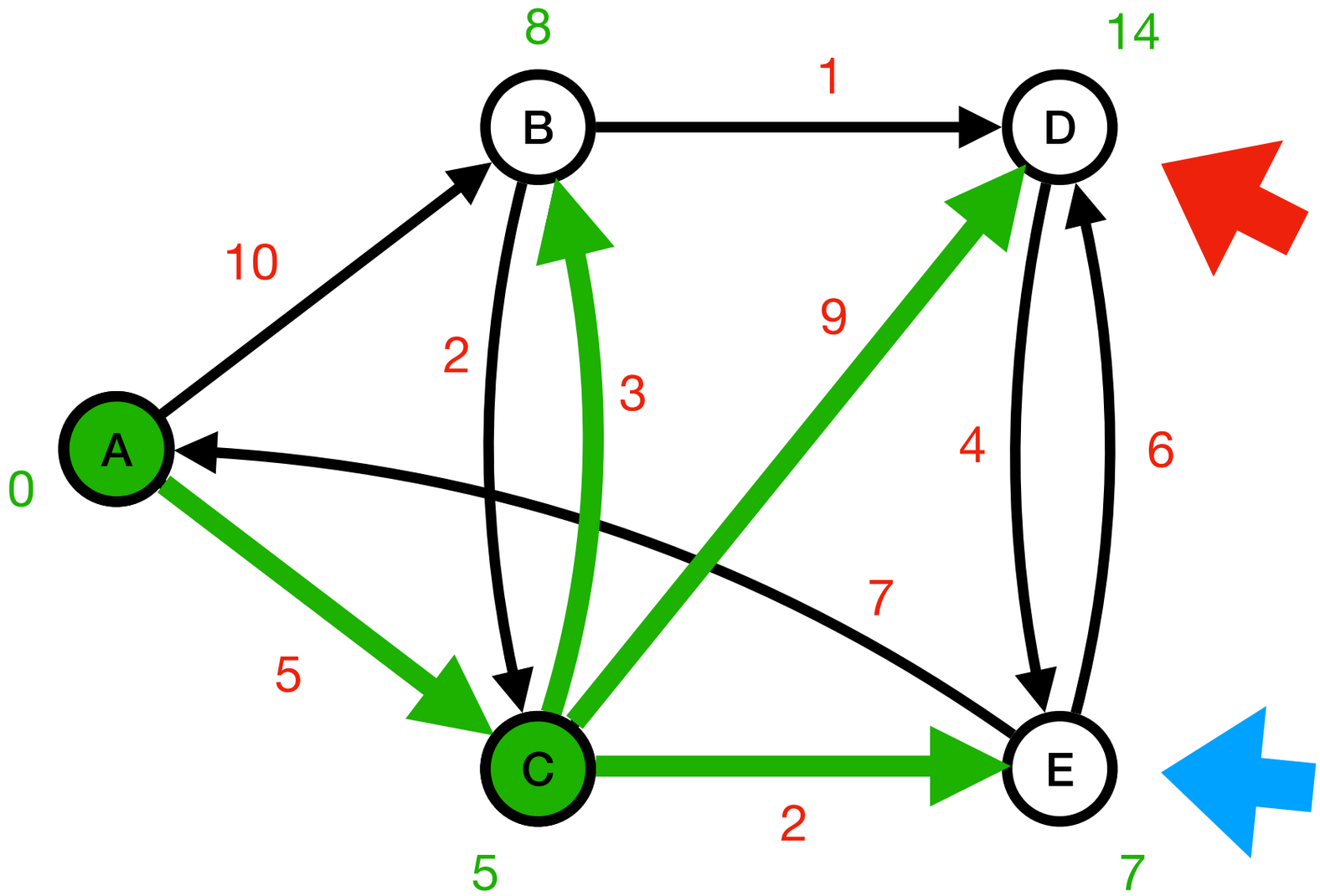

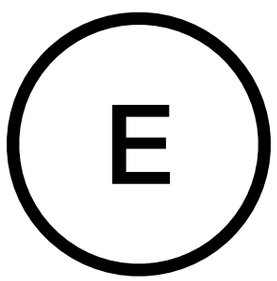

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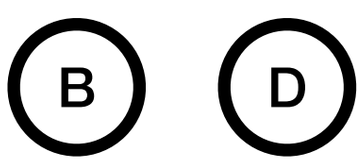

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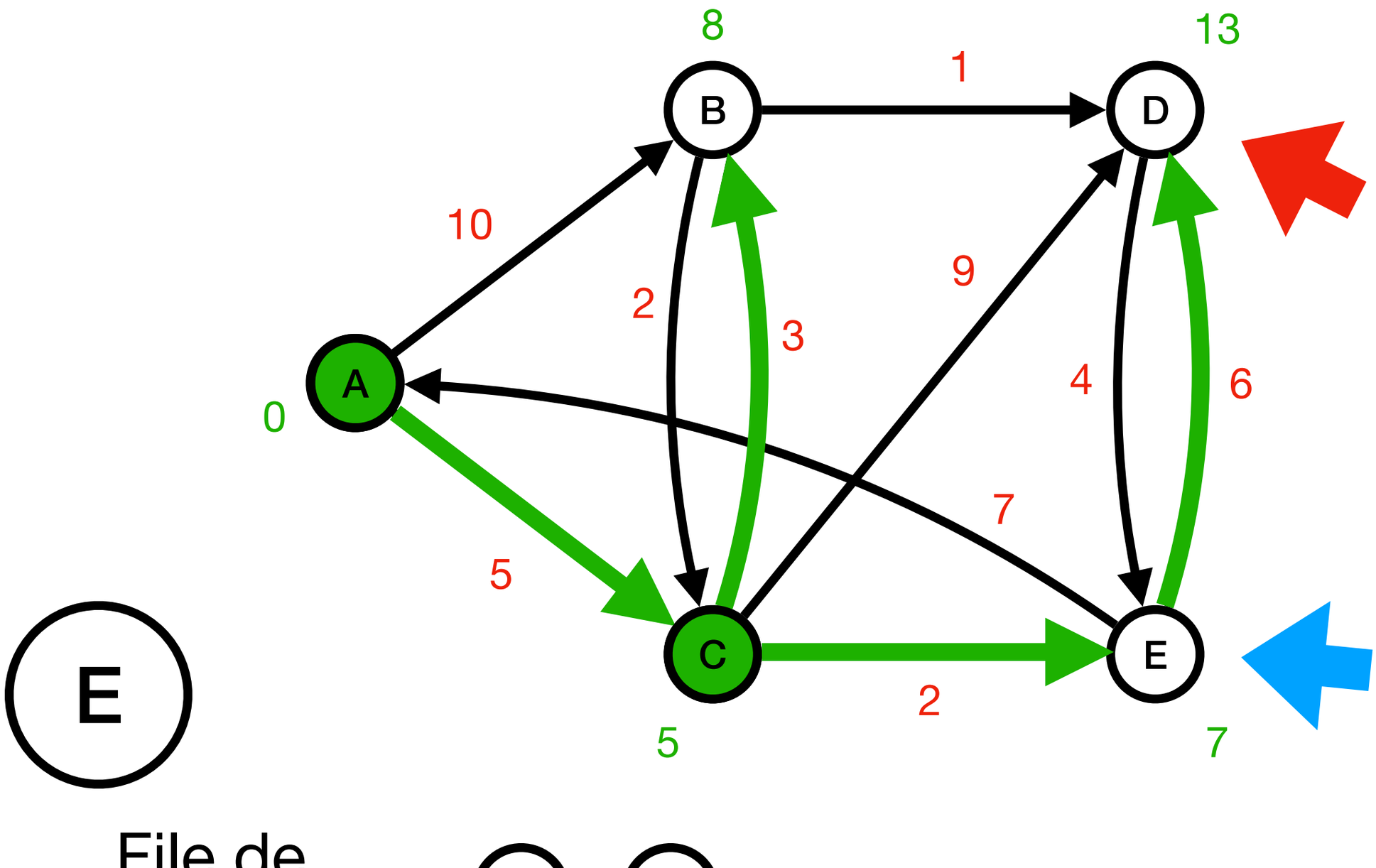

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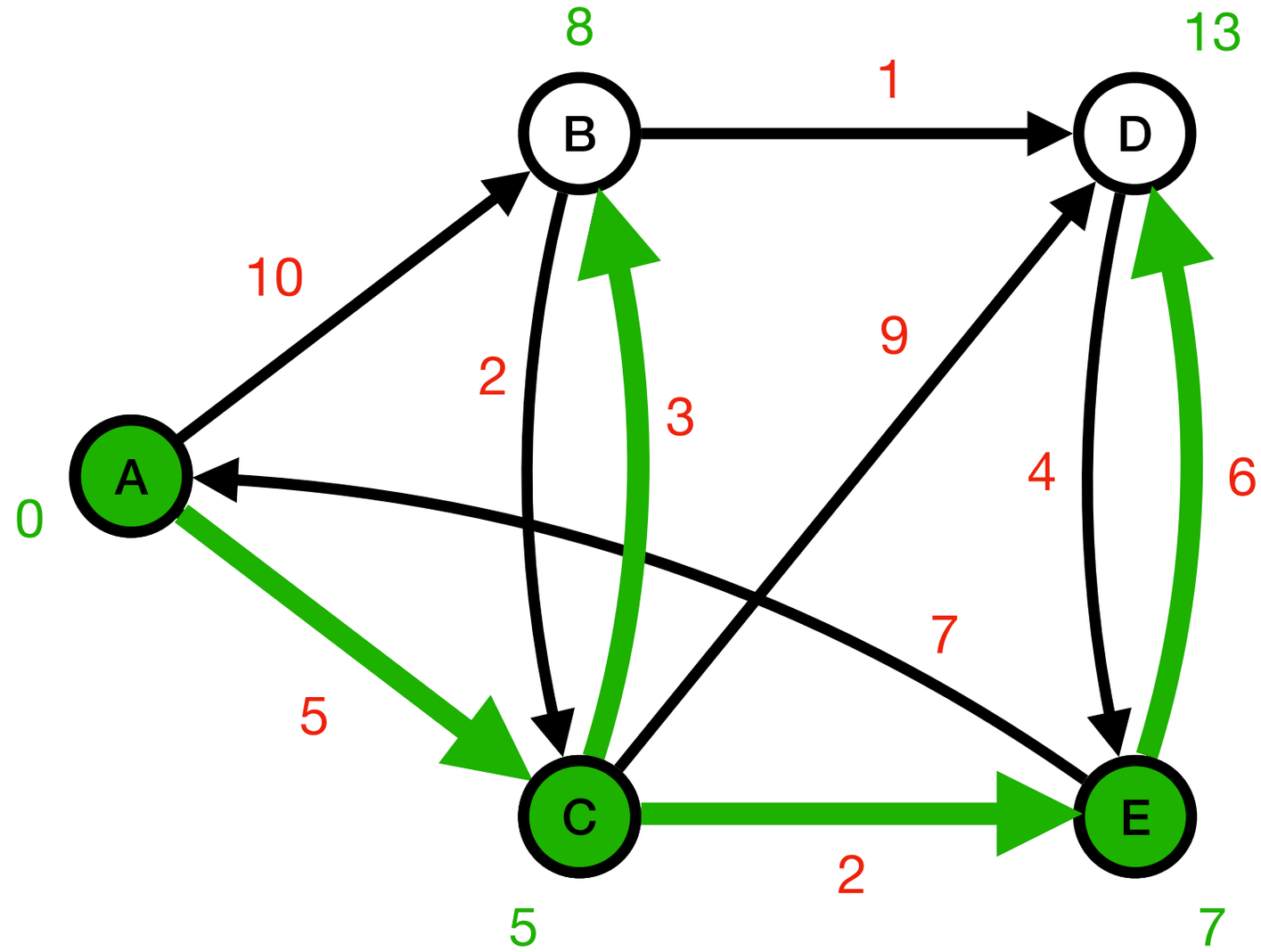
File de priorité → B D

8 13

Algorithme de Dijkstra



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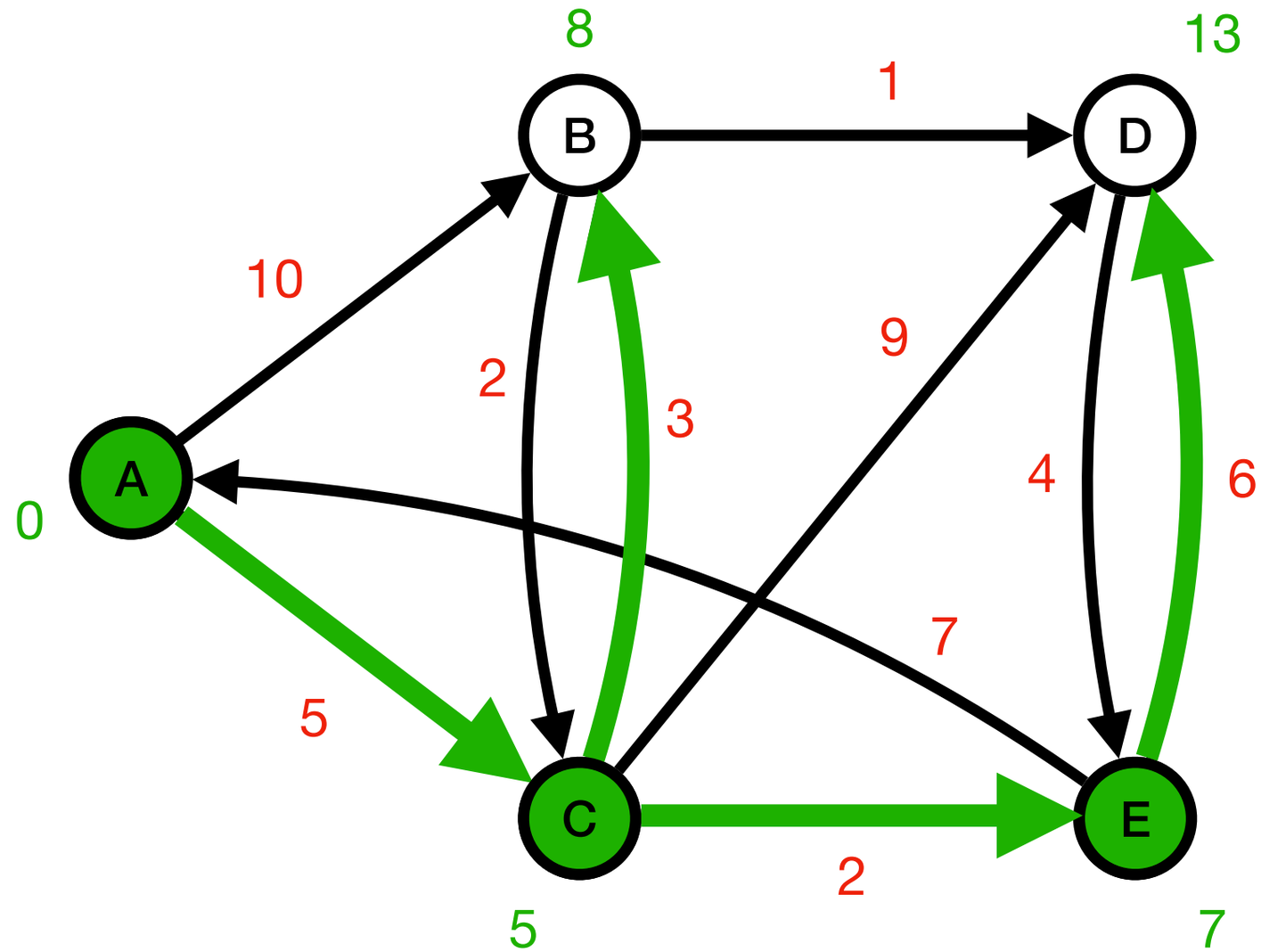
8 13

Algorithme de Dijkstra



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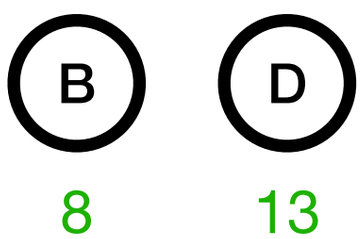

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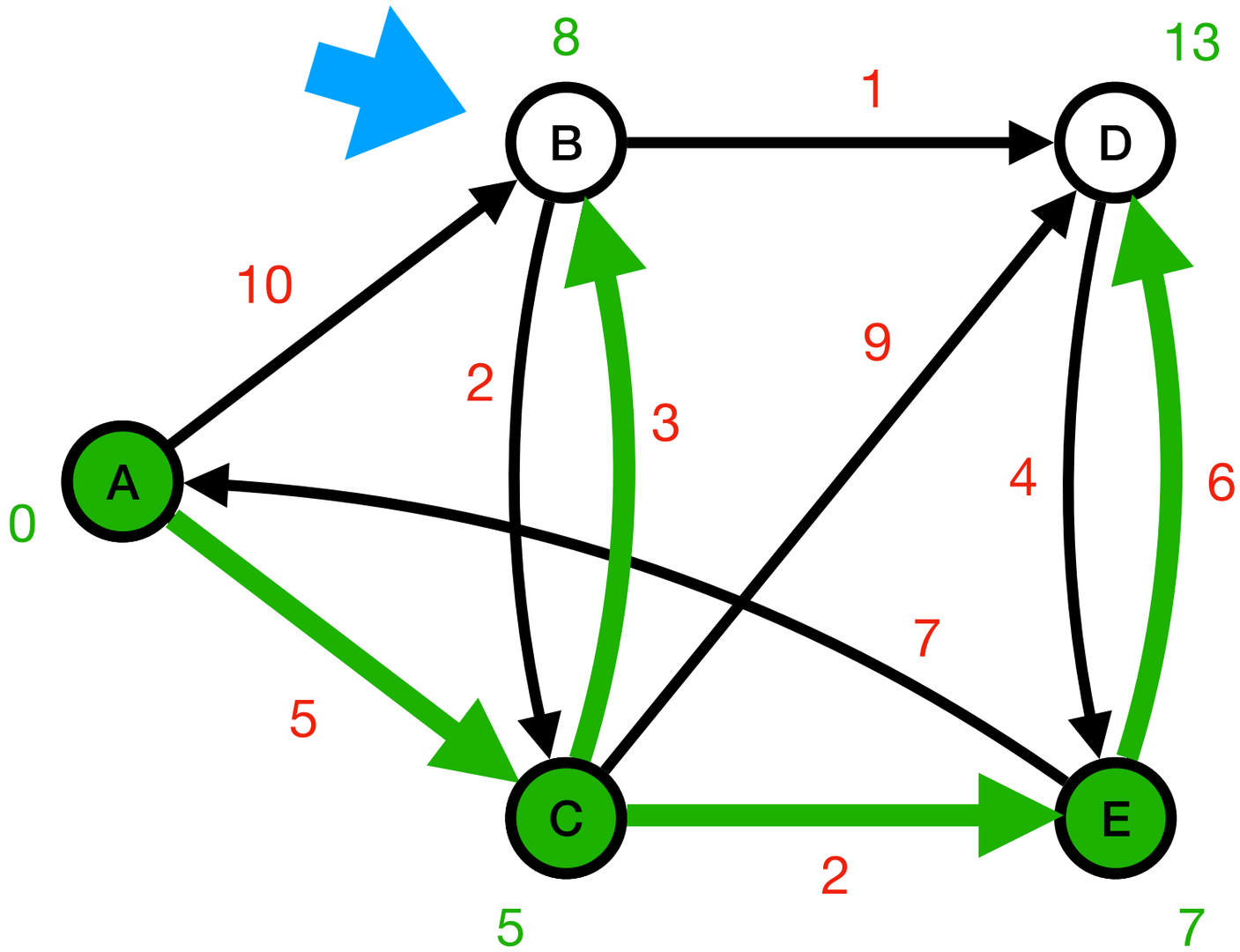

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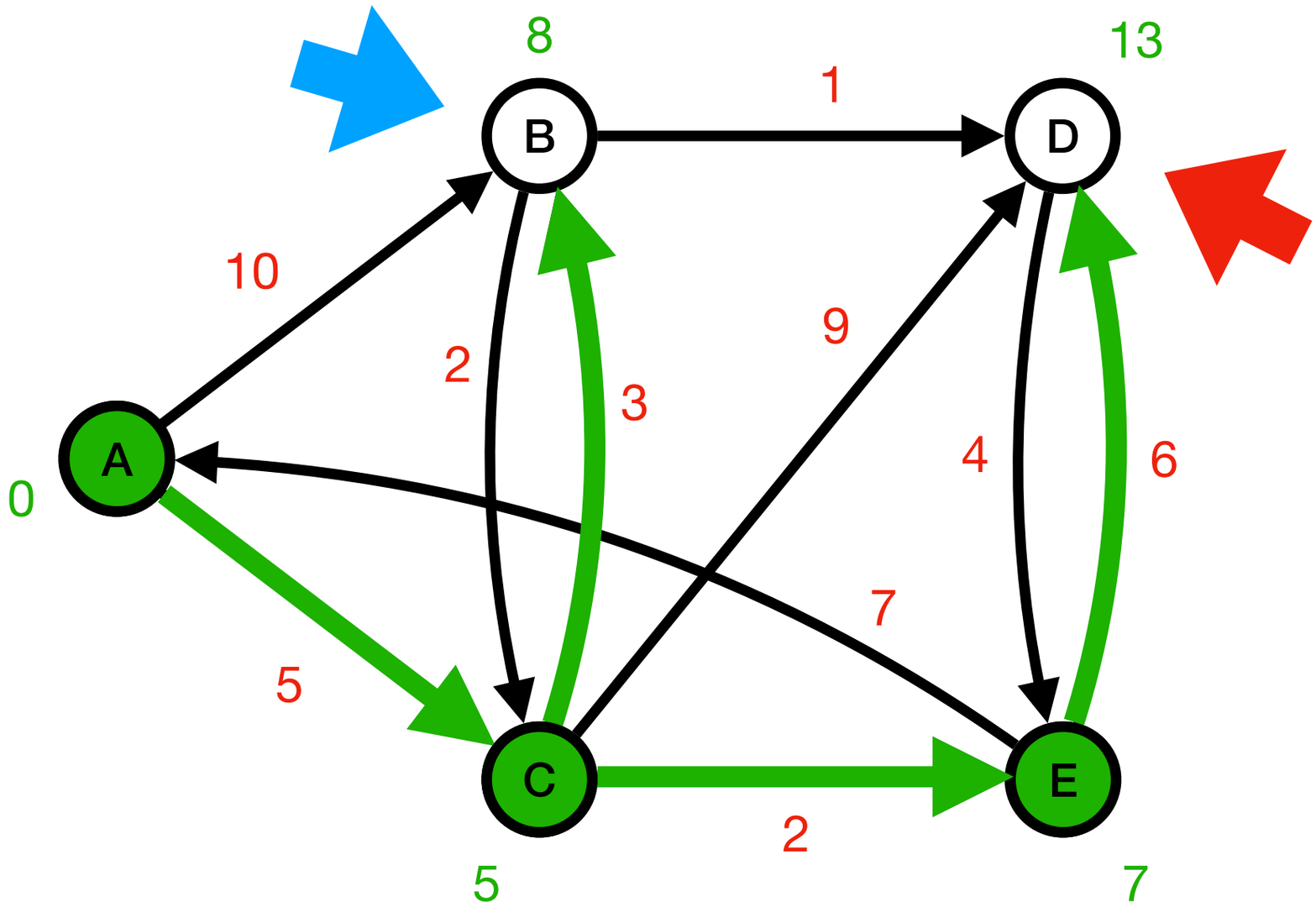
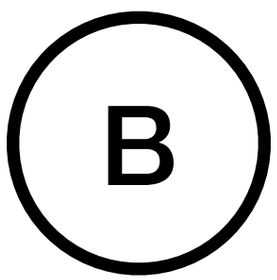
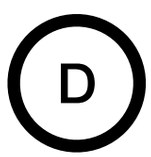




File de priorité → 

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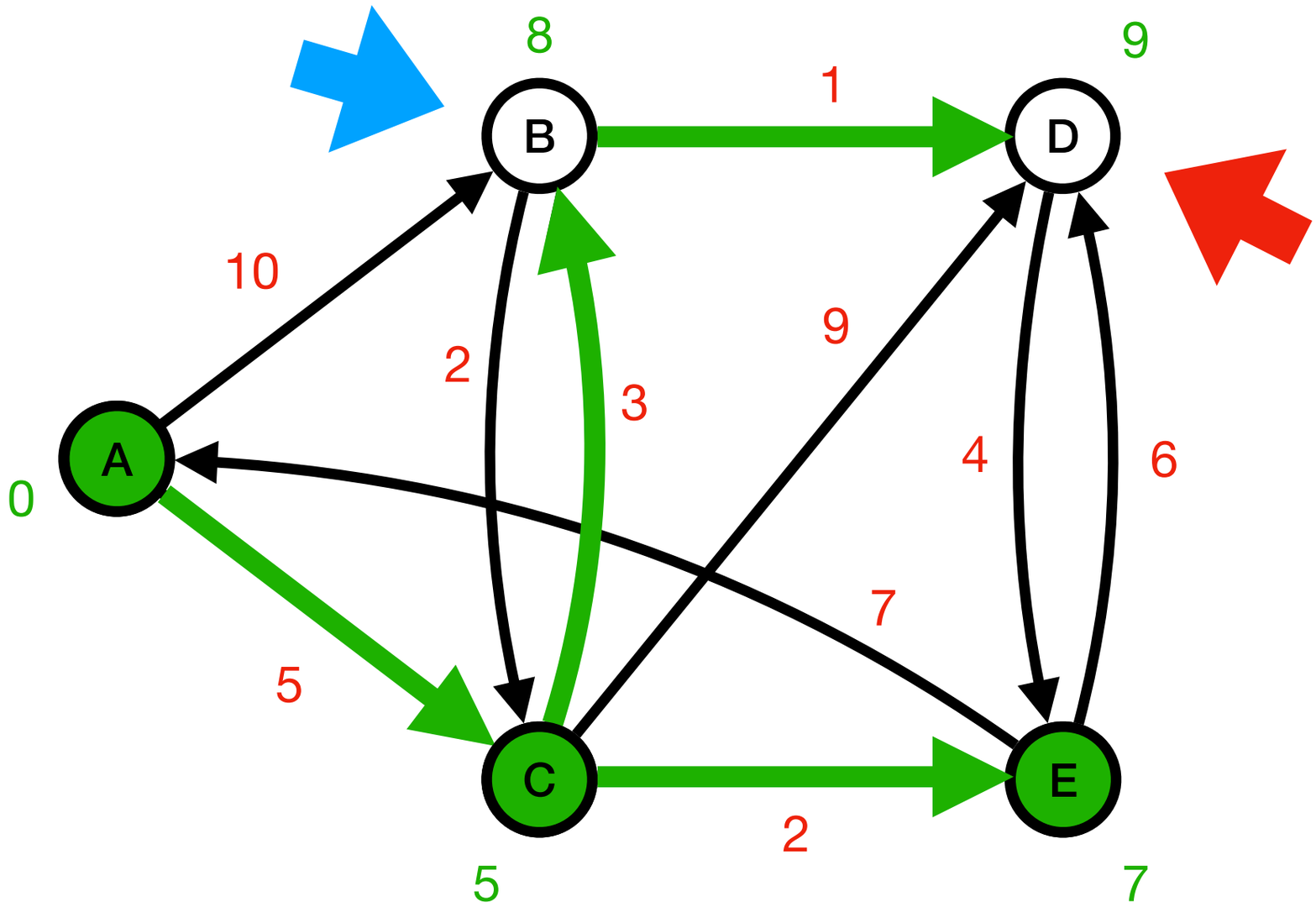
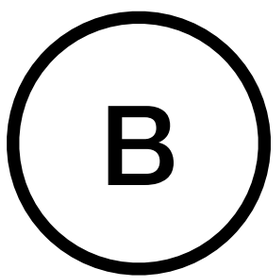
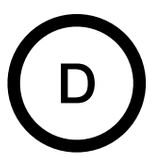




File de priorité → 

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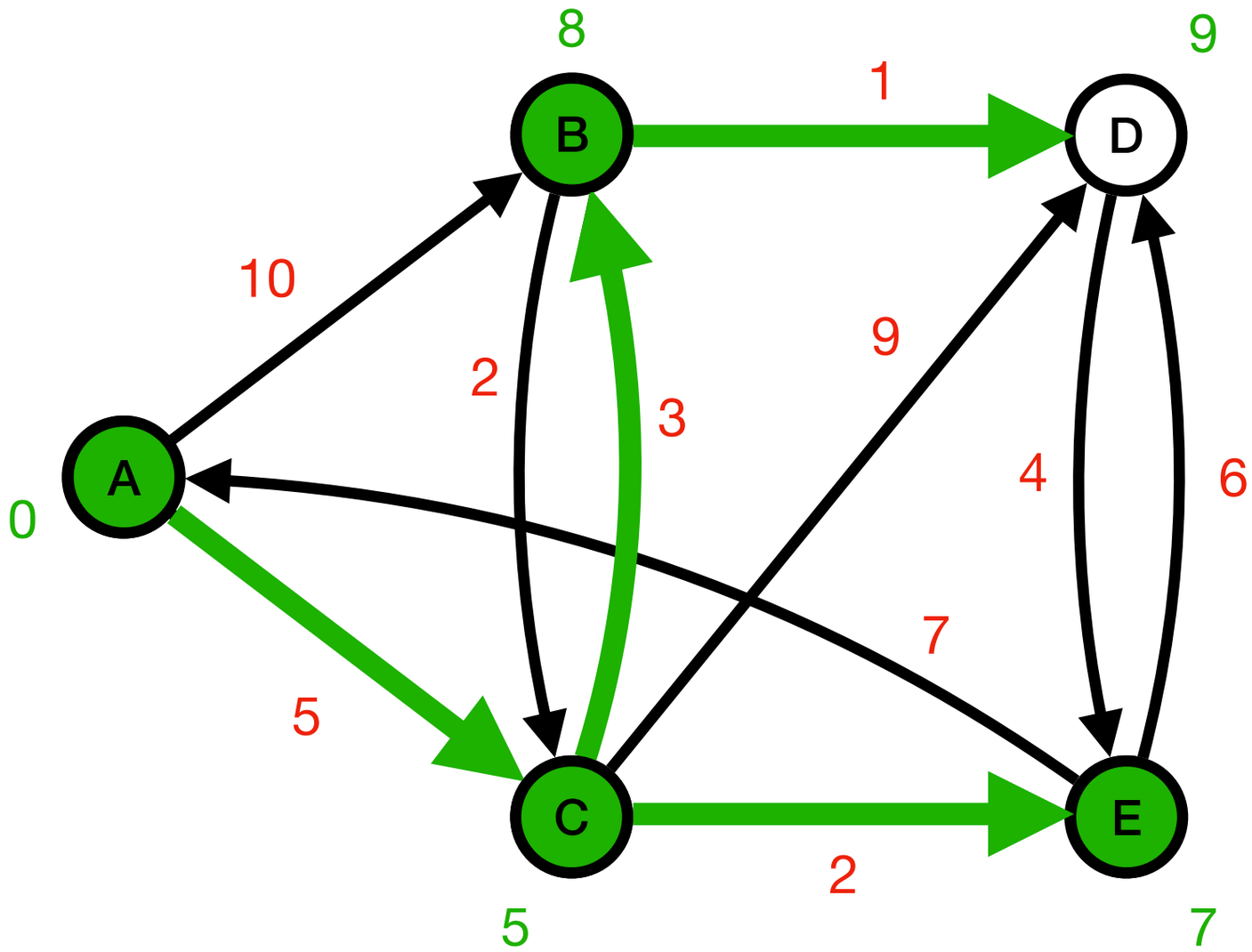
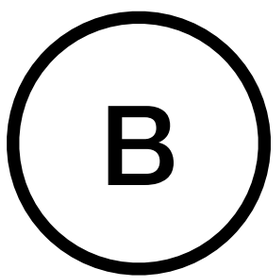
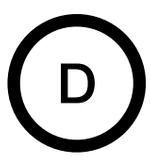
Algorithme de Dijkstra





File de priorité → 

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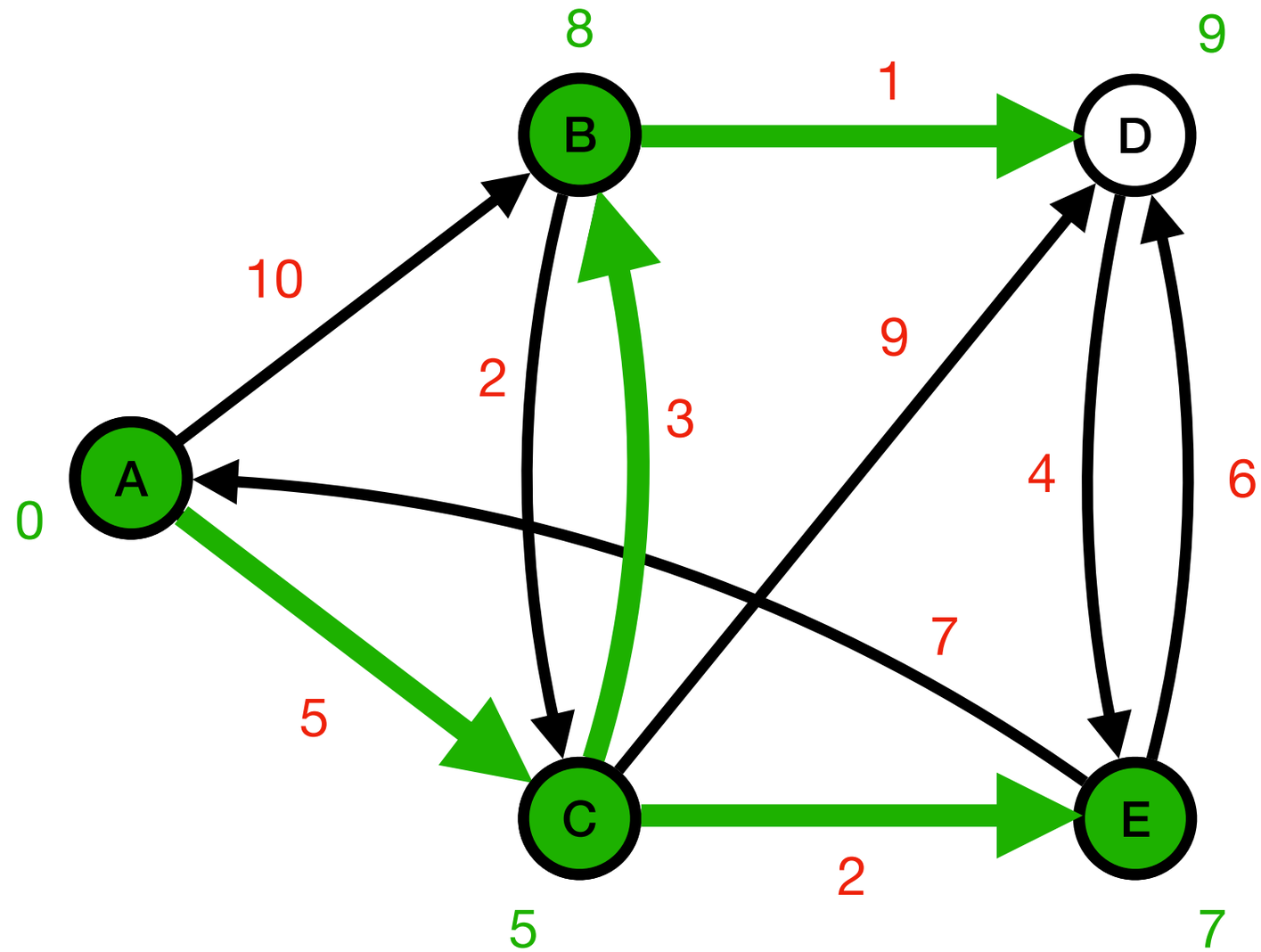

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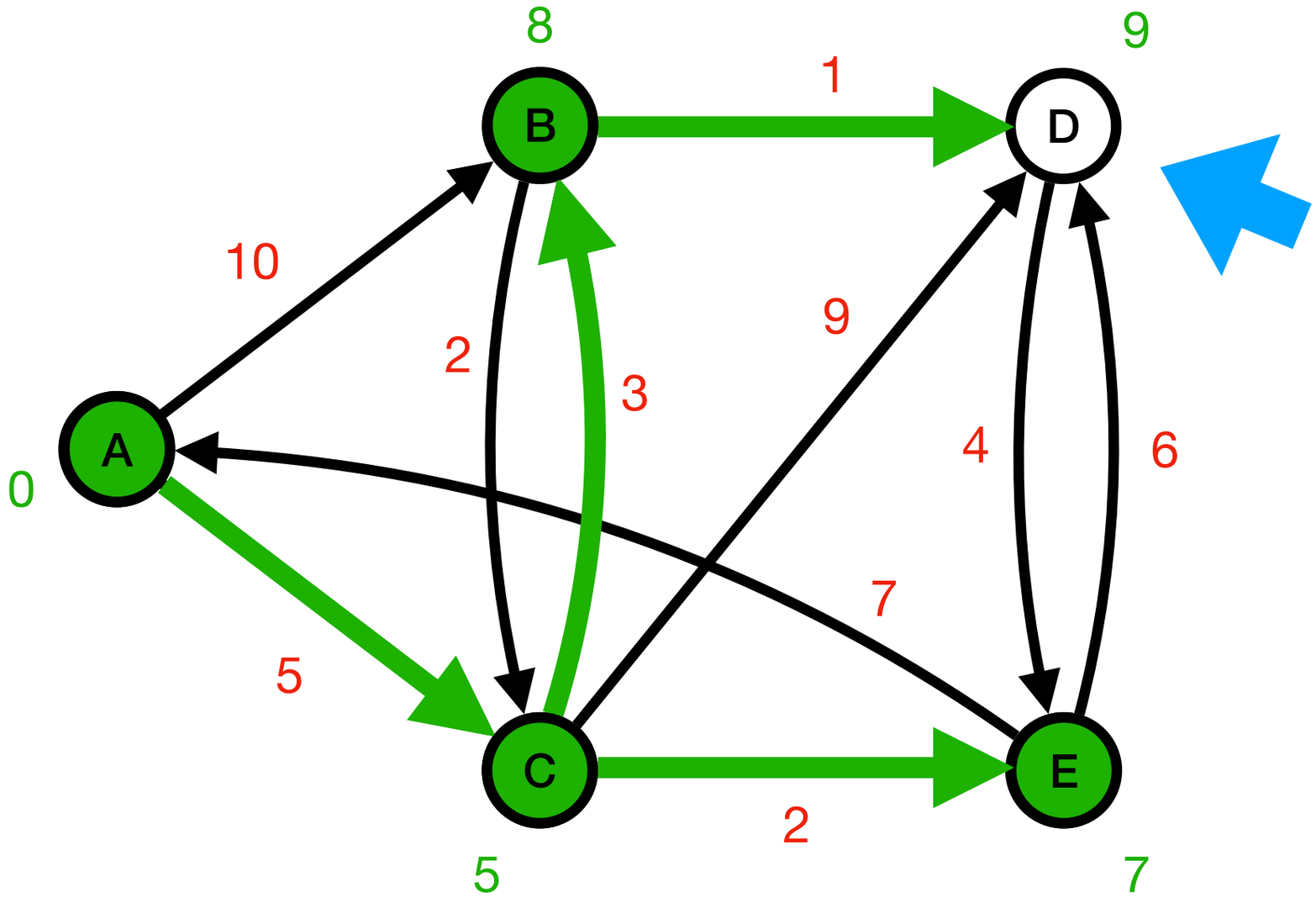

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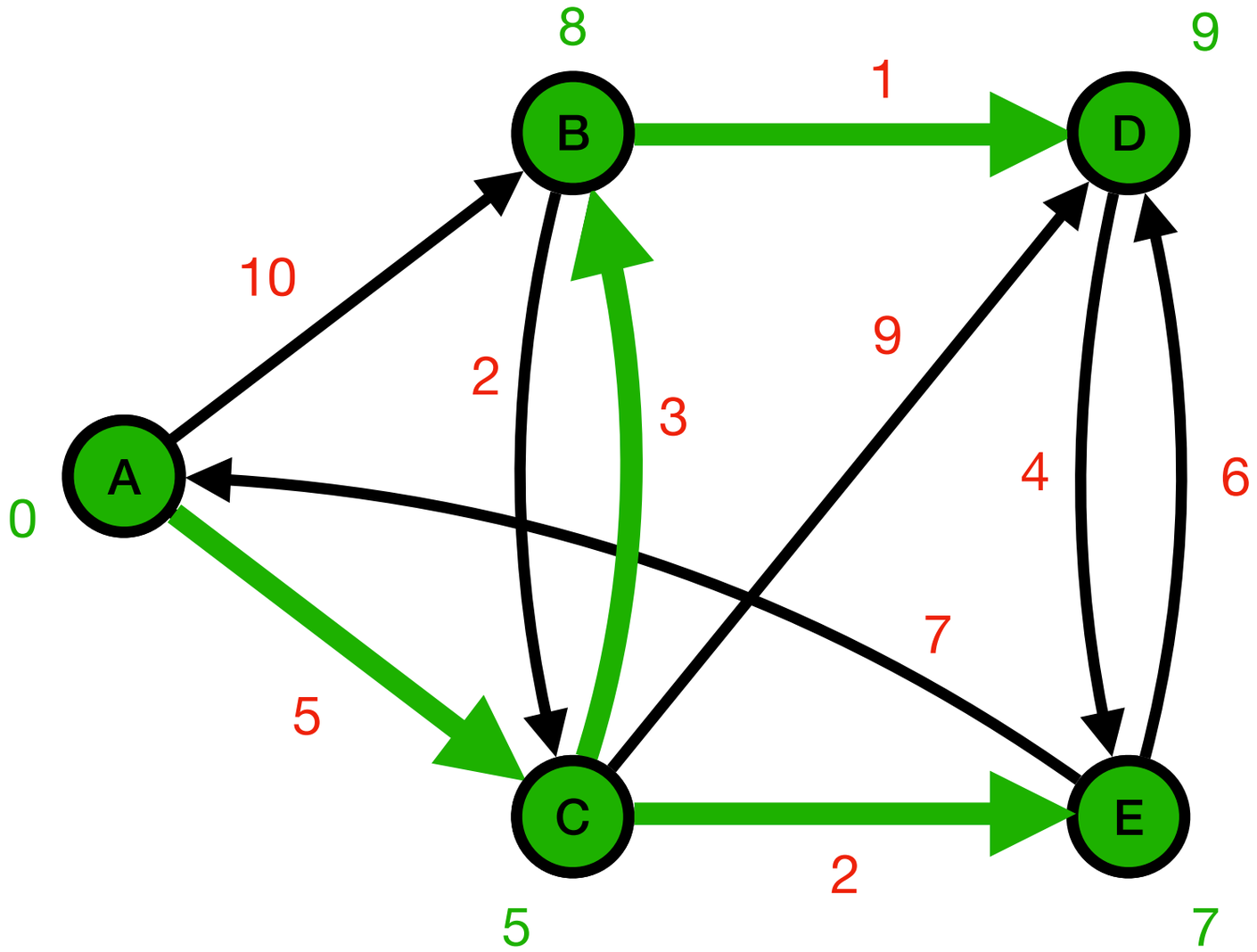

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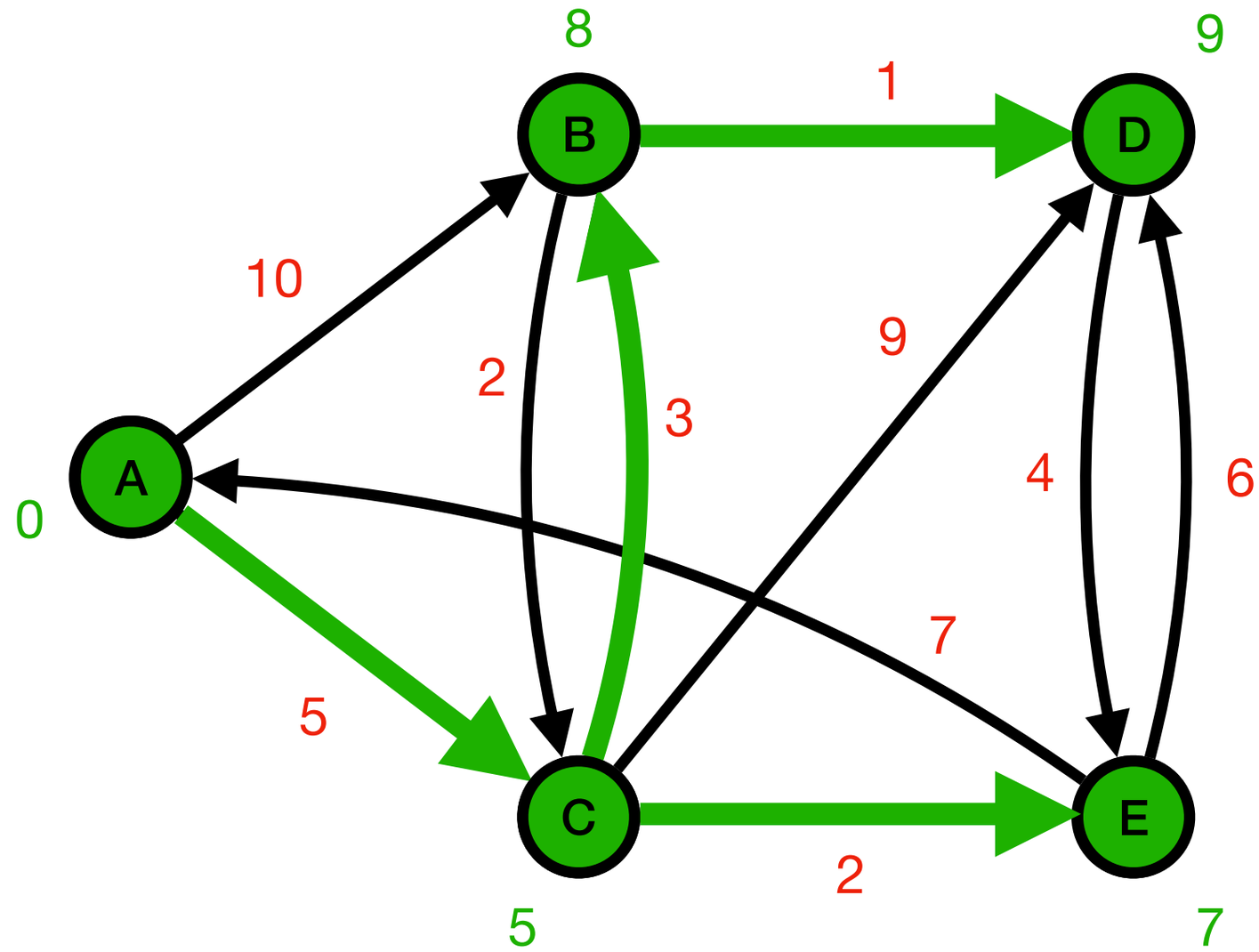

