Introduction à la science informatique

Semaine 6 (1 séance de 2h)

Récursivité

La factorielle

$$n! = \prod_{i=1}^{n} i = 1 \times 2 \times \cdots \times (n-1) \times n$$

$n!$ = nb. permutations de n objets

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$n!$ = nb. permutations de n objets

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$2! = 2$

$n!$ = nb. permutations de n objets

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$2! = 2$

$n!$ = nb. permutations de n objets

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$2! = 2$

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$n!$ = nb. permutations de n objets

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$2! = 2$

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$3! = 6$

$n!$ = nb. permutations de n objets

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$2! = 2$

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$n!$ = nb. permutations de n objets

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$2! = 2$

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[🥐]

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$2! = 2$ $3! = 6$ $1! = 1$

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$2! = 2$ $3! = 6$ $1! = 1$ $0! = 1$

Algorithme itératif pour $n!$

$$n! = 1 \times 2 \times \cdots \times (n-1) \times n$$

Algorithme itératif pour $n!$

$$n! = 1 \times 2 \times \cdots \times (n-1) \times n$$

```
def fact_iter(n):
    r = 1
    for i in range(1, n + 1):
        r = r * i
    return r
```

La factorielle

$$n! = \prod_{i=1}^{n} i = 1 \times 2 \times \cdots \times (n-1) \times n$$

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$$n \times (n-1)!$$

remarque : $0! = \prod_{i=1}^{0} i = 1$

Algorithme récursif pour $n!$

$n! = n \times (n-1)!$ avec $0! = 1$

Algorithme récursif pour $n!$

$n! = n \times (n-1)!$ avec $0! = 1$

```
def fact(n):
```

Algorithme récursif pour $n!$

$$n! = n \times (n-1)! \text{ avec } 0! = 1$$

```
def fact(n):
    if n == 0:
```

Algorithme récursif pour $n!$

$$n! = n \times (n-1)! \text{ avec } 0! = 1$$

```
def fact(n):
    if n == 0:
        return 1
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Algorithme récursif pour $n!$

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Algorithme récursif pour $n!$

$$n! = n \times (n-1)! \text{ avec } 0! = 1$$