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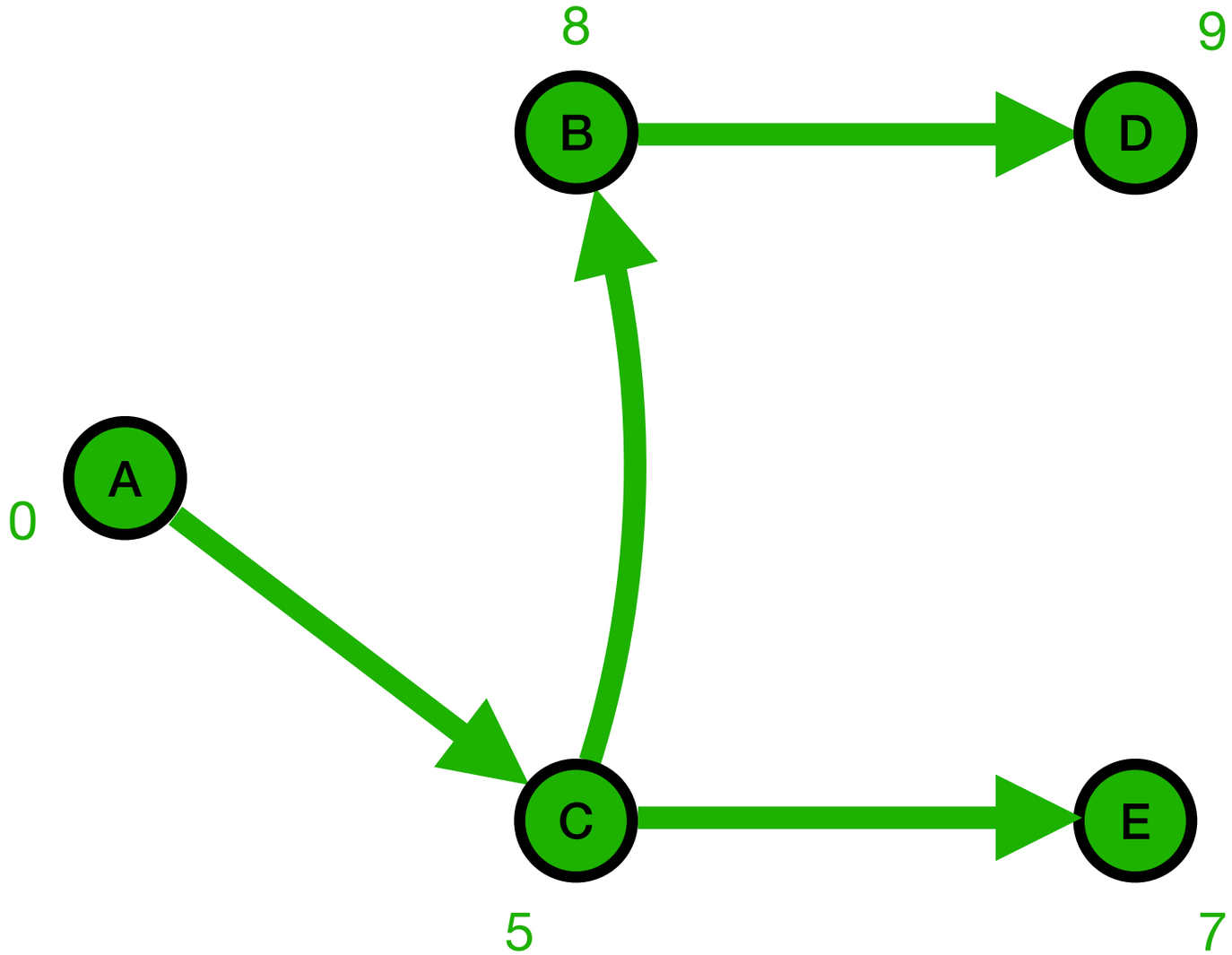
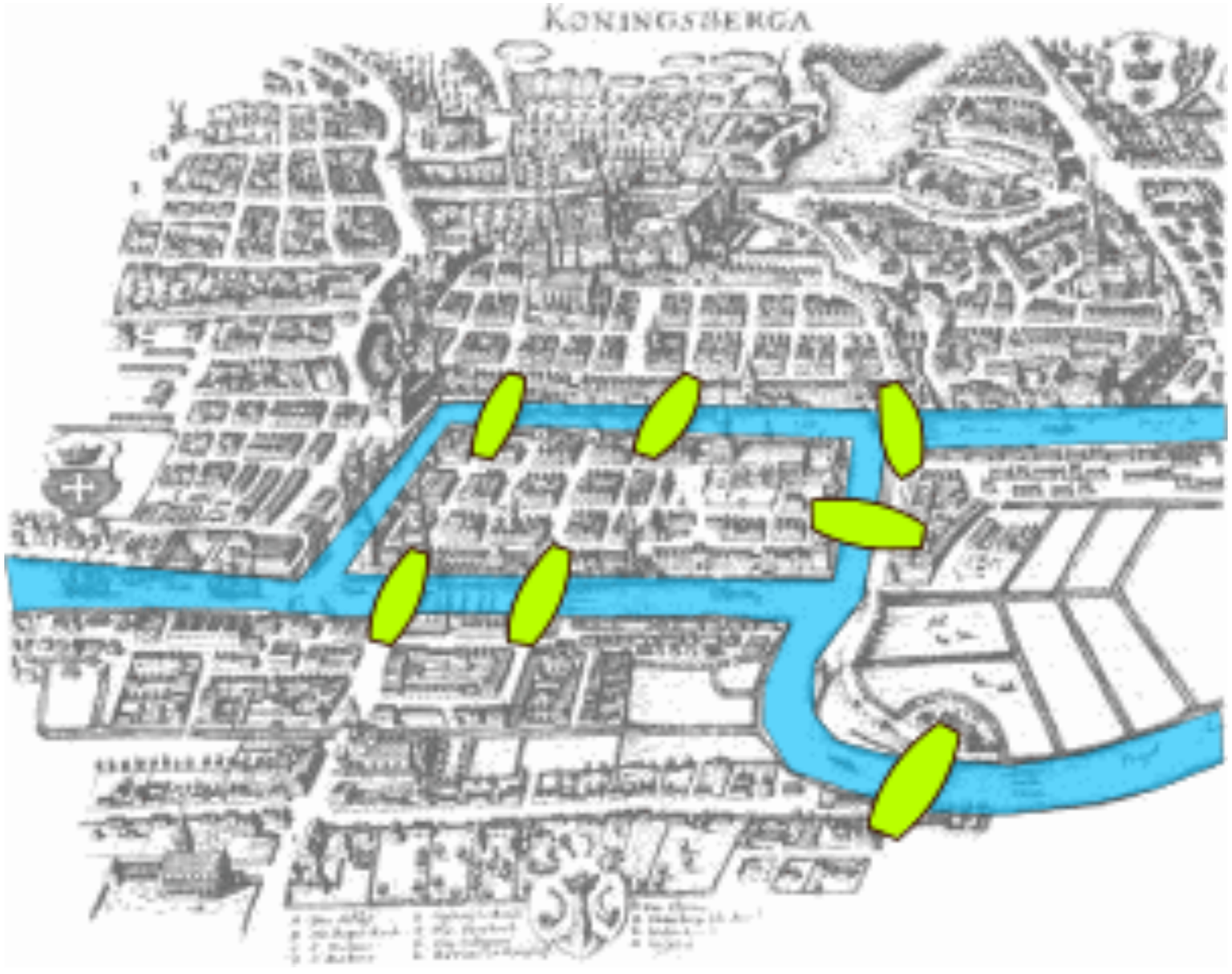


Graphes eulériens

Problème des sept ponts de Königsberg



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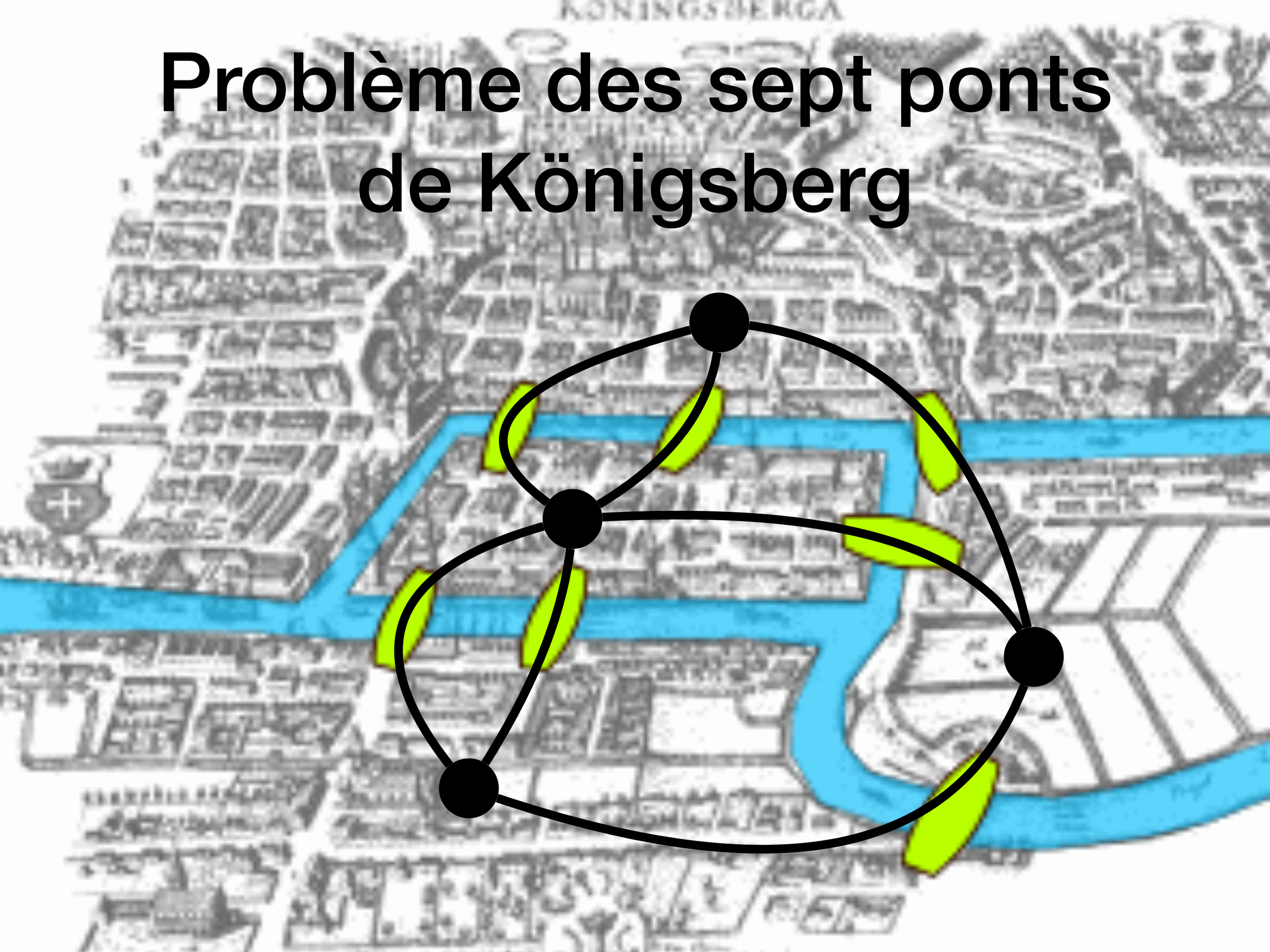

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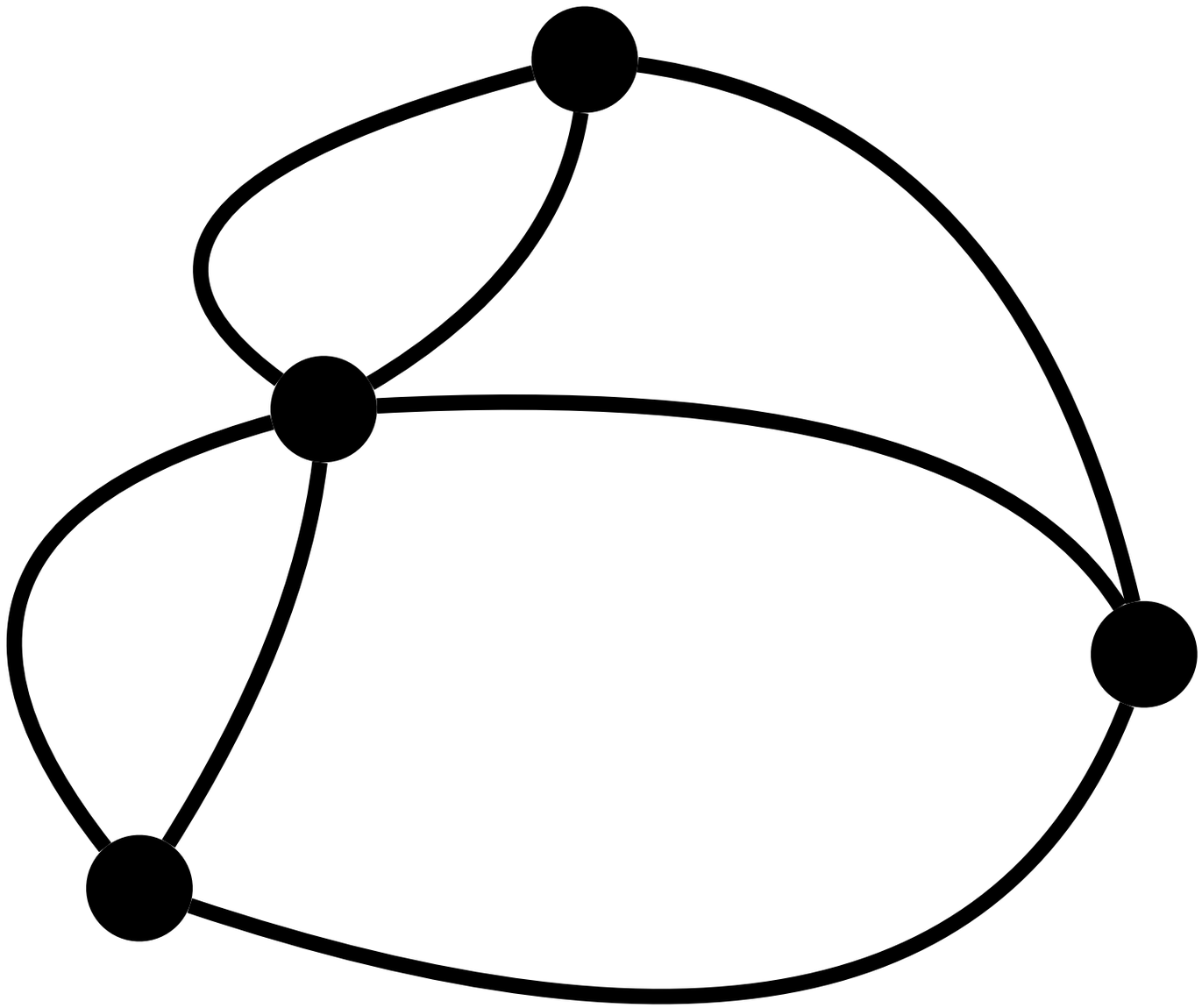

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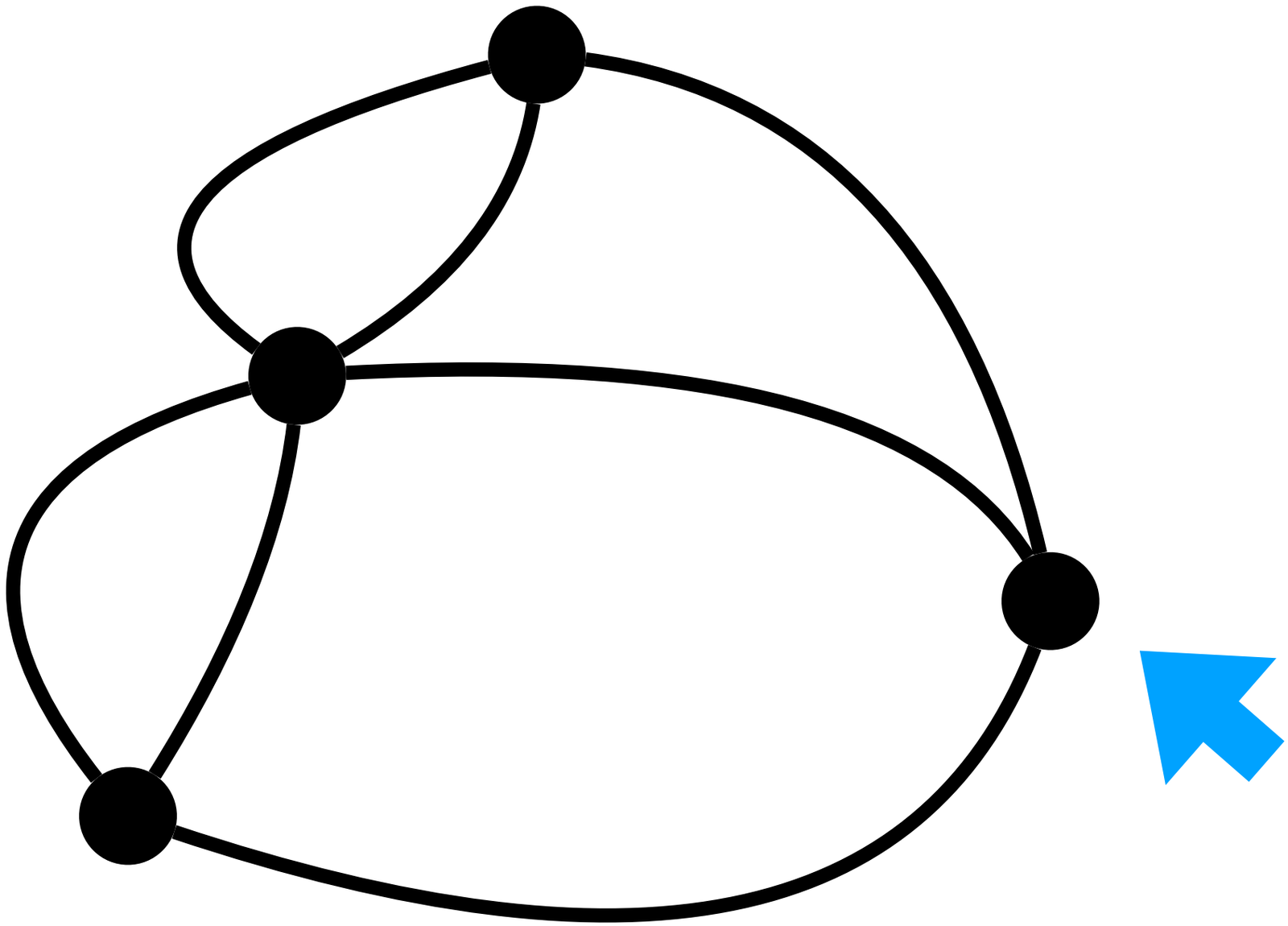

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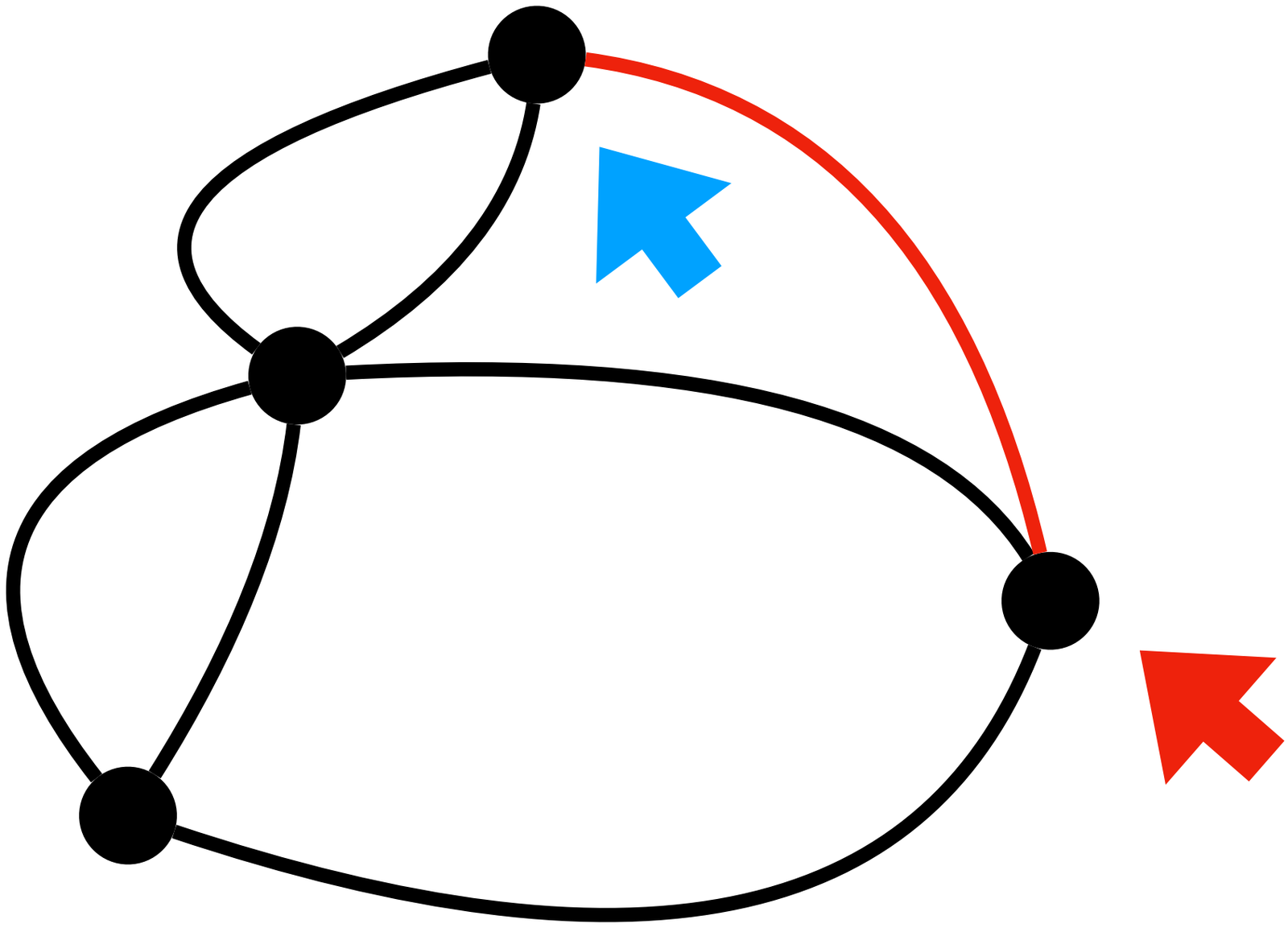

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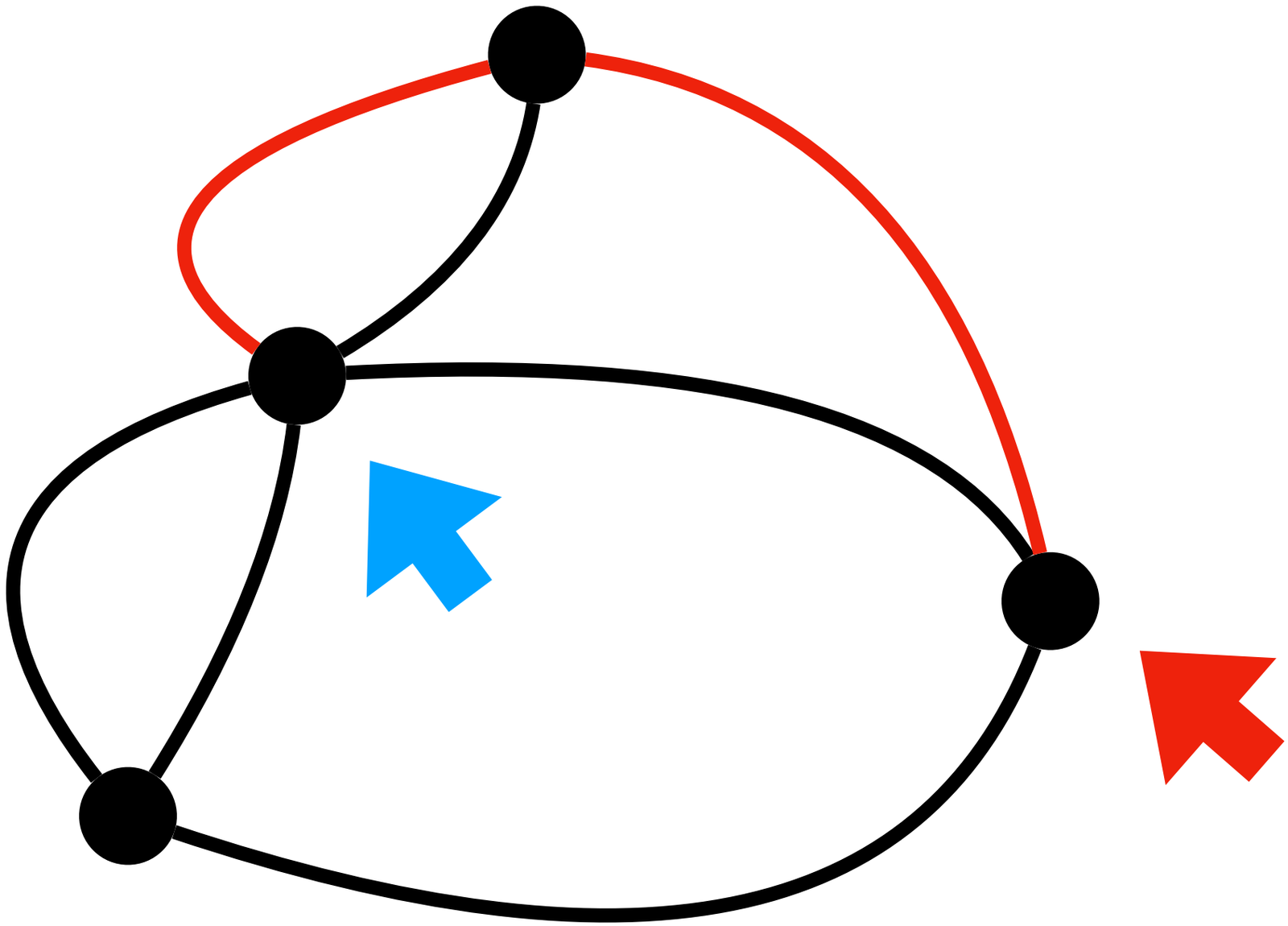

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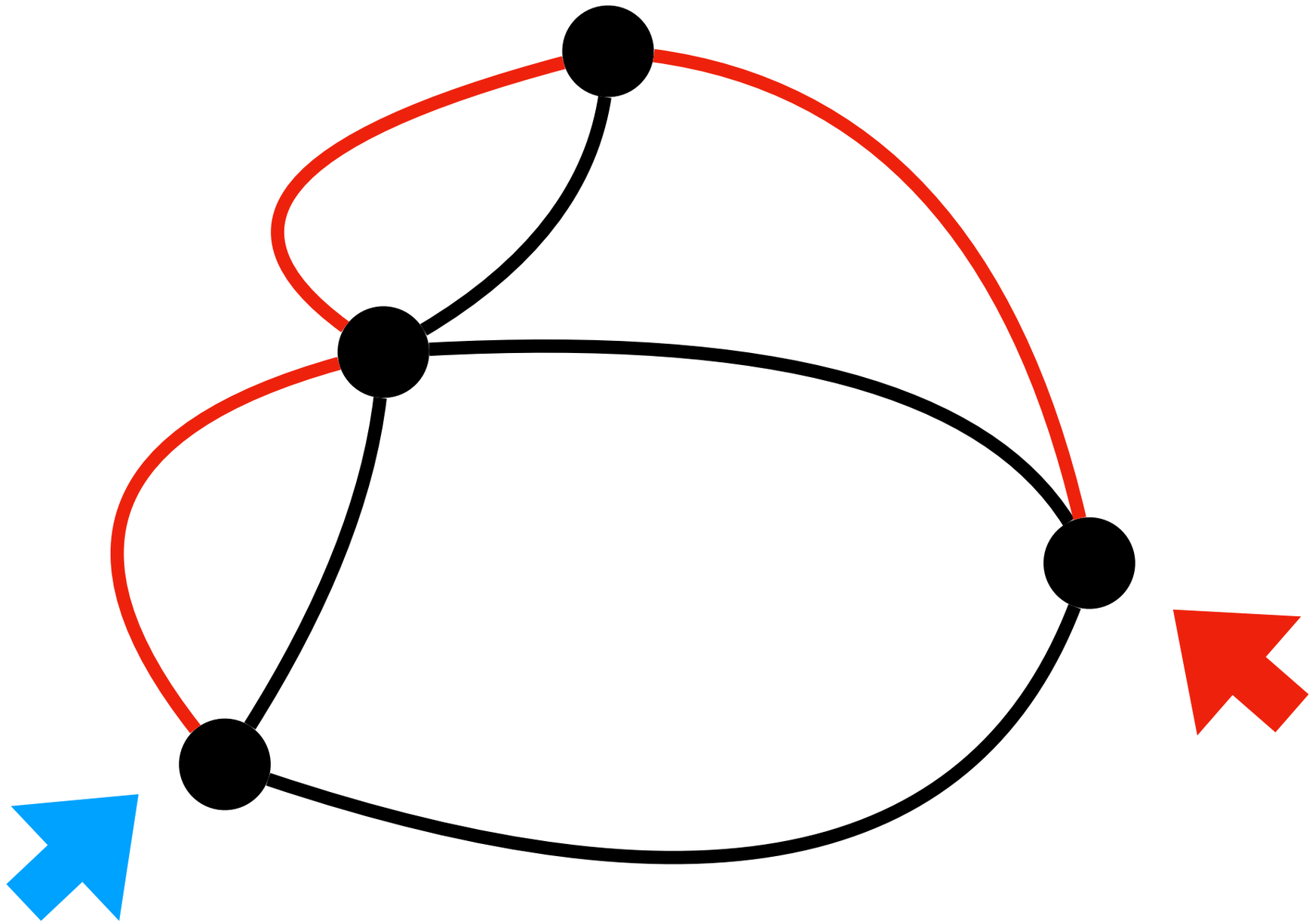

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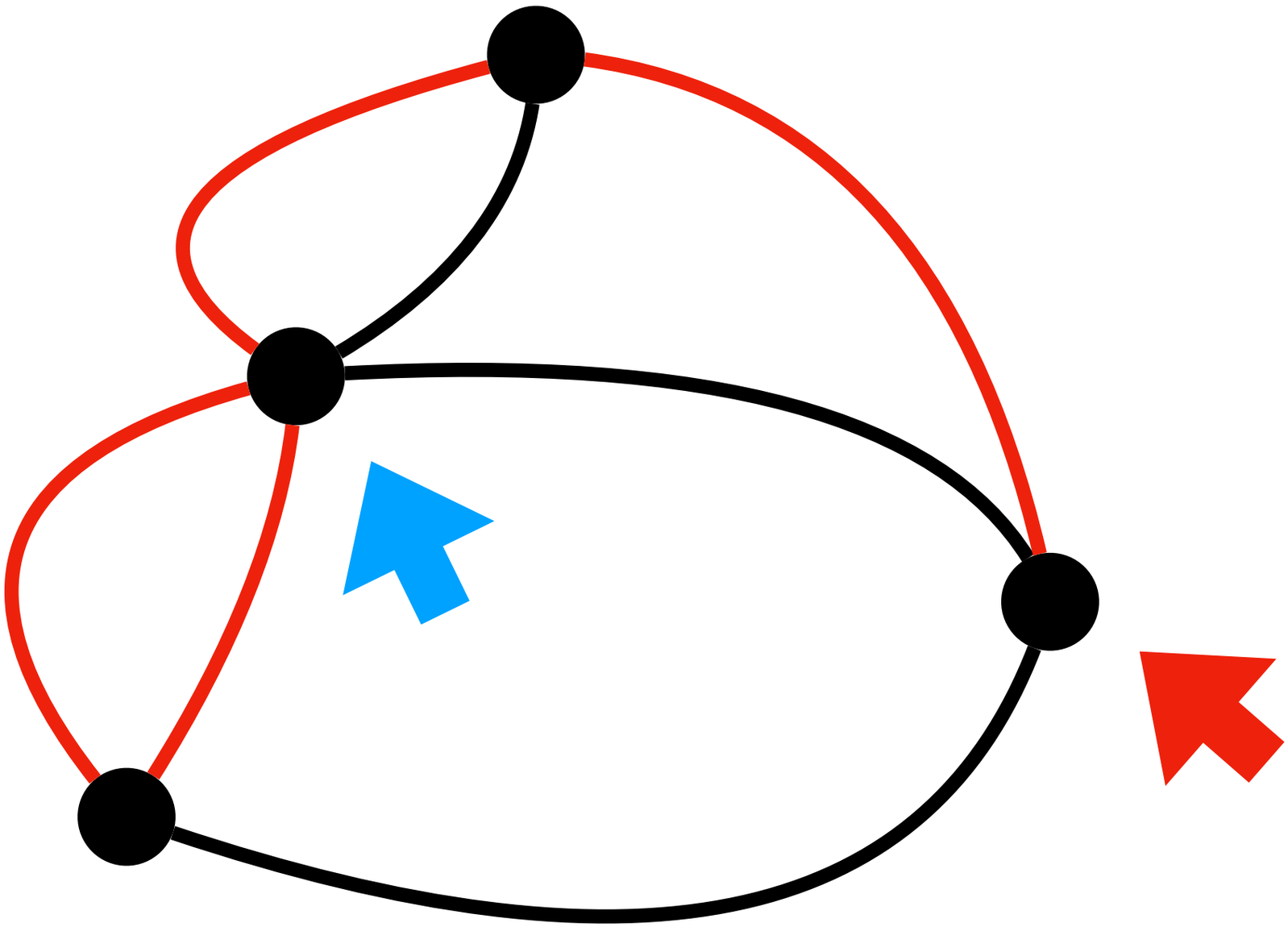

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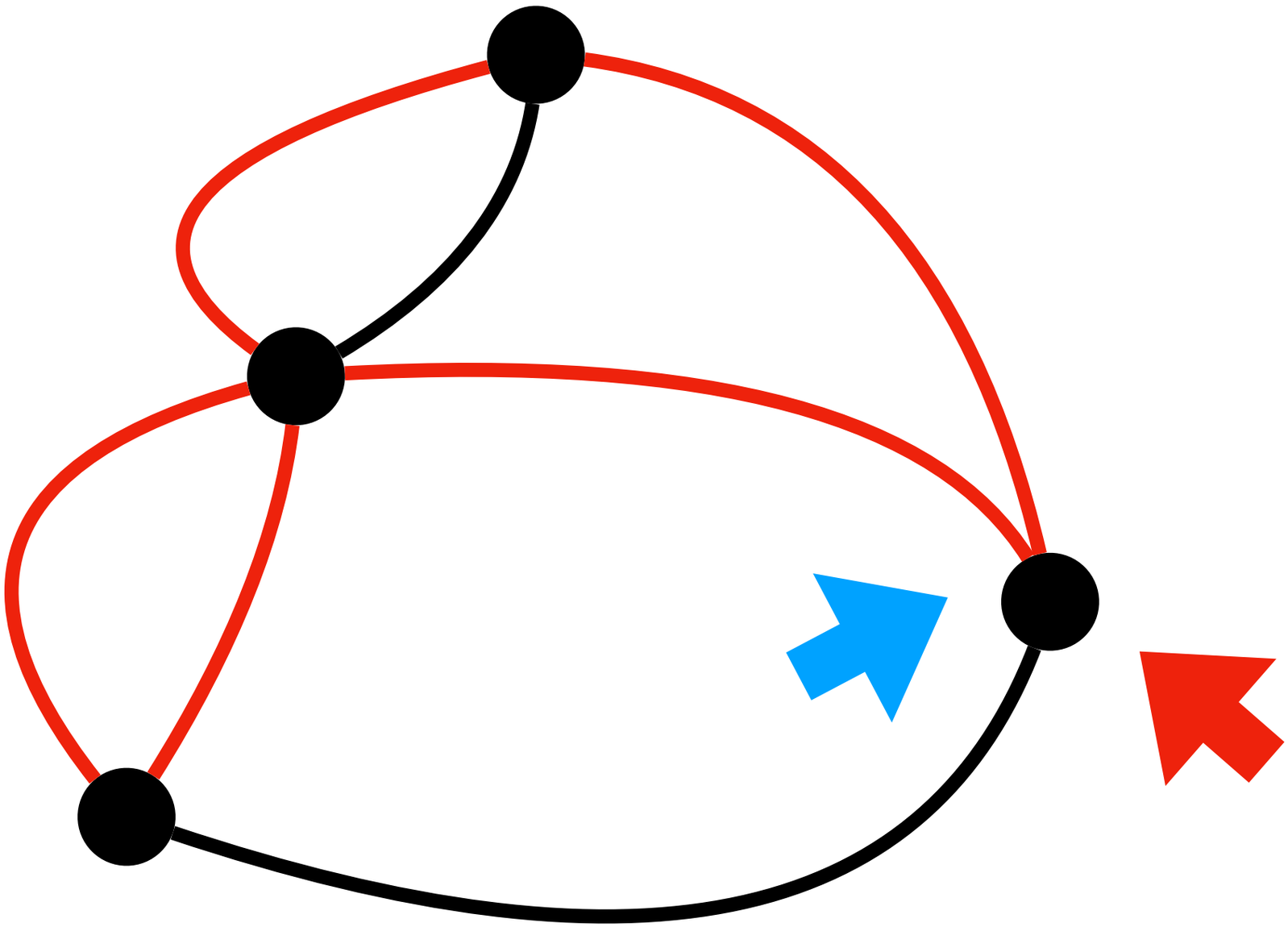

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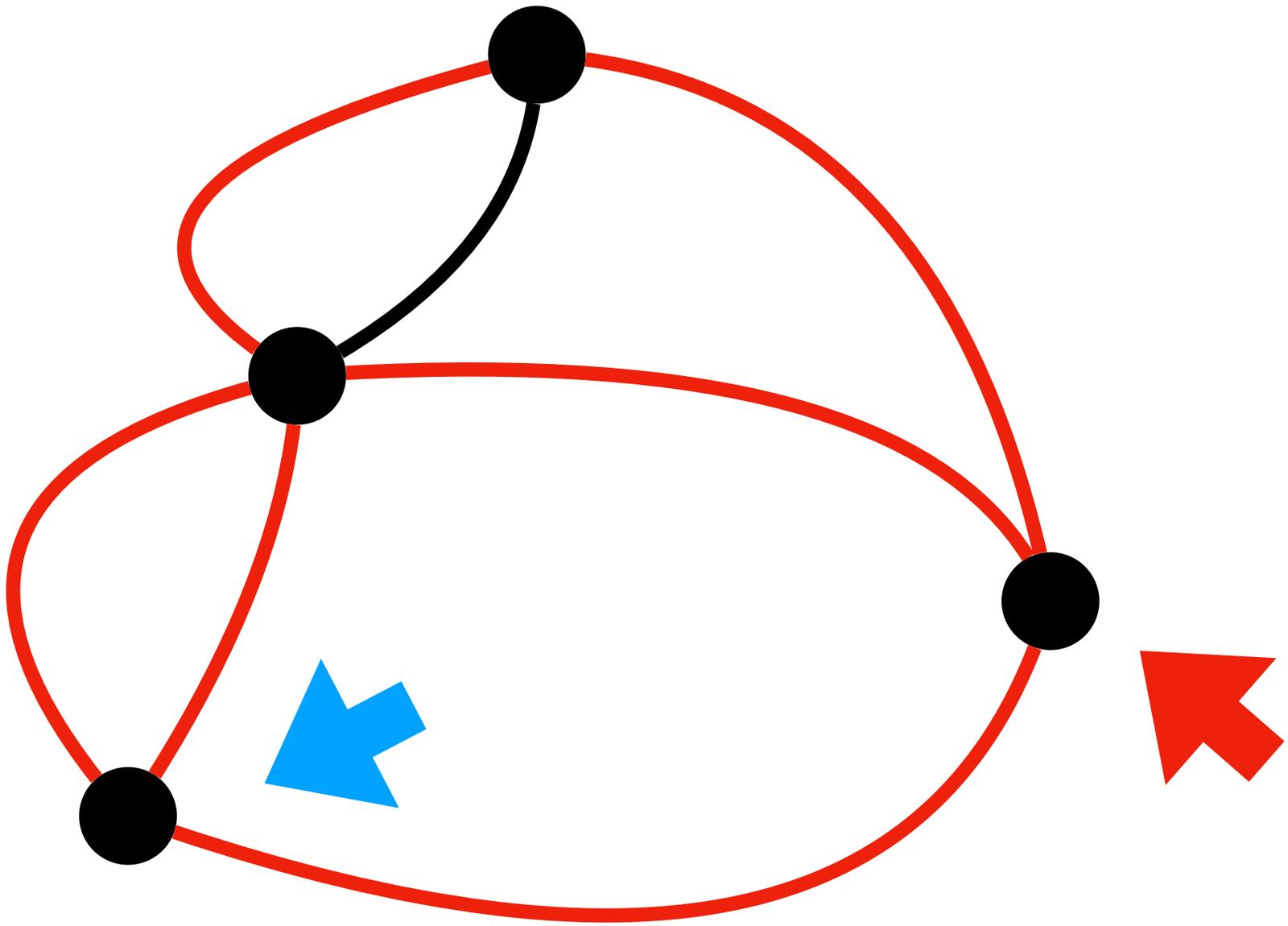

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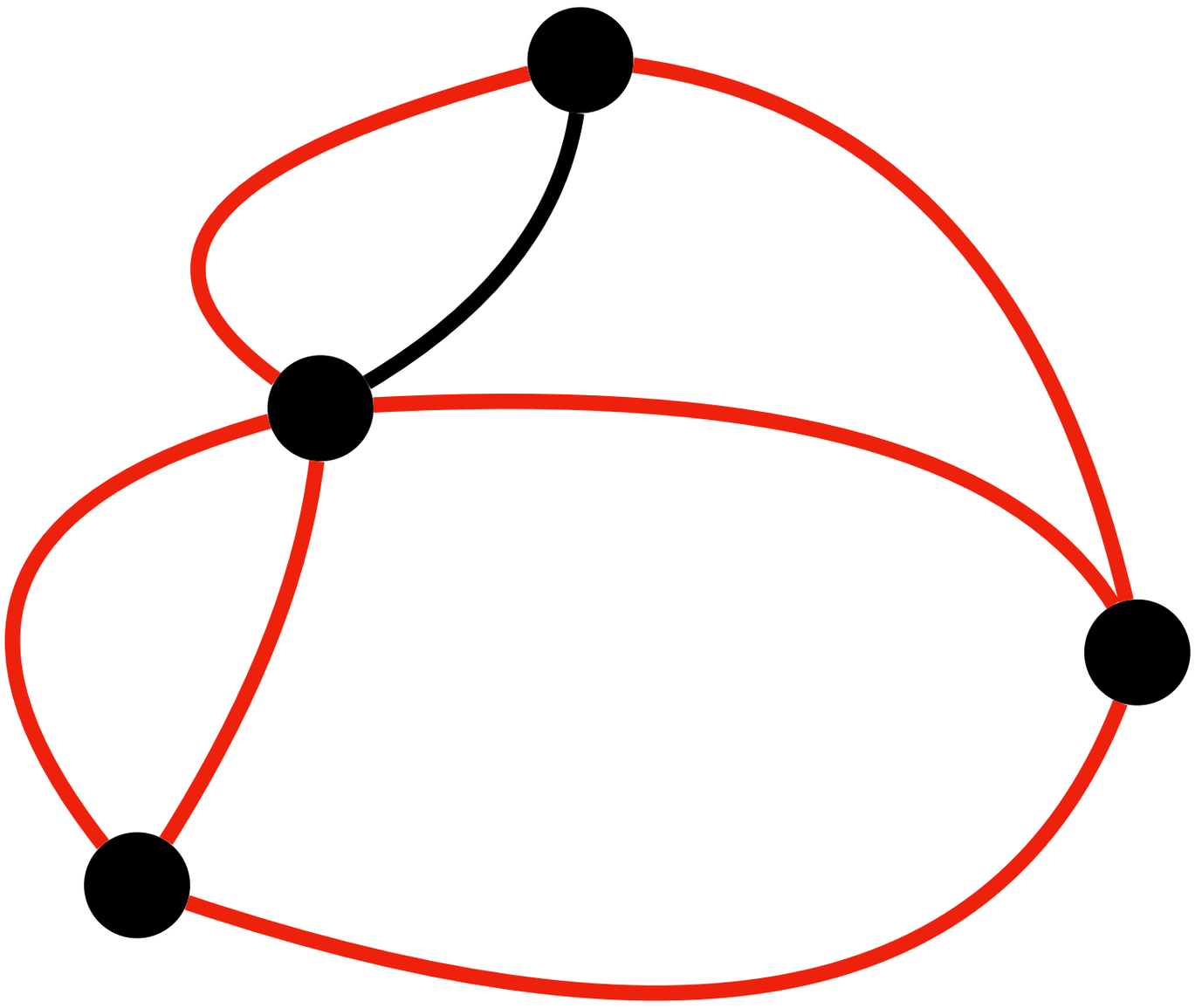
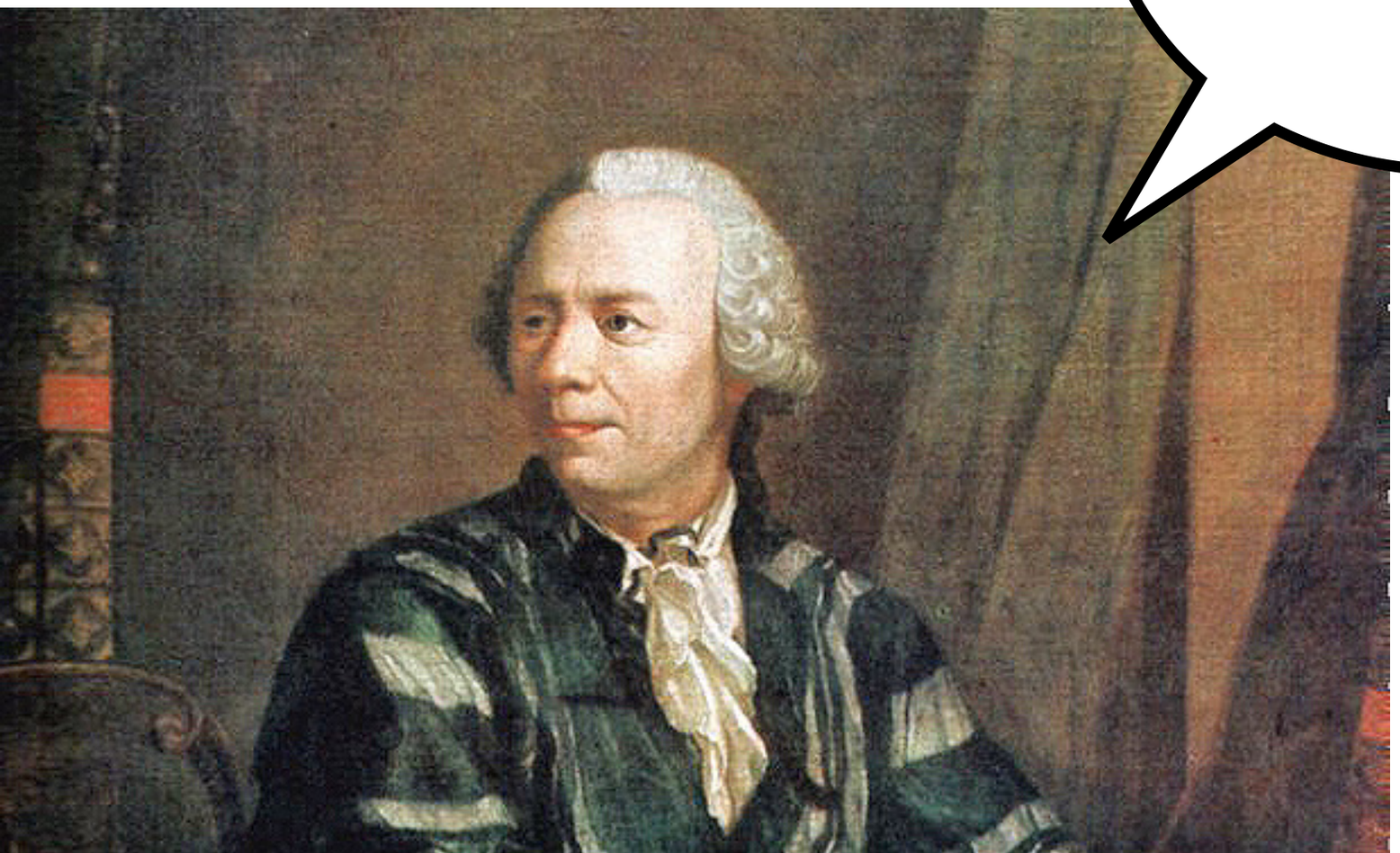


Problème des sept ponts de Königsberg

Il n'ya pas de solution !