```
def fact(n):
    if n == 0:
        return 1
    else:
        return n * fact(n - 1)
```

Exécution d’un algorithme récursif

Calcul de $4! = 24$

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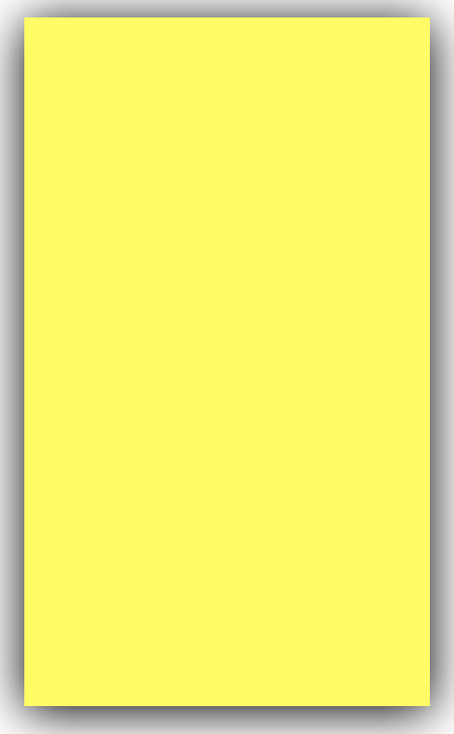

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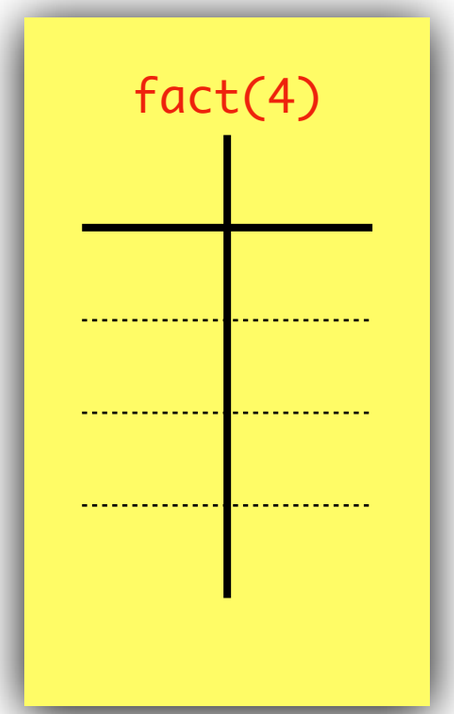

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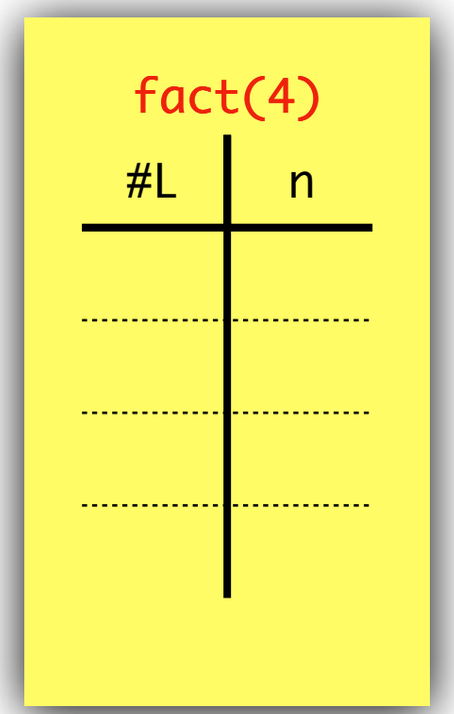

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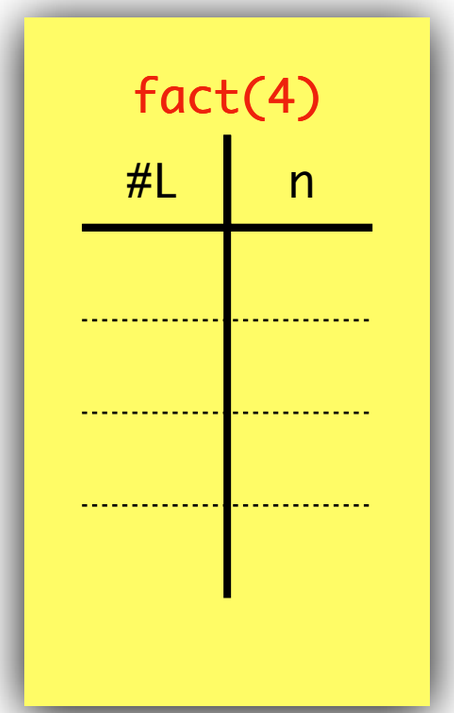

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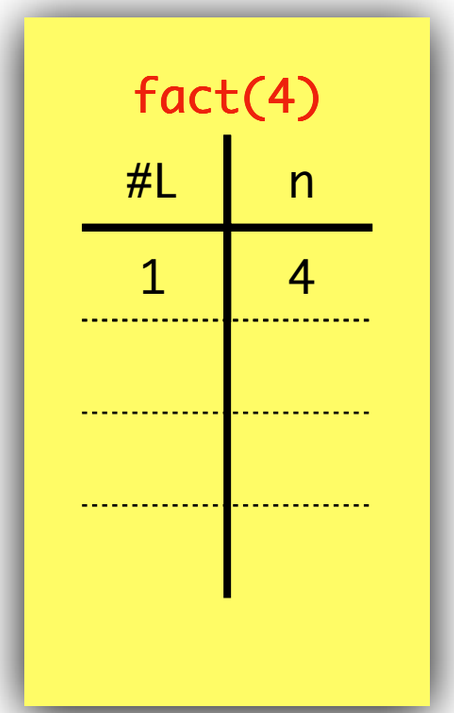

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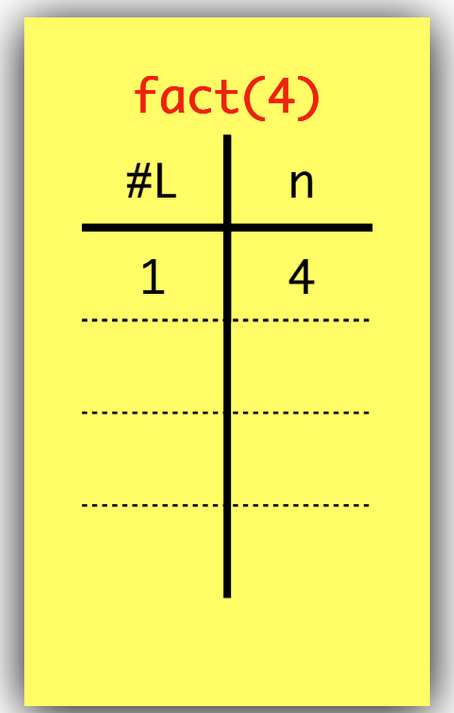

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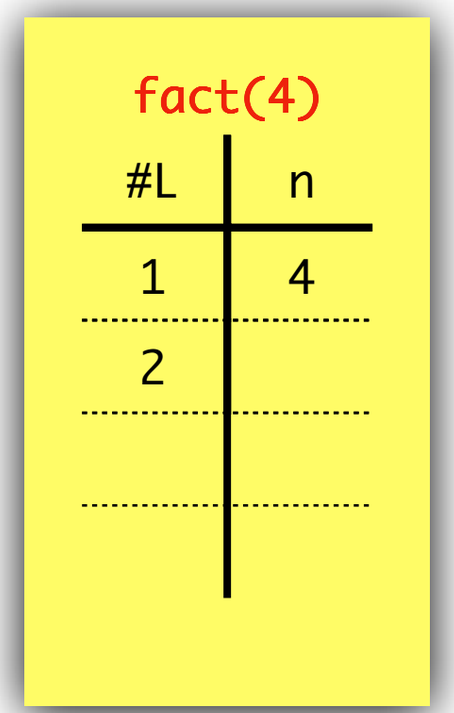

Calcul de $4! = 24$

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```

👉 4

fact(4)

#L	n
1	4
2	

Calcul de $4! = 24$

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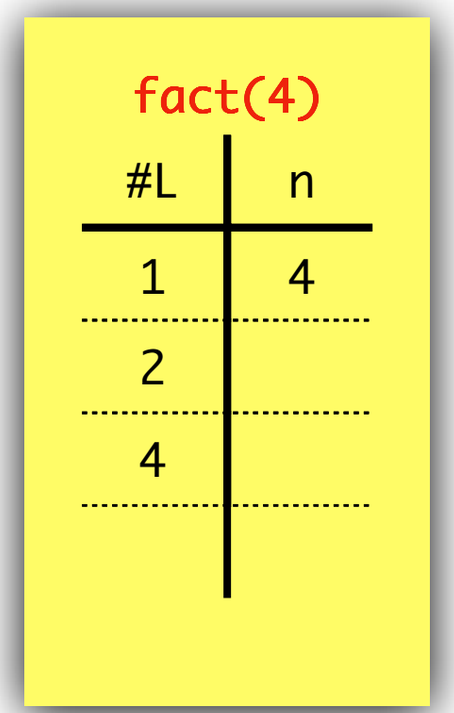

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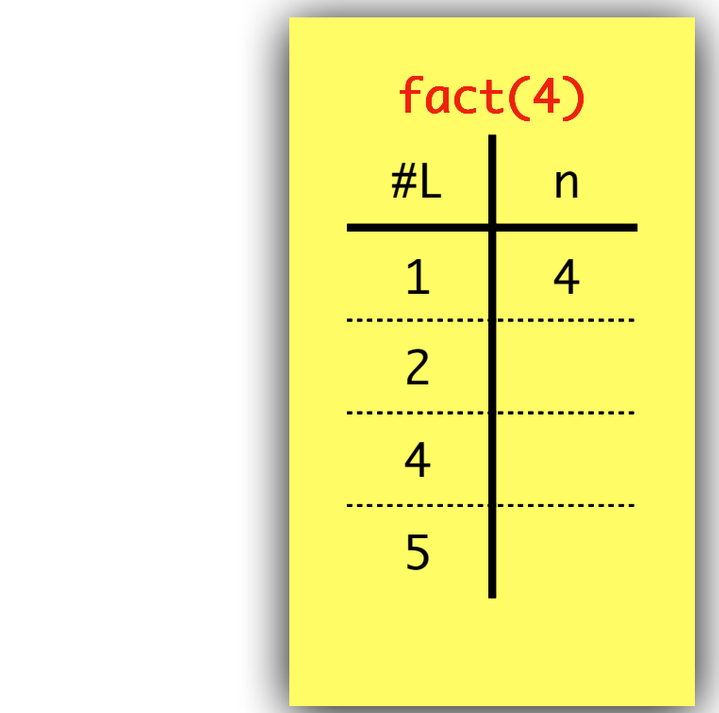

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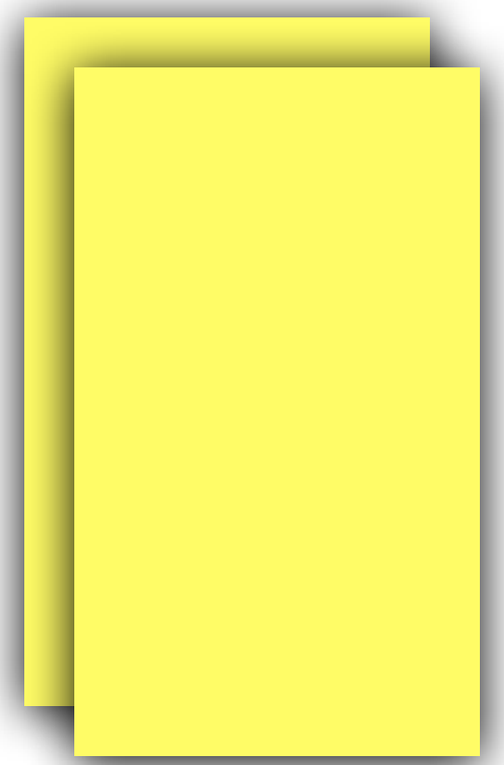

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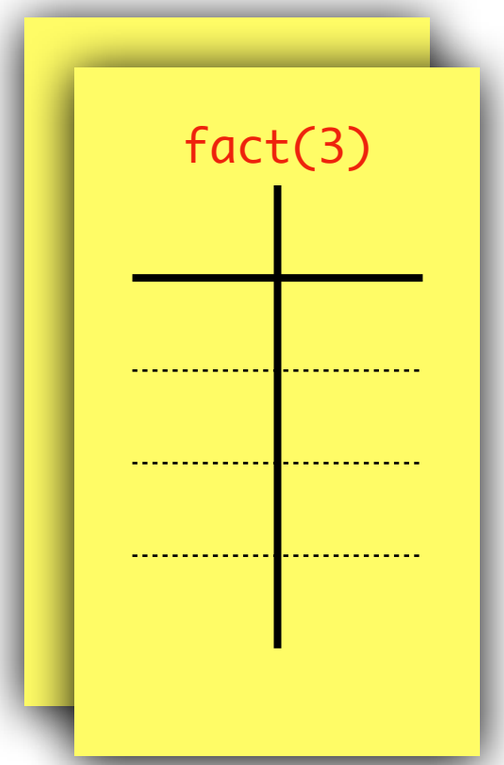