← Leonhard Euler

Théorème d’Euler (1736)

Un graphe non orienté connexe admet un cycle
qui traverse chaque arête une et une seule fois
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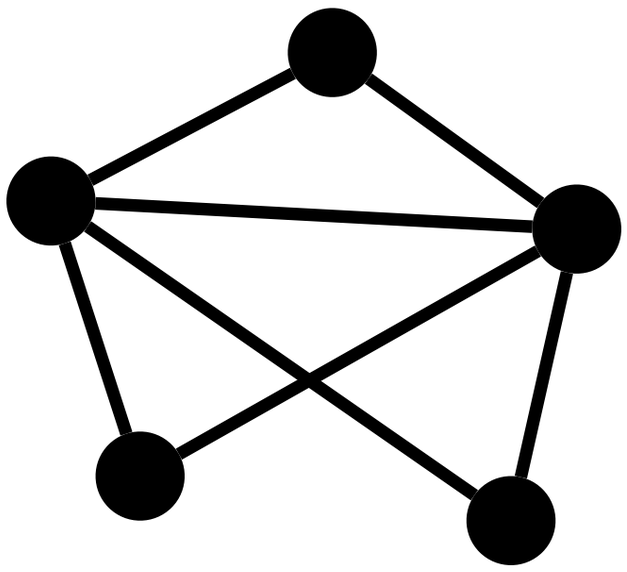

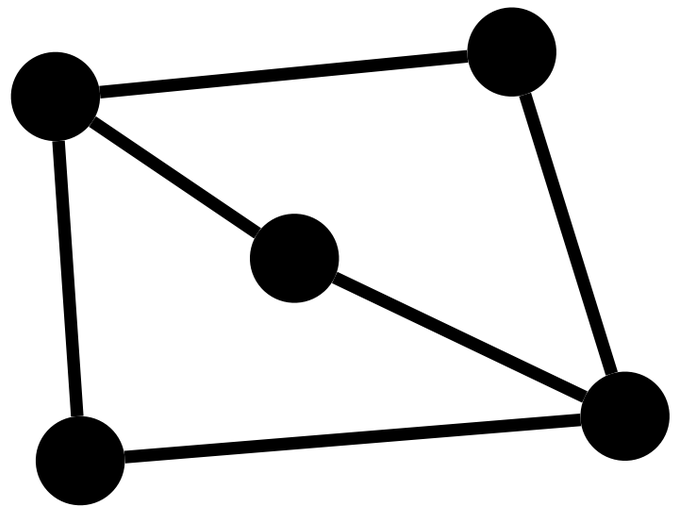

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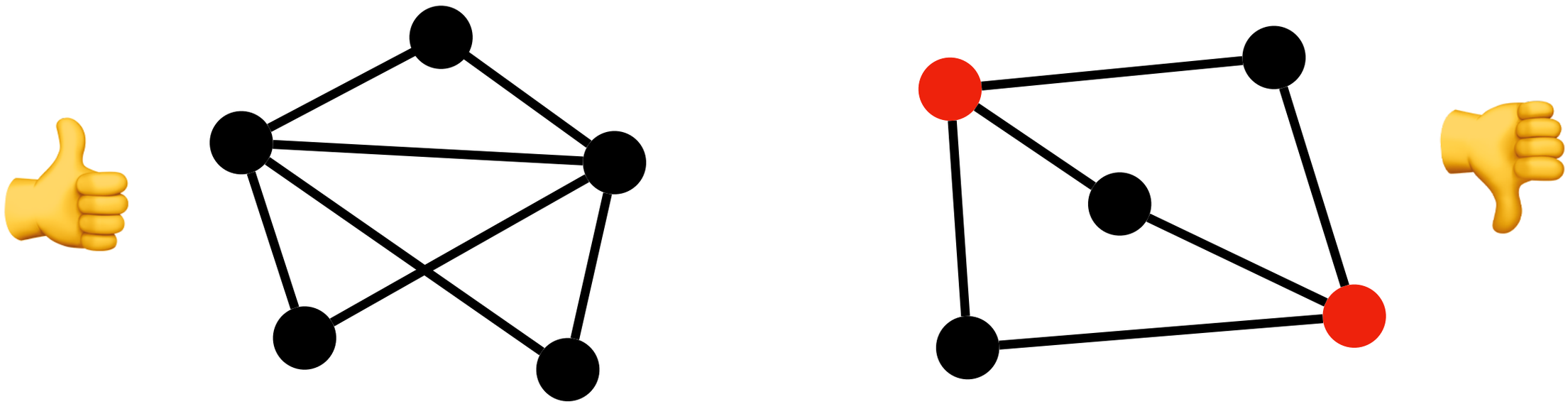
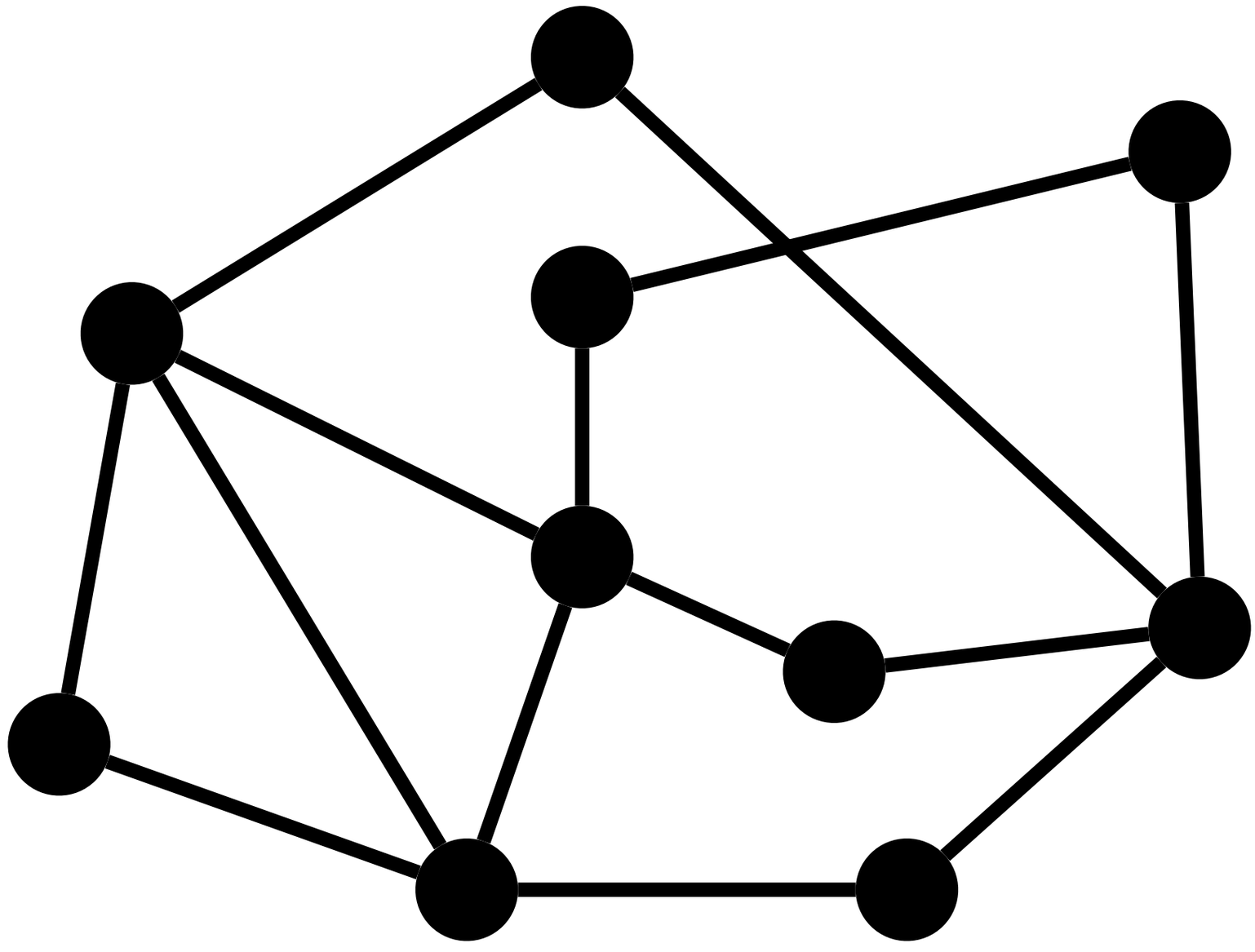
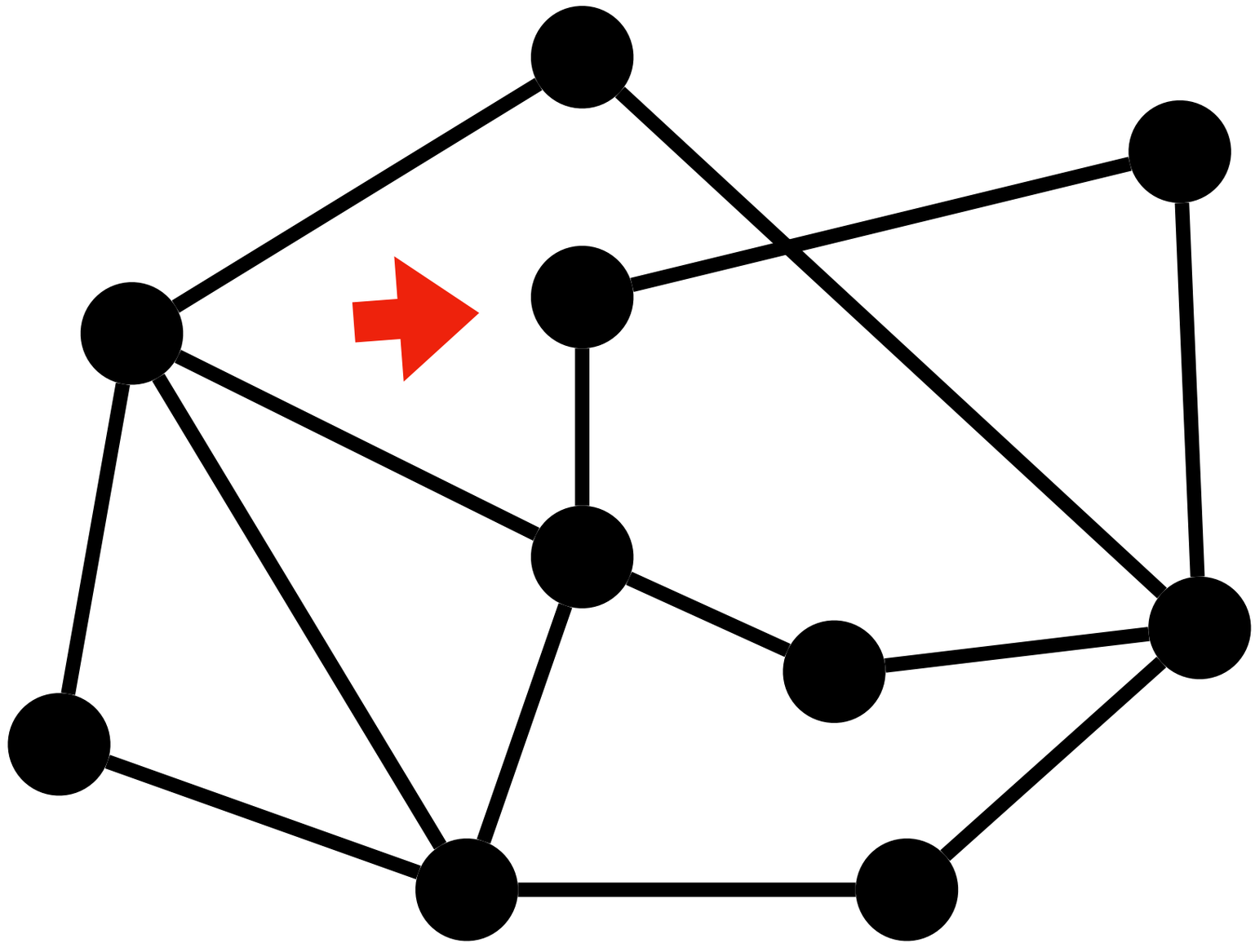


Algorithme de Hierholzer



Algorithme de Hierholzer



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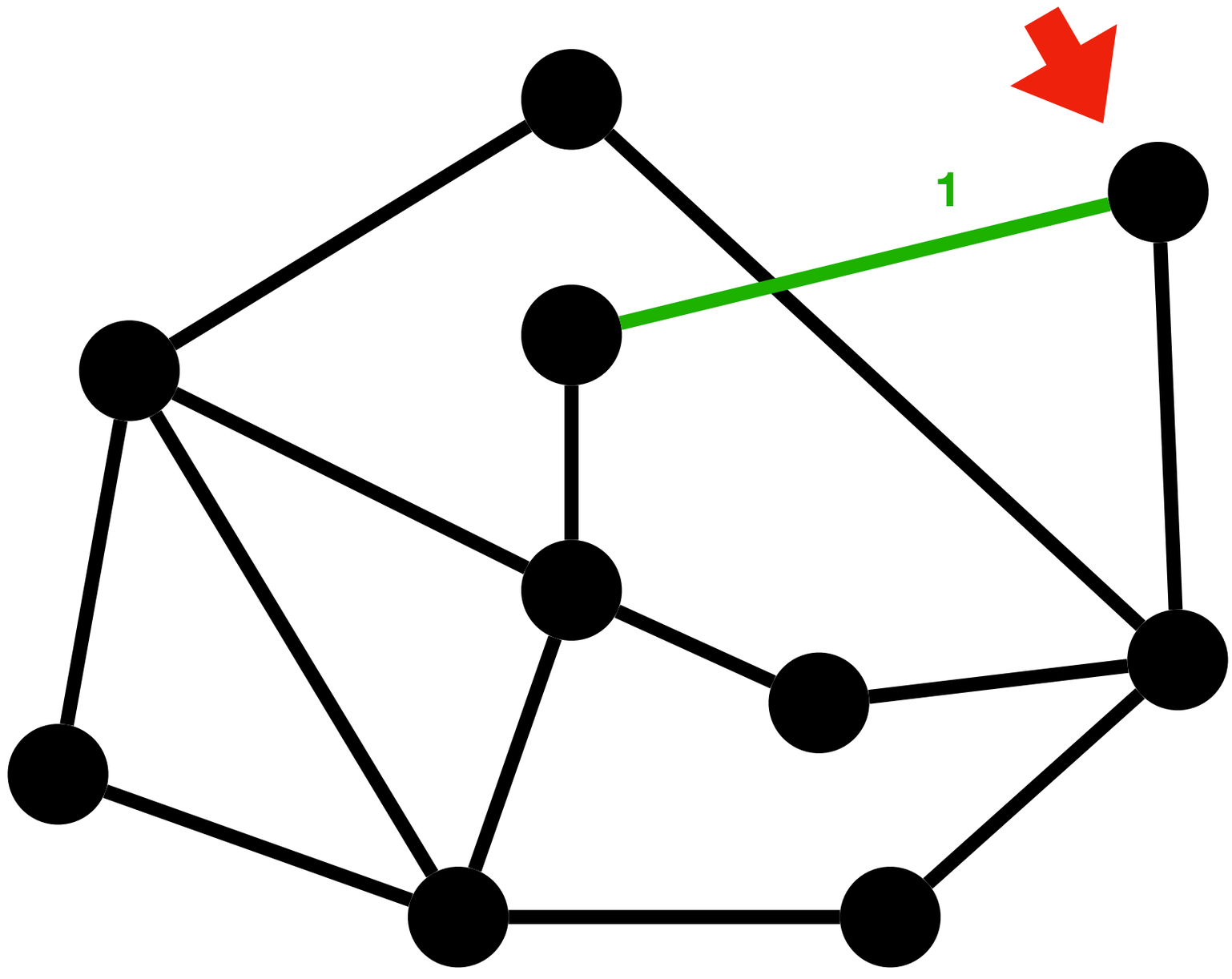

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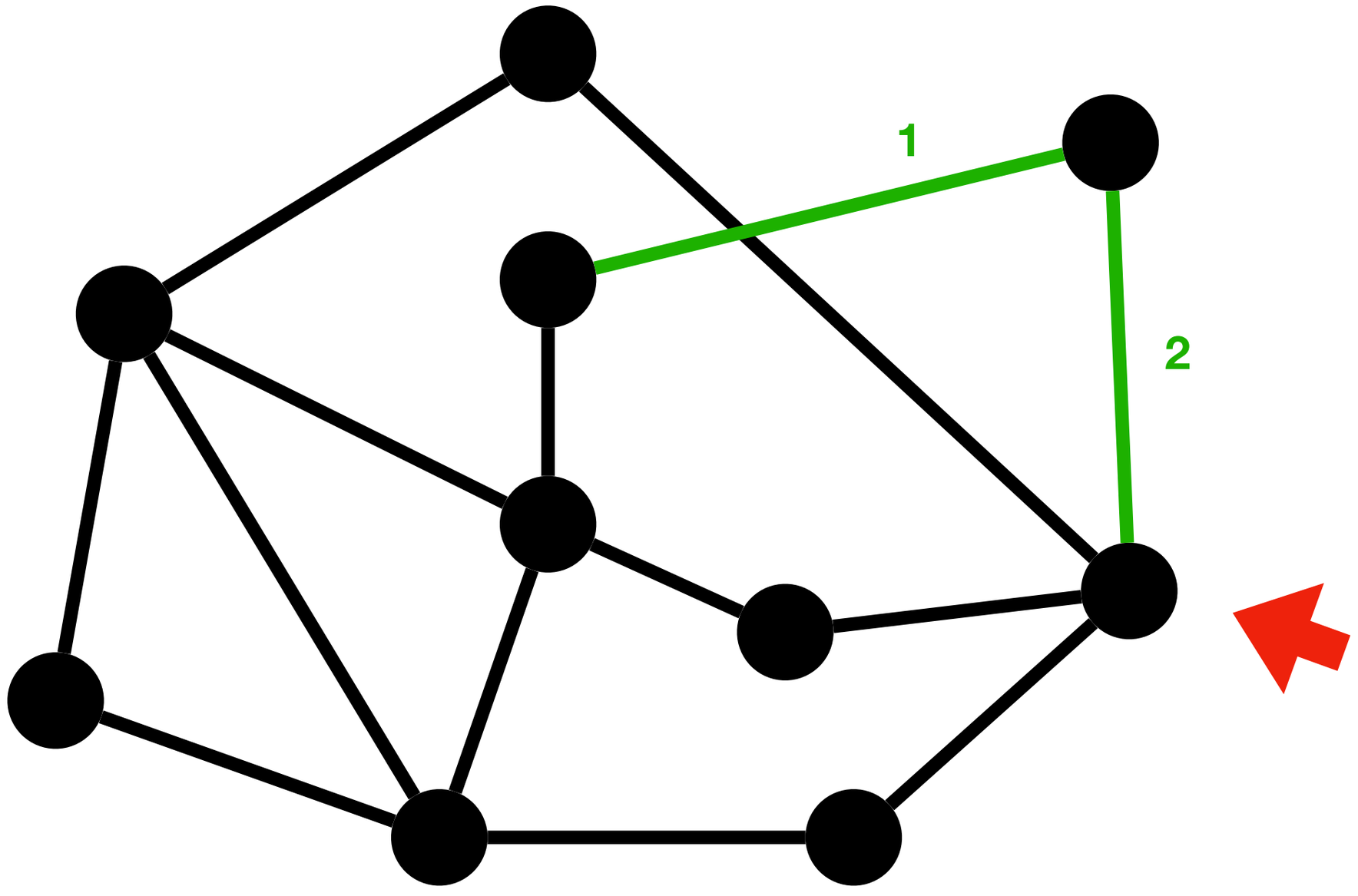

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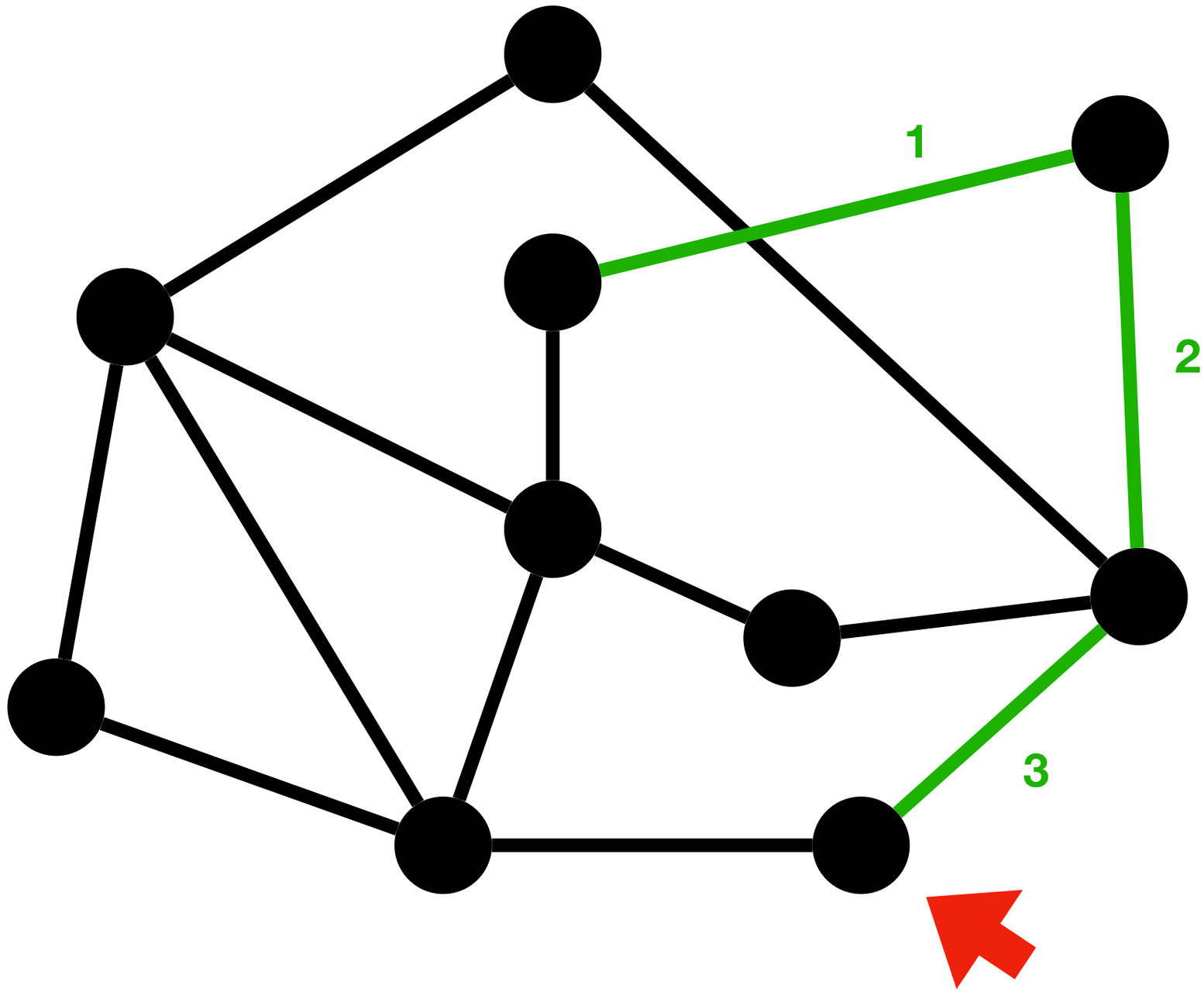

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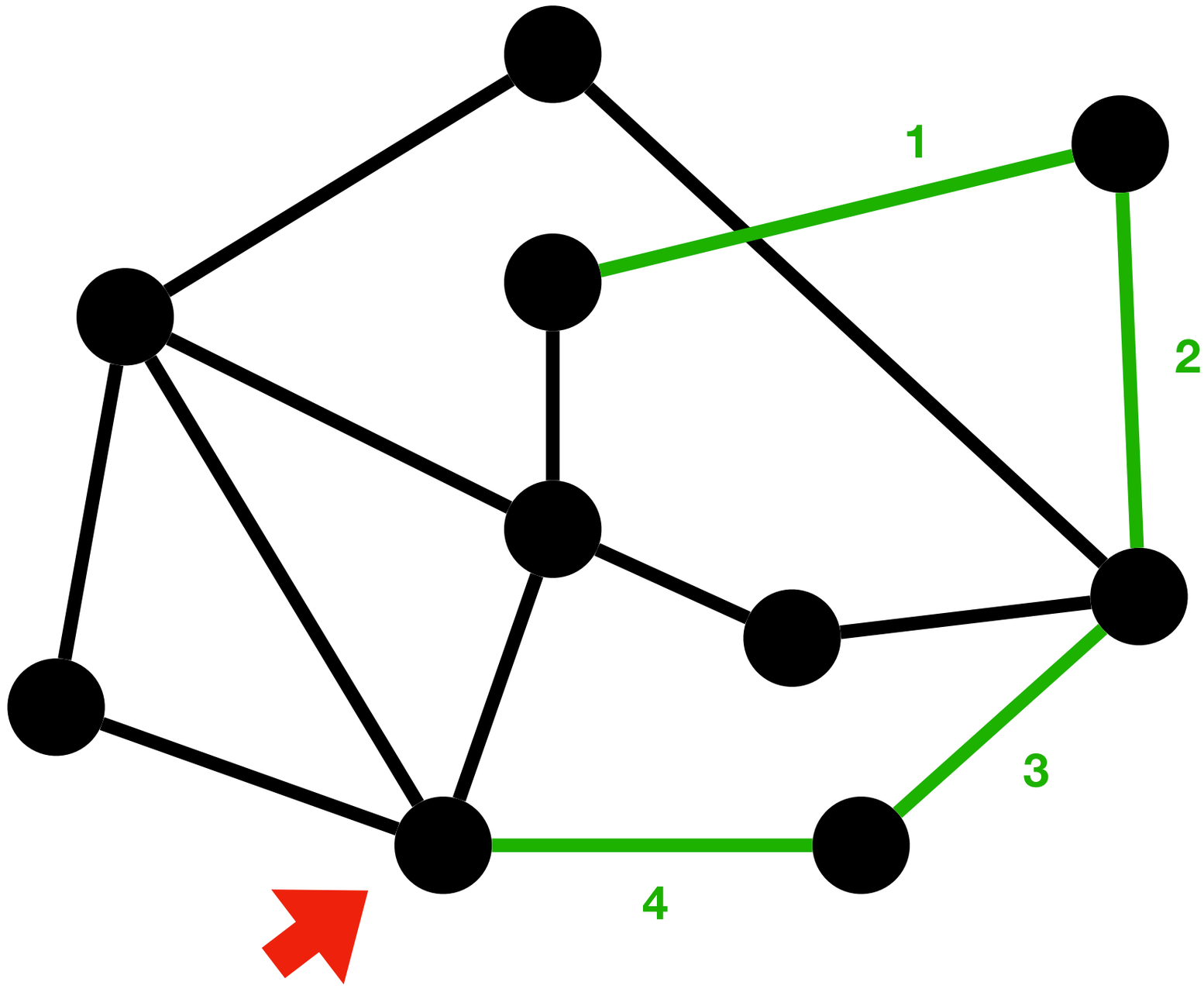

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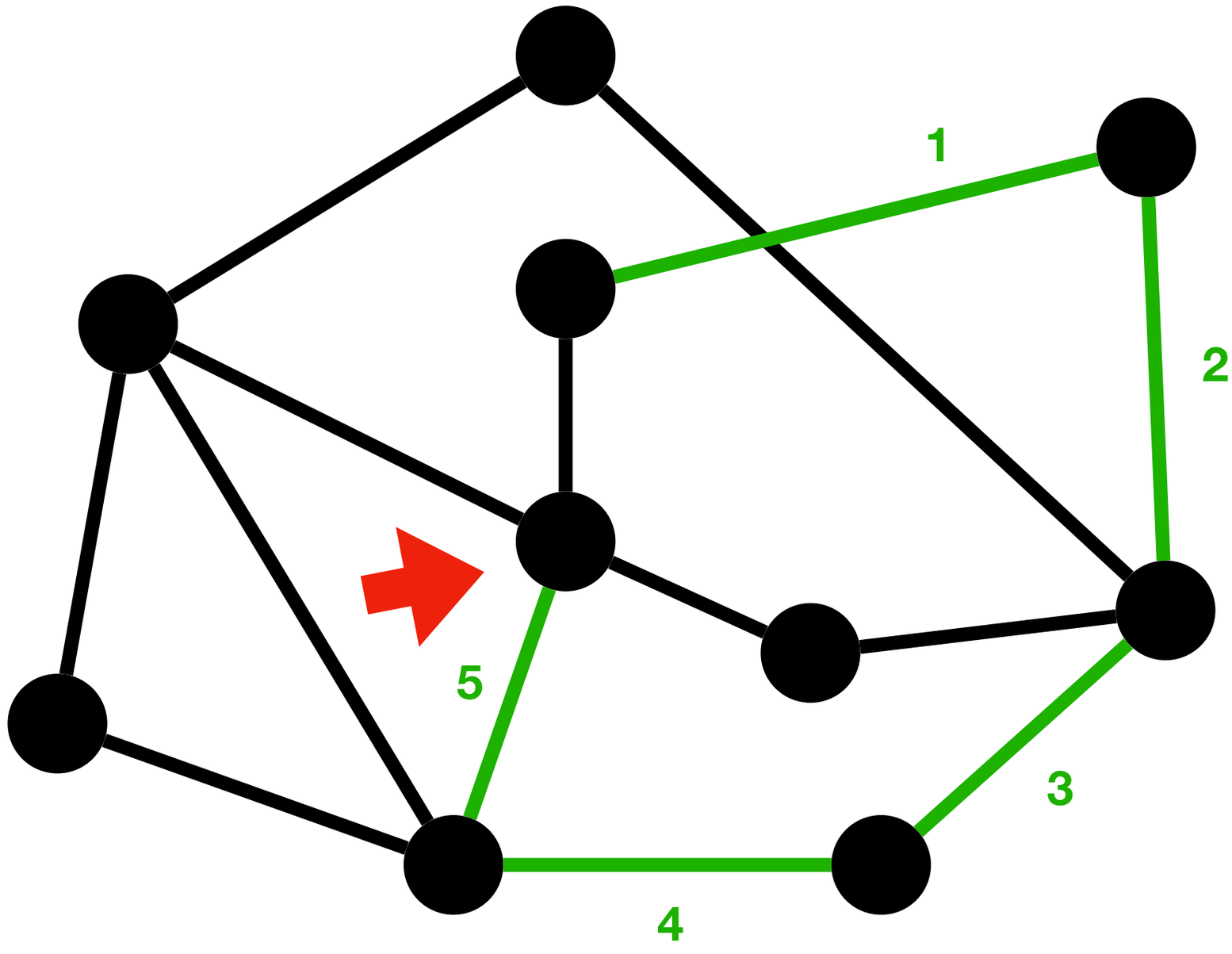

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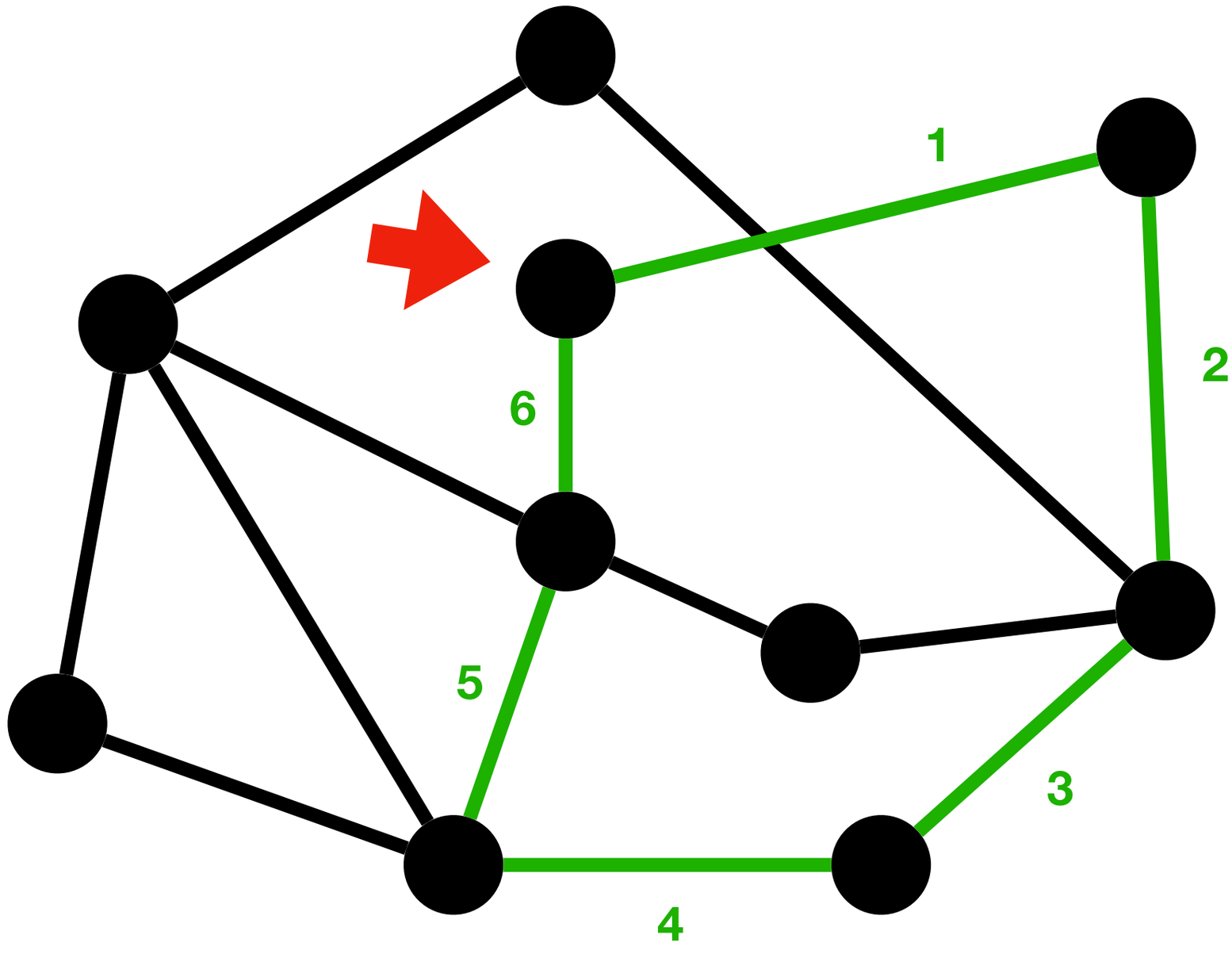

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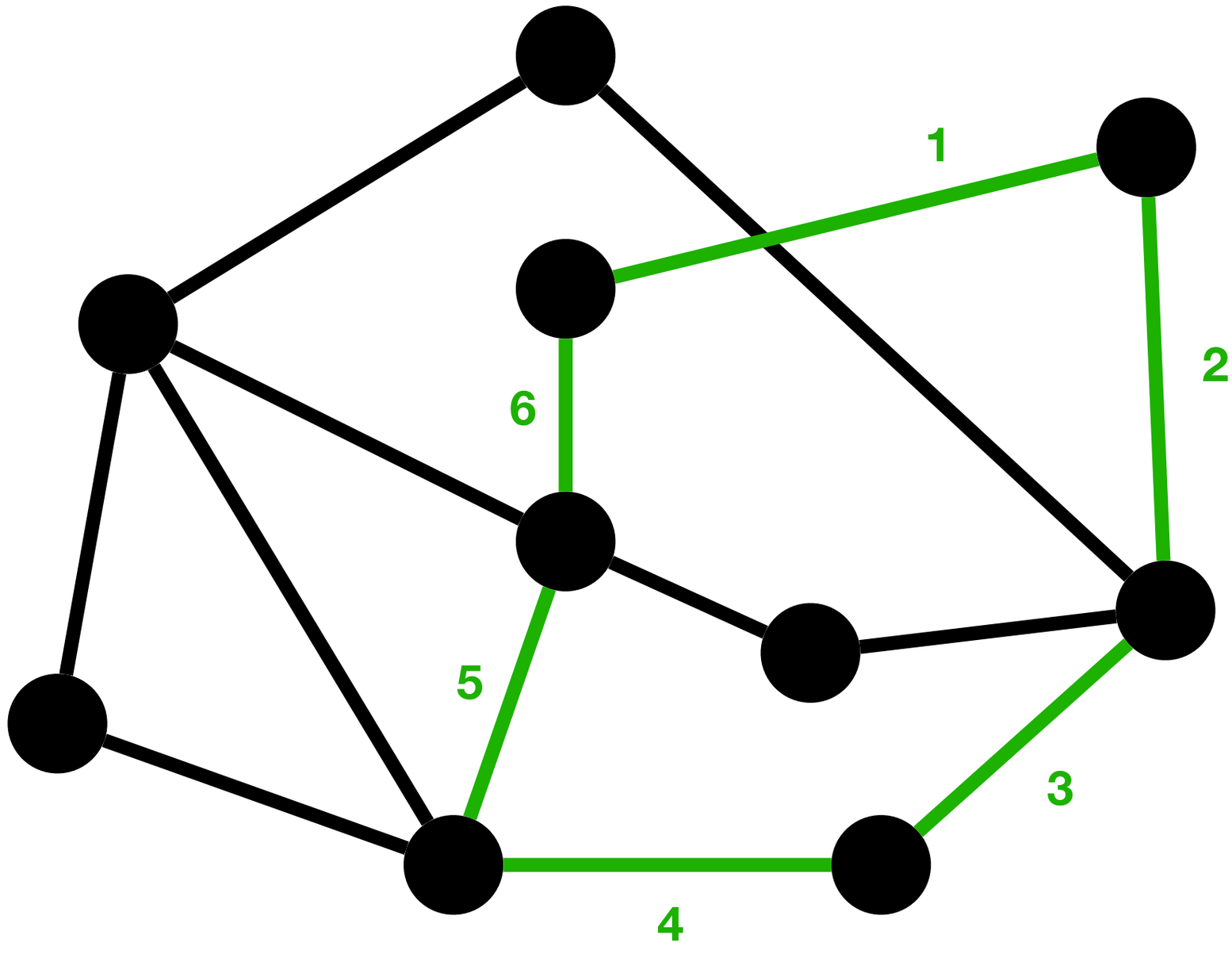

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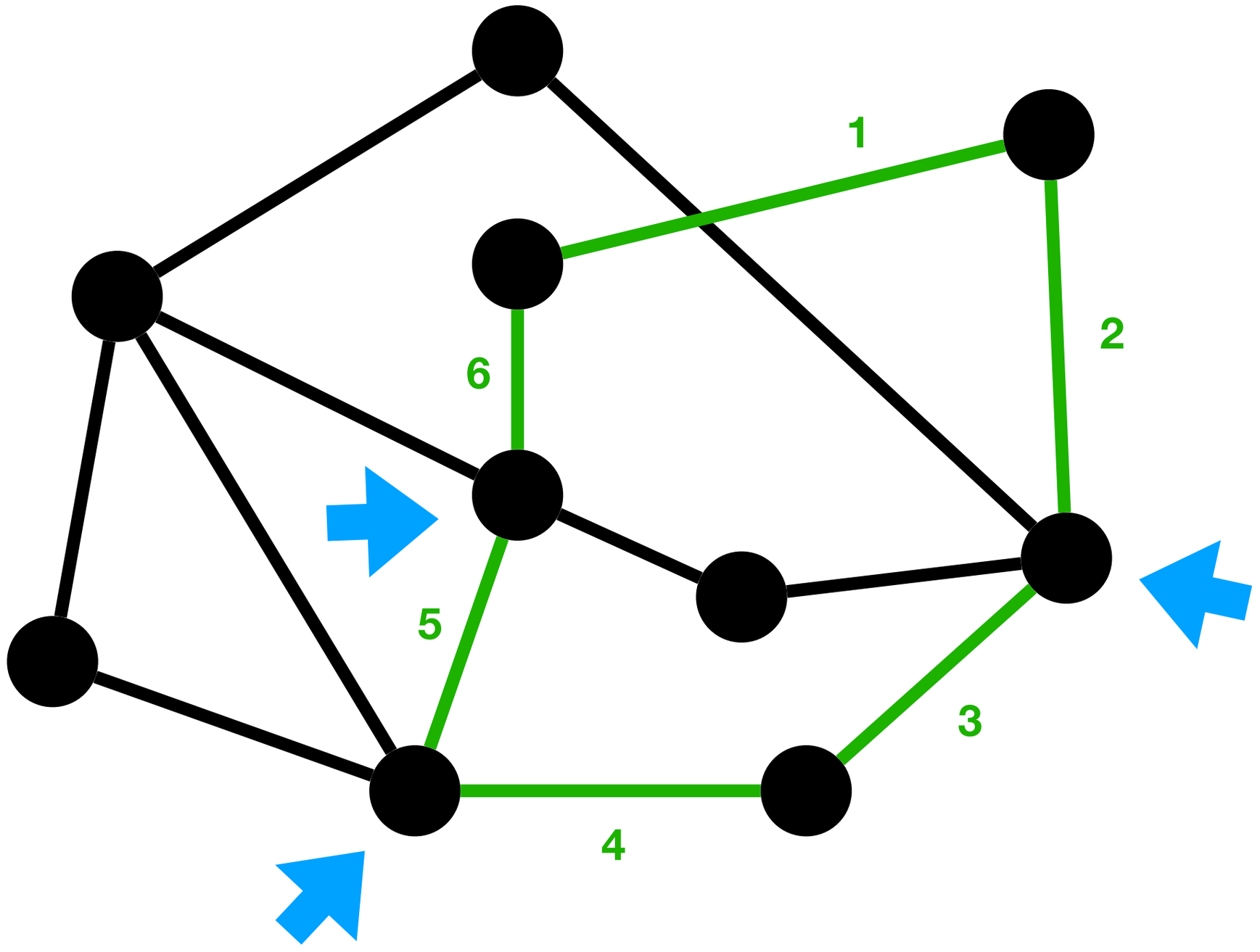