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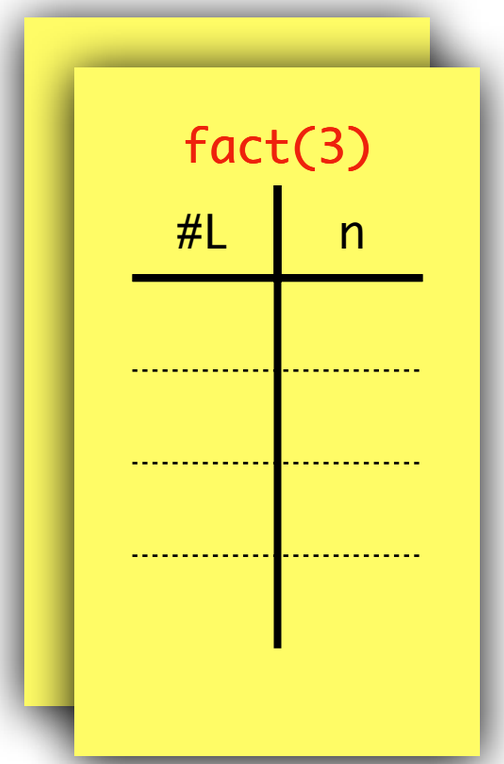

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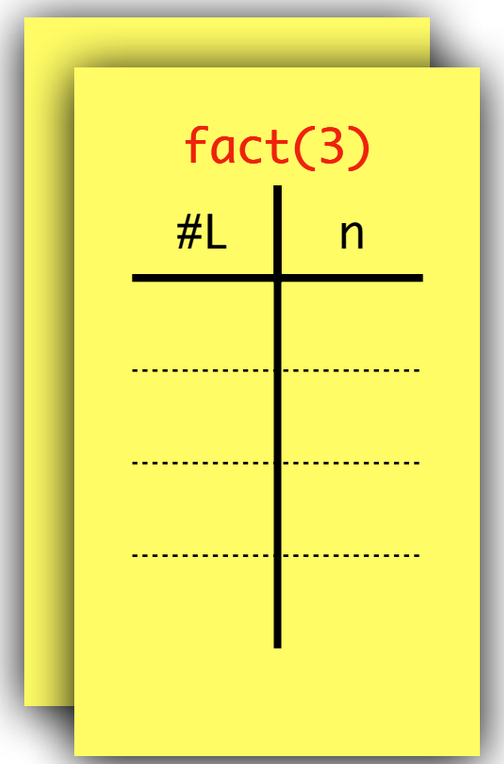

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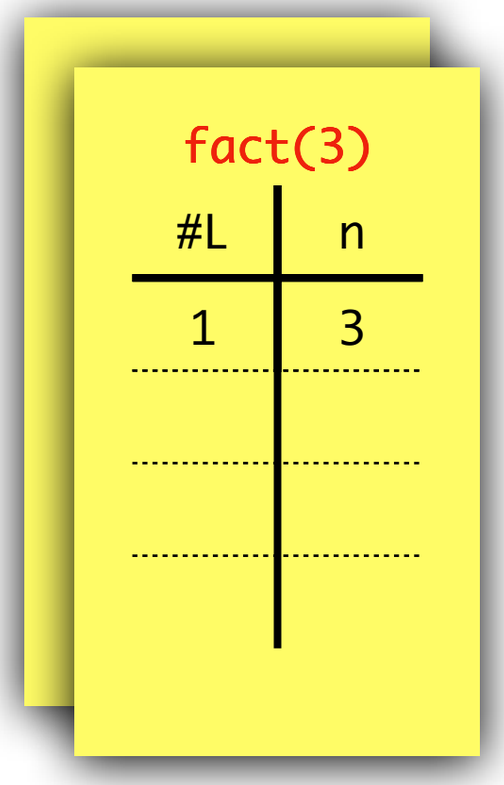

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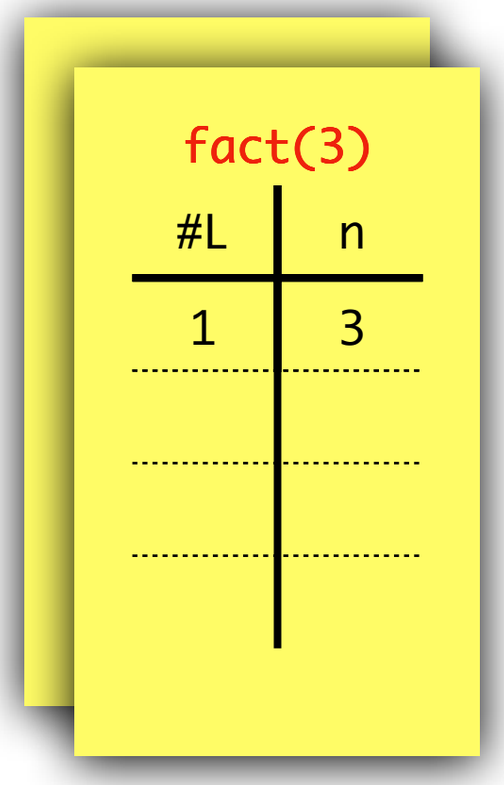

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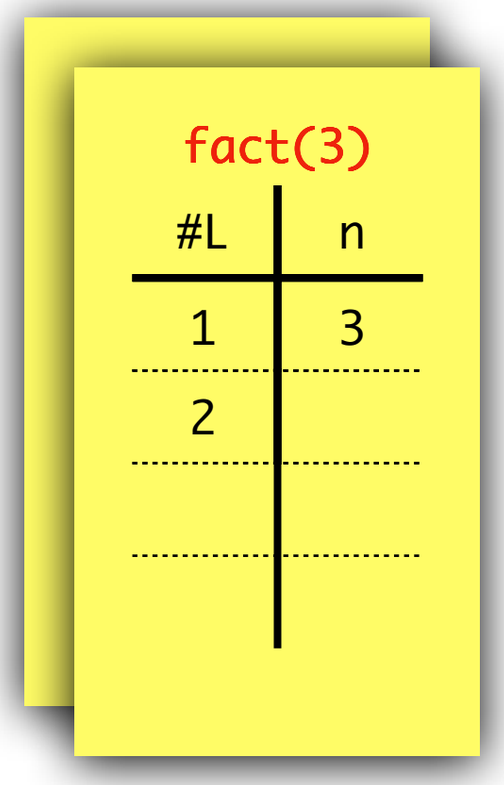

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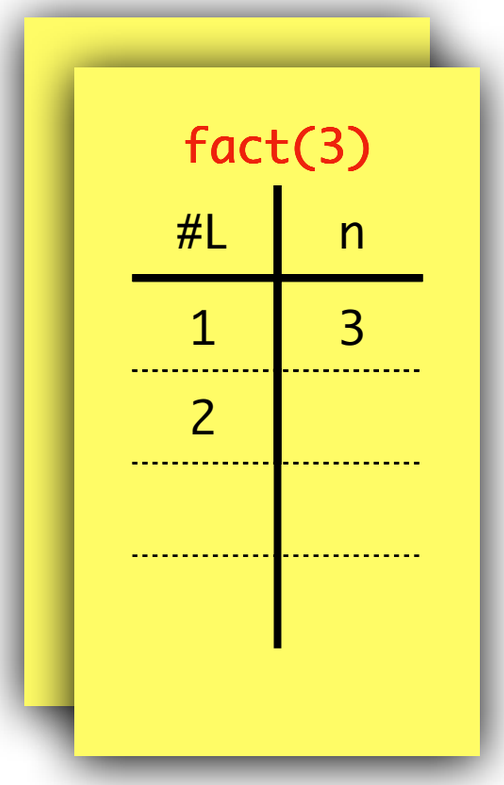

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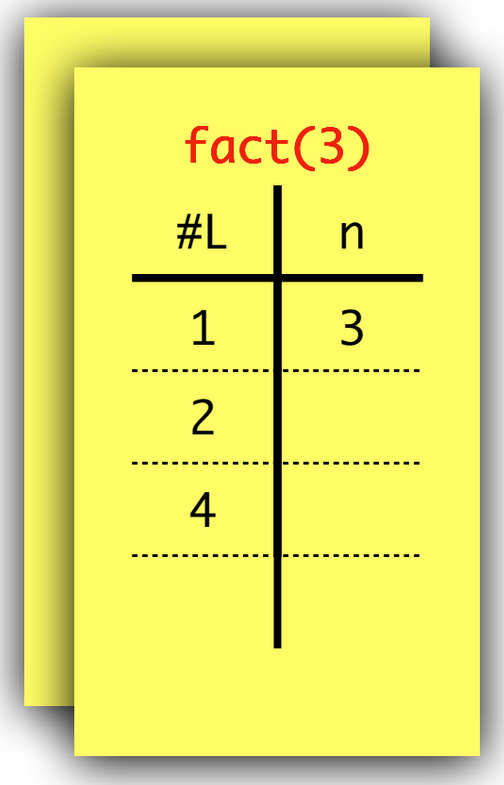

Calcul de $4! = 24$

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```

👉

fact(3)

#L	n
1	3
2	
4	

Calcul de $4! = 24$

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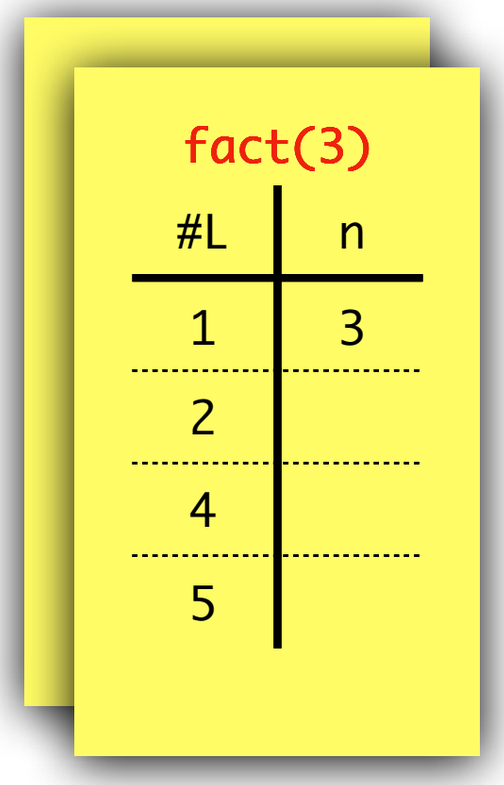

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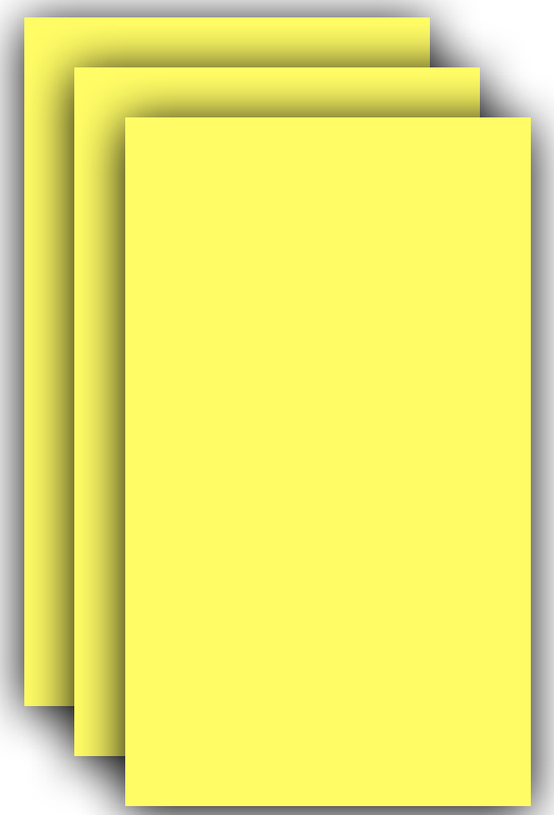

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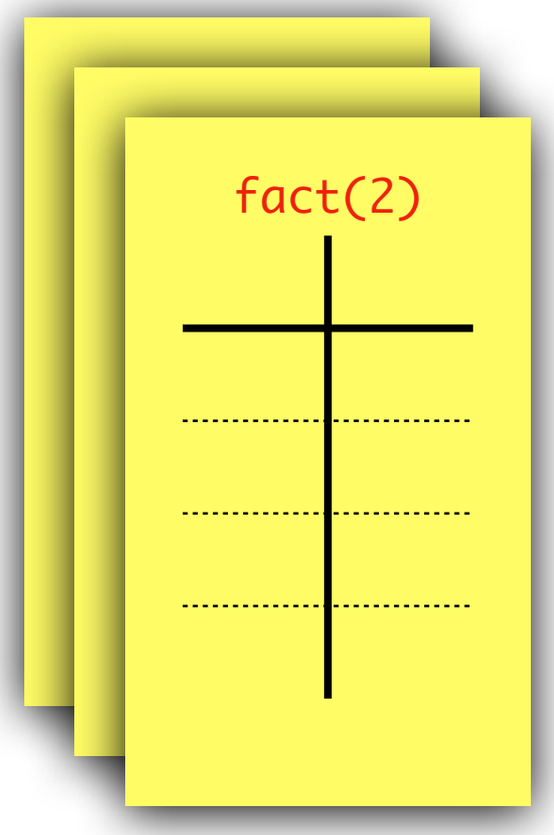

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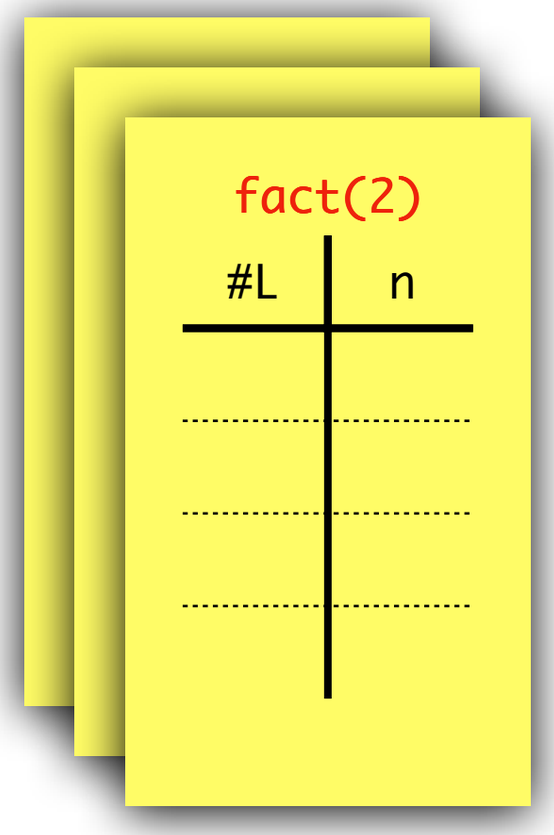

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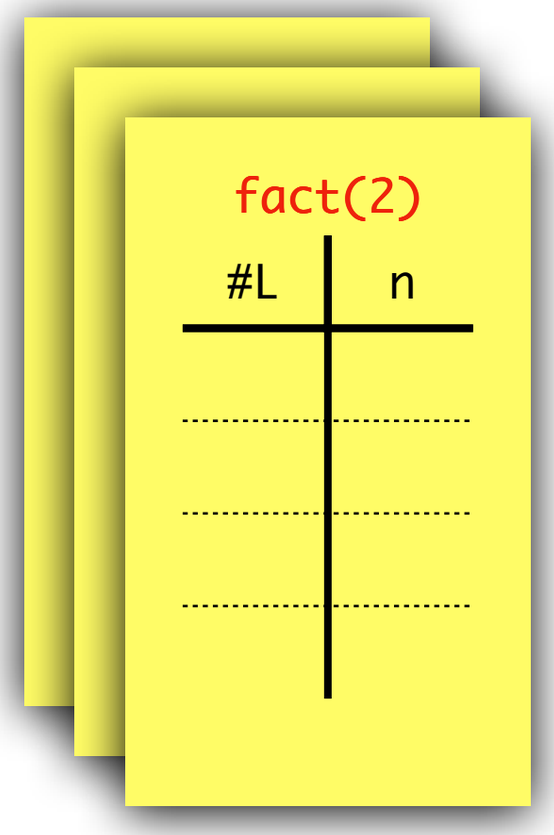