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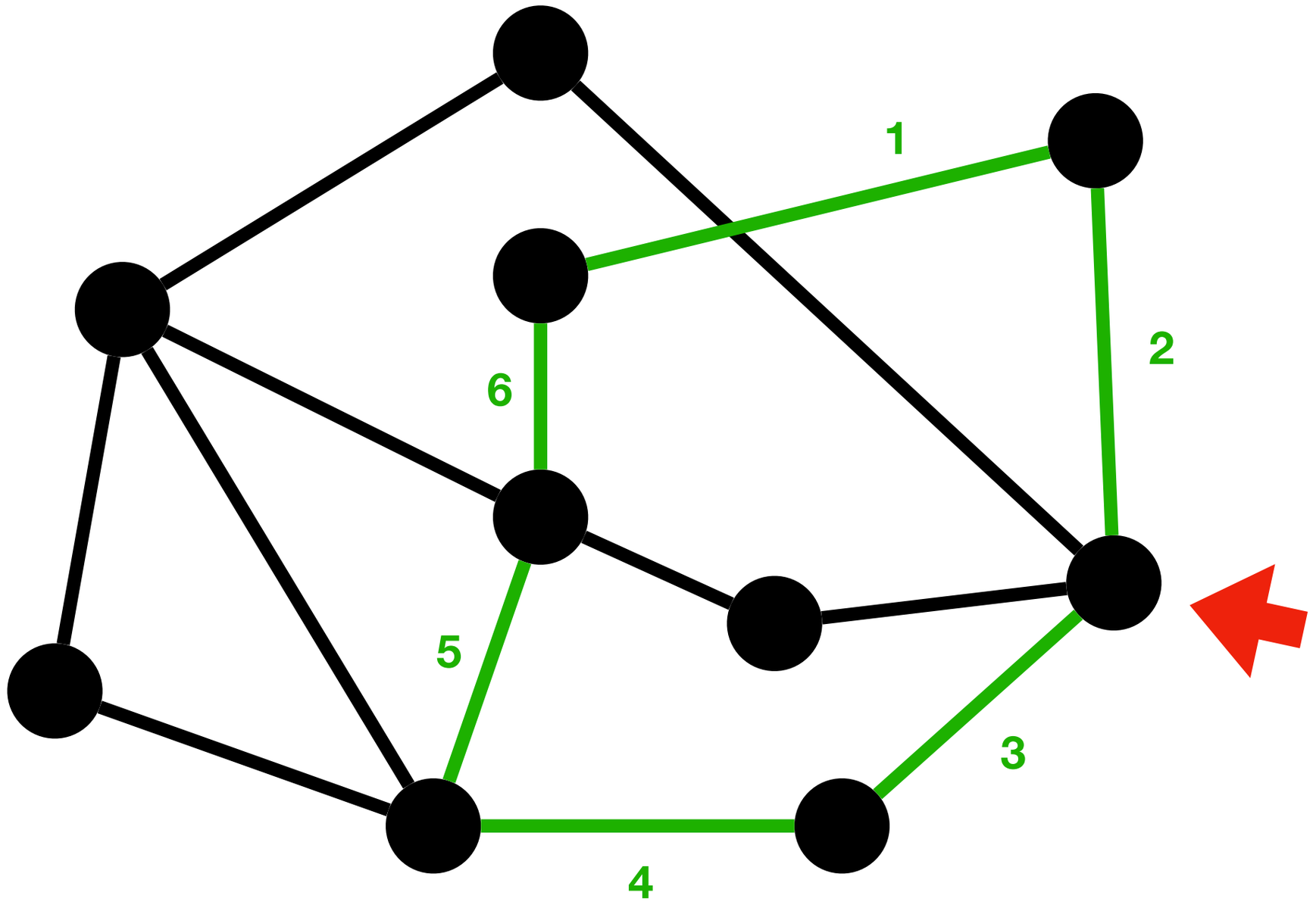

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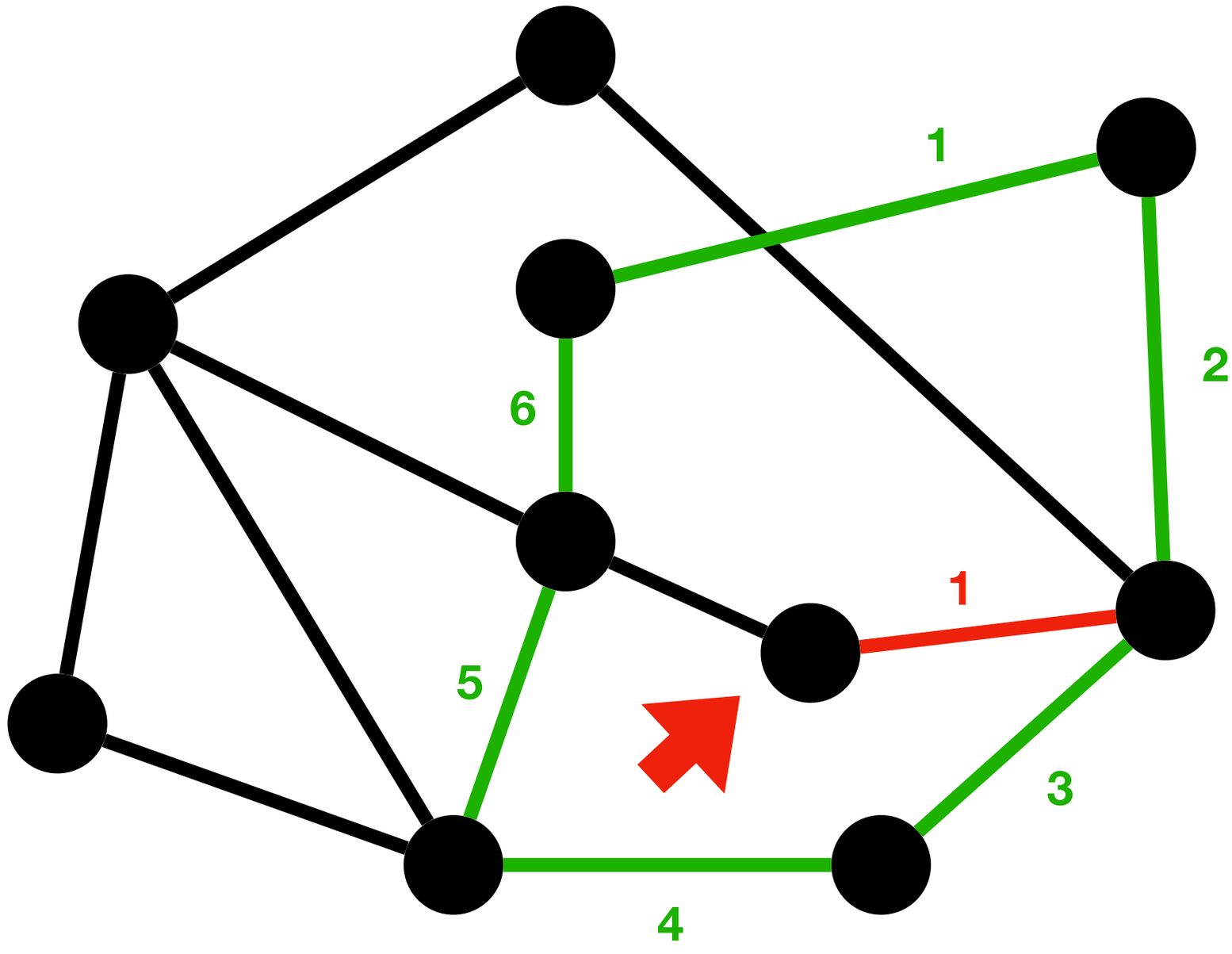

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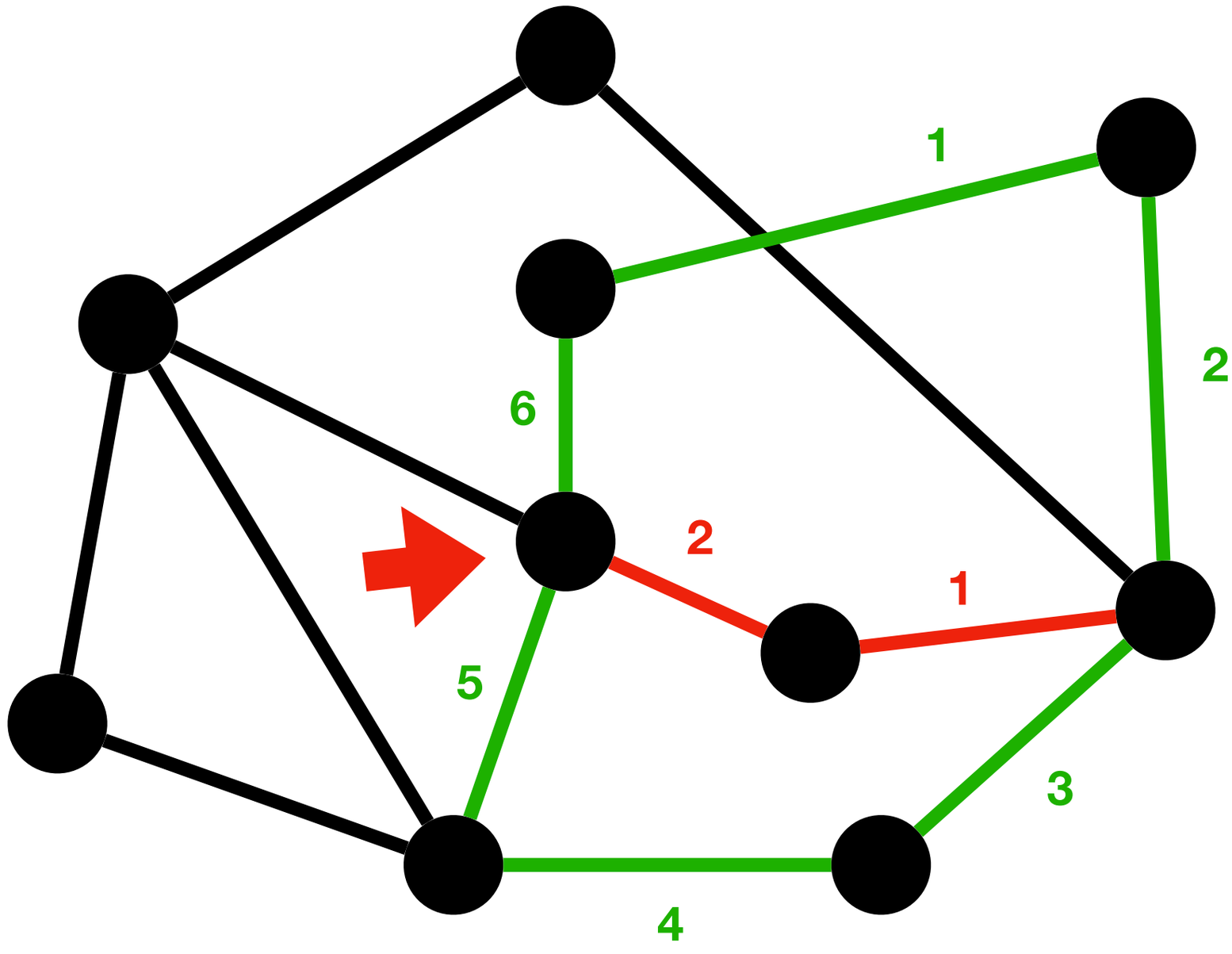
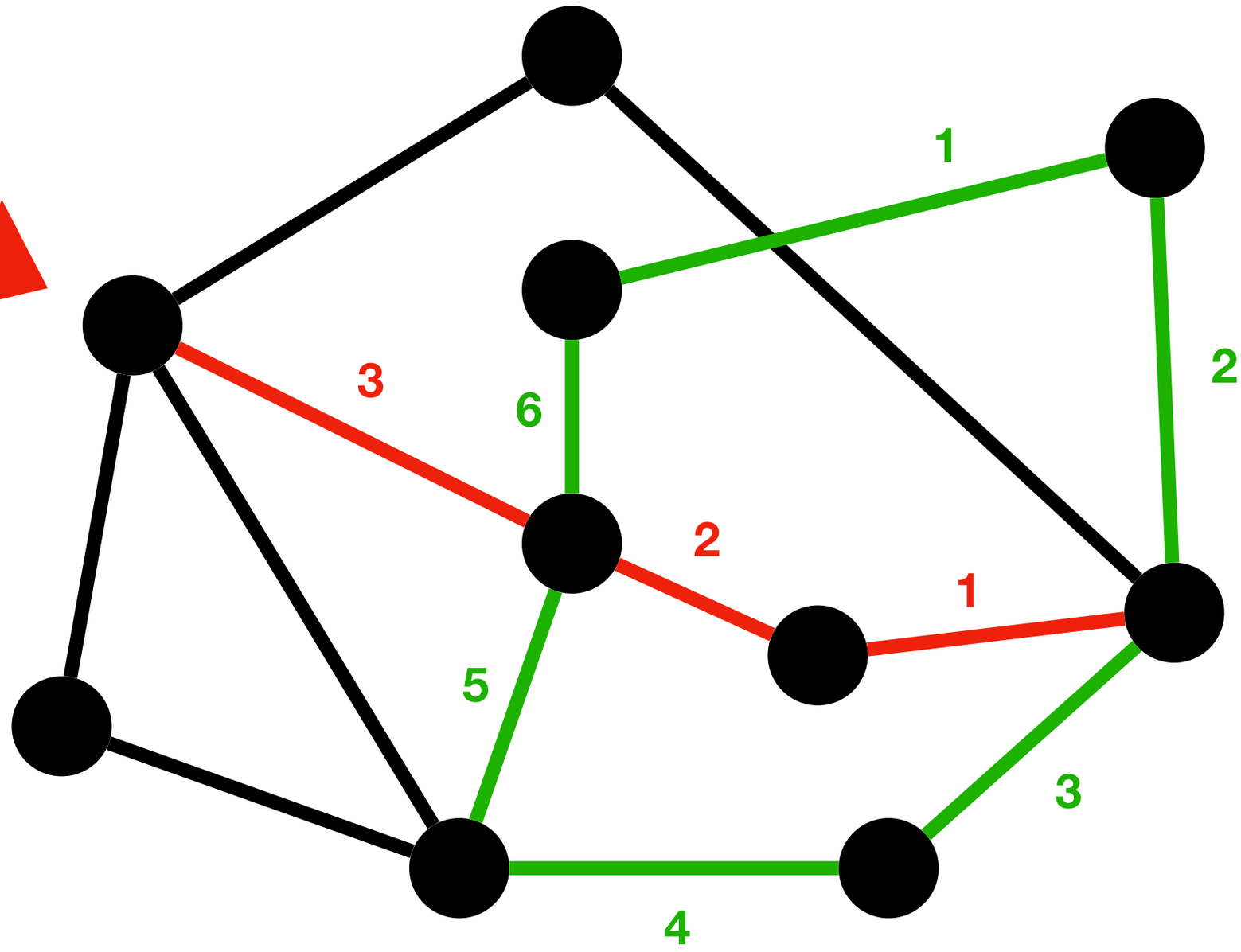


Algorithme de Hierholzer



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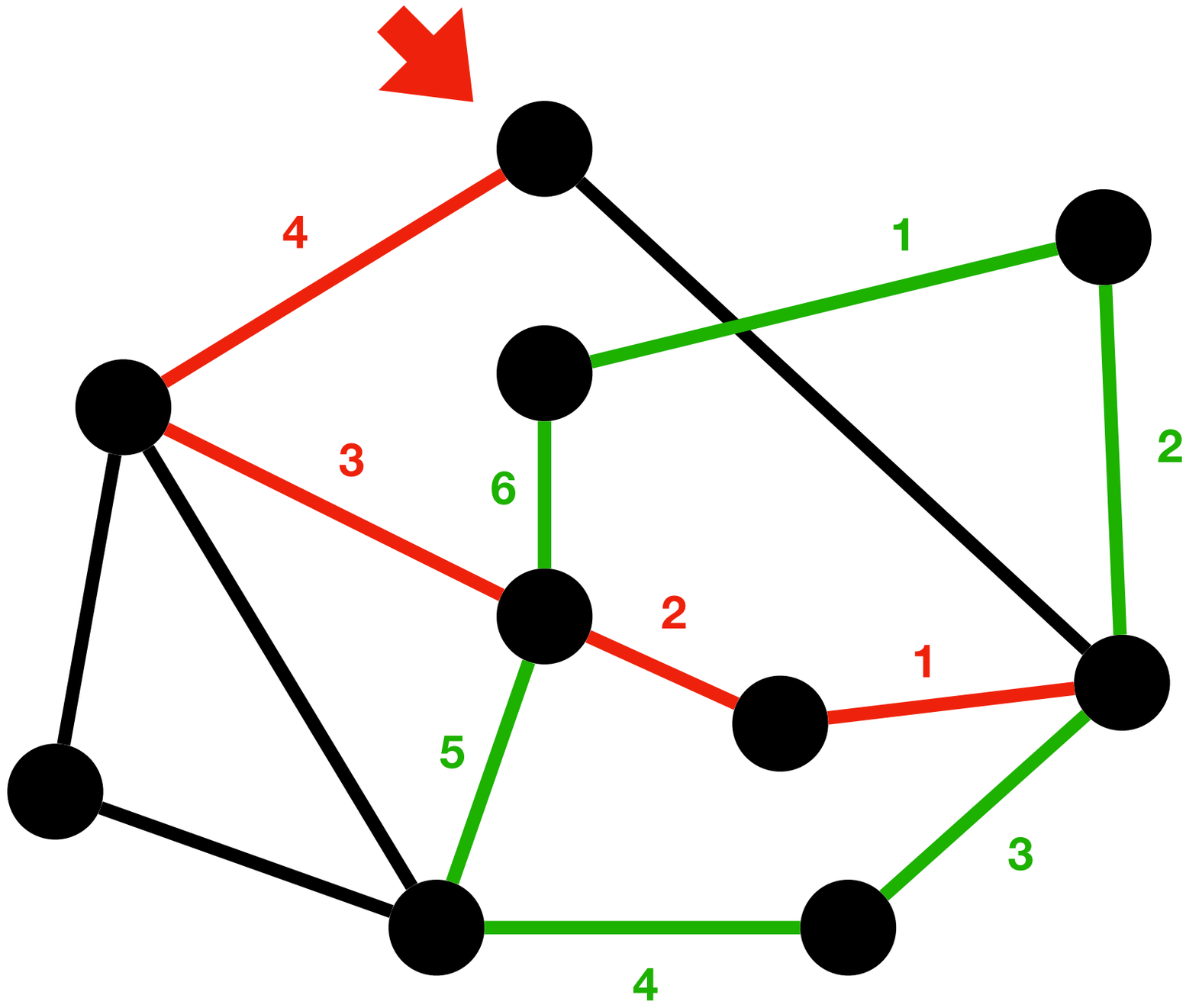

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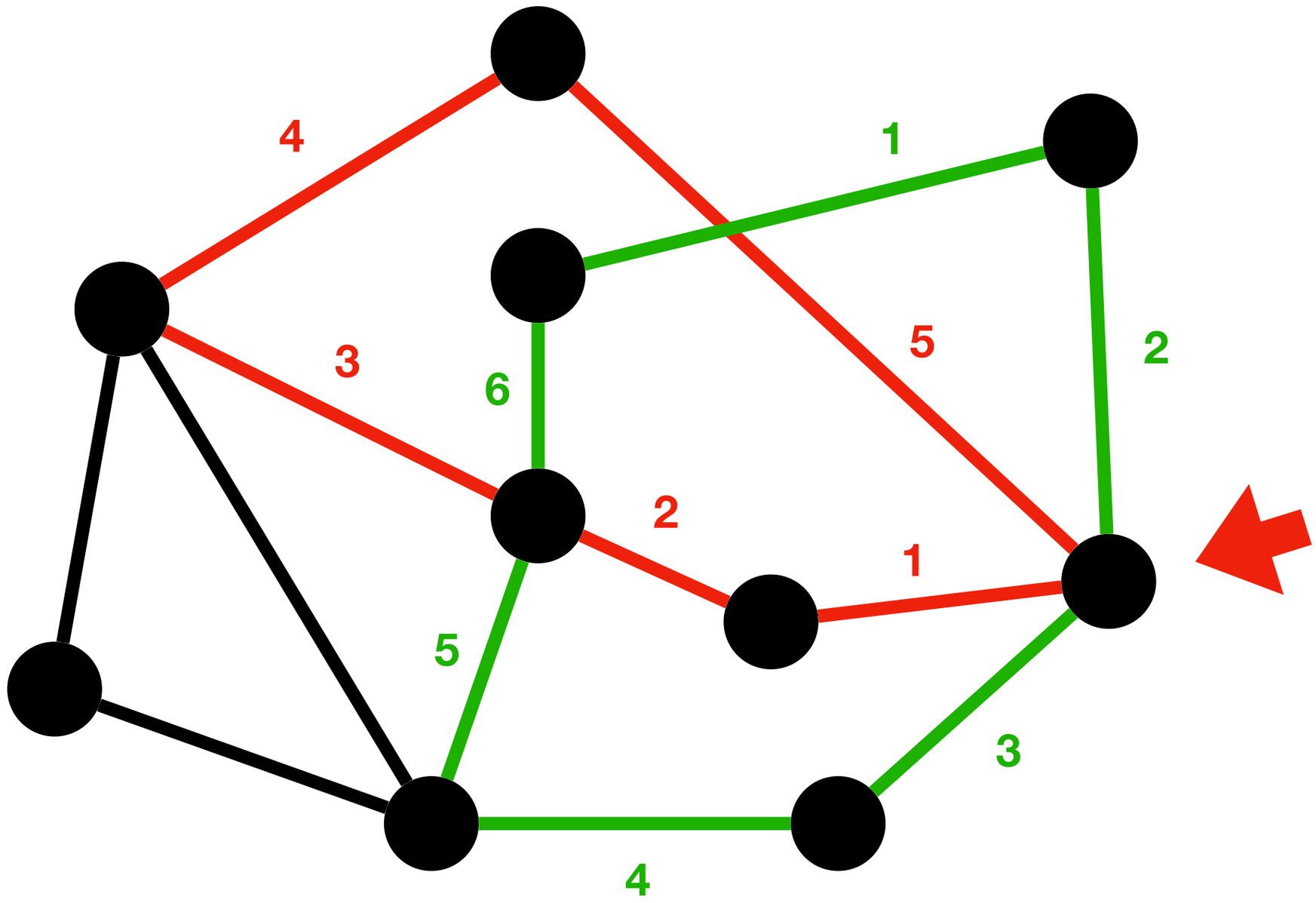

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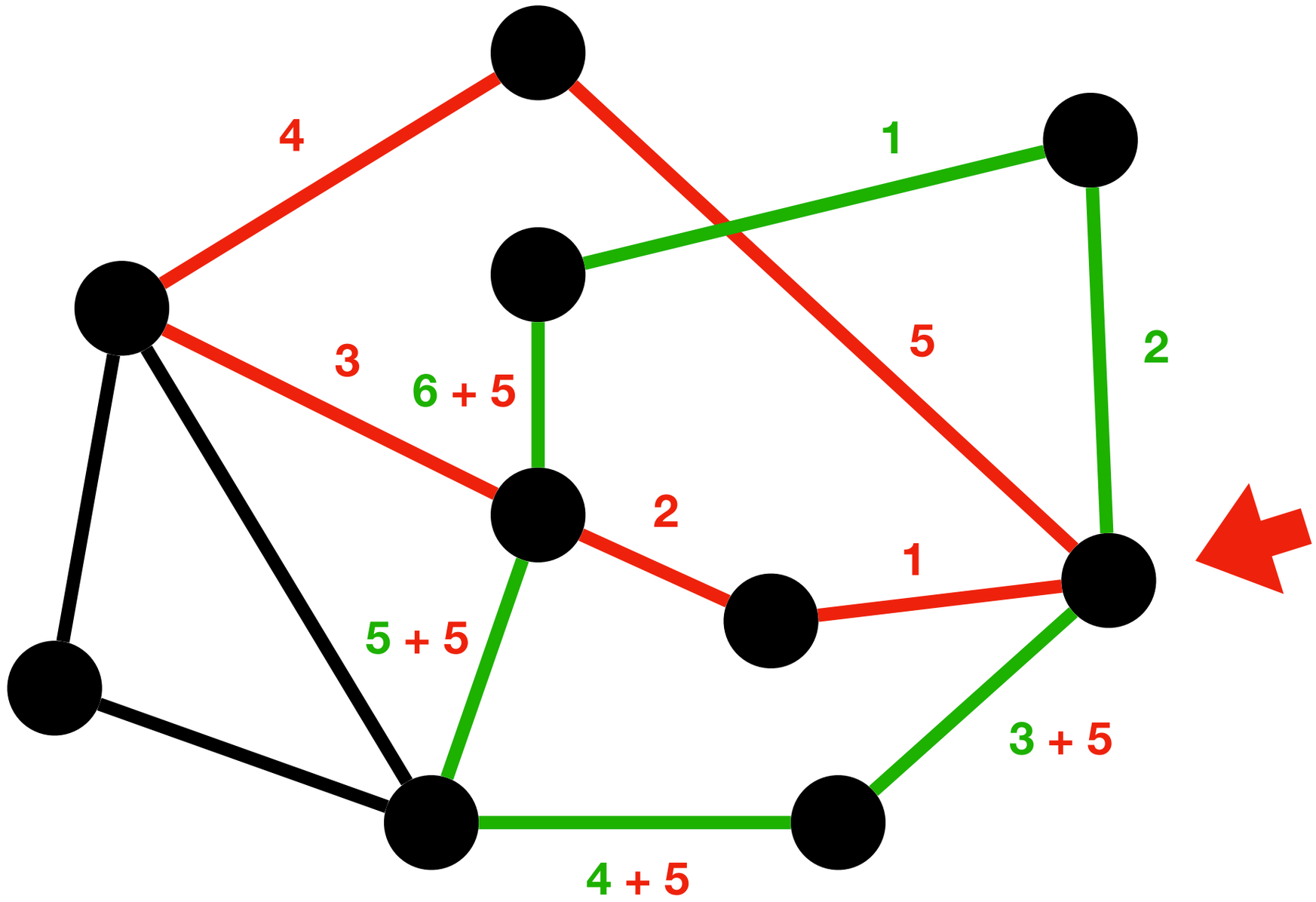

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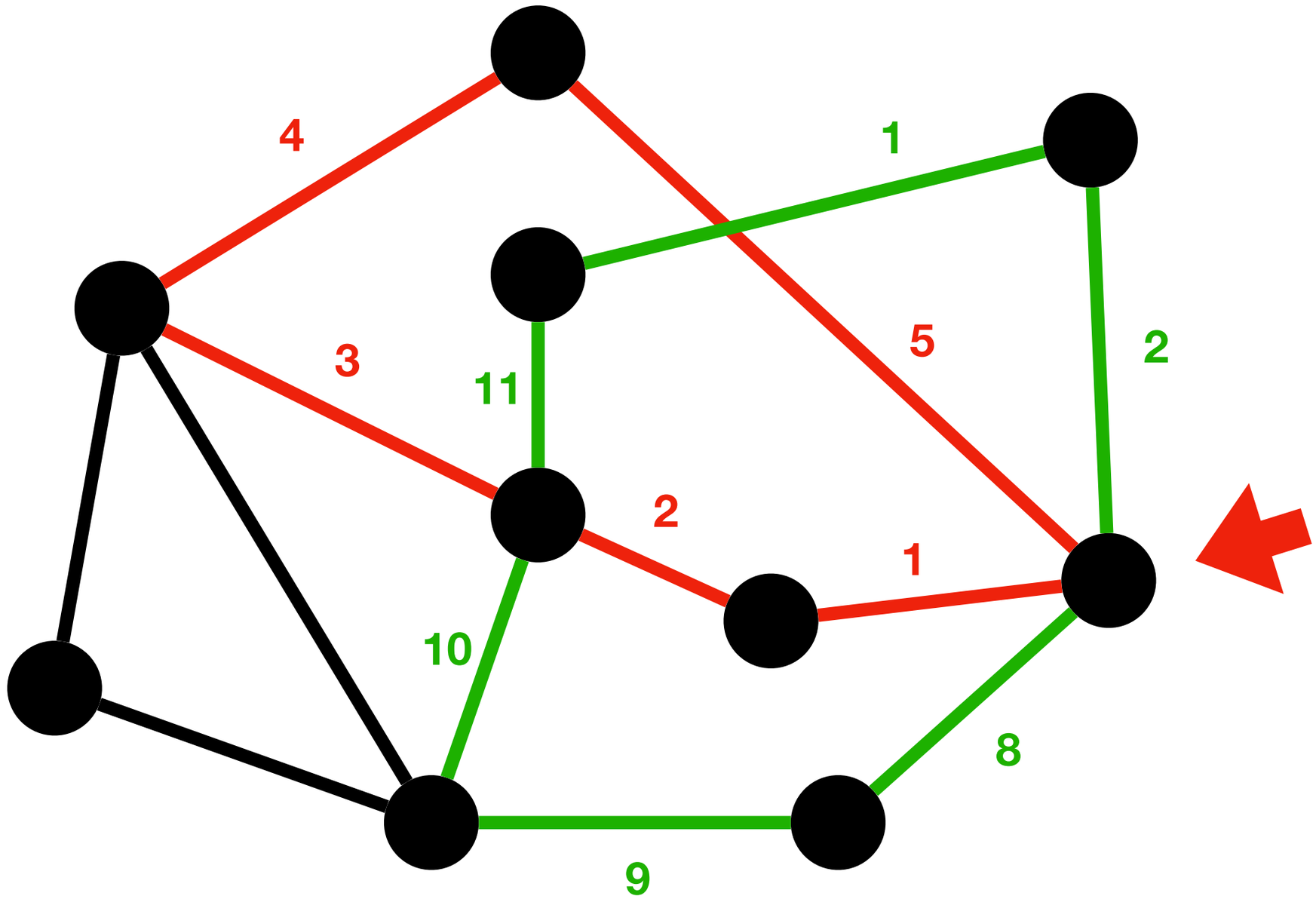

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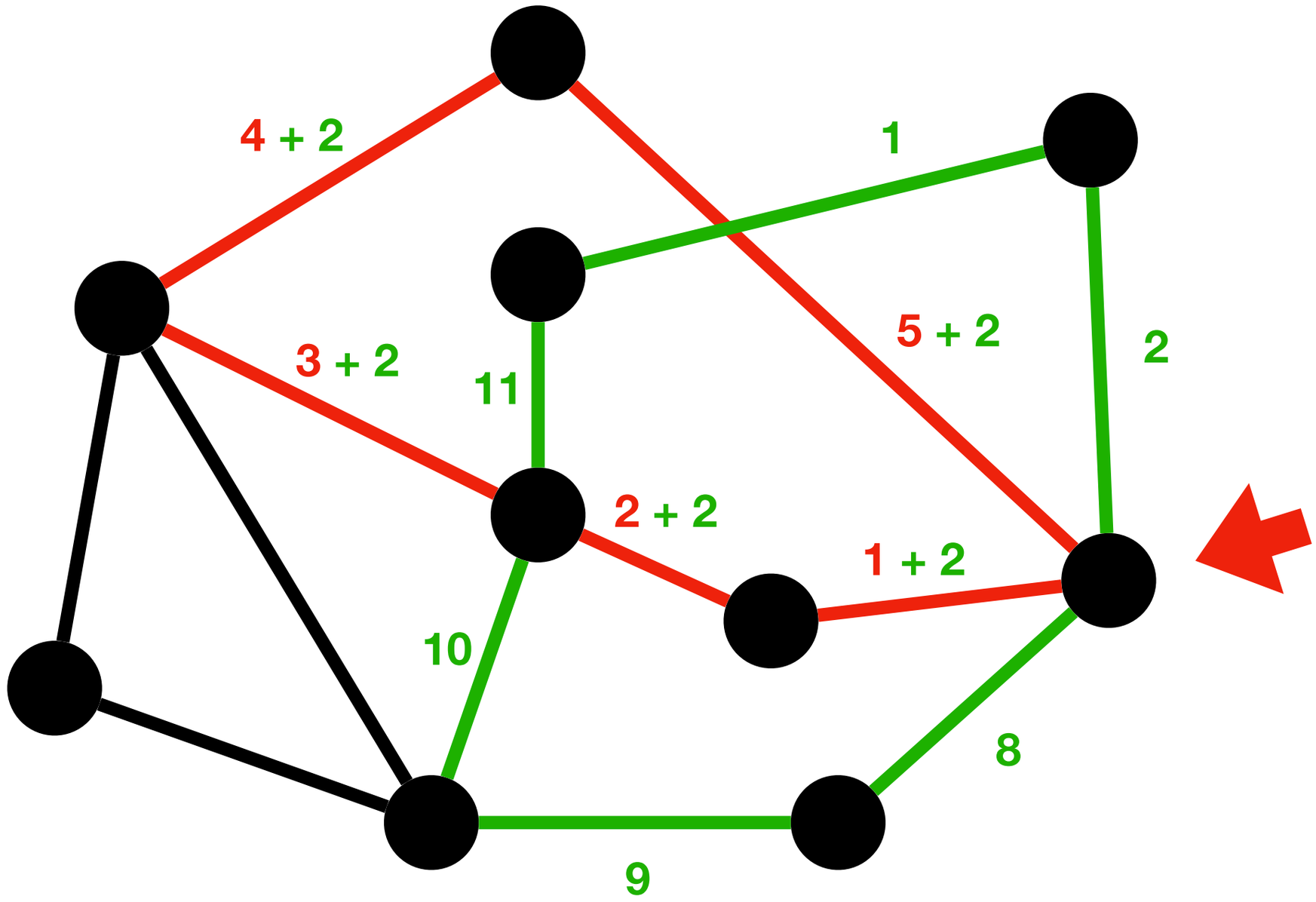

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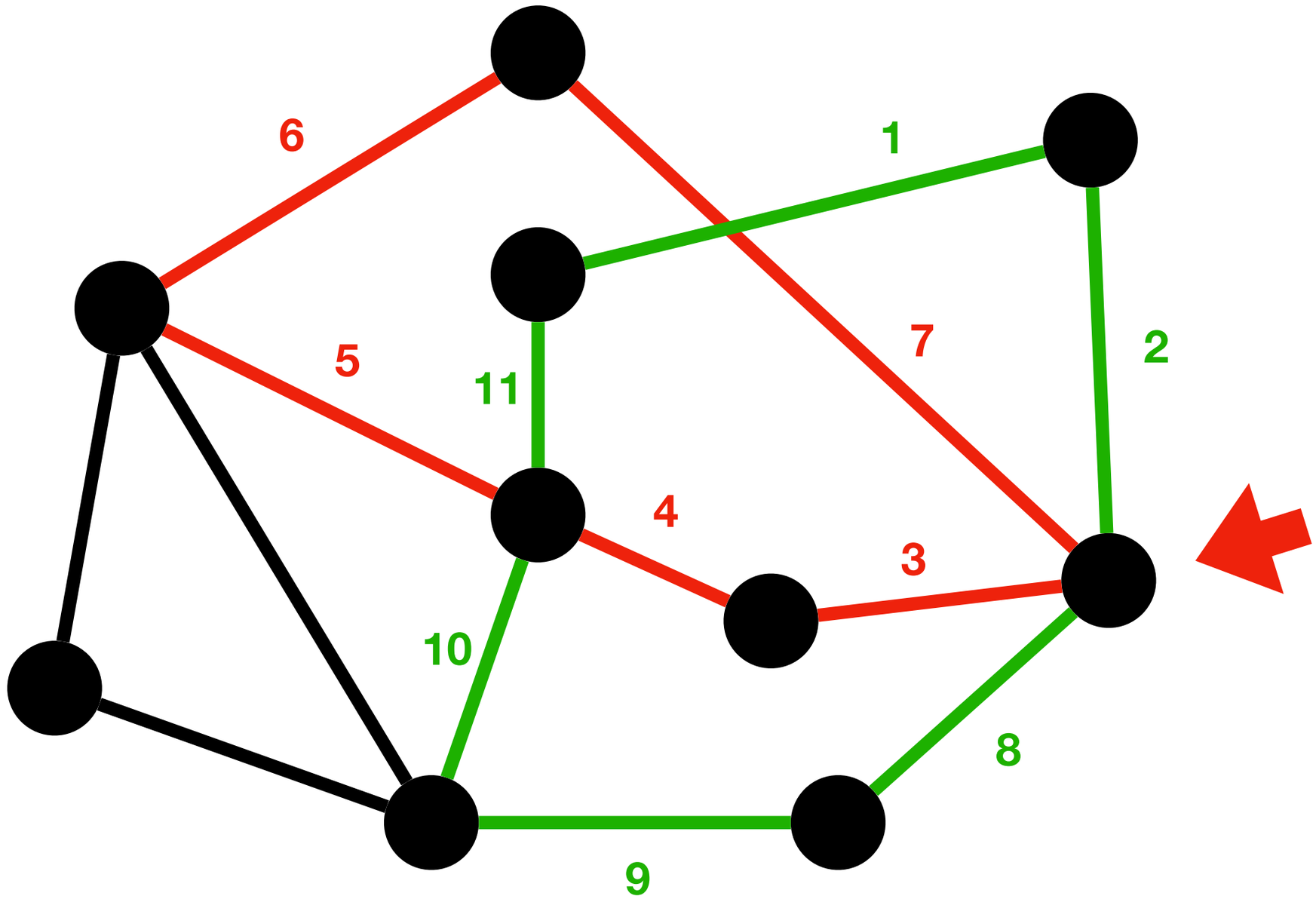

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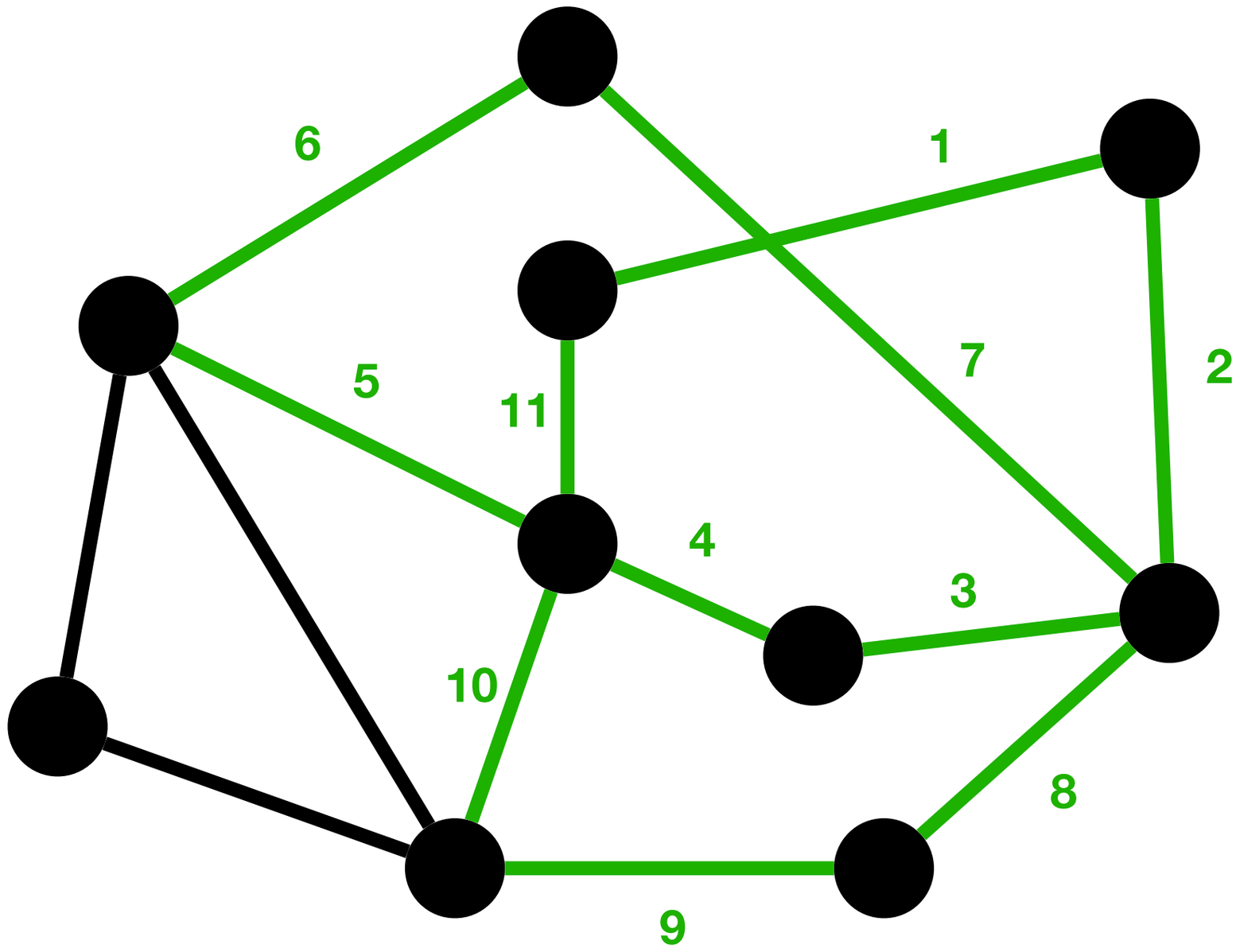

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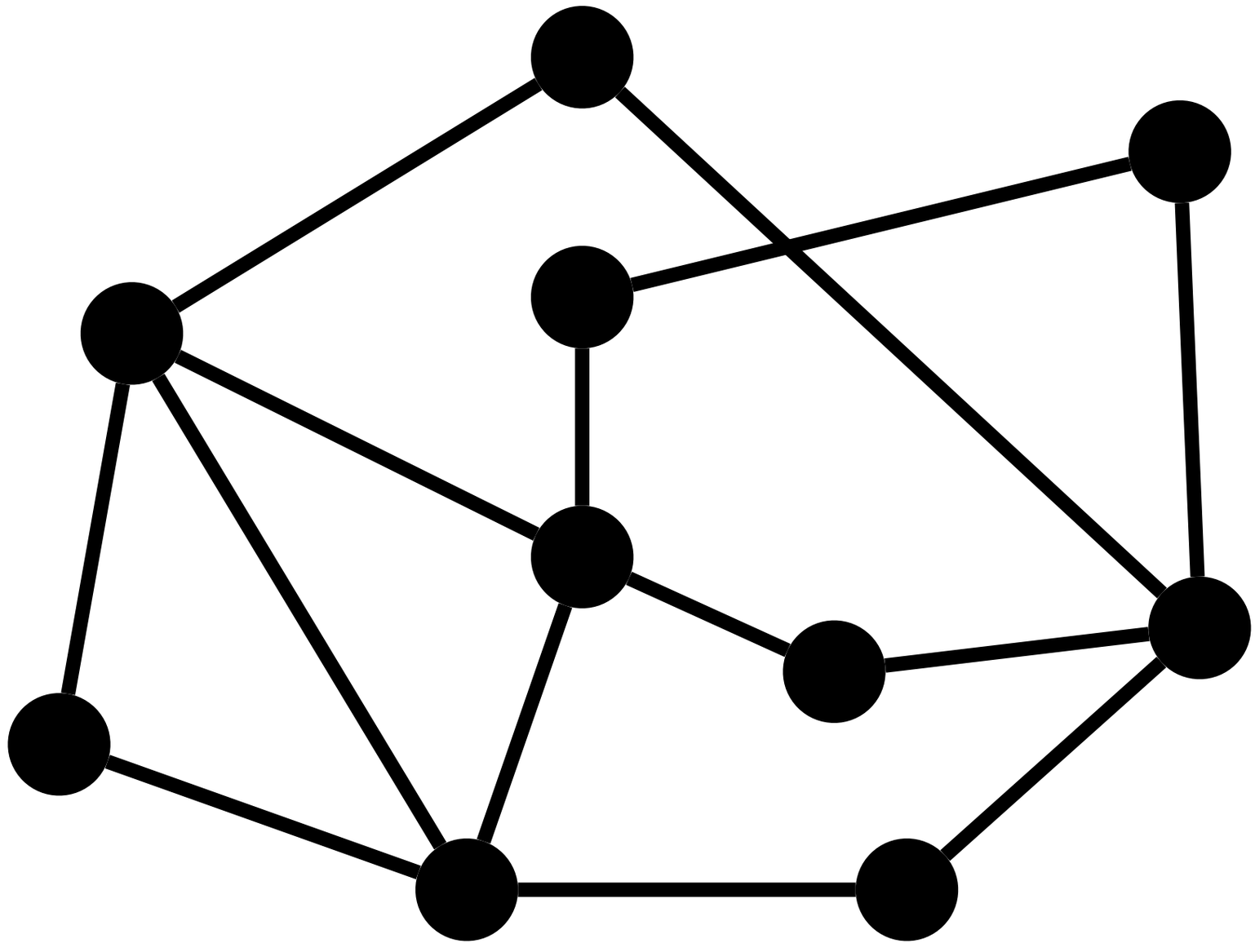

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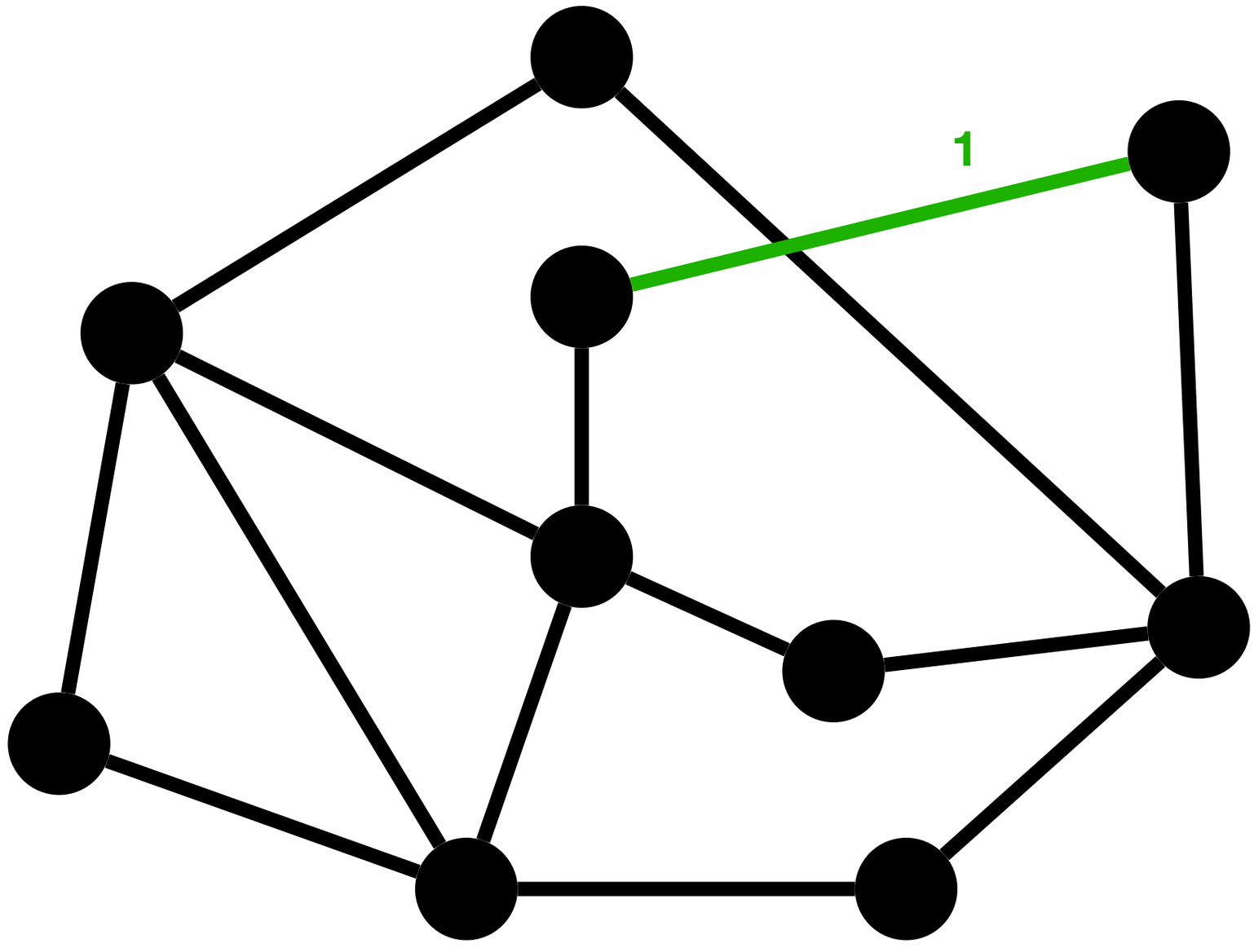