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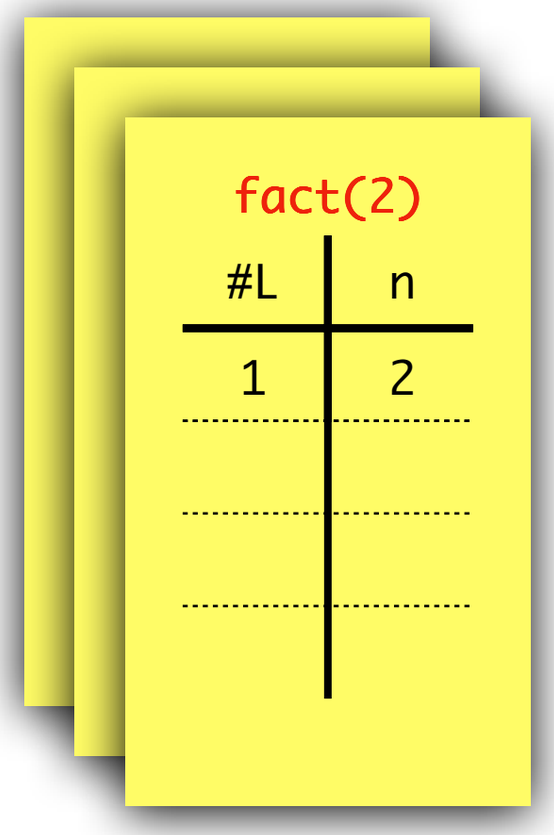

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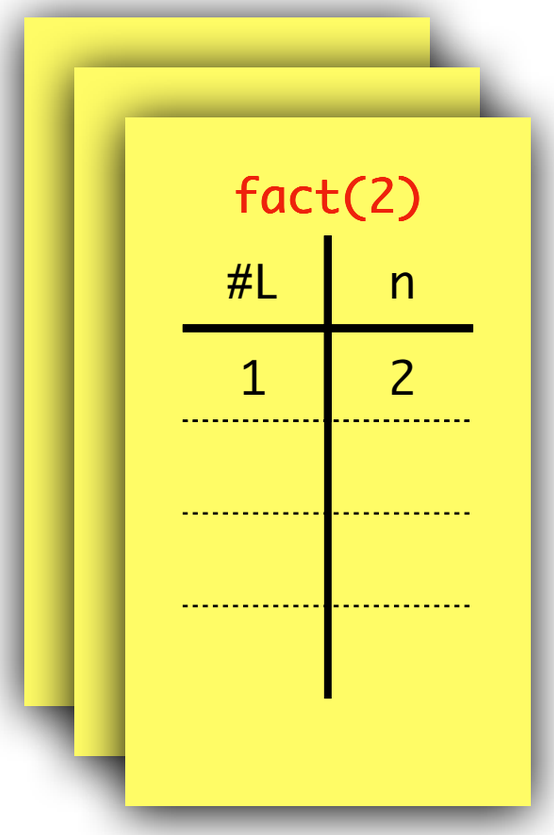

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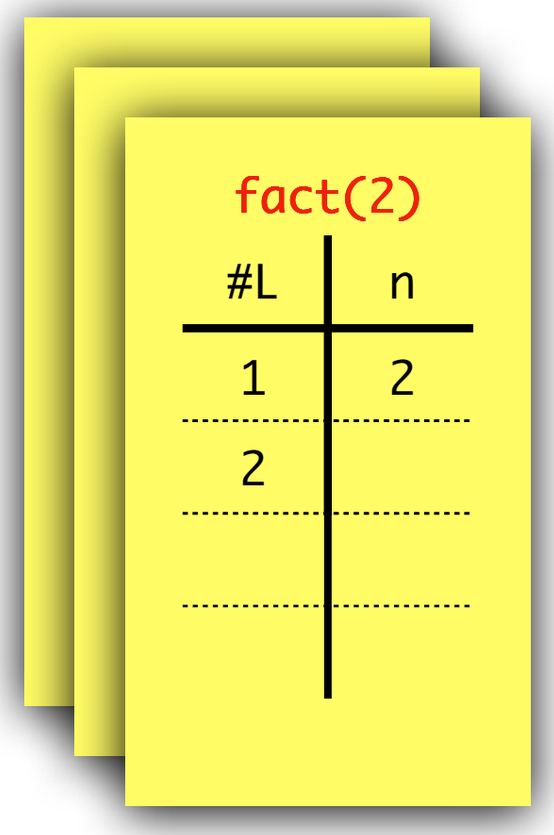

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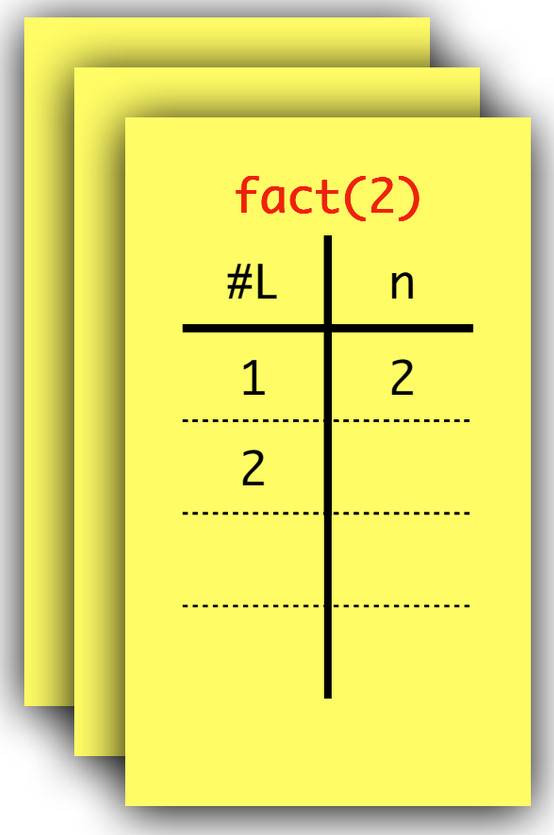

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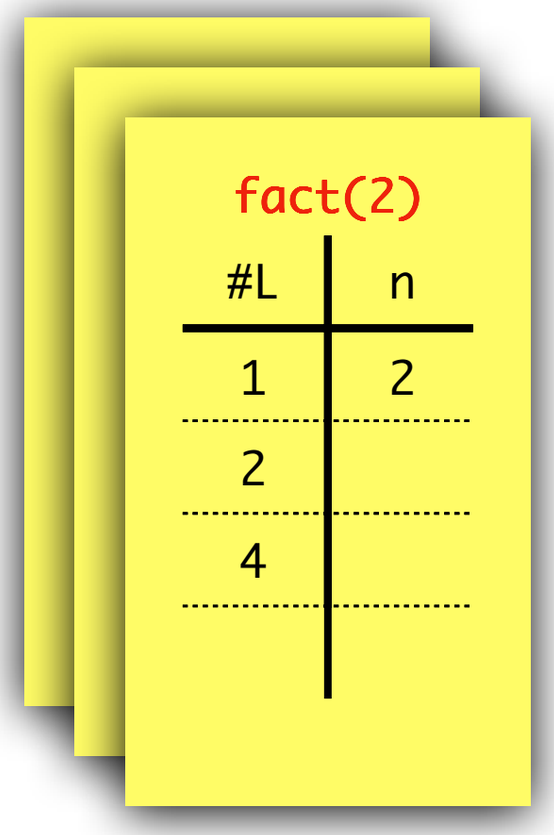

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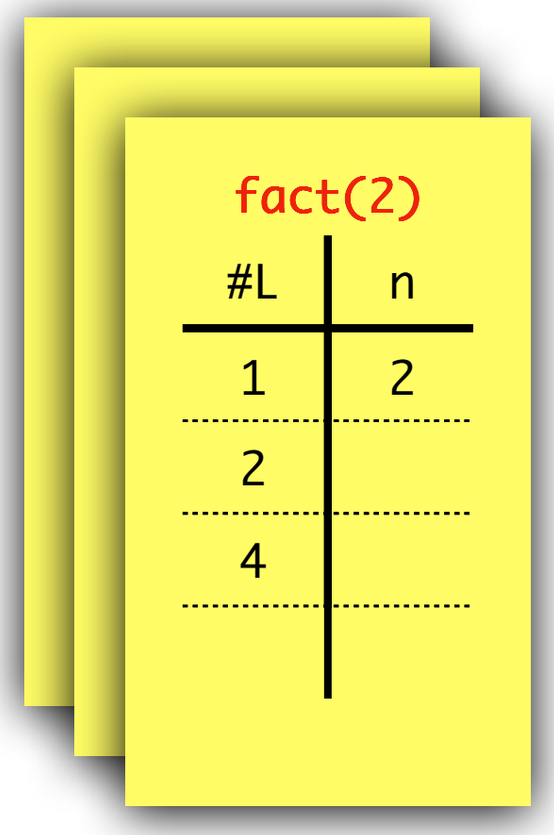

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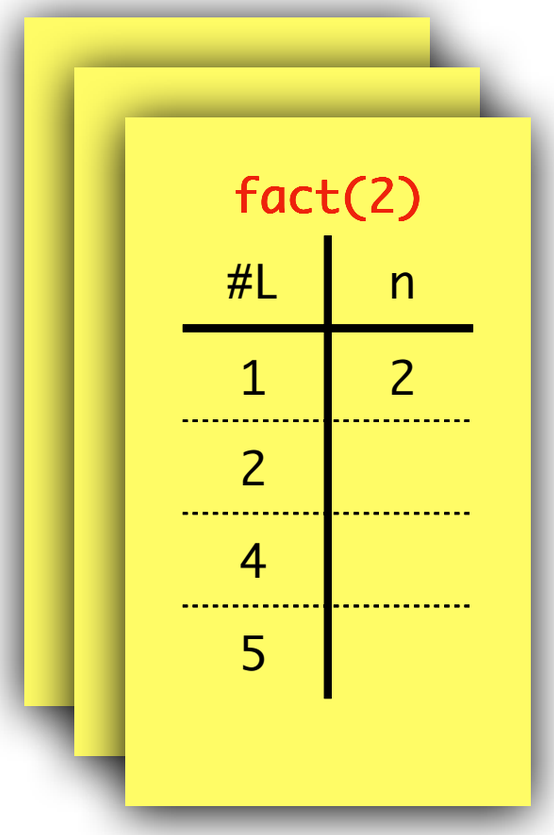

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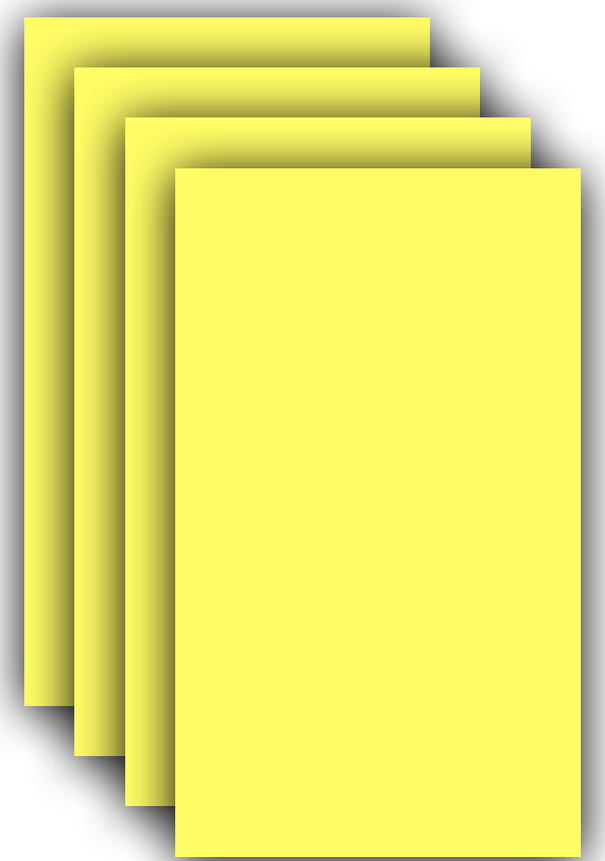

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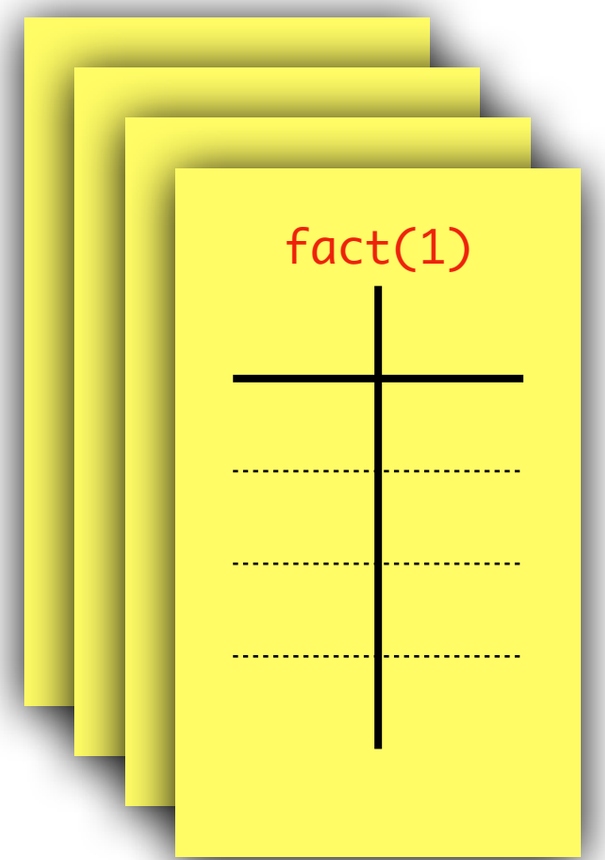

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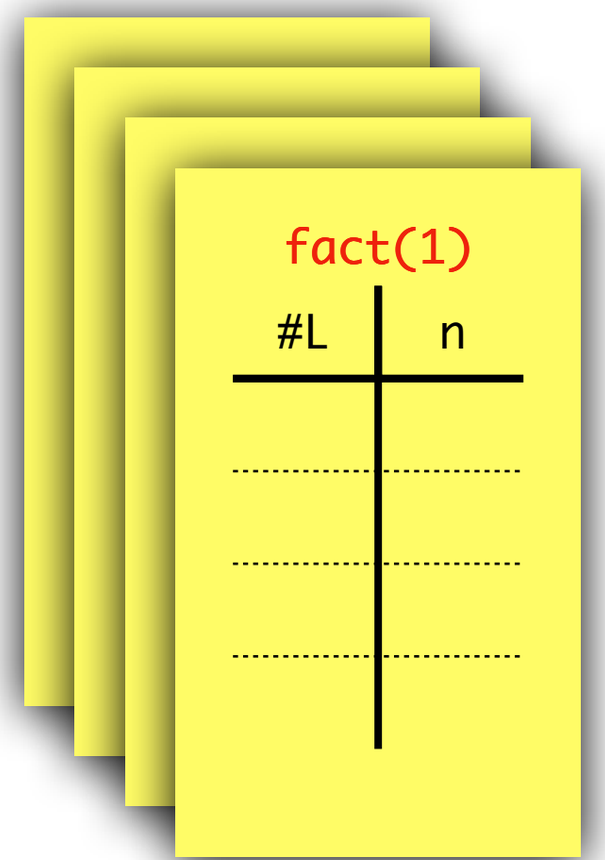

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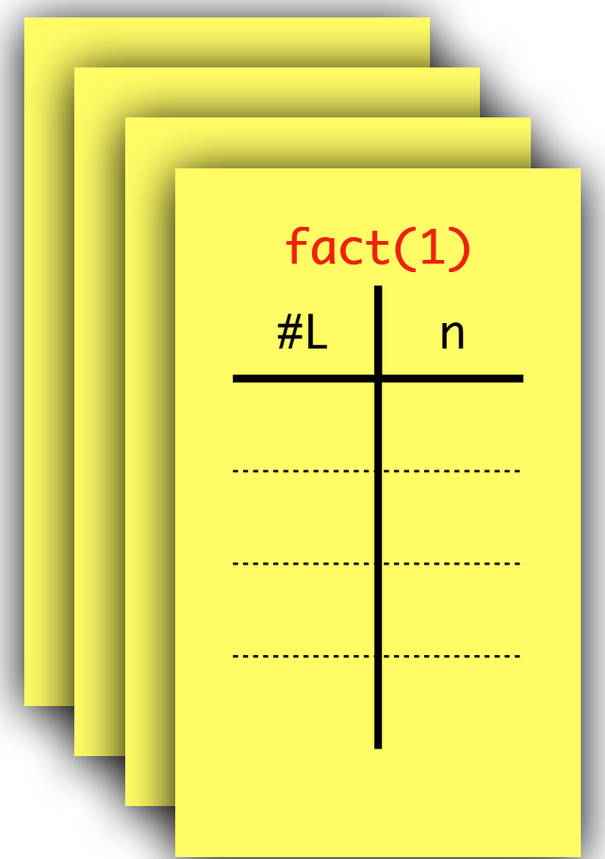


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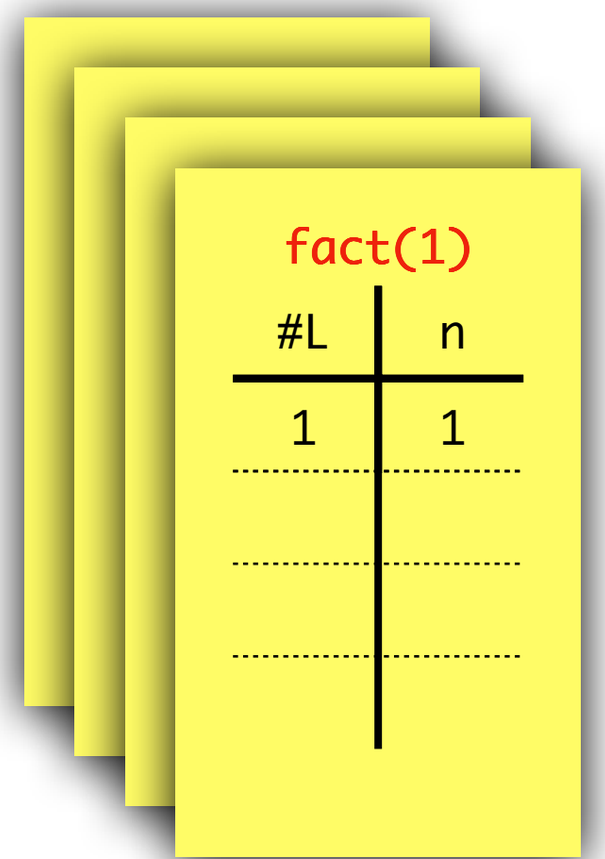

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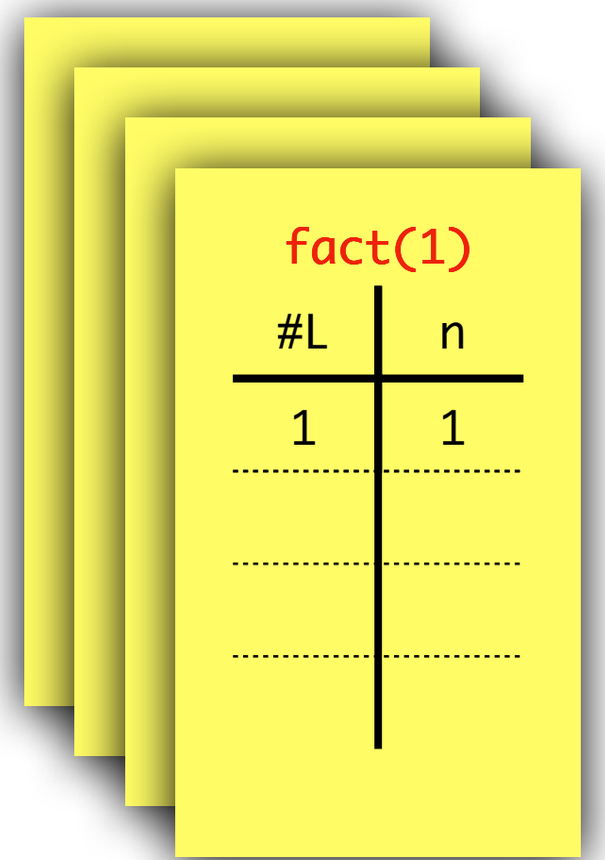

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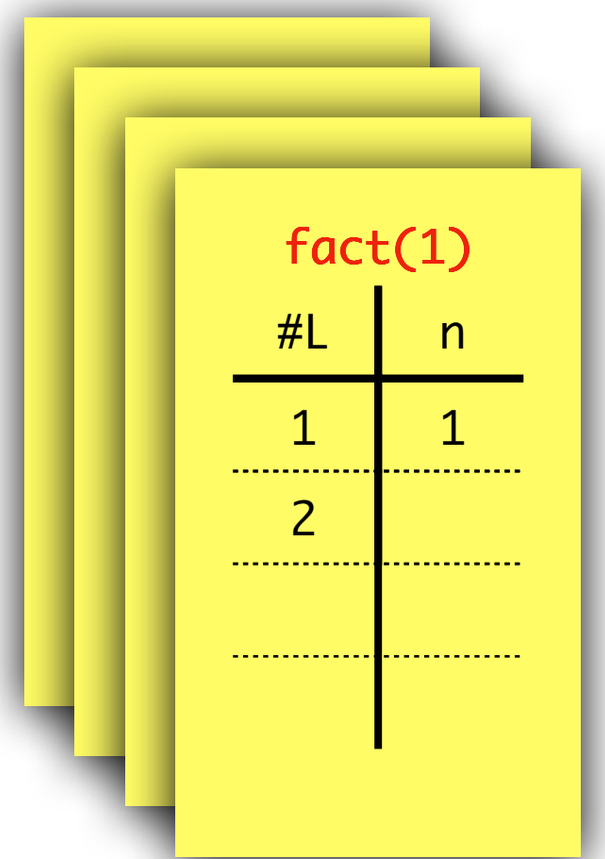