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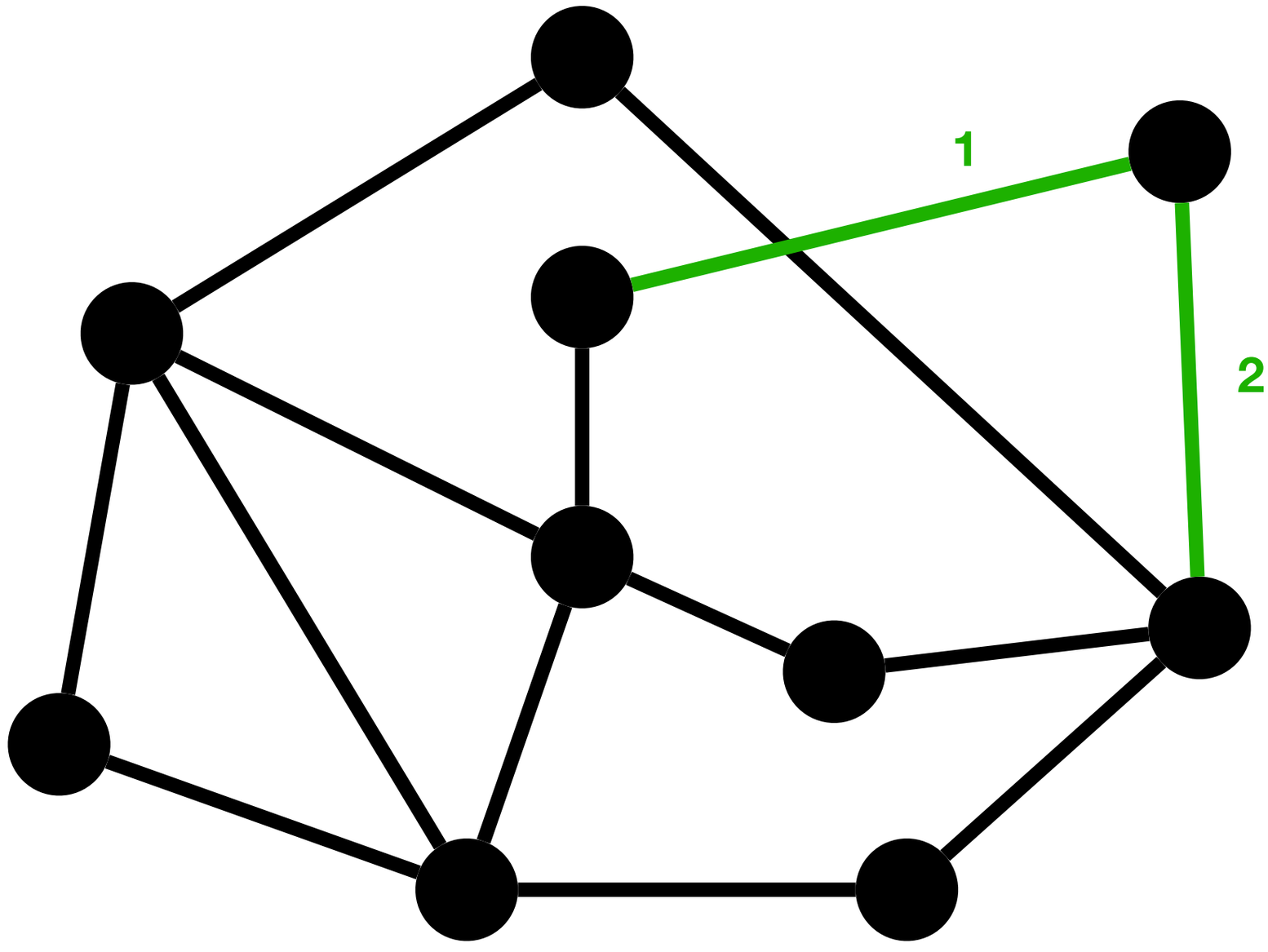

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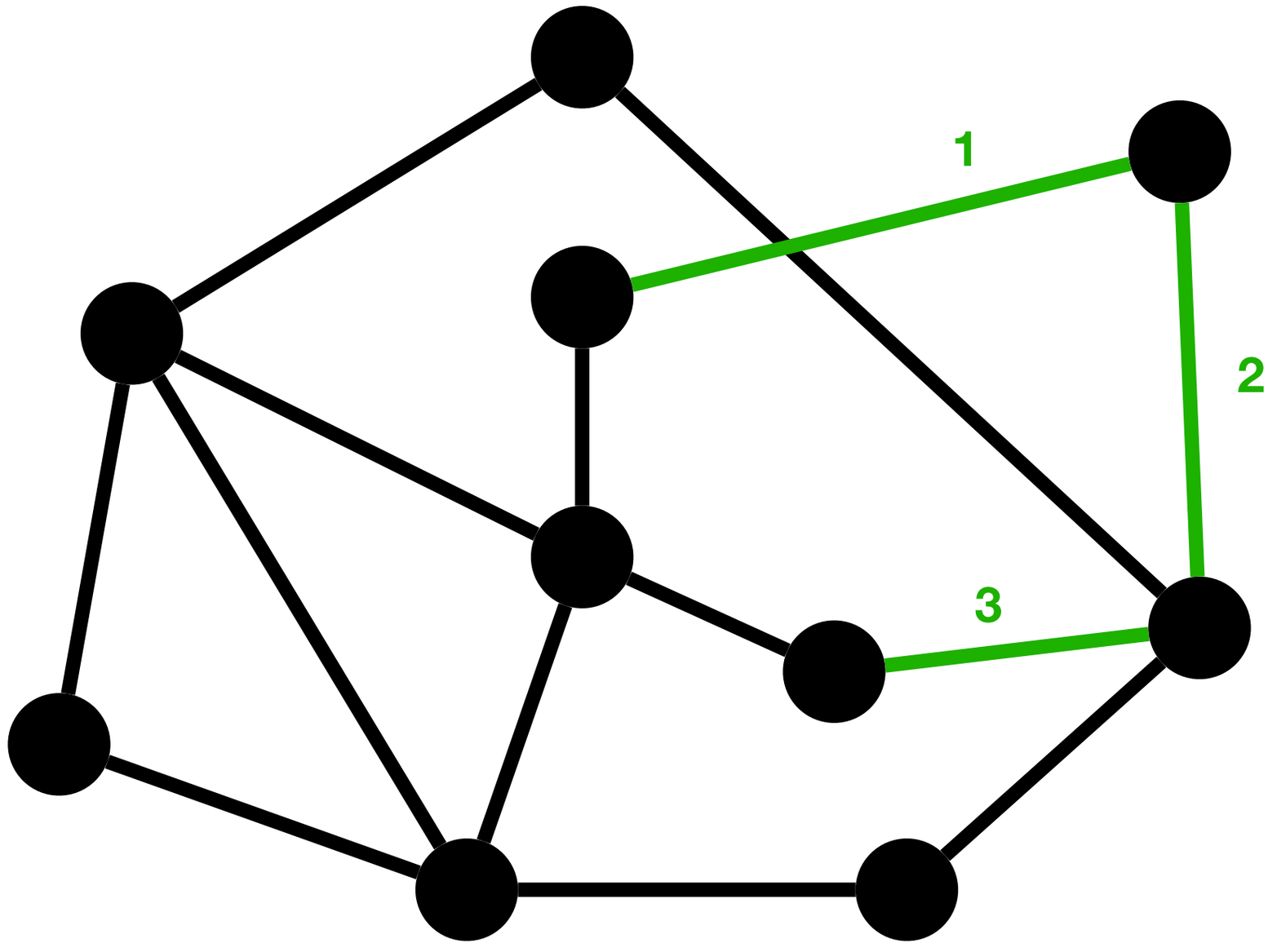

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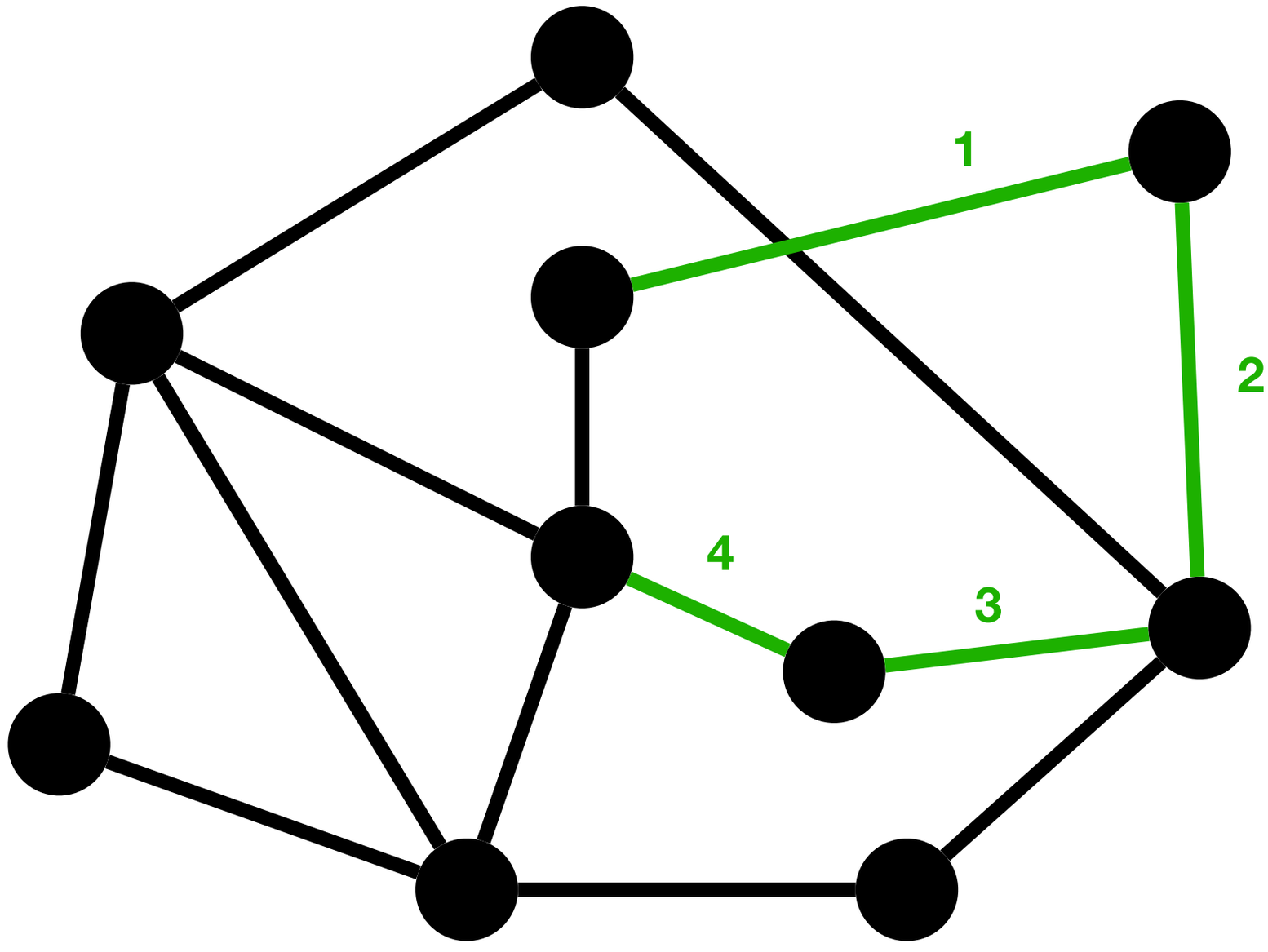

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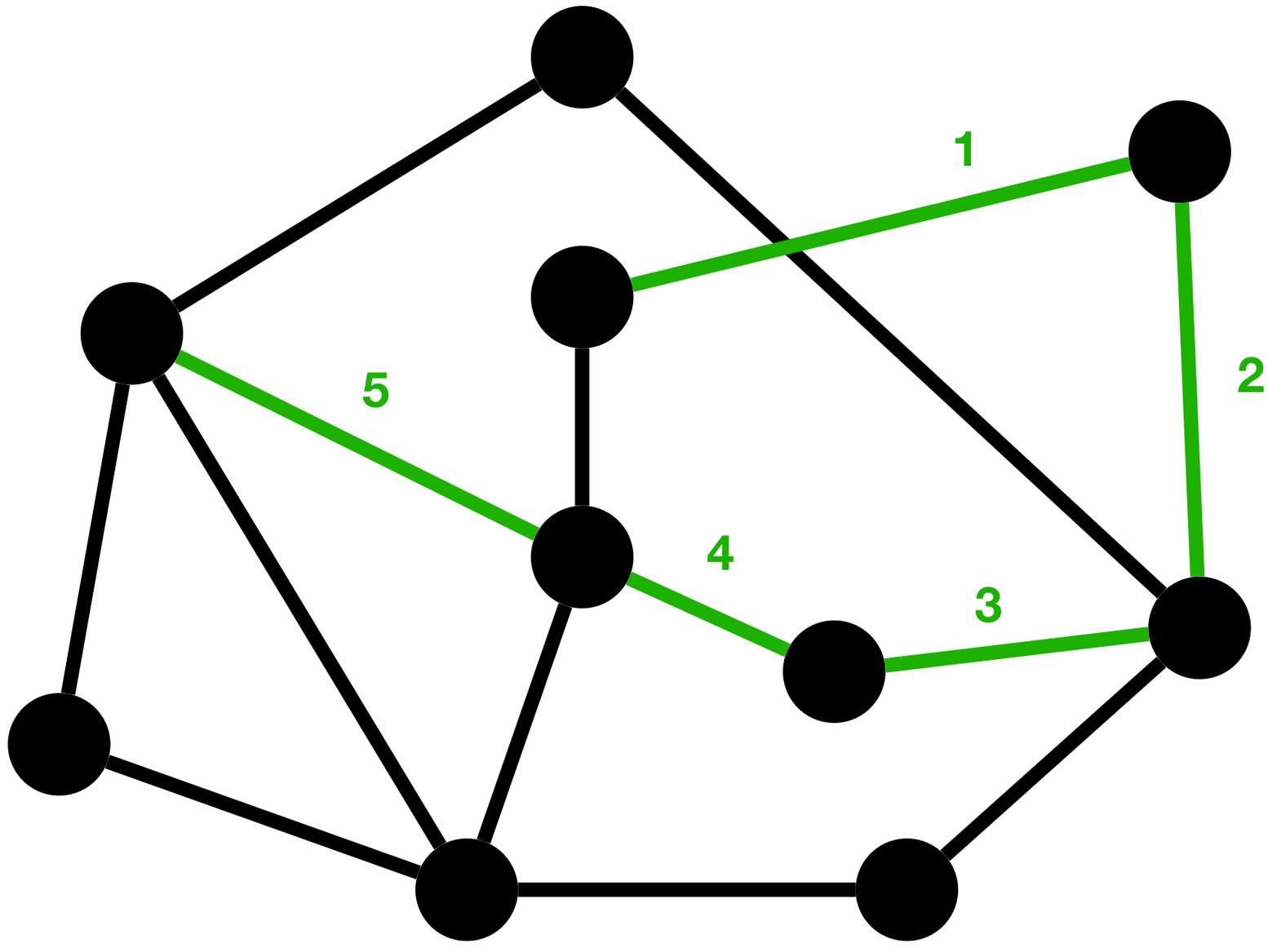

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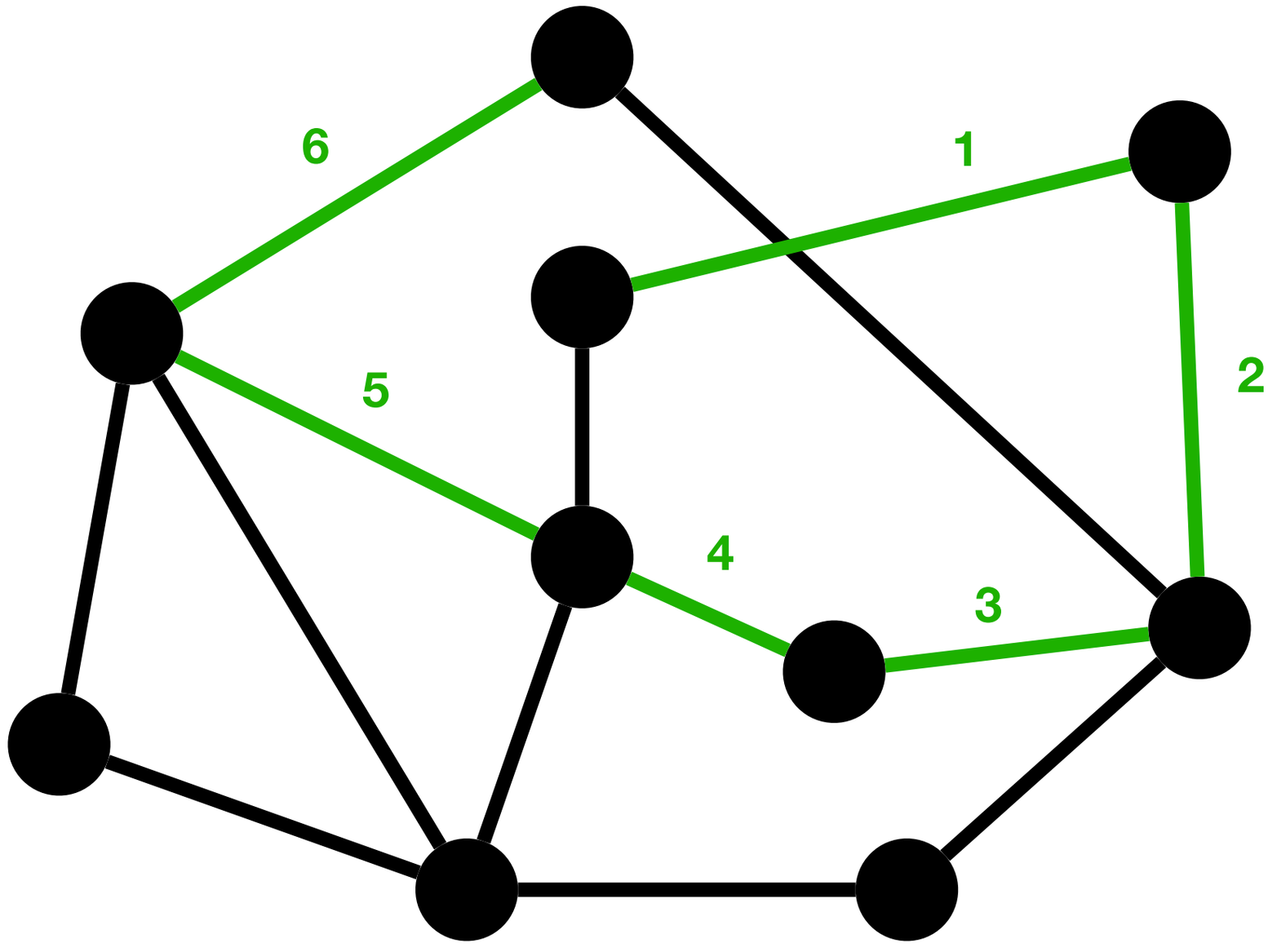

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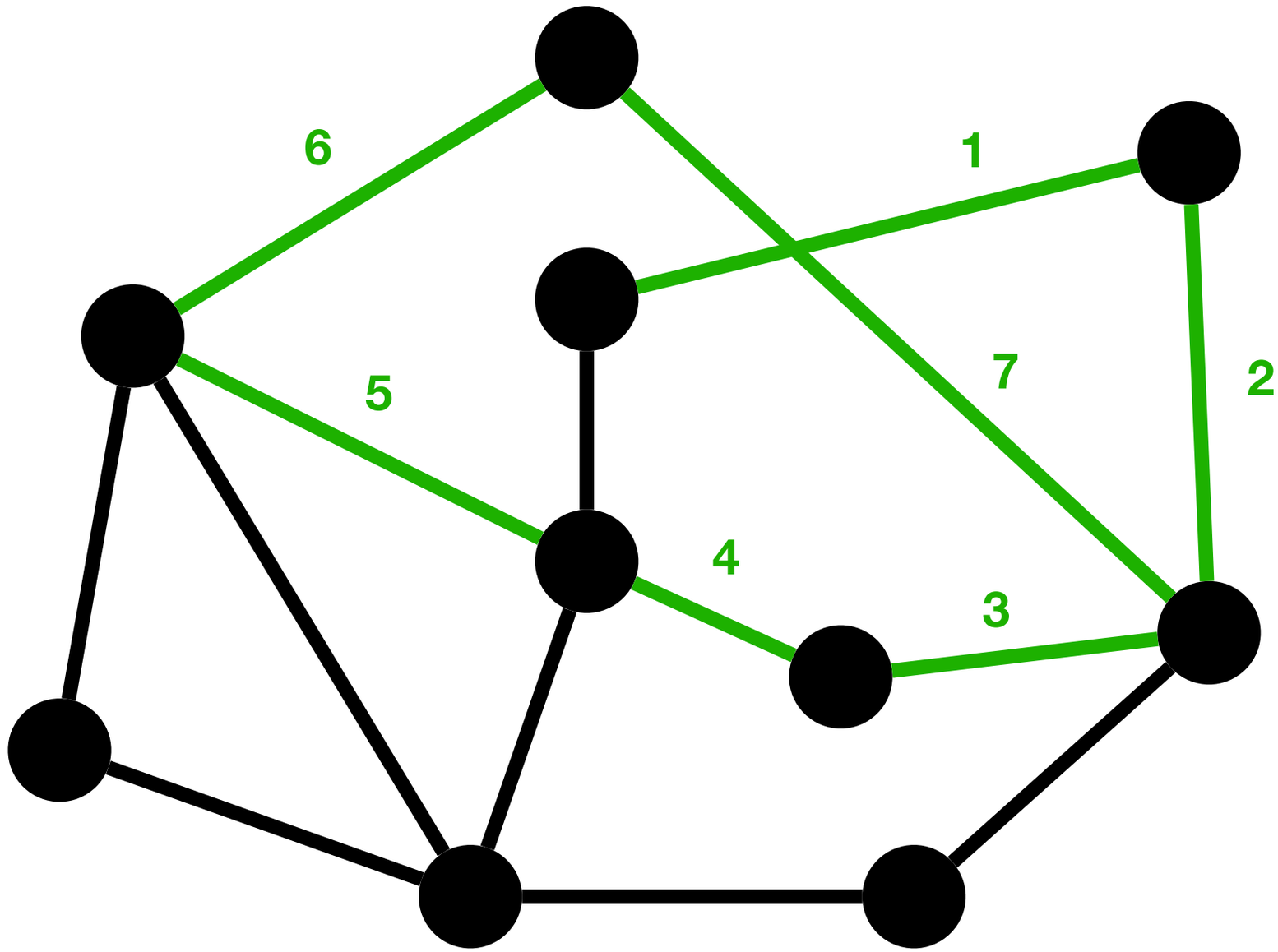

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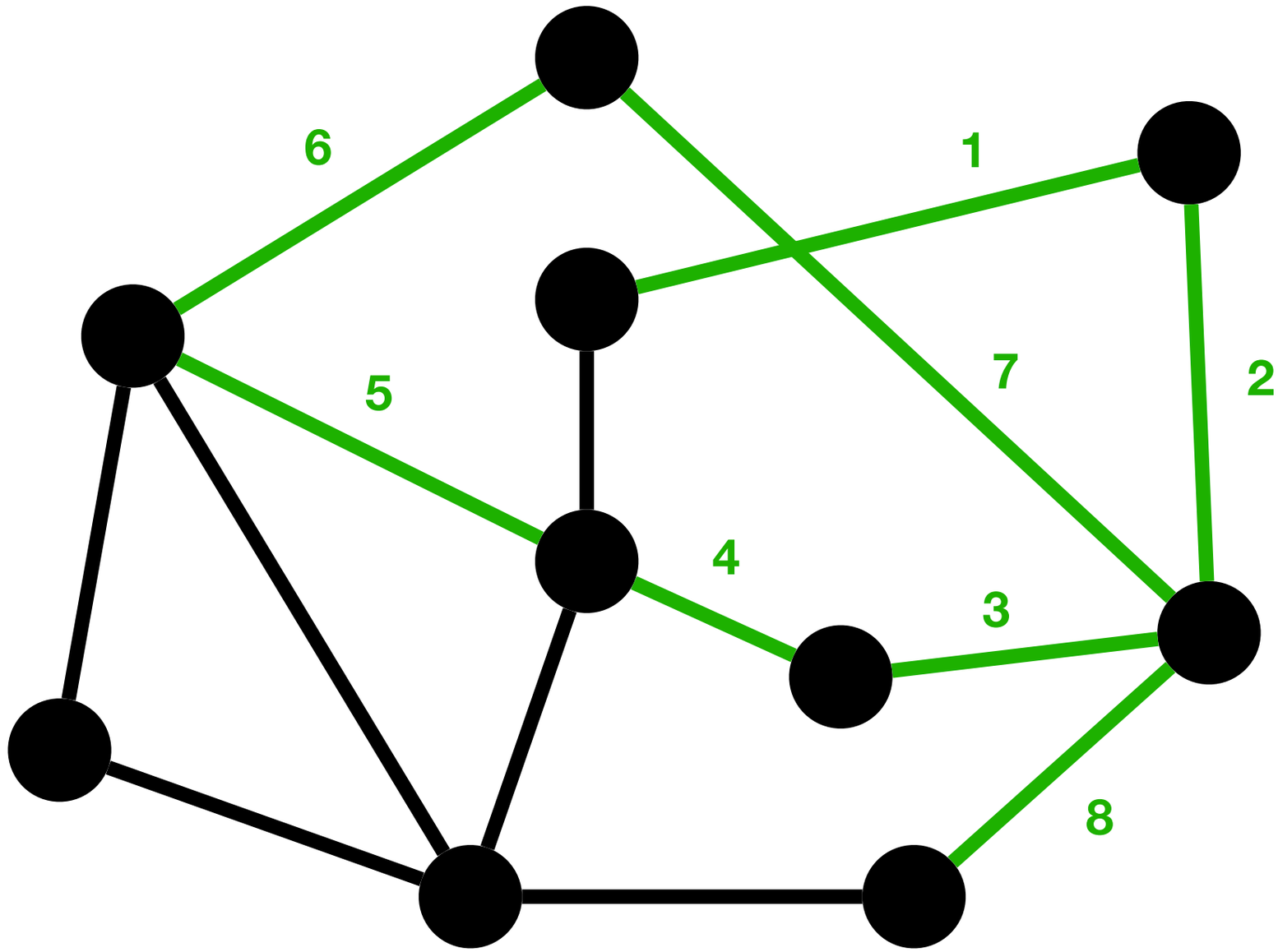

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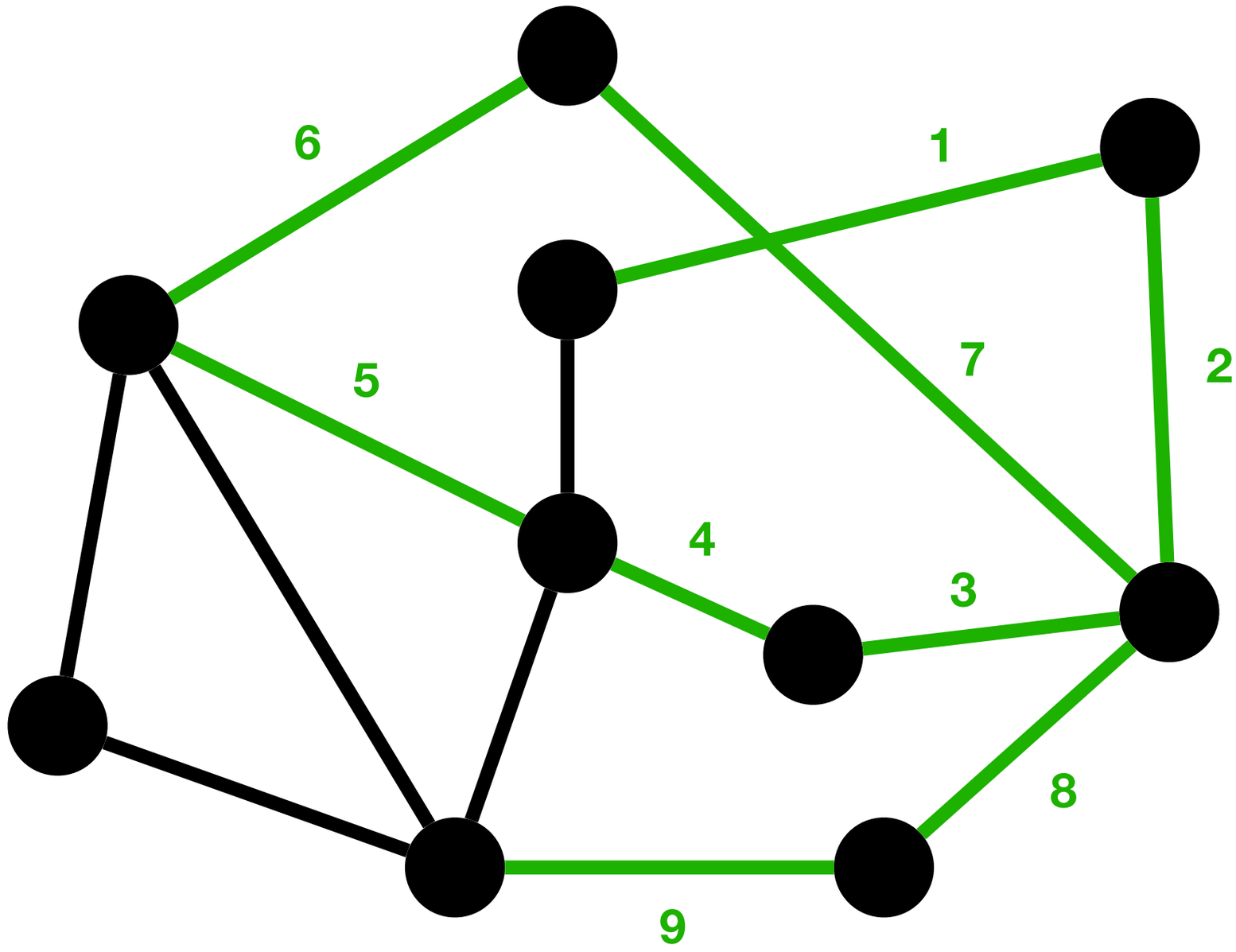

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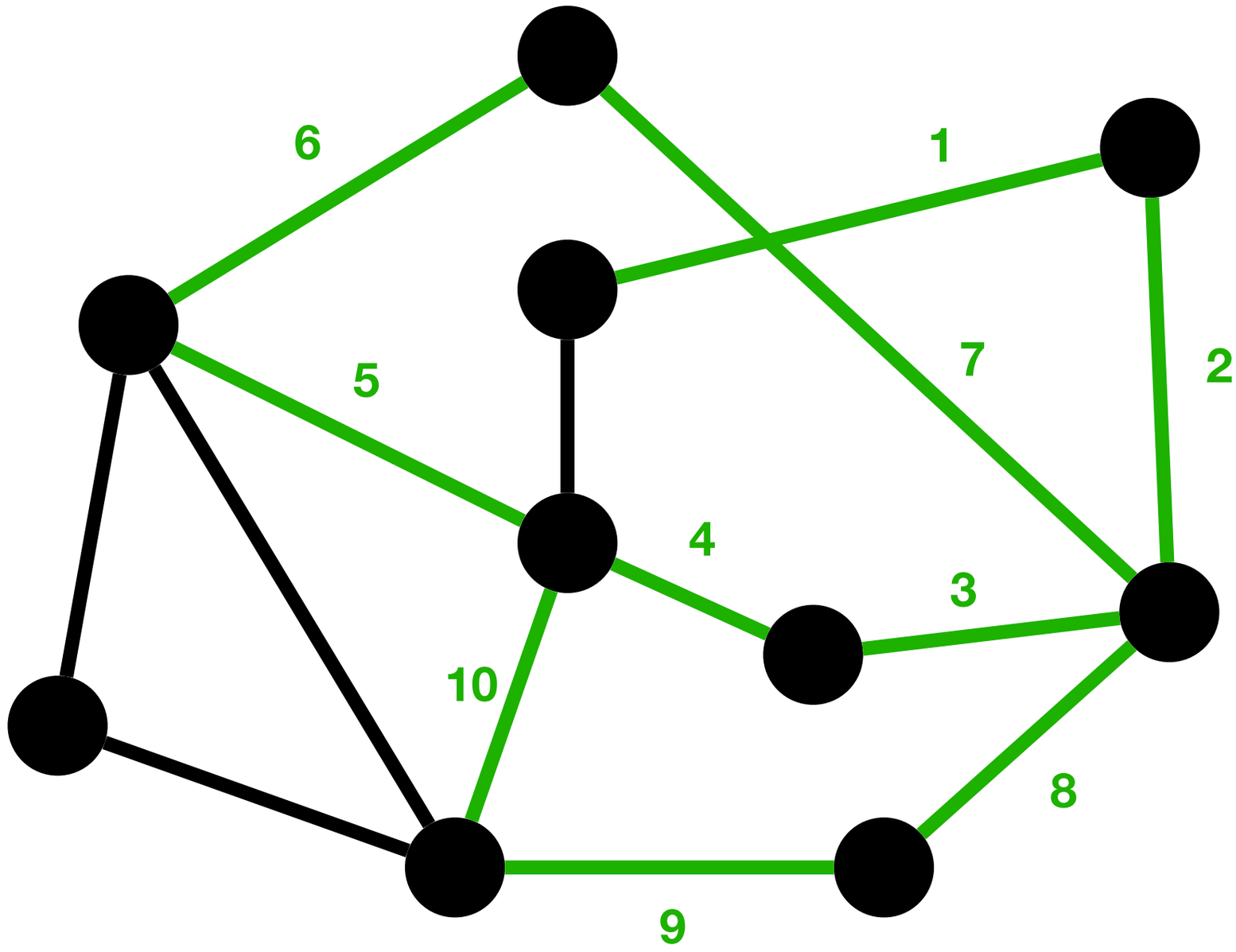

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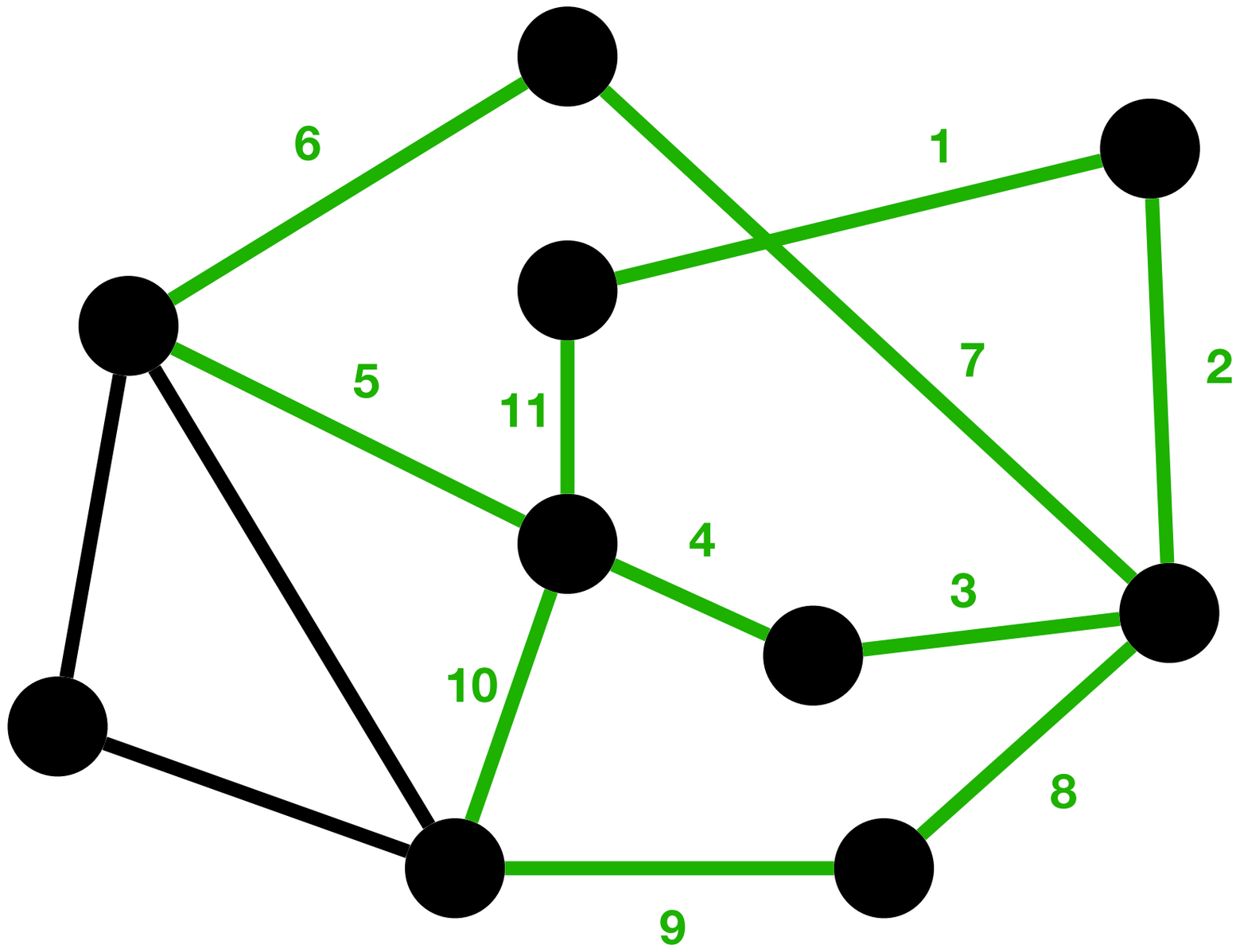

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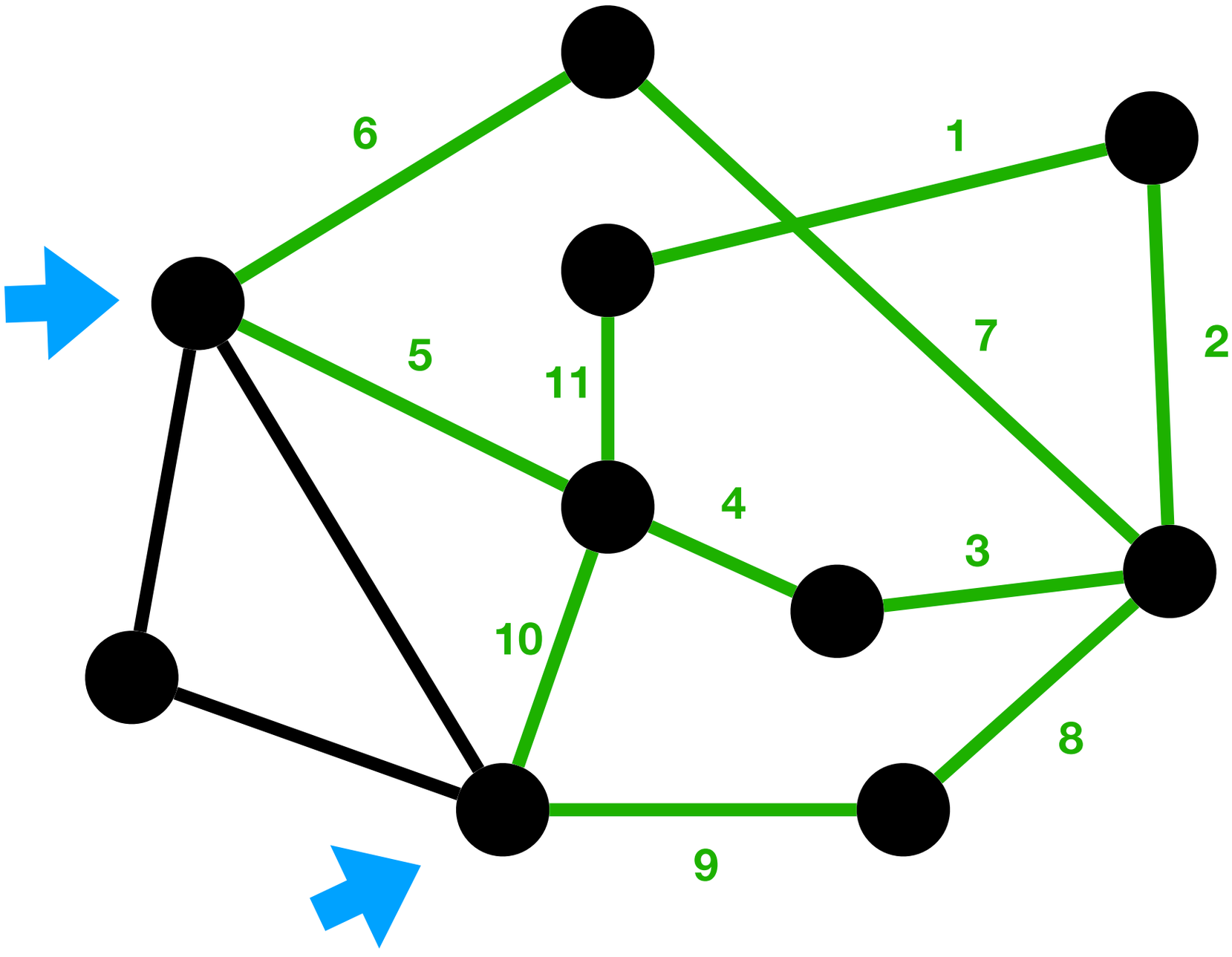

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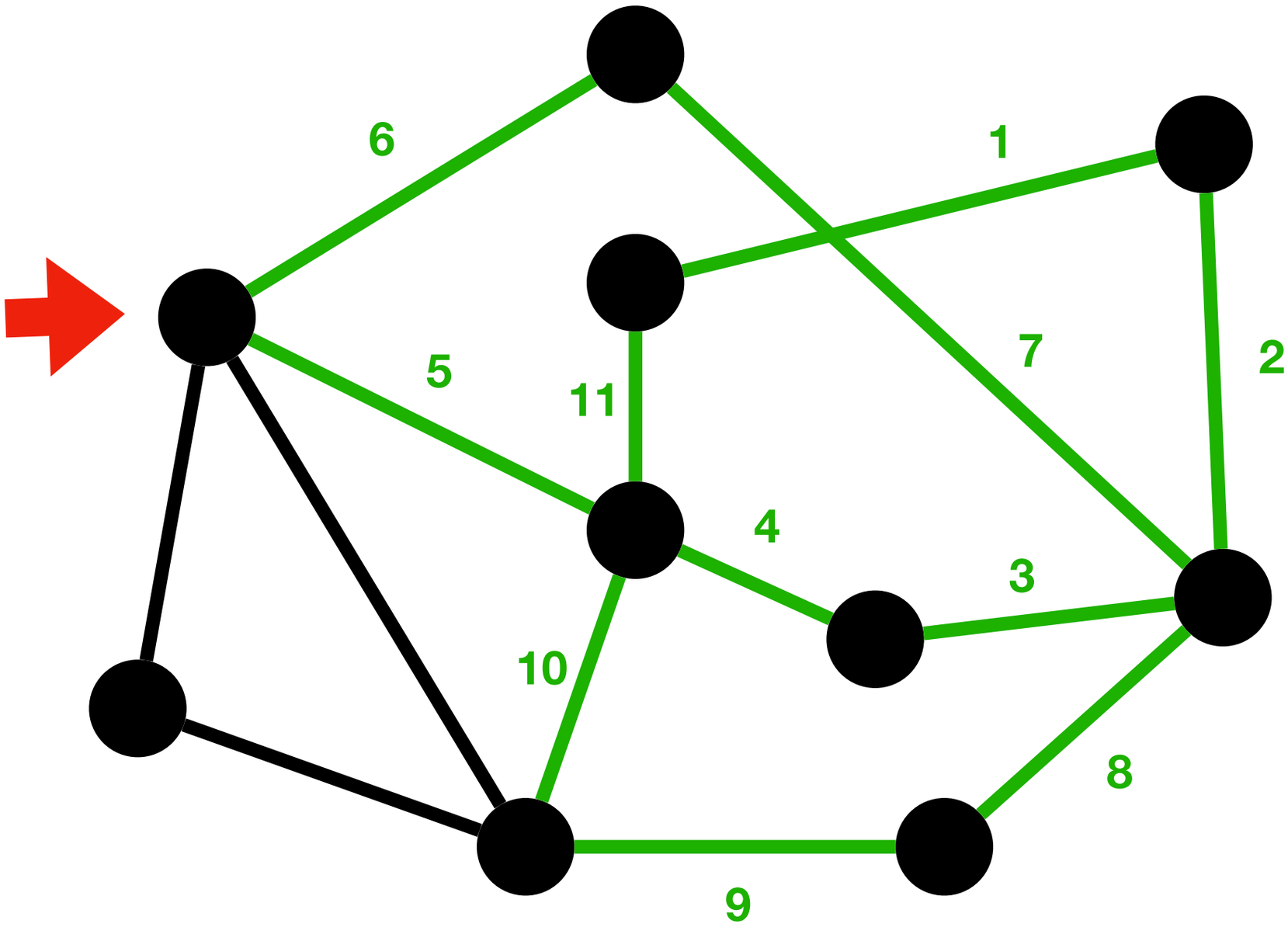

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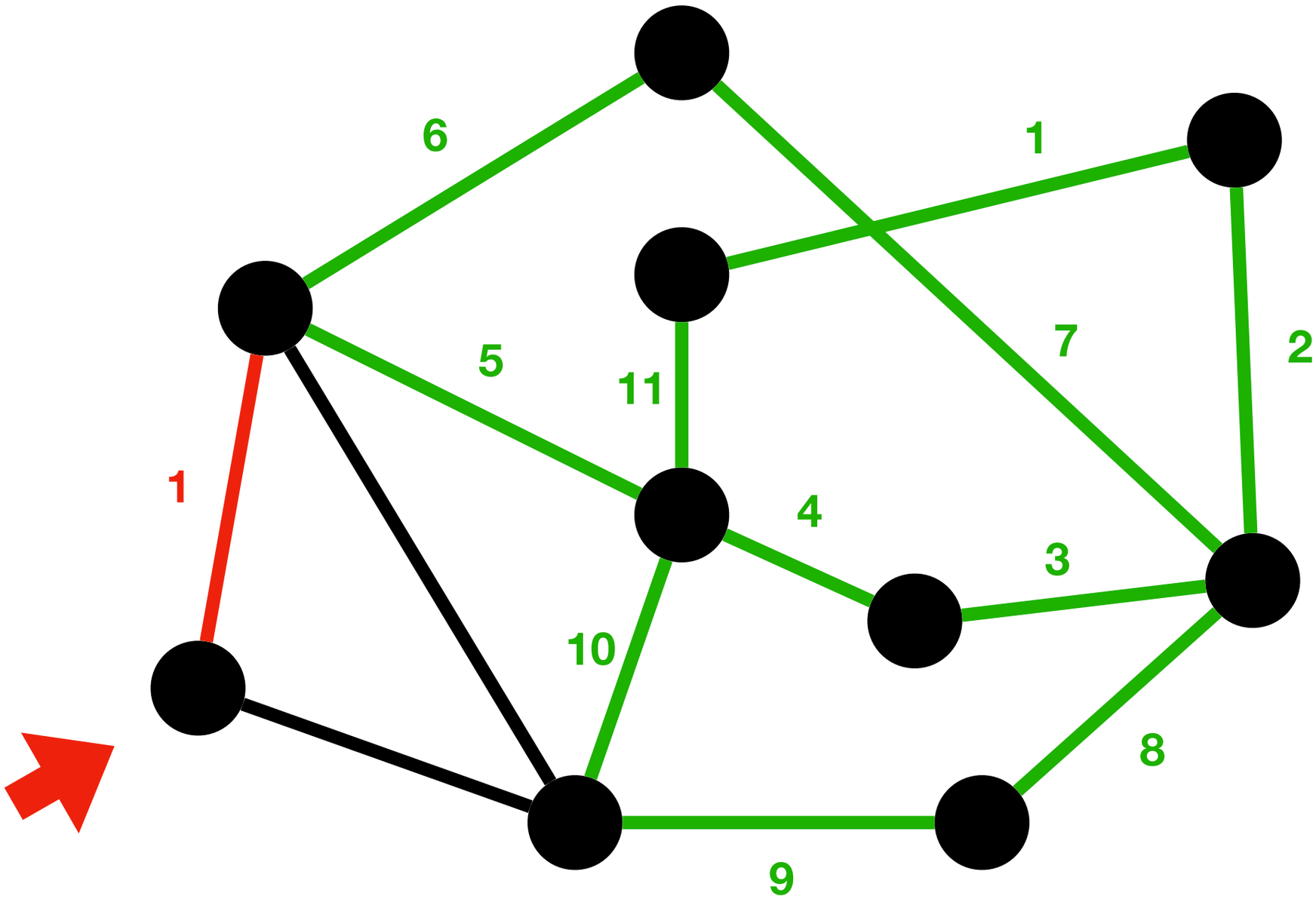

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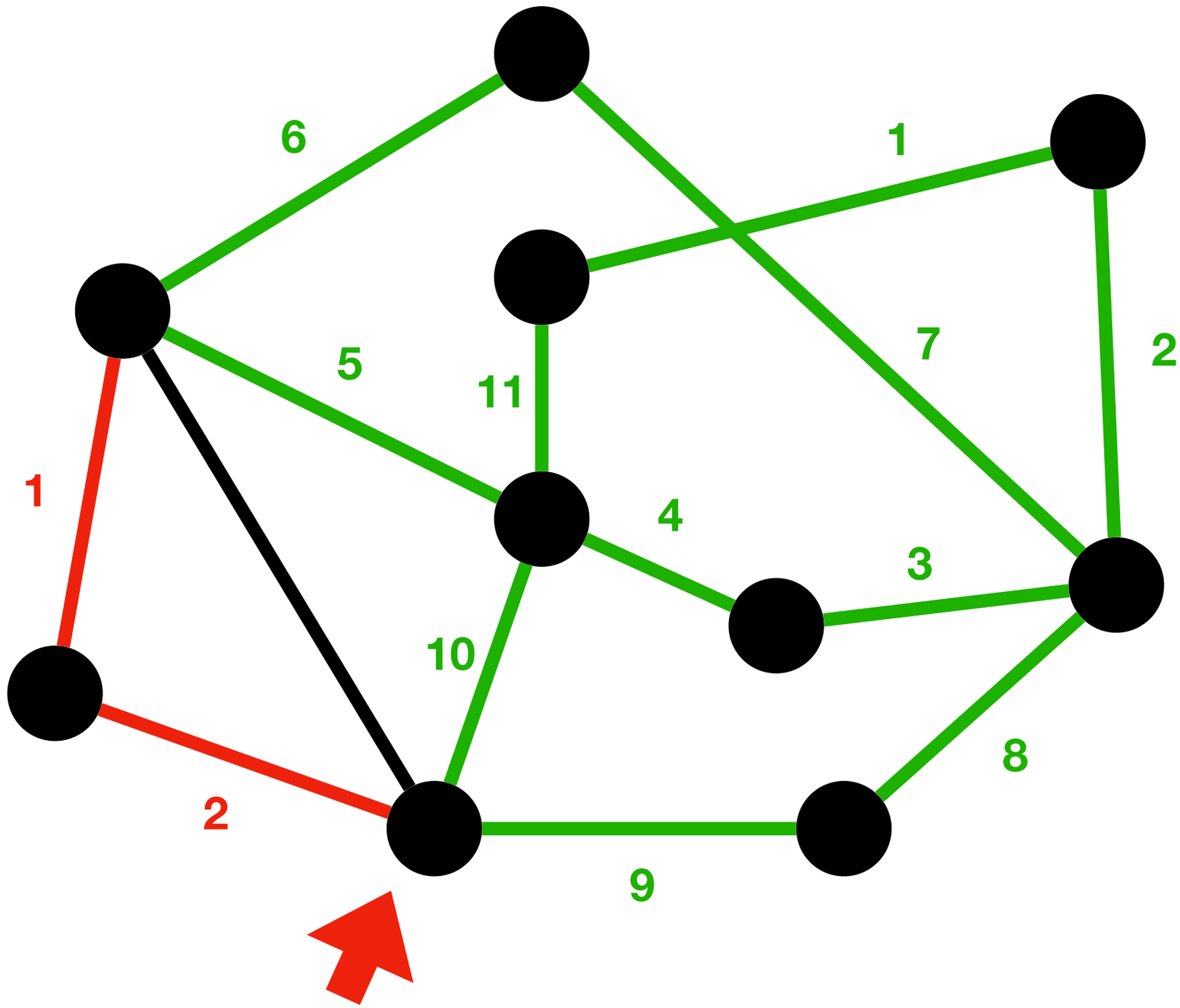

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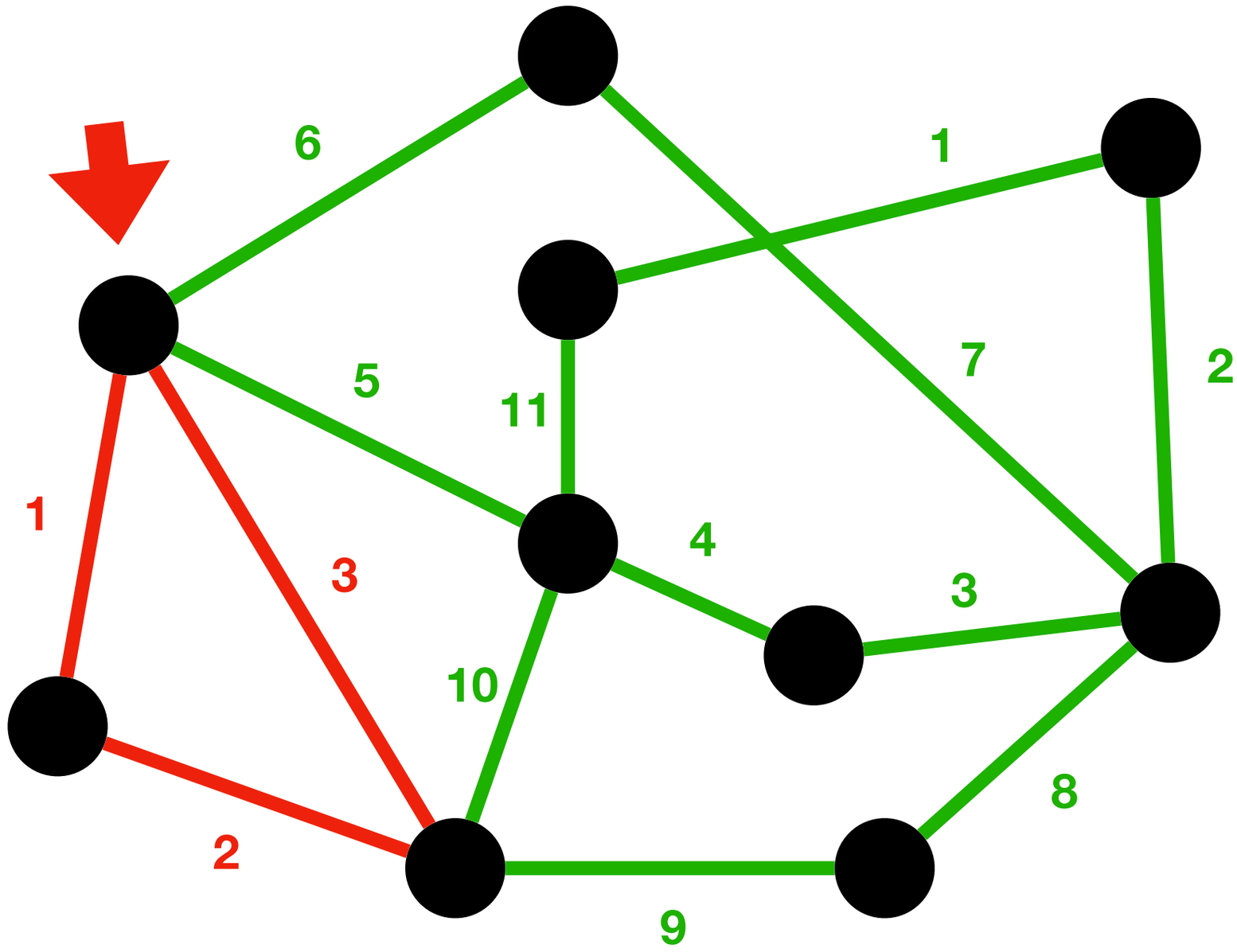

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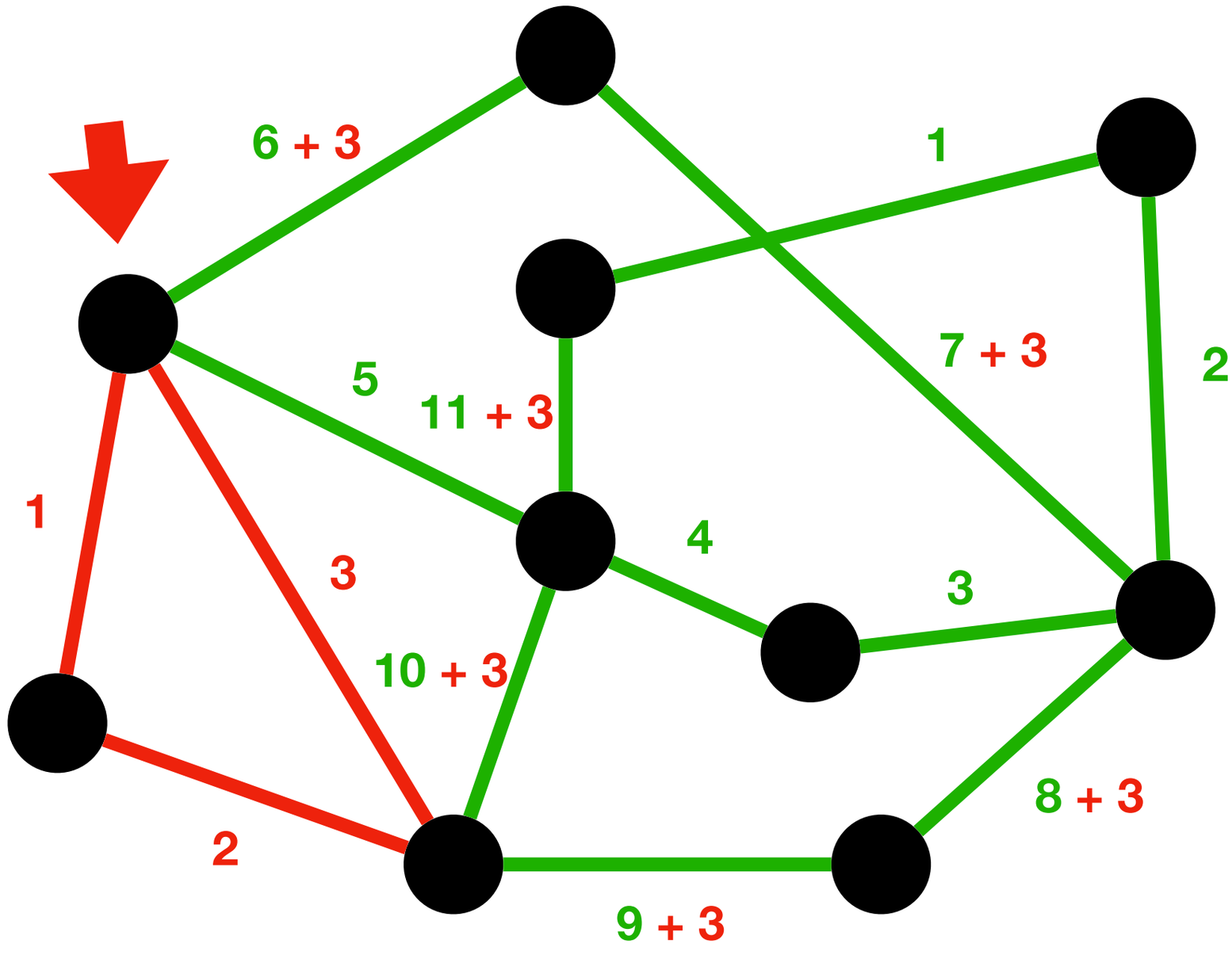

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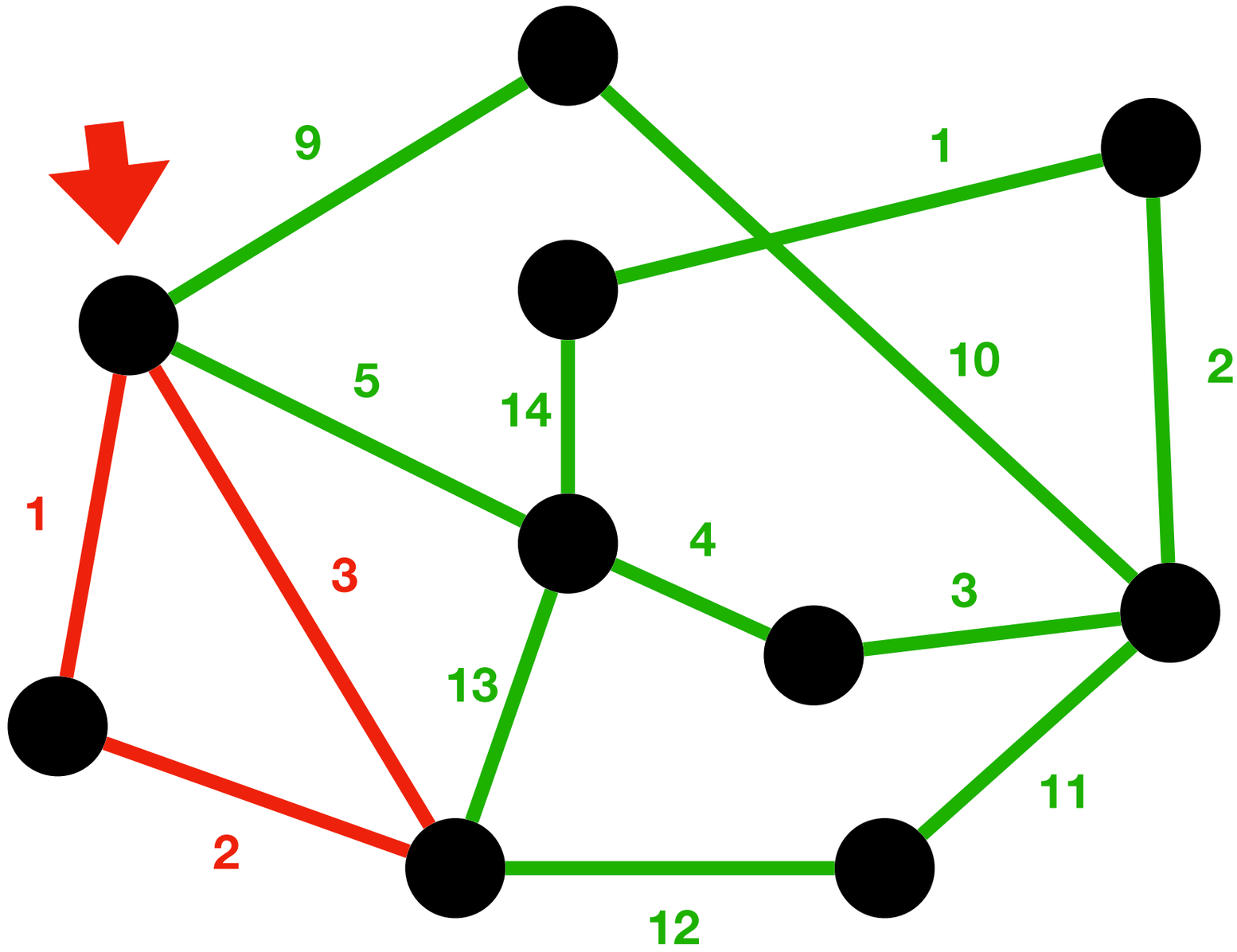

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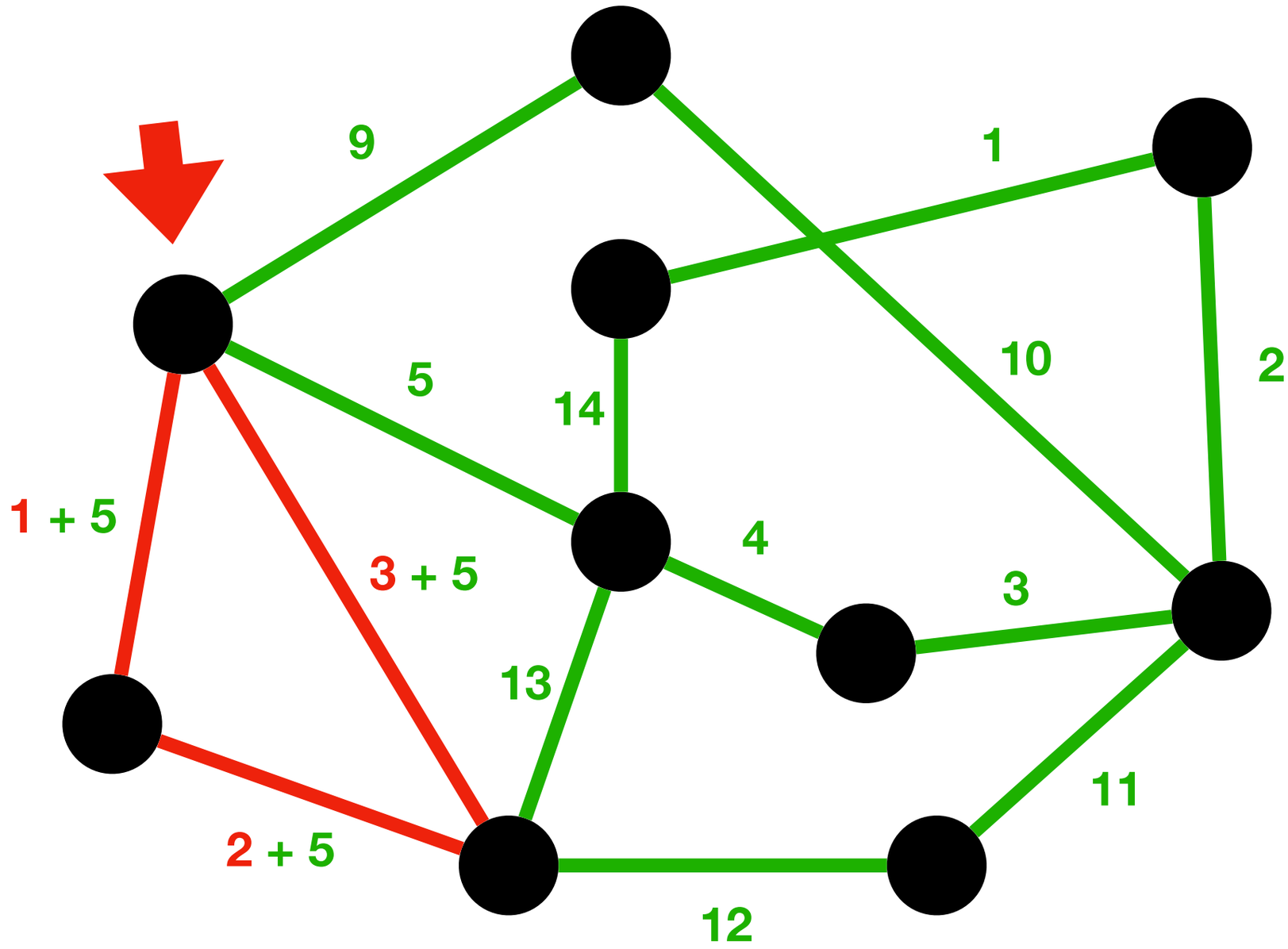

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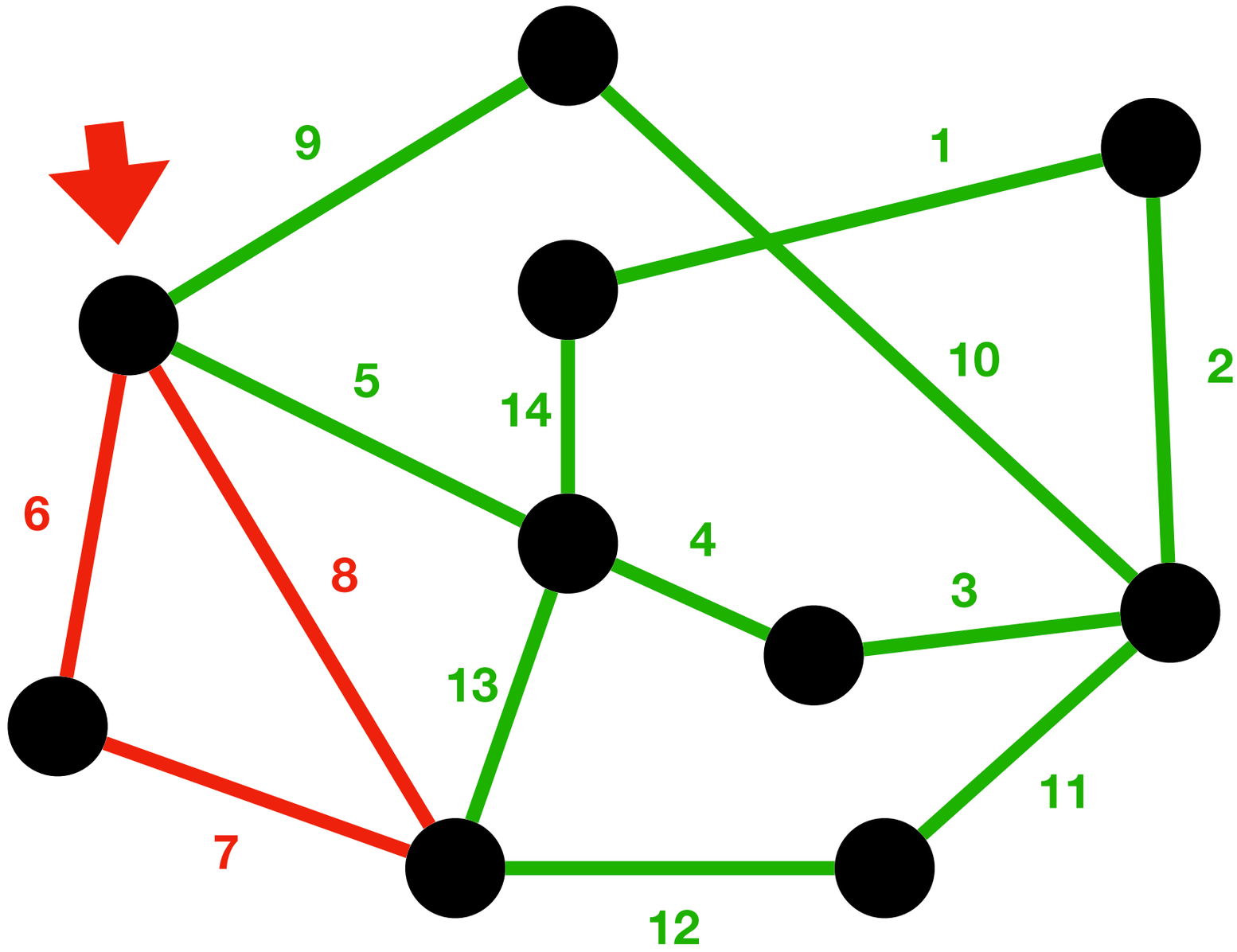

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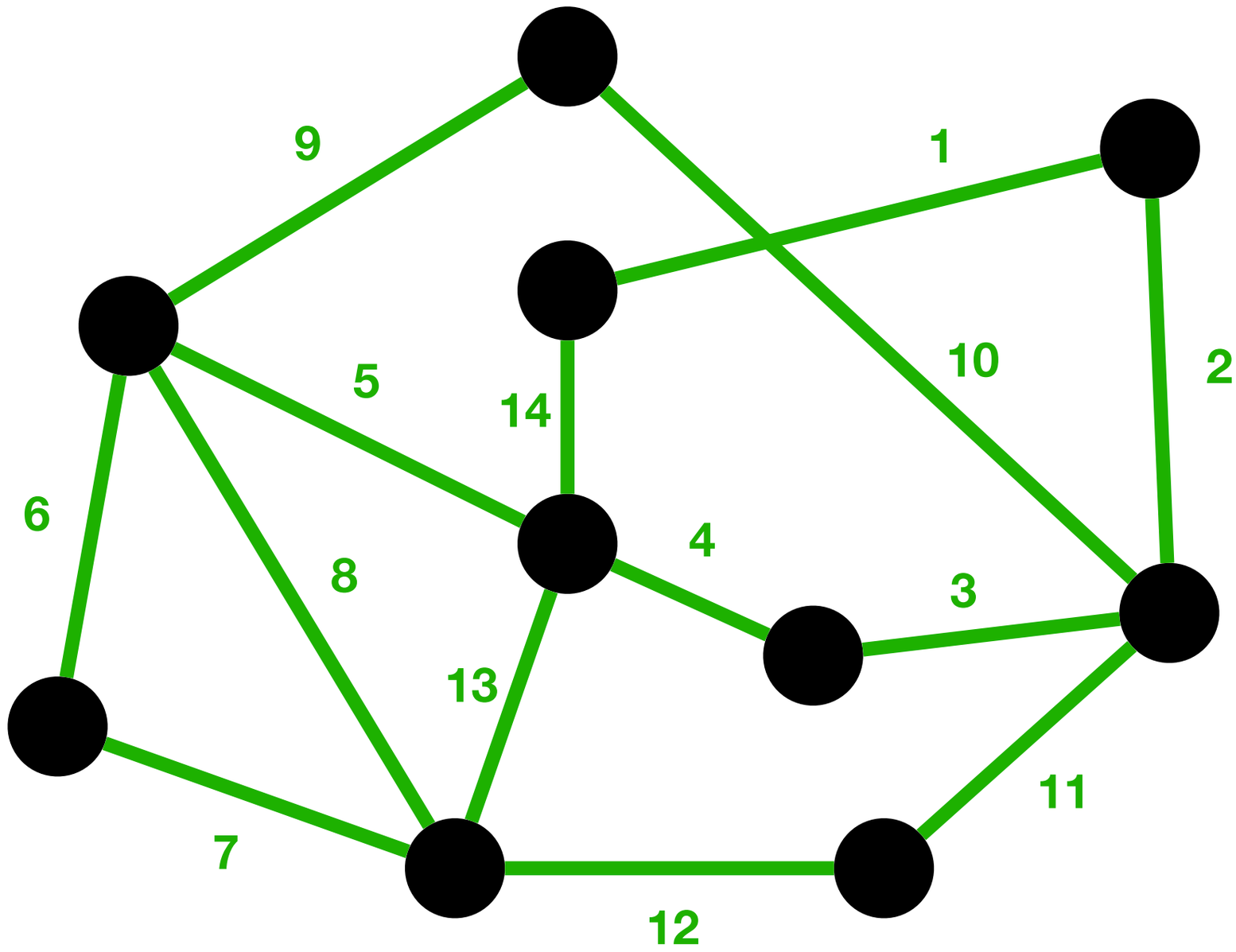

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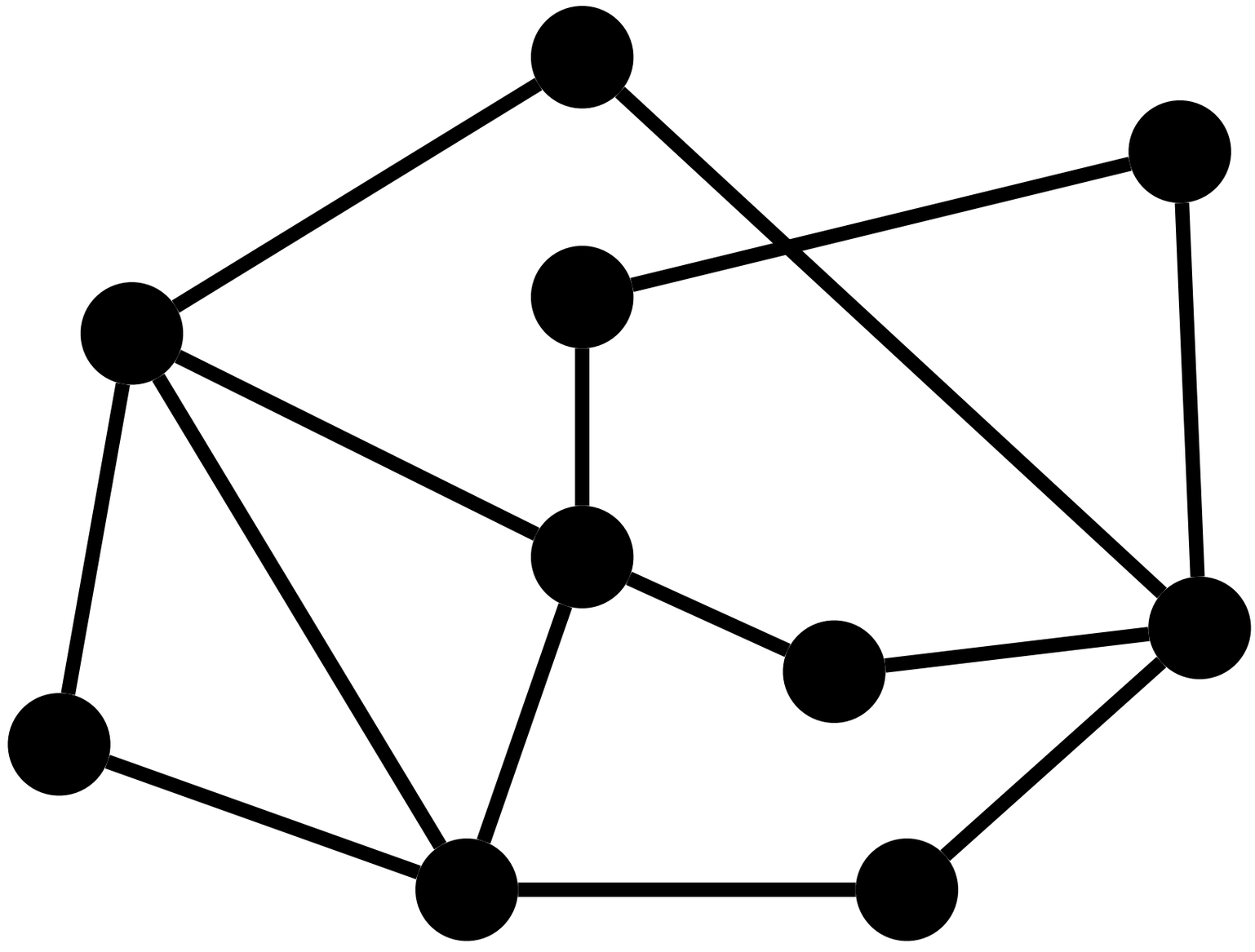

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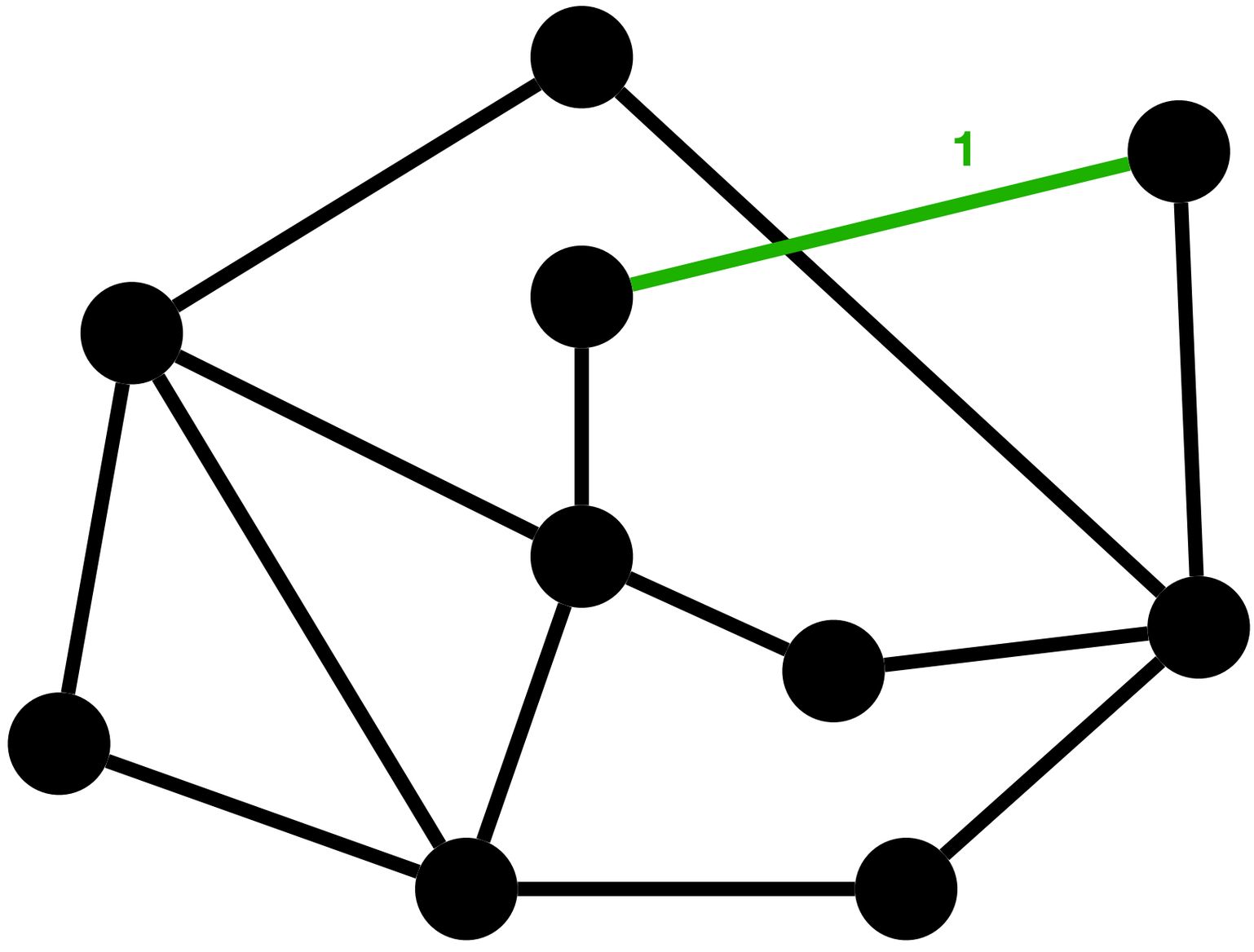

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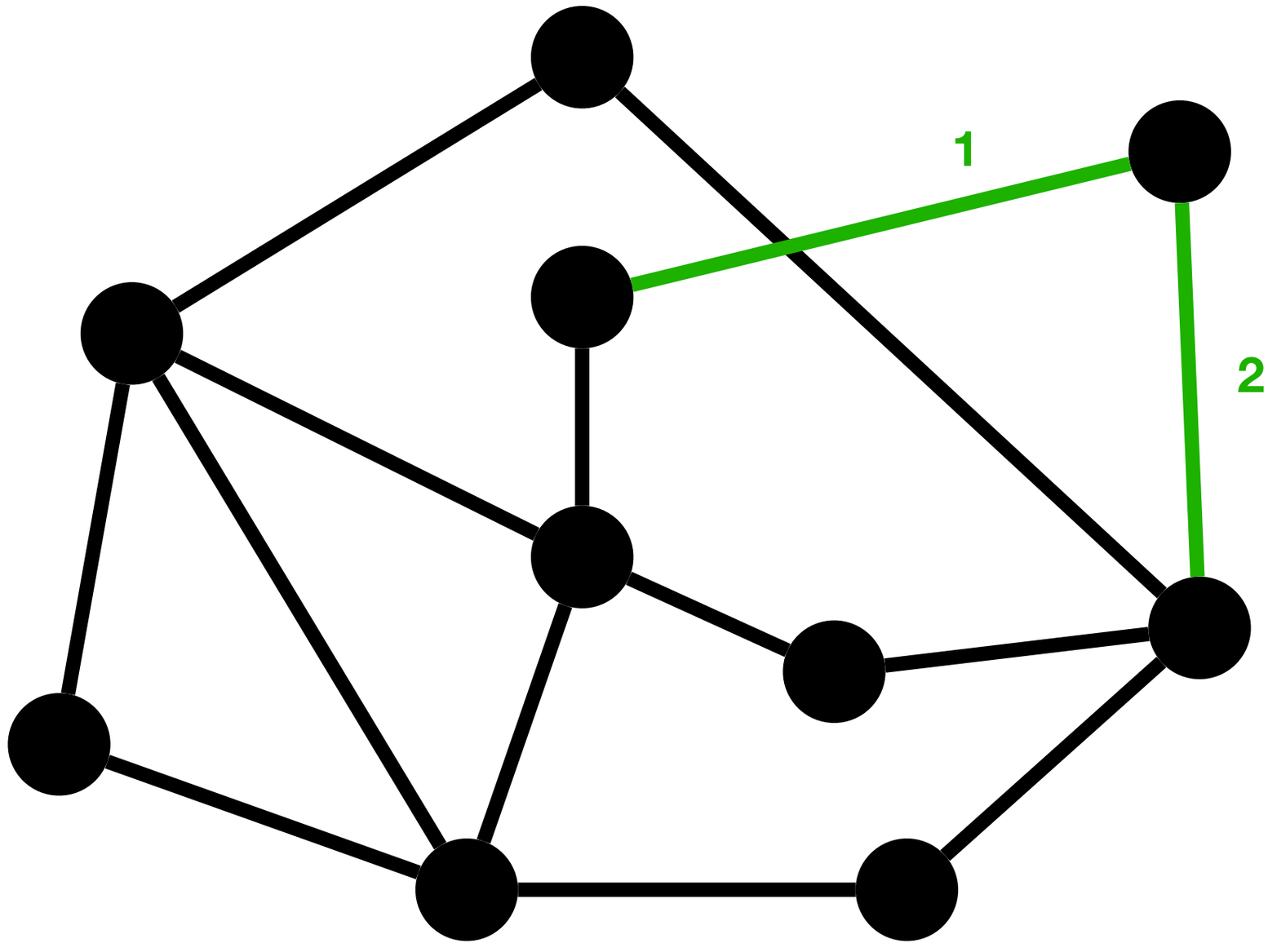

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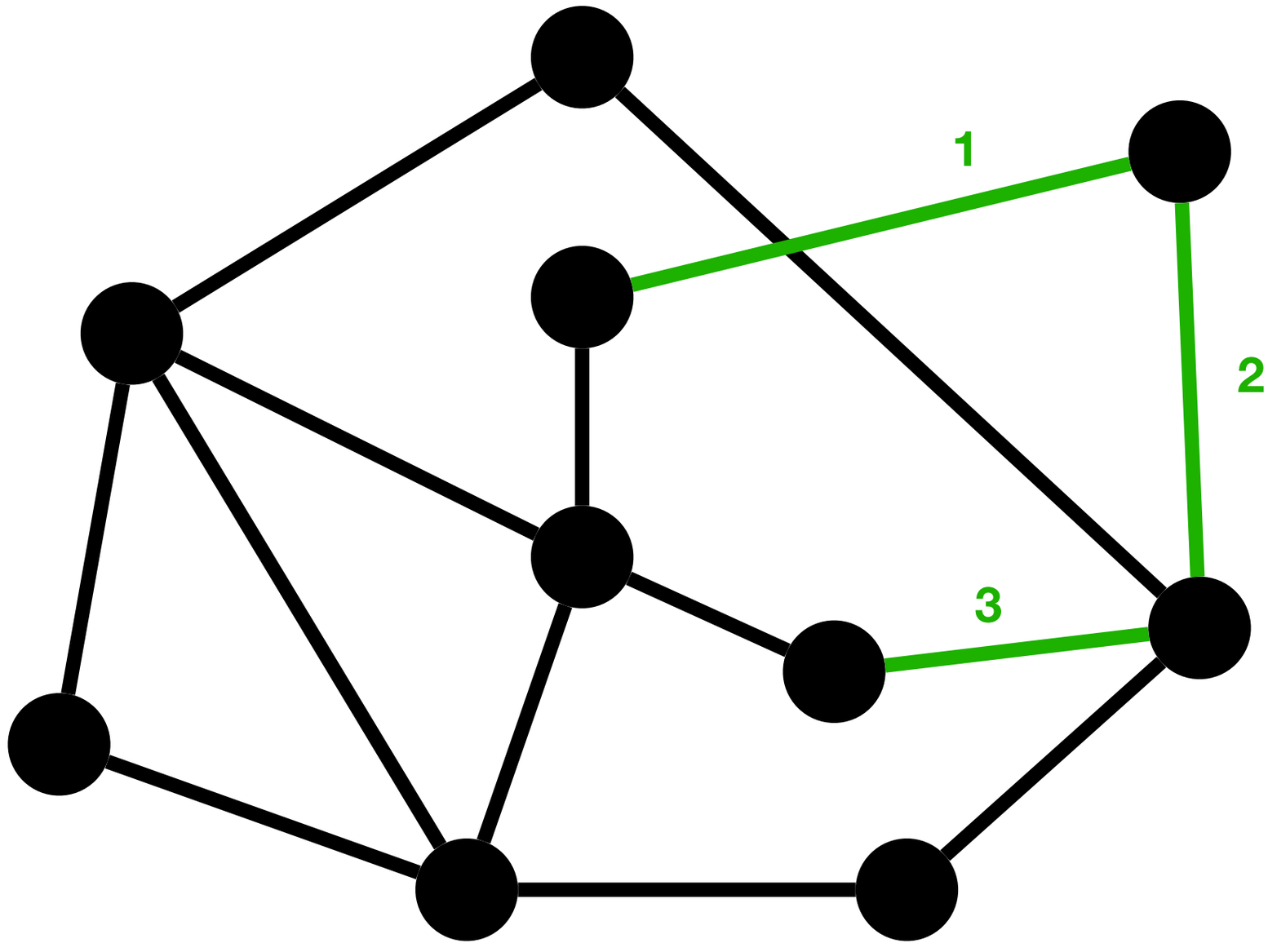

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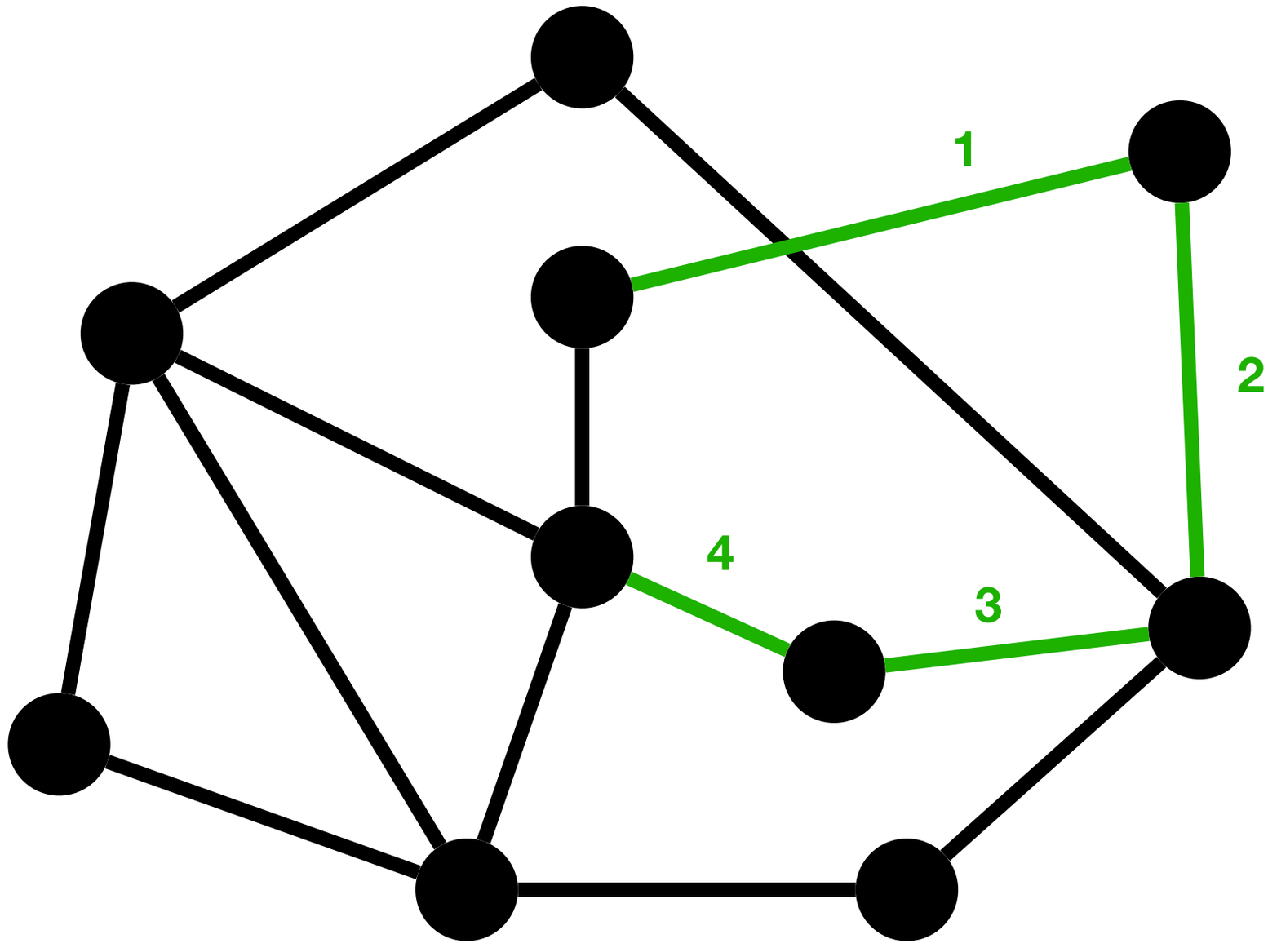

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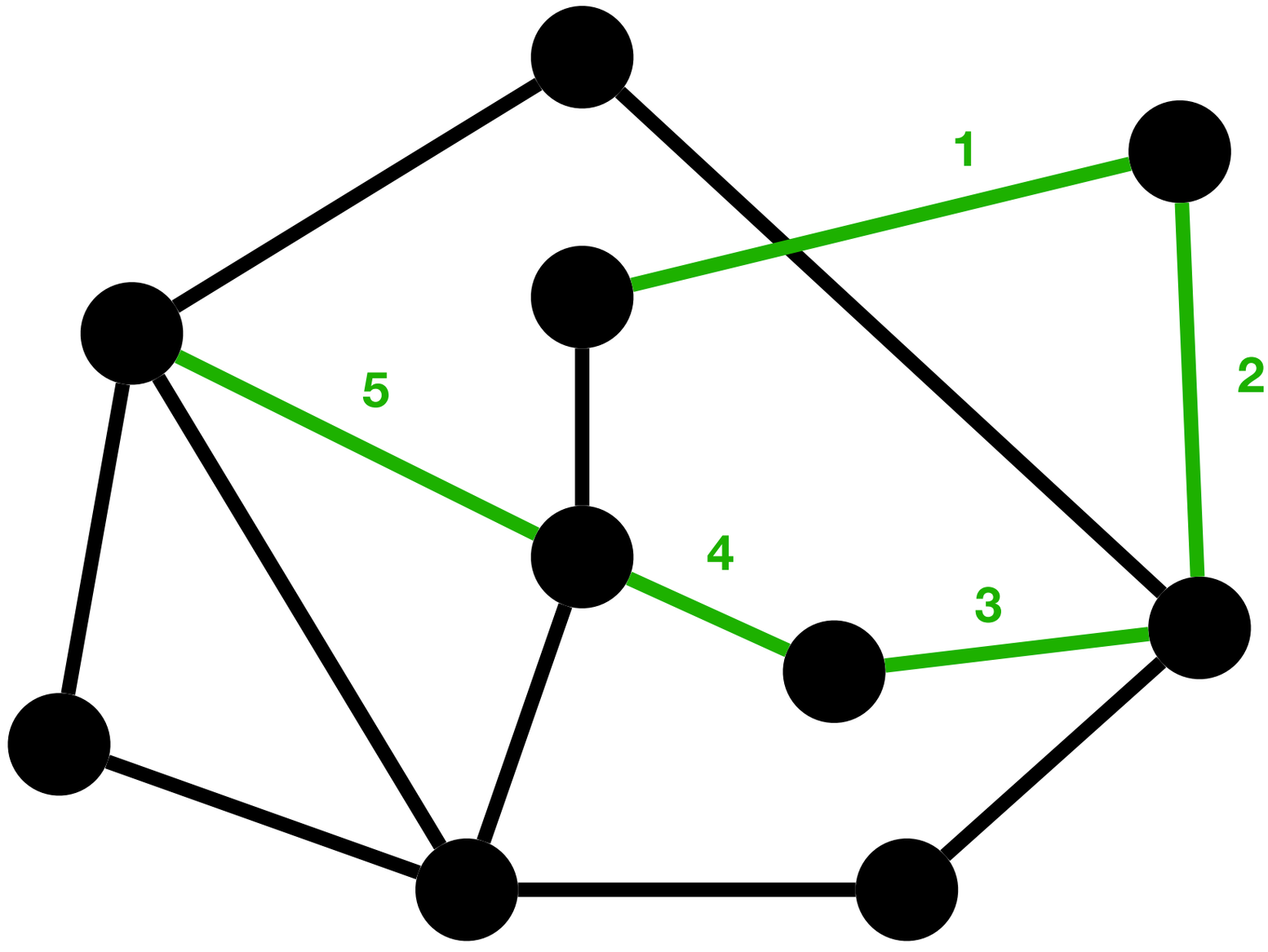

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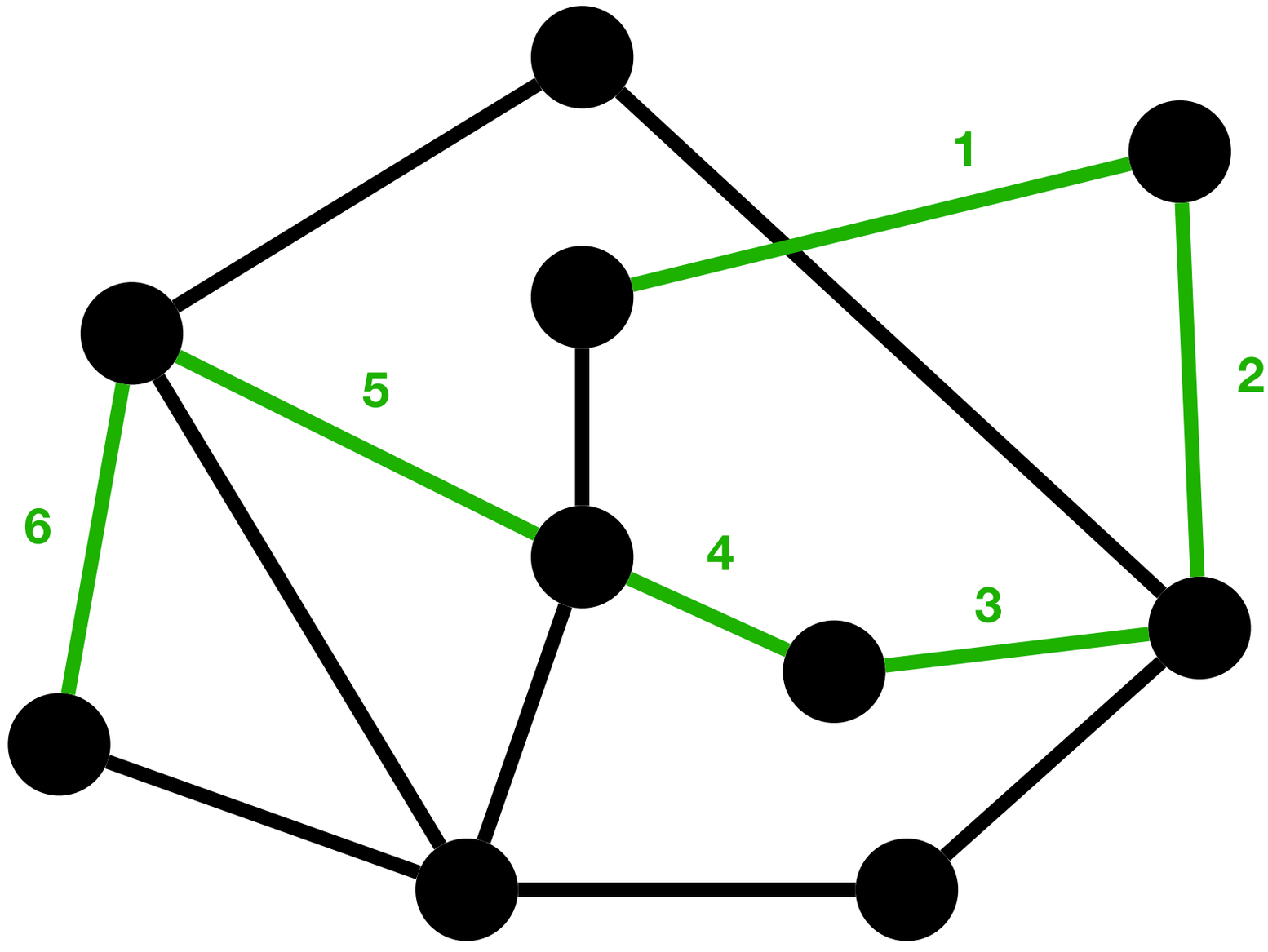

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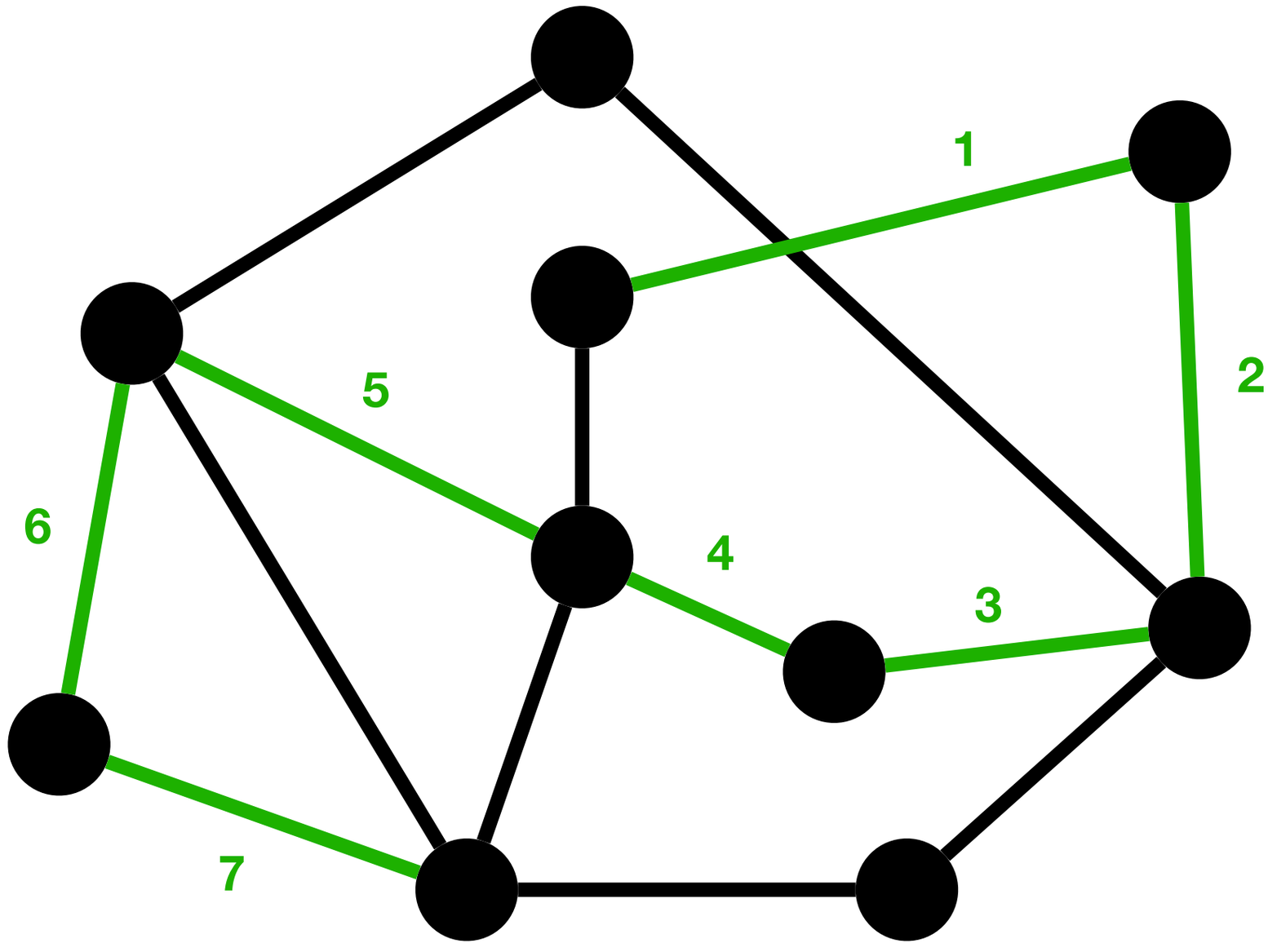

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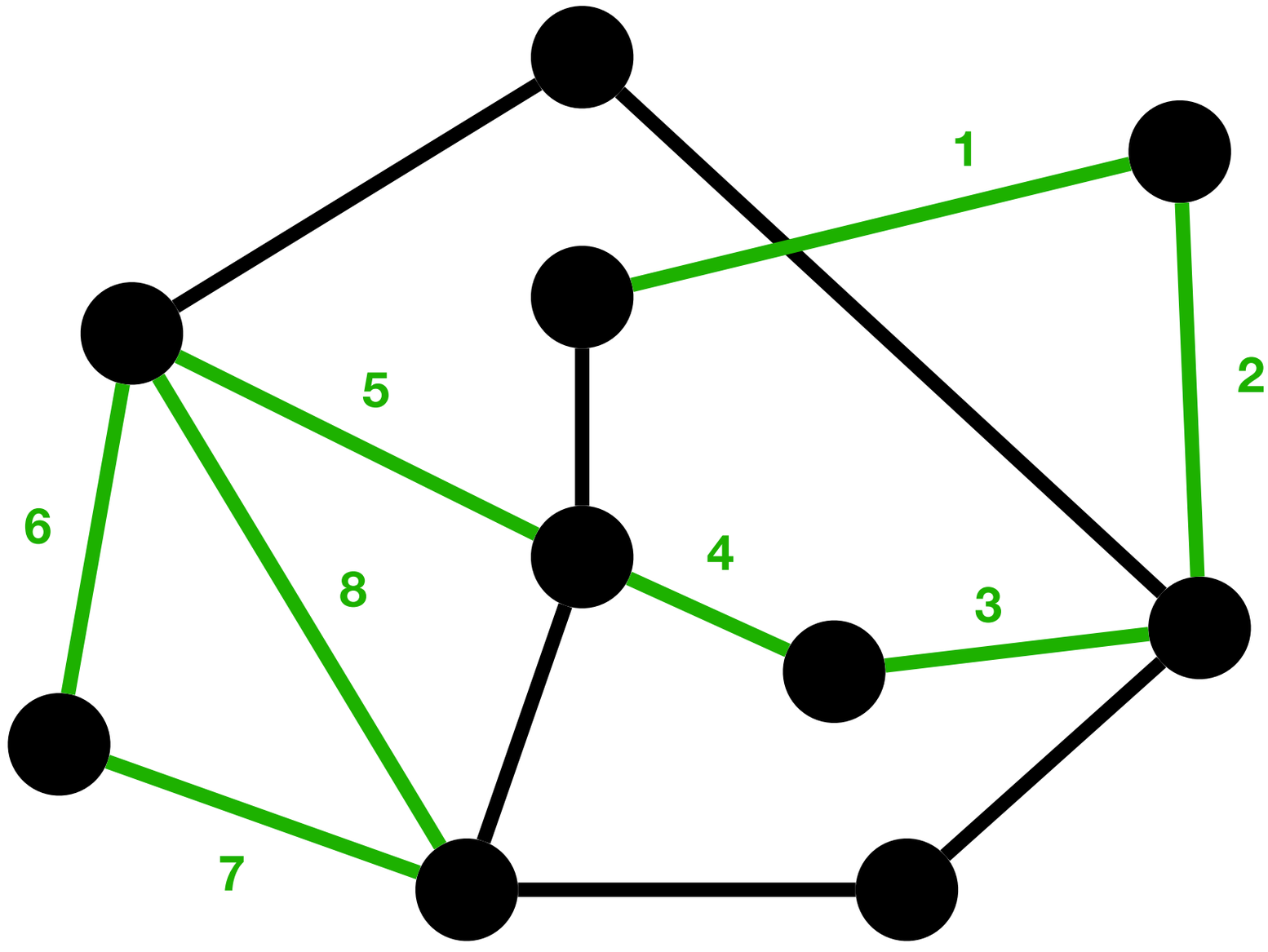

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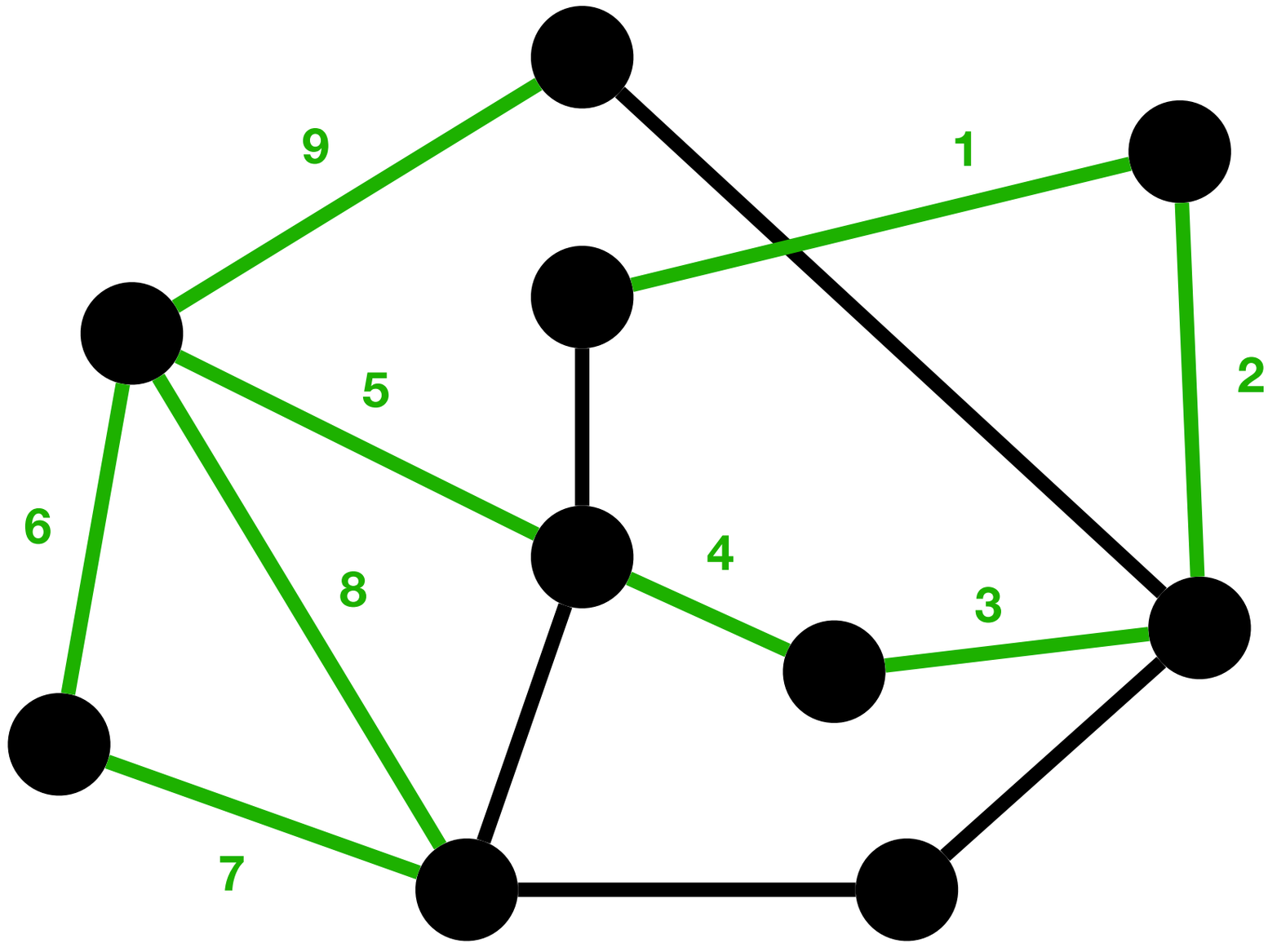

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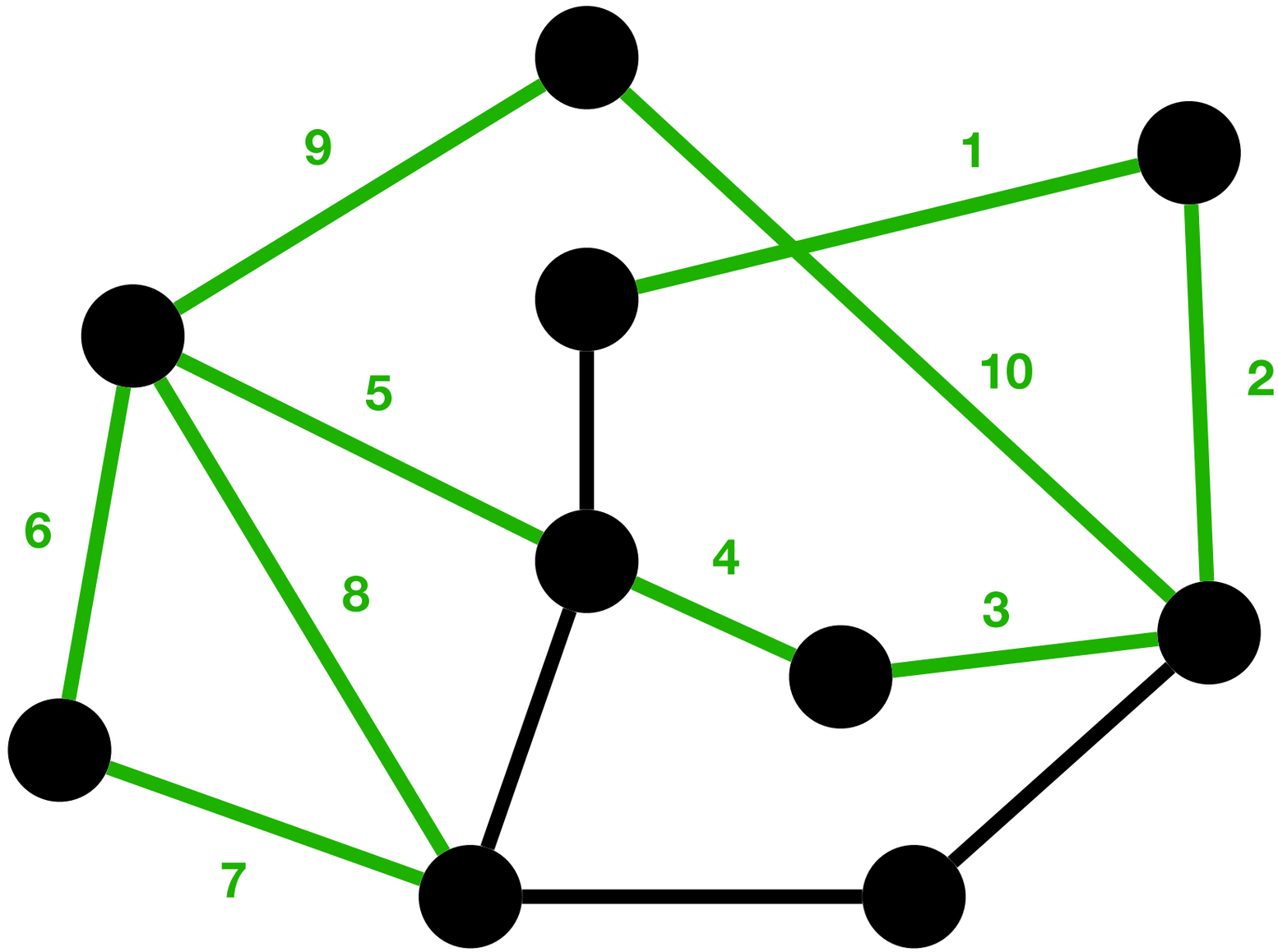

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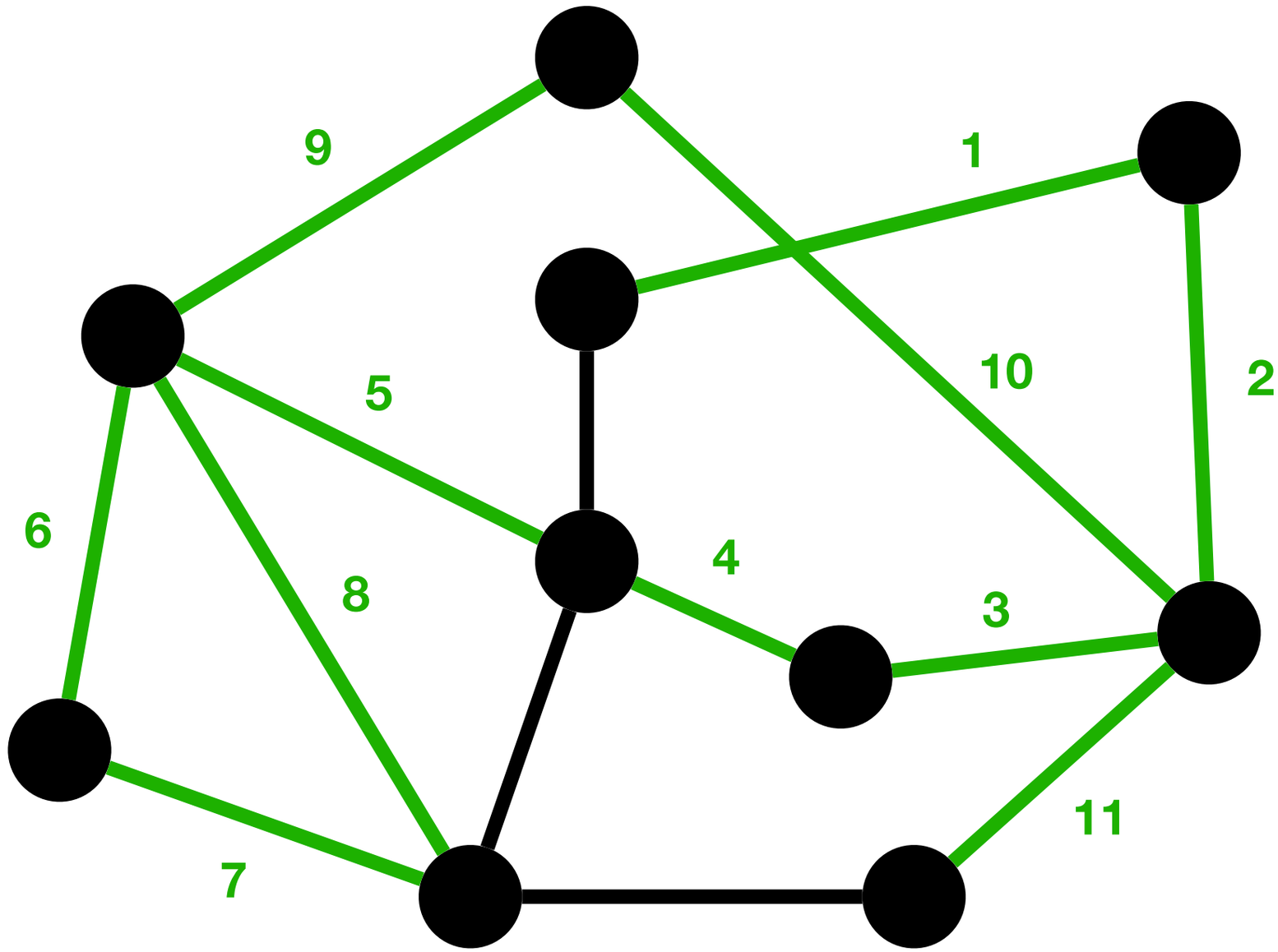

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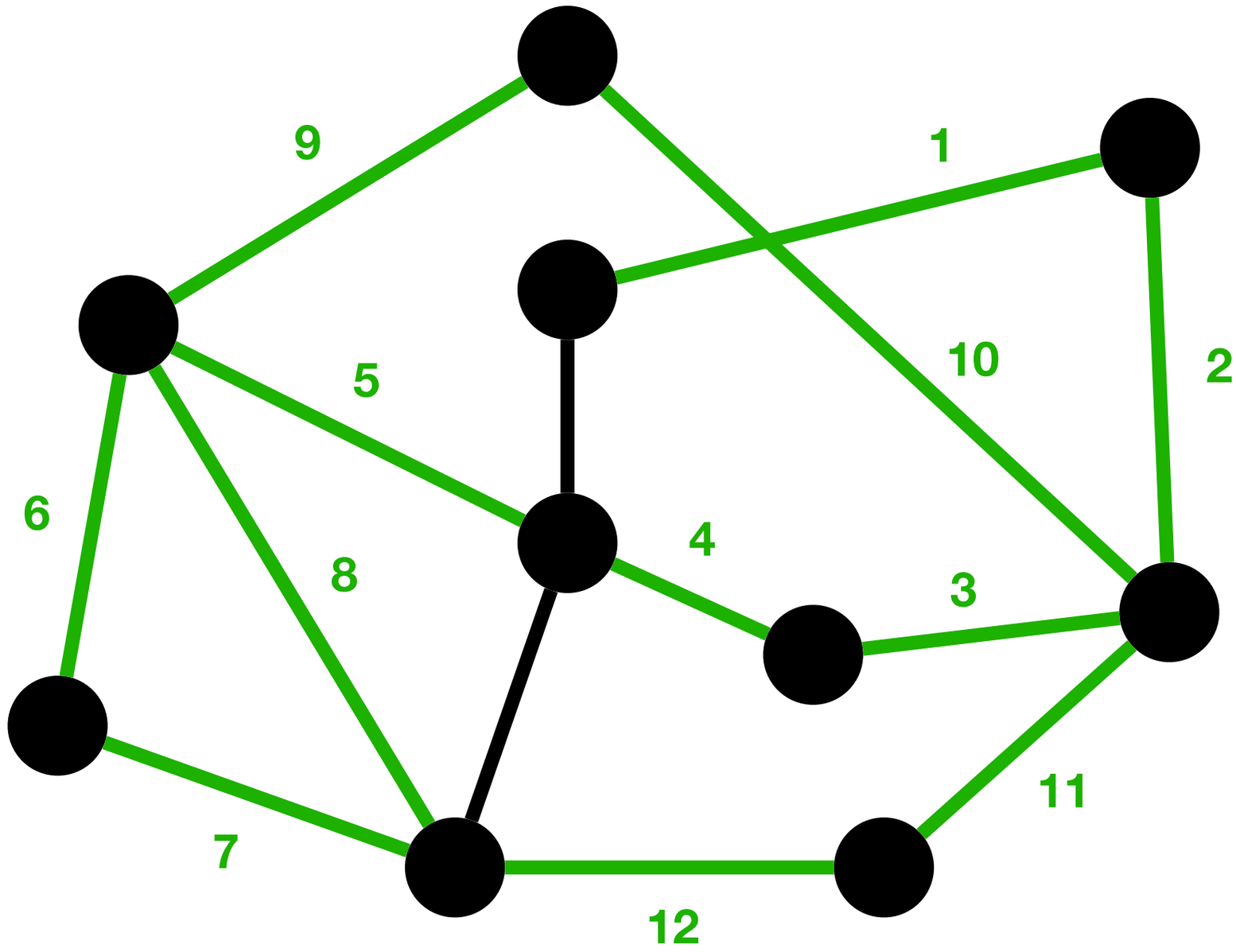

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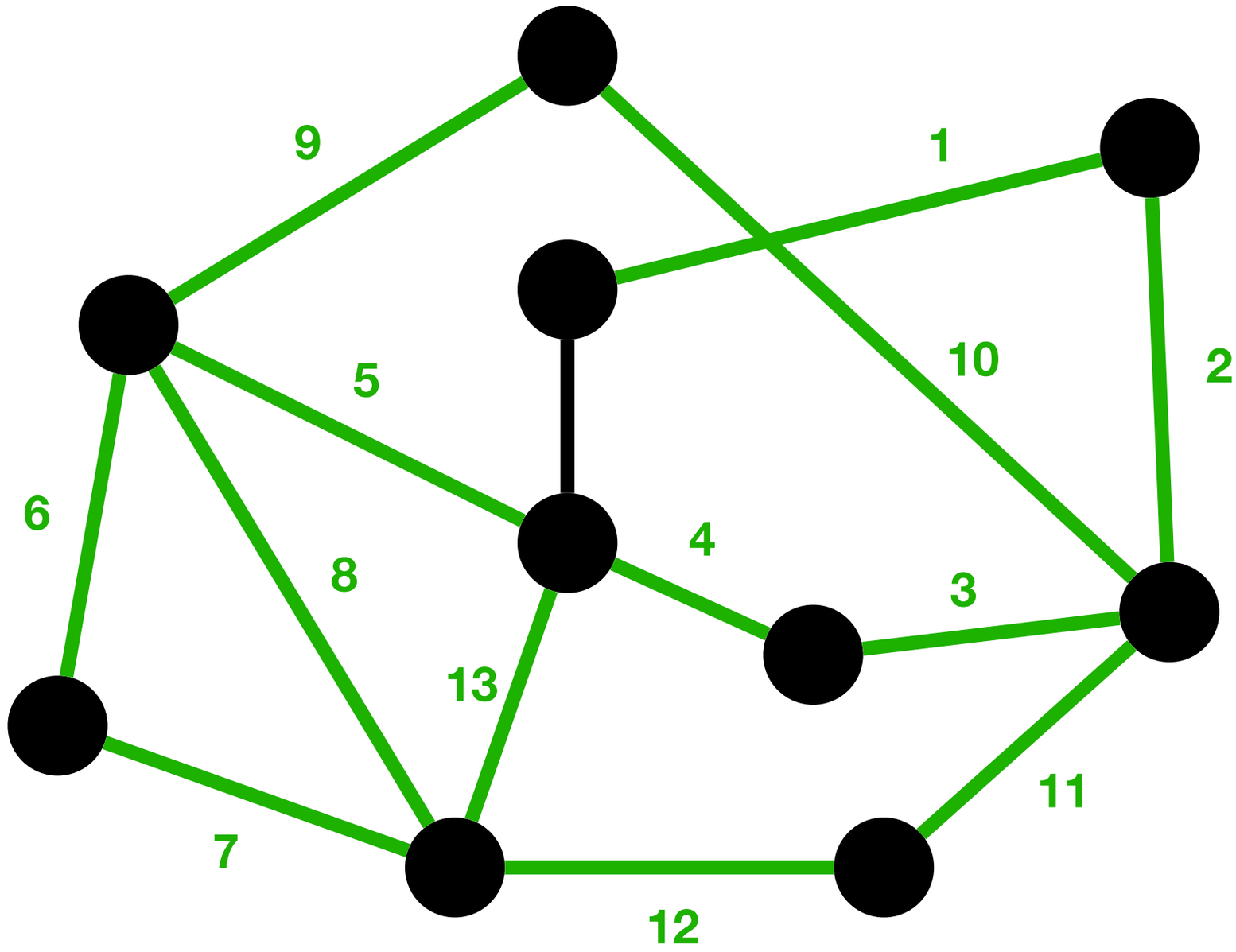

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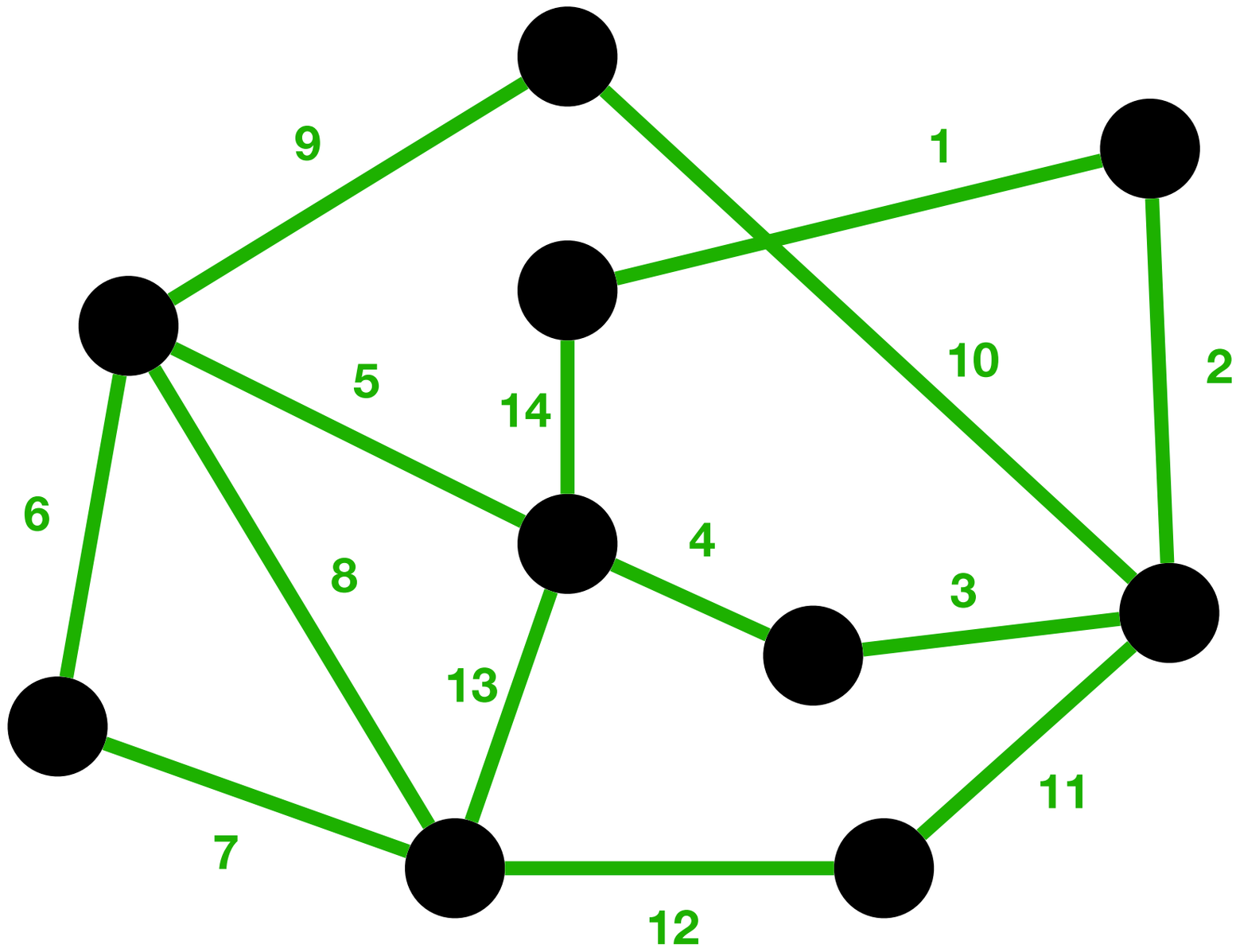

Algorithme de Hierholzer

- Choisir n'importe quel sommet initial *v*

- Suivre un chemin arbitraire d'arêtes jusqu'à retourner à *v*, en traversant chaque arête une fois au plus, obtenant ainsi un cycle partiel *c*

- **Tant qu**'il y a des sommets *u* dans le cycle *c* avec des arêtes qu'on n'a pas encore choisi **faire**

    - Suivre un chemin de sommets à partir de *u* jusqu'à retourner à *u*, obtenant un cycle *c'*

    - Prolonger le cycle *c* par *c'*

Terminaison et correction de l’algorithme de Hierholzer

Terminaison et correction de l’algorithme de Hierholzer

- « Suivre un chemin arbitraire d’arêtes jusqu’à retourner à *v*, en traversant chaque arête une fois au plus, obtenant ainsi un cycle partiel *c* »

Terminaison et correction de l'algorithme de Hierholzer

- « Suivre un chemin arbitraire d'arêtes jusqu'à retourner à *v*, en traversant chaque arête une fois au plus, obtenant ainsi un cycle partiel *c* »

- On ne peut pas rester bloqué dans aucun sommet different de *v*, puisque ils ont toujours un nombre pair d'arêtes pas encore explorées (« si on entre dans un sommet, on peut toujours en sortir »)

Terminaison et correction de l'algorithme de Hierholzer

- « Suivre un chemin arbitraire d'arêtes jusqu'à retourner à *v*, en traversant chaque arête une fois au plus, obtenant ainsi un cycle partiel *c* »

- On ne peut pas rester bloqué dans aucun sommet different de *v*, puisque ils ont toujours un nombre pair d'arêtes pas encore explorées (« si on entre dans un sommet, on peut toujours en sortir »)

- Et, tôt ou tard, on retourne à *v*, puisque dans le pire des cas on épuise l'ensemble des autres arêtes, et *v* a un nombre **impair** d'arêtes inexplorées (donc il reste une arête pour y entrer)

Terminaison et correction de l'algorithme de Hierholzer

- « Suivre un chemin arbitraire d'arêtes jusqu'à retourner à *v*, en traversant chaque arête une fois au plus, obtenant ainsi un cycle partiel *c* »

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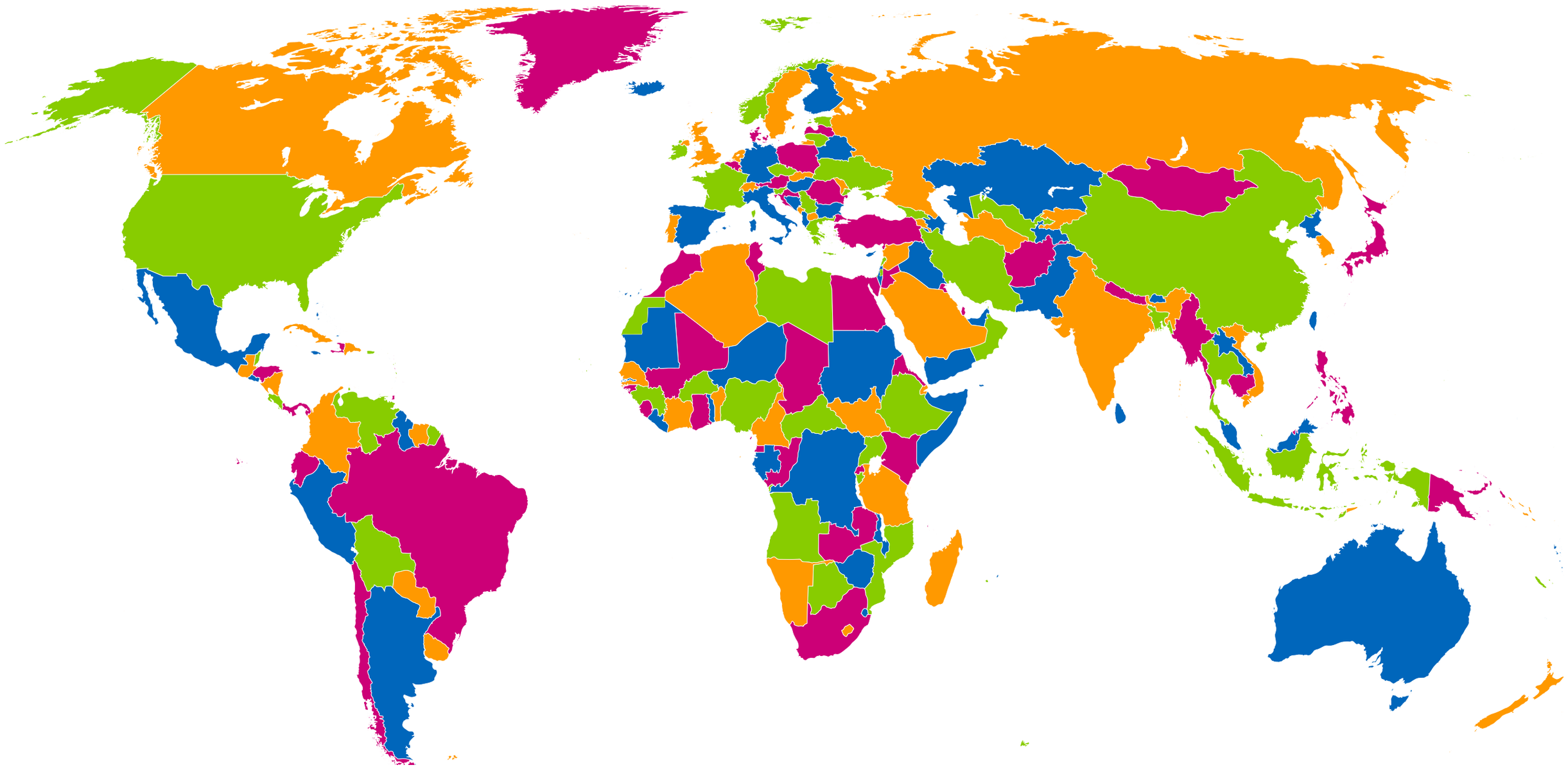
- Et, tôt ou tard, on retourne à *v*, puisque dans le pire des cas on épuise l'ensemble des autres arêtes, et *v* a un nombre **impair** d'arêtes inexplorées (donc il reste une arête pour y entrer)

- Ça est également vrais pour chaque sommet *u* choisi après *v*

Coloration des graphes

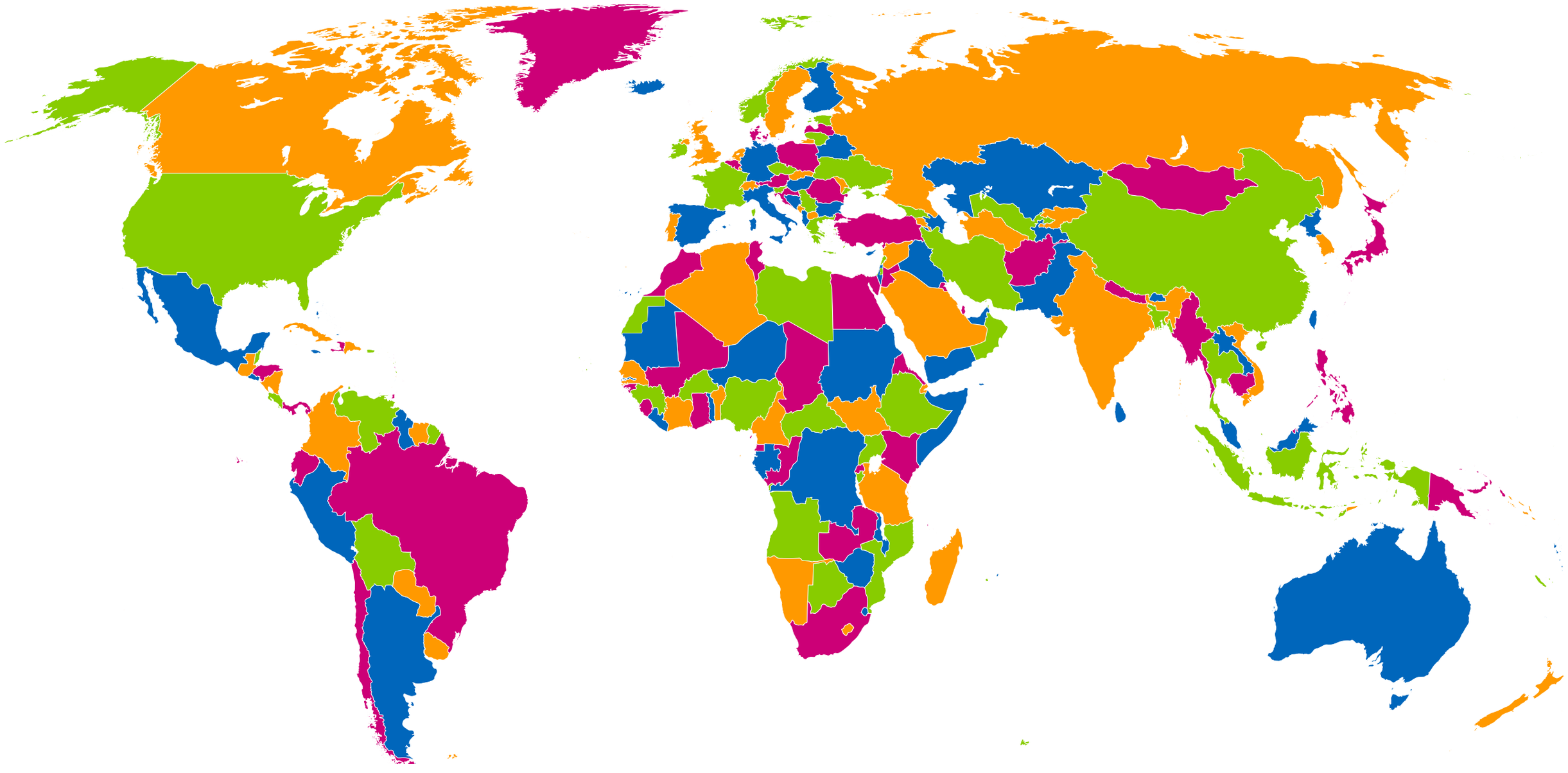
Coloration de cartes



Théorème des quatre couleurs

On peut colorier n’importe quelle carte
avec un maximum de 4 couleurs
de sorte que les pays (régions, etc.) adjacents
reçoivent toujours deux couleurs distinctes

Coloration de graphes



Coloration de graphes

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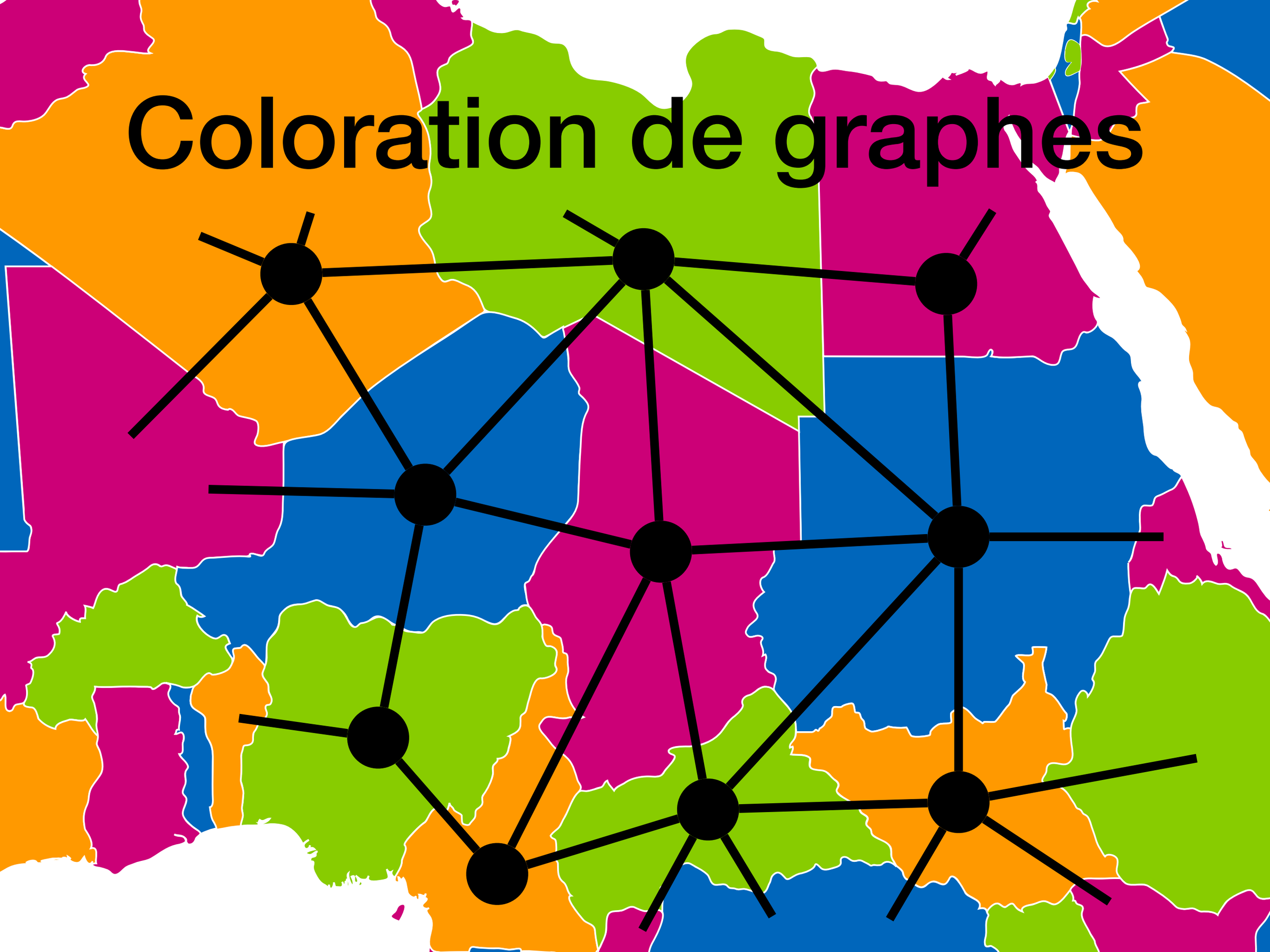

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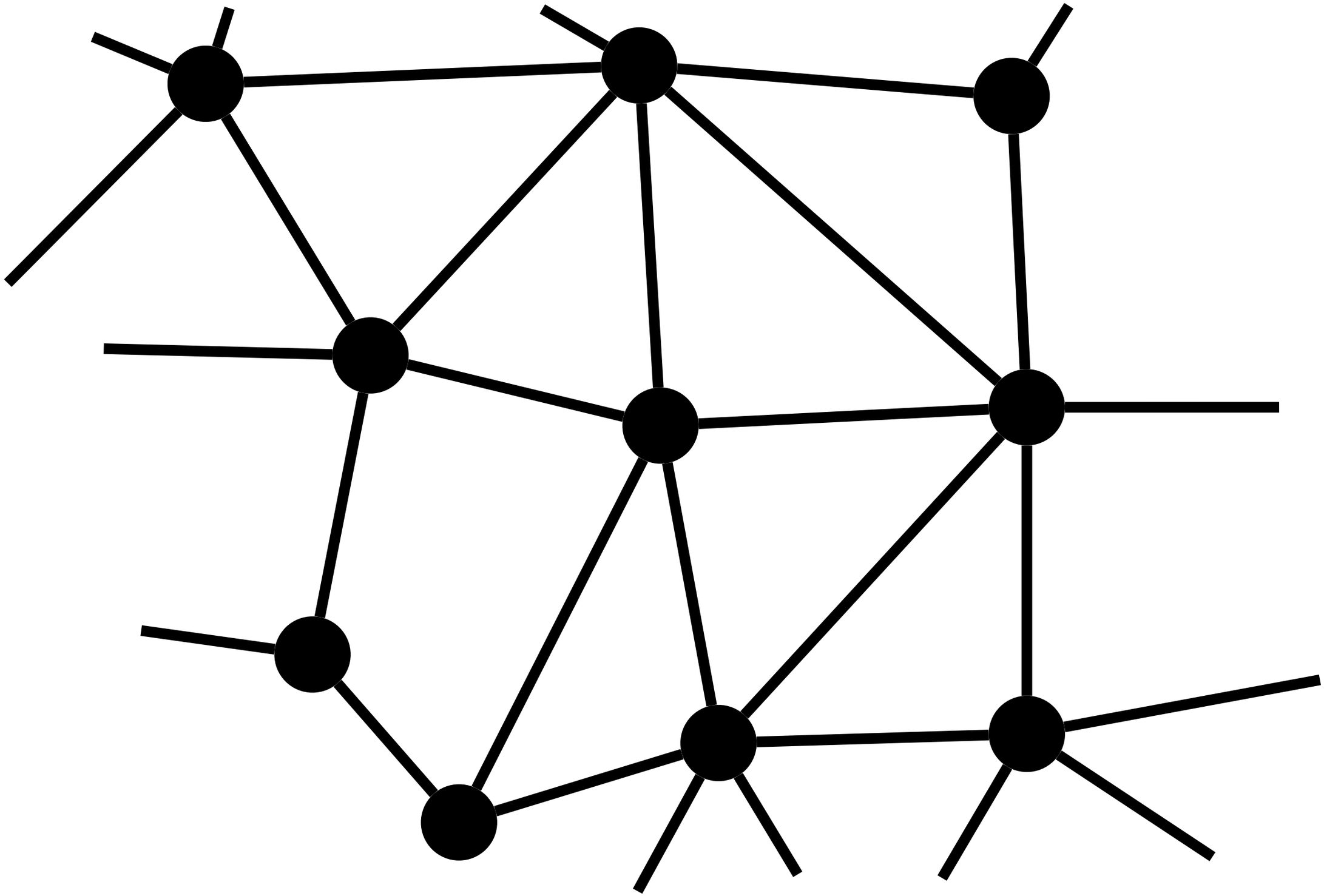
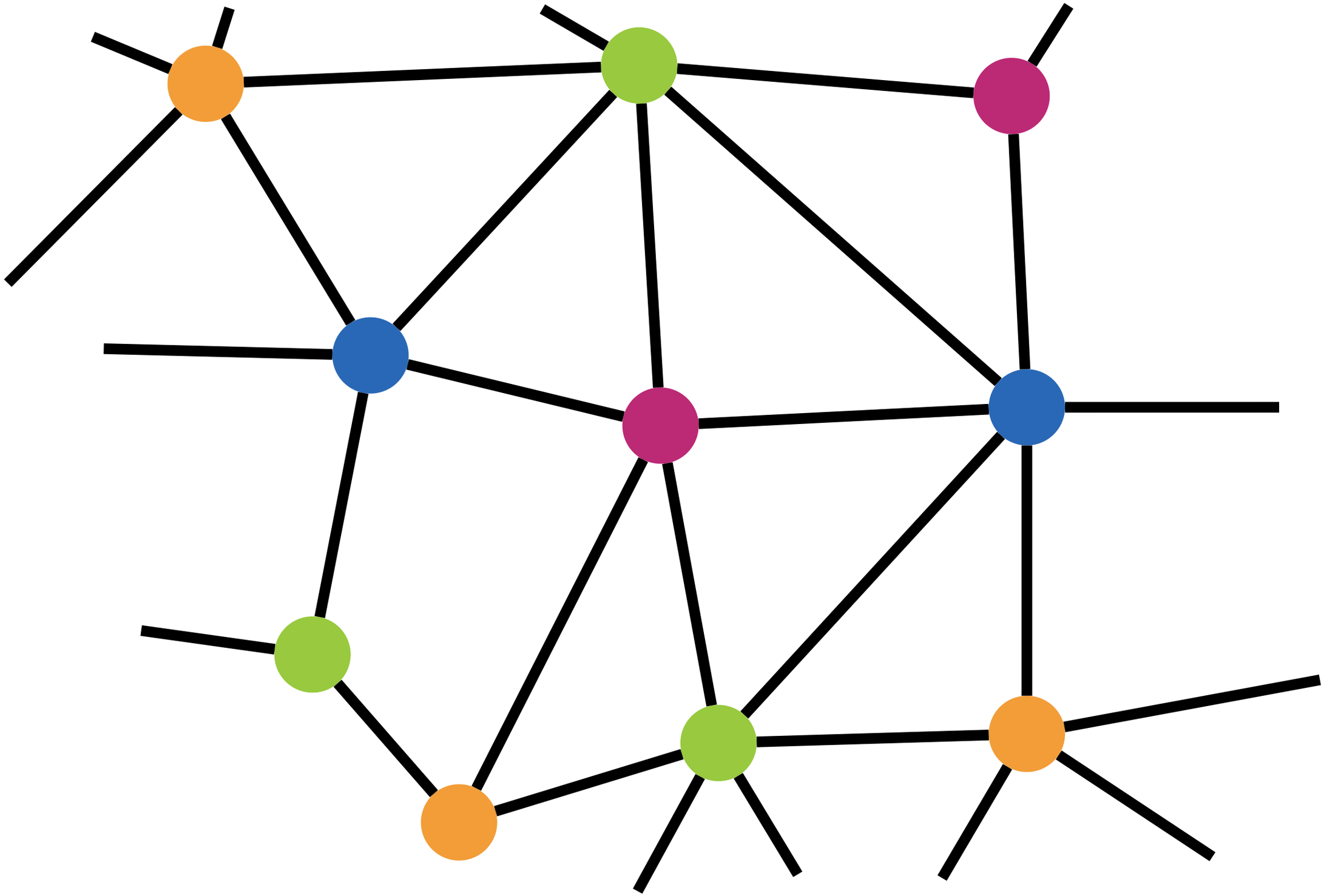


Coloration de graphes

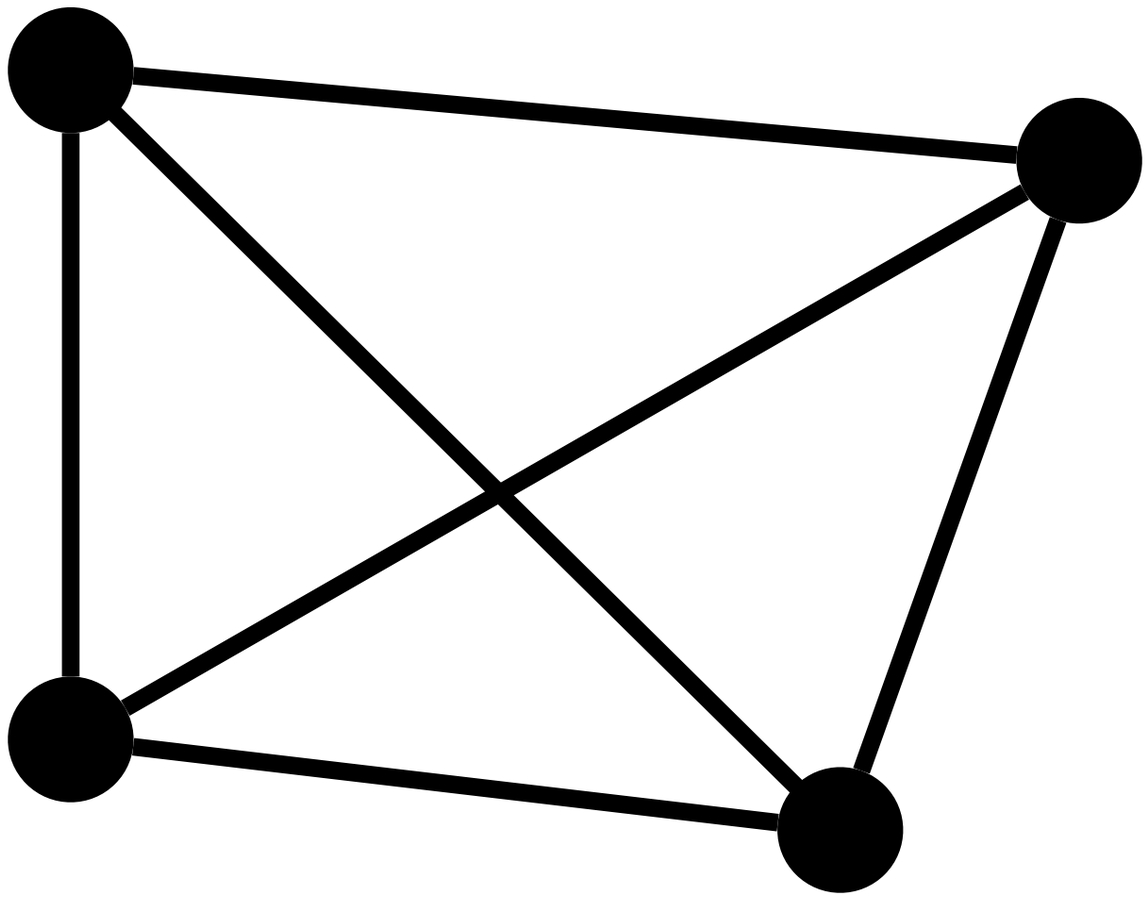


Graphes planaires

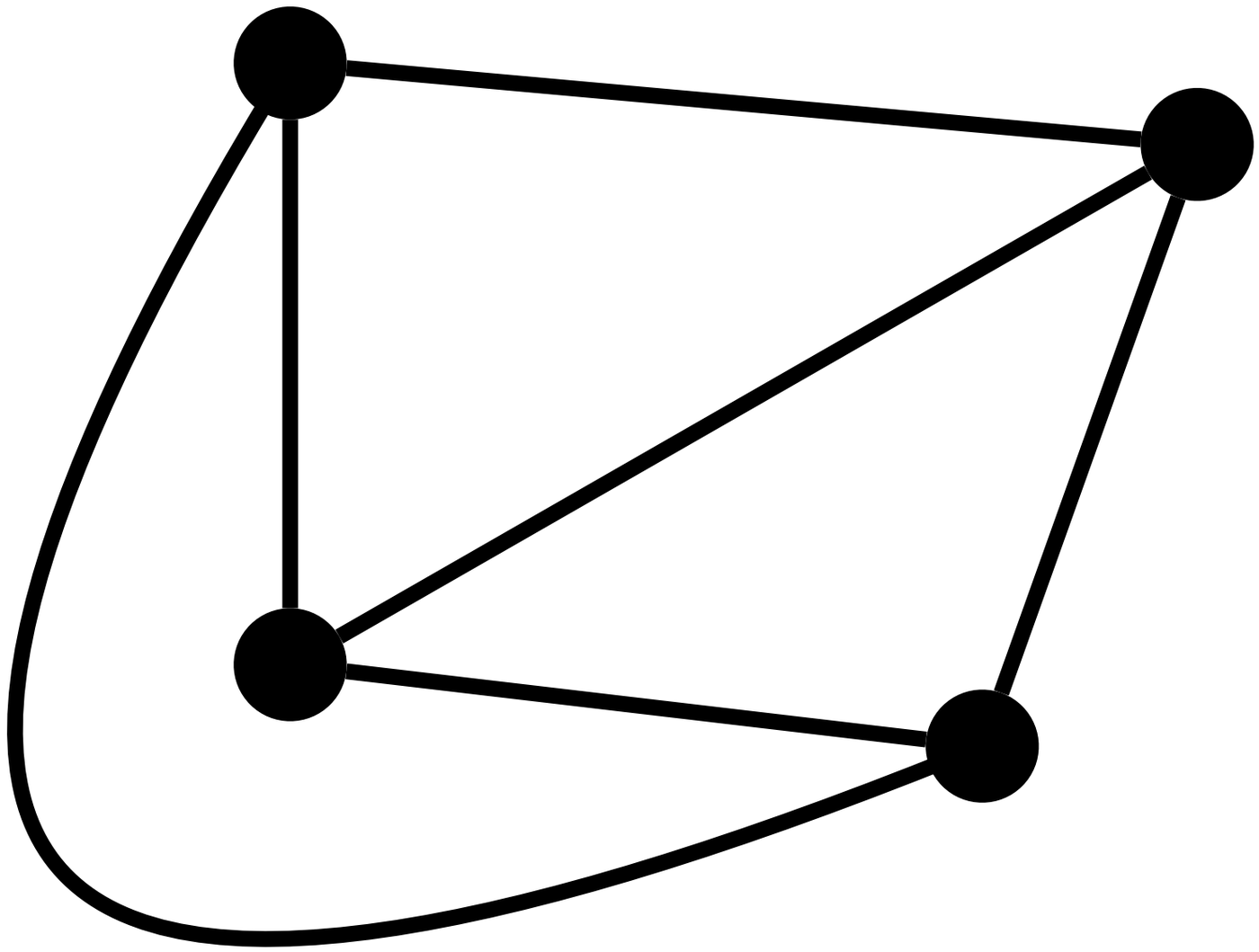
- Un graphe (non orienté) planaire est un graphe qu'on peut dessiner sur un plan (une feuille de papier) sans qu'aucune arête n'en croise une autre

- Un graphe non orienté *G* est planaire si et seulement si il existe une carte ayant *G* comme graphe associé

Graphes qui ne semblent pas planaires



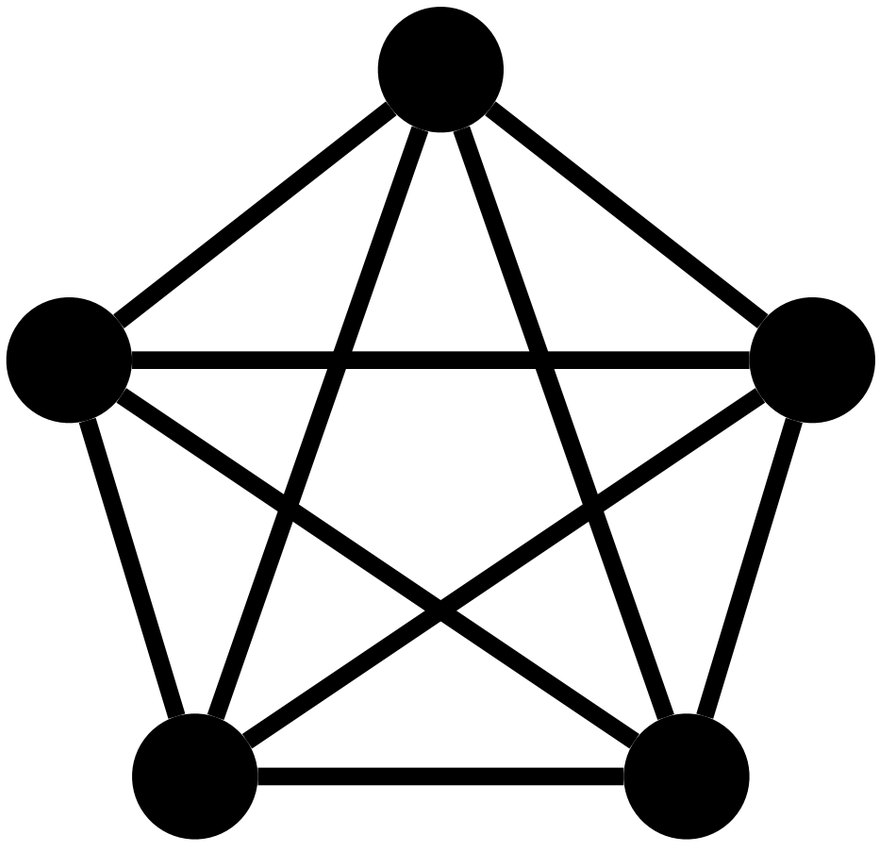
Graphes qui ne semblent pas planaires



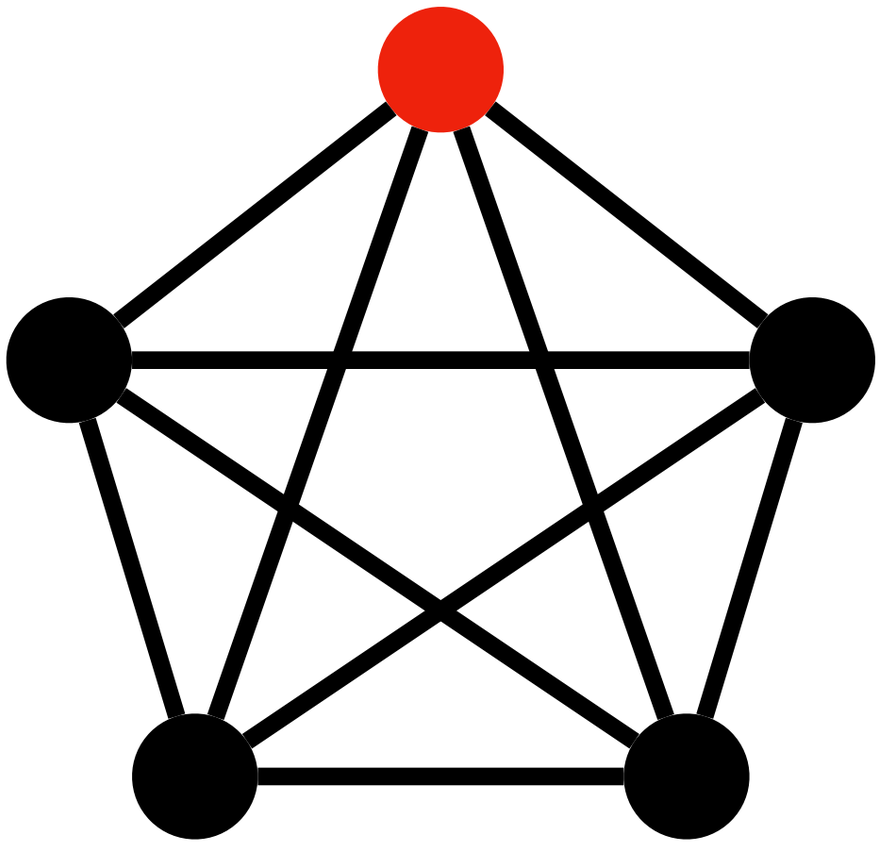
Théorème des quatre couleurs

On peut colorier les sommets de n’importe
quel graphe non orienté planaire
avec un maximum de 4 couleurs
de sorte que les sommets adjacents
reçoivent toujours deux couleurs distinctes

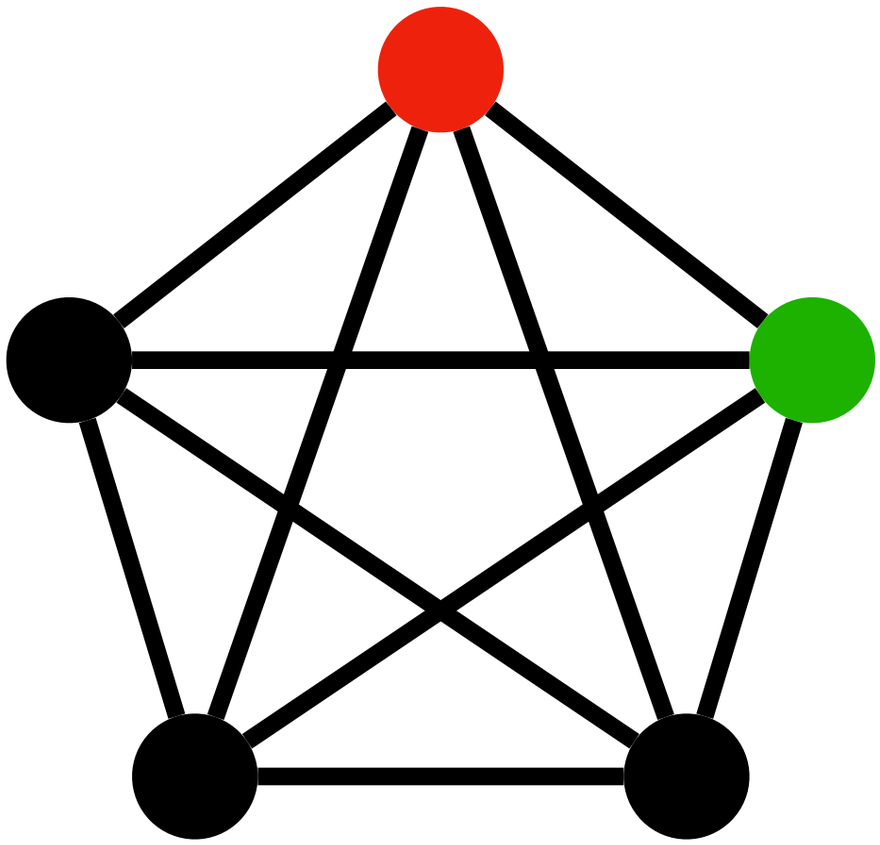
Graphes non planaires



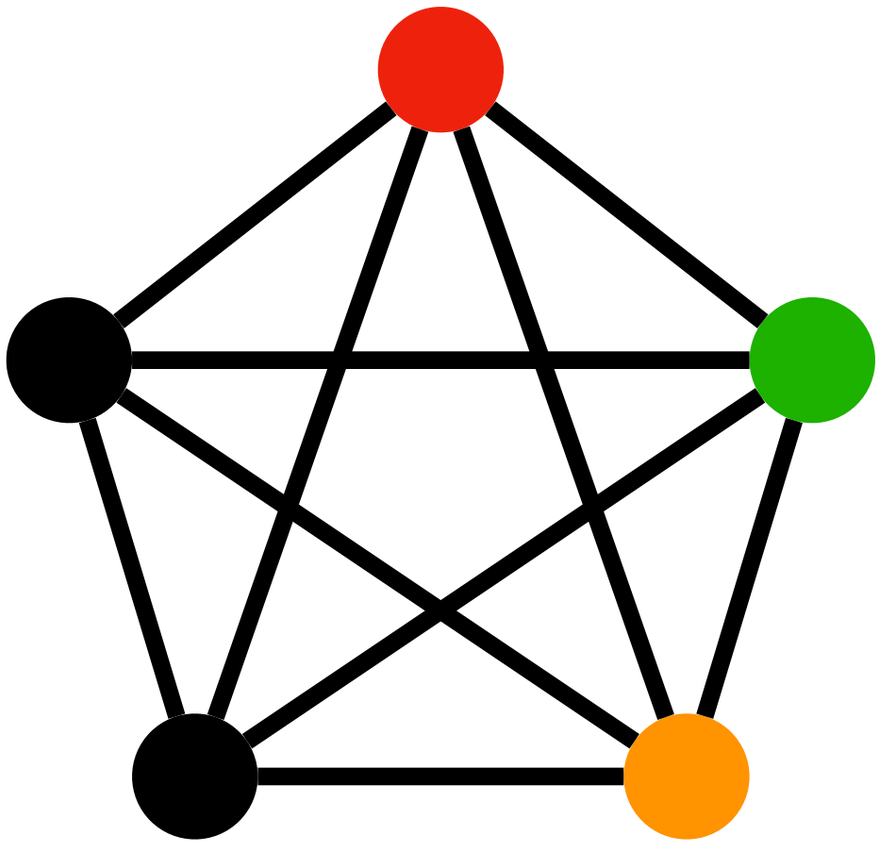
Graphes non planaires



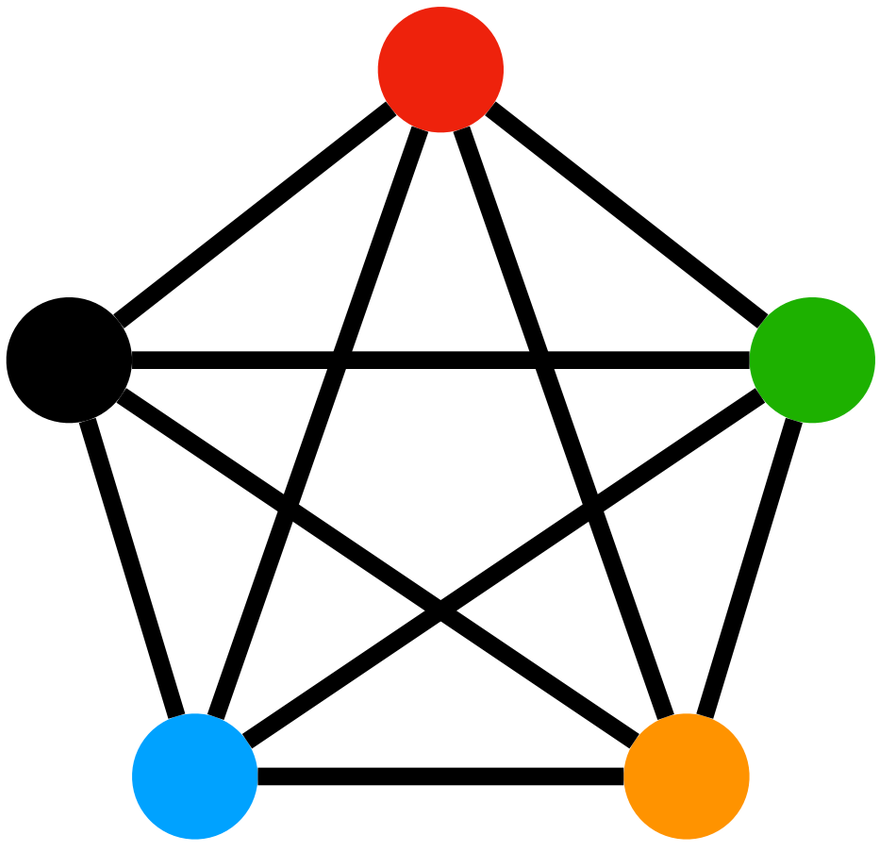
Graphes non planaires



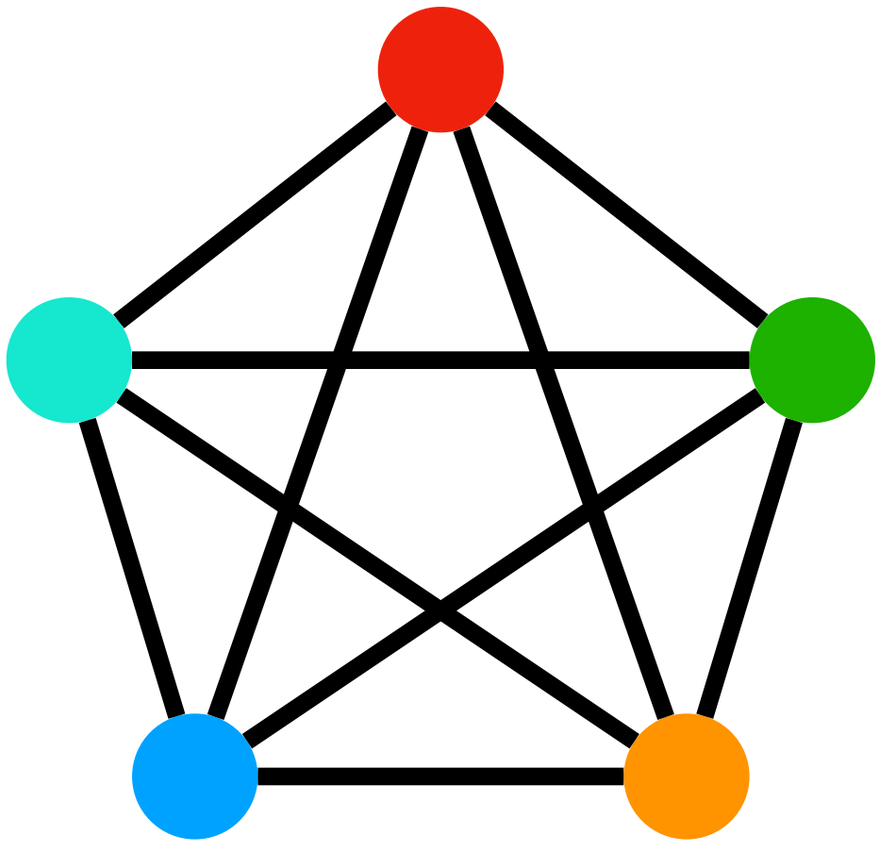
Graphes non planaires



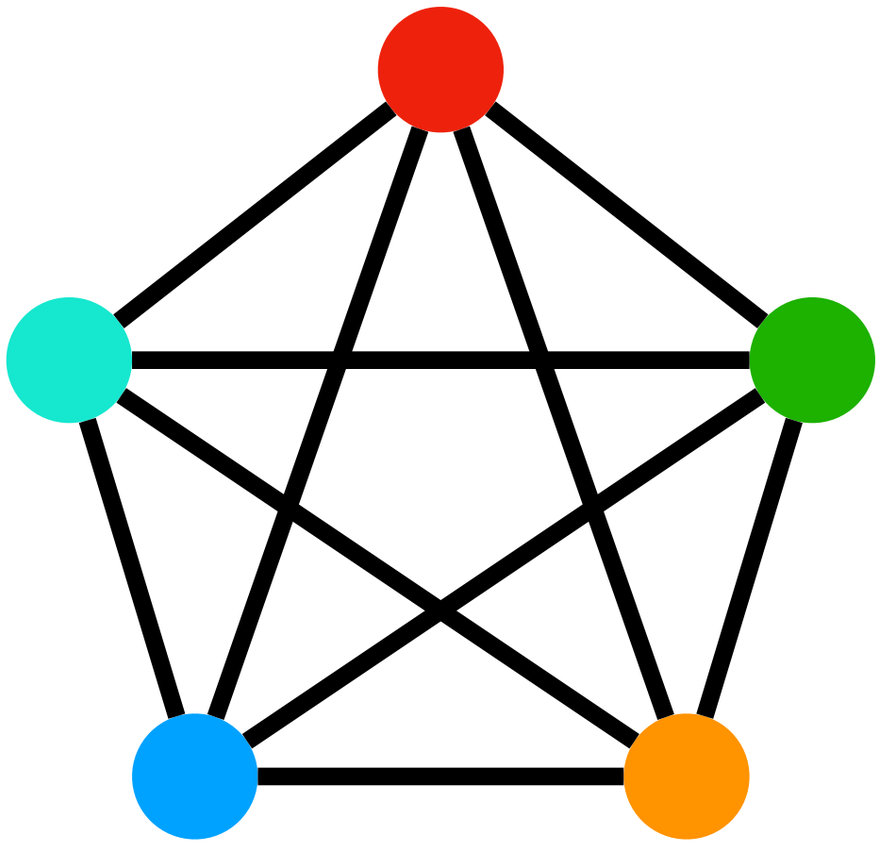
Graphes non planaires

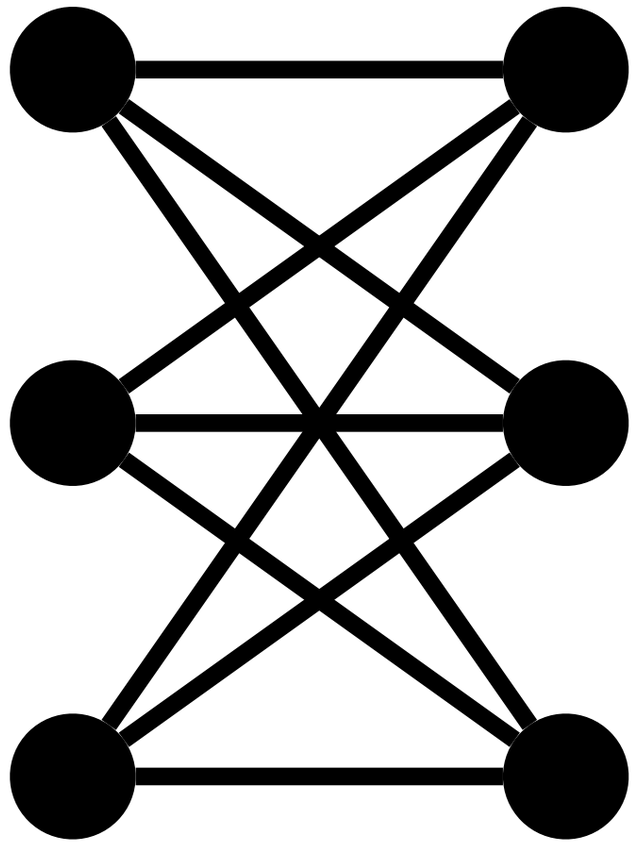


Graphes non planaires

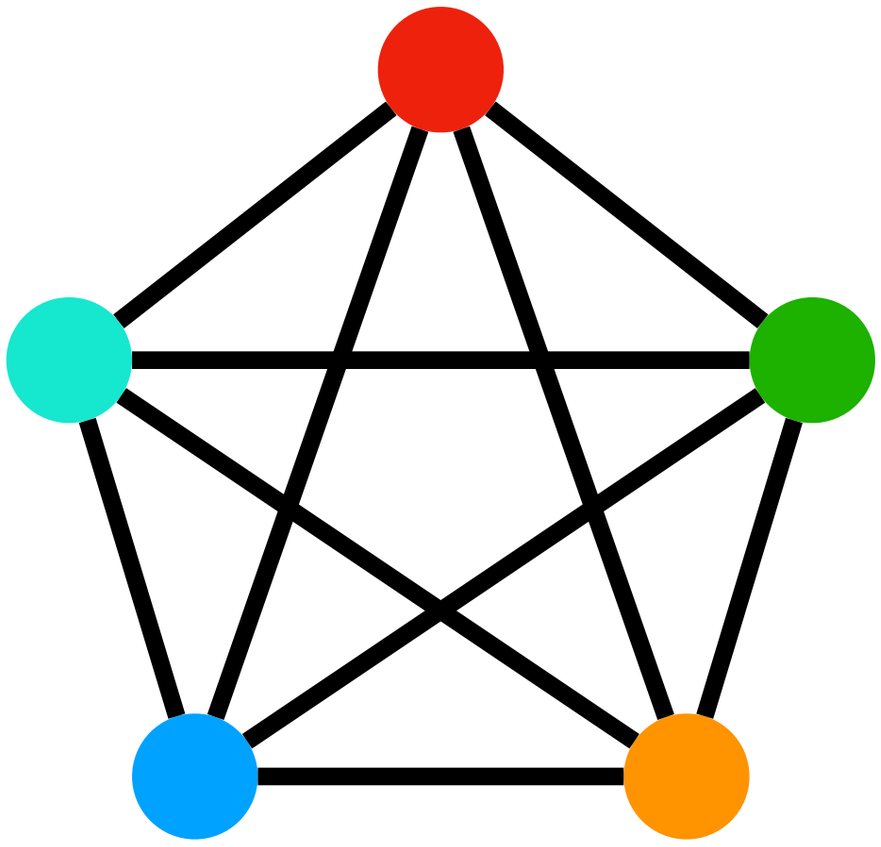


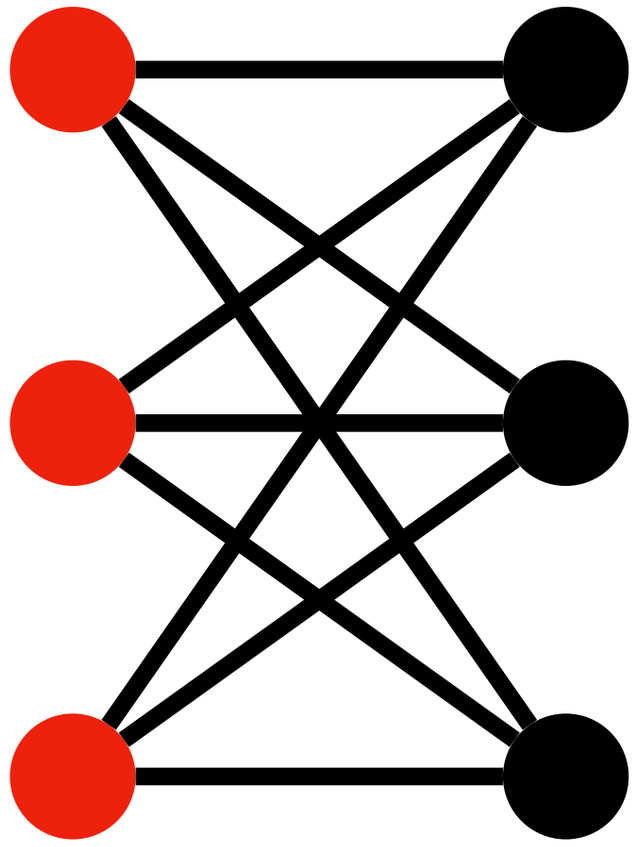
Graphes non planaires





Graphes non planaires





Graphes non planaires