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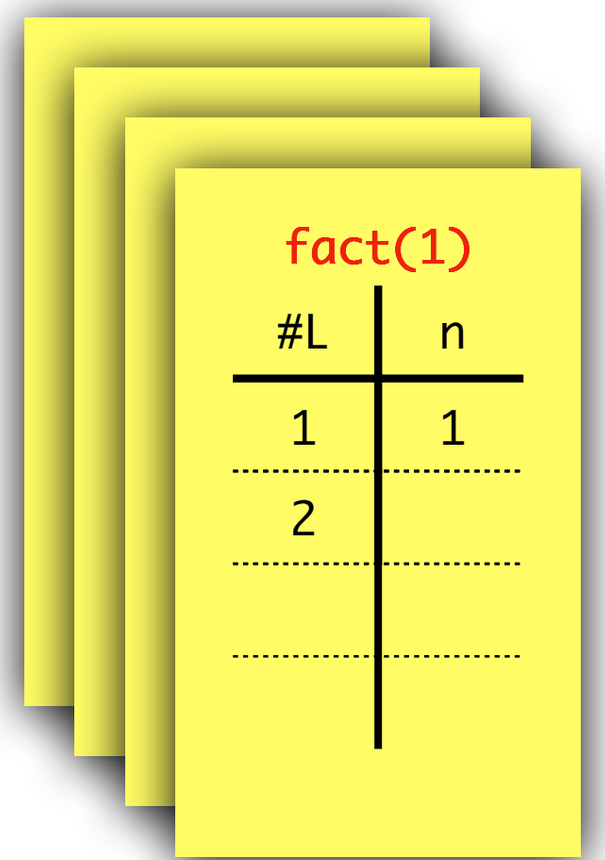

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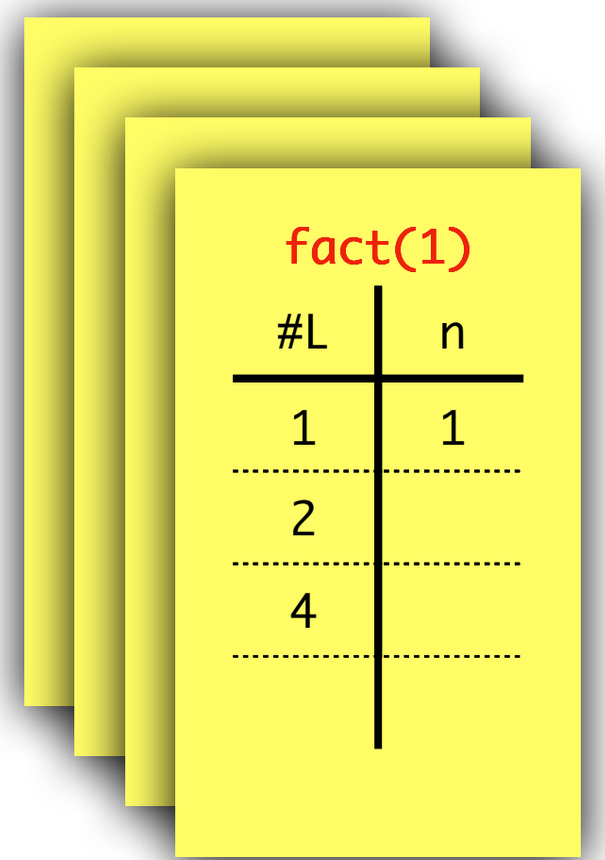

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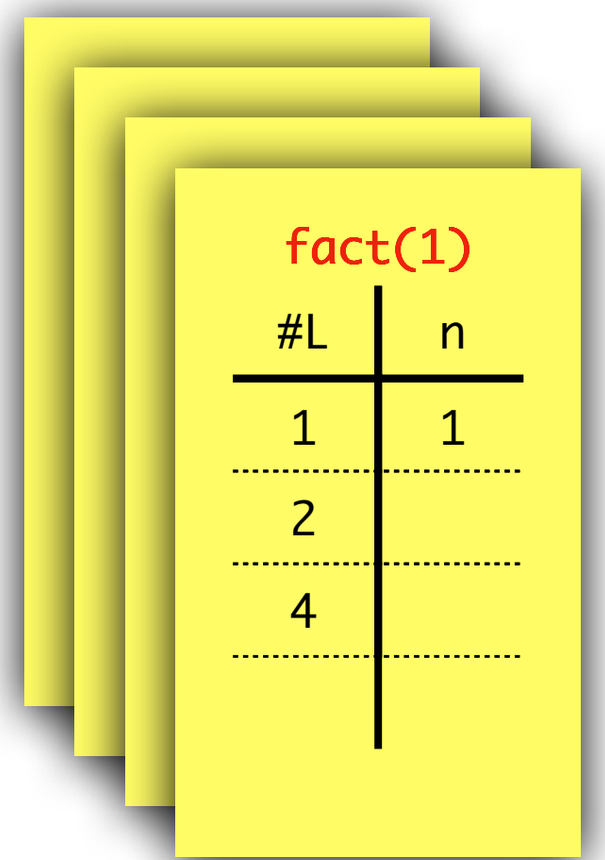

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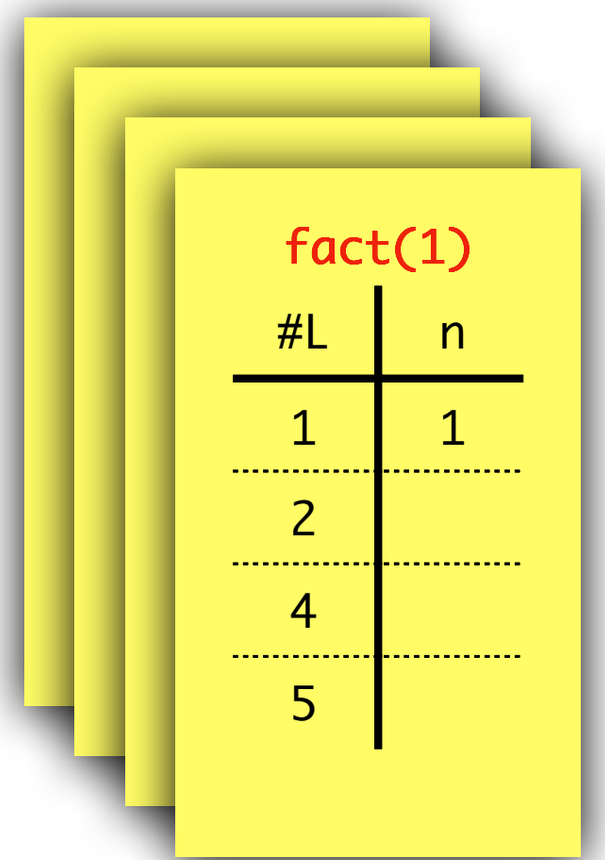

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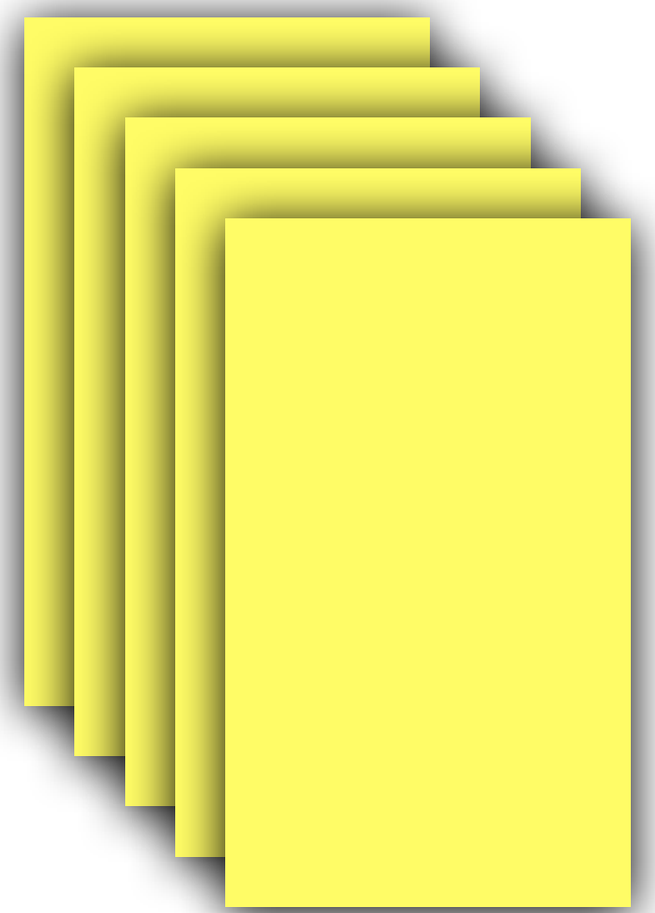

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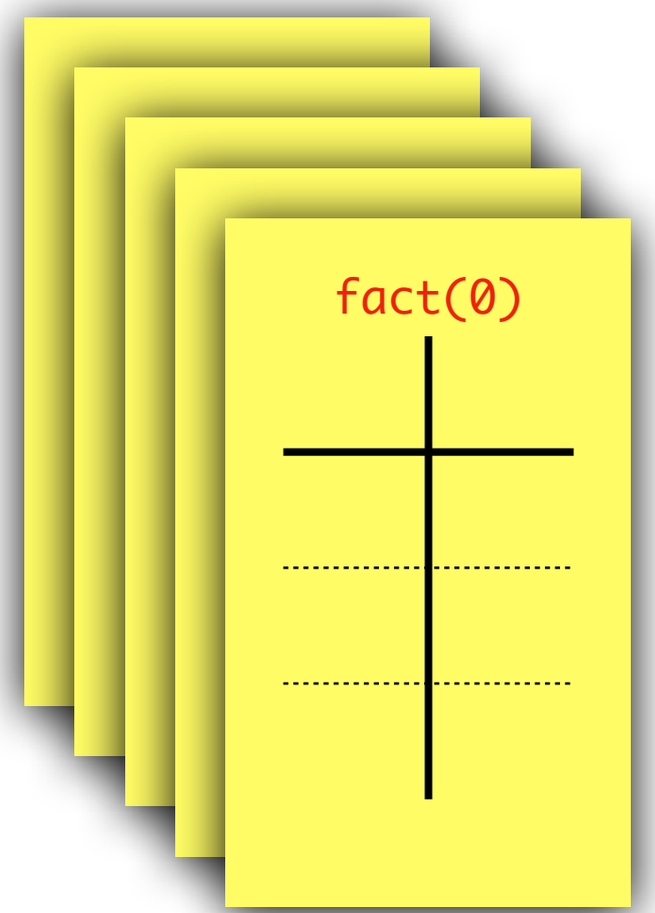

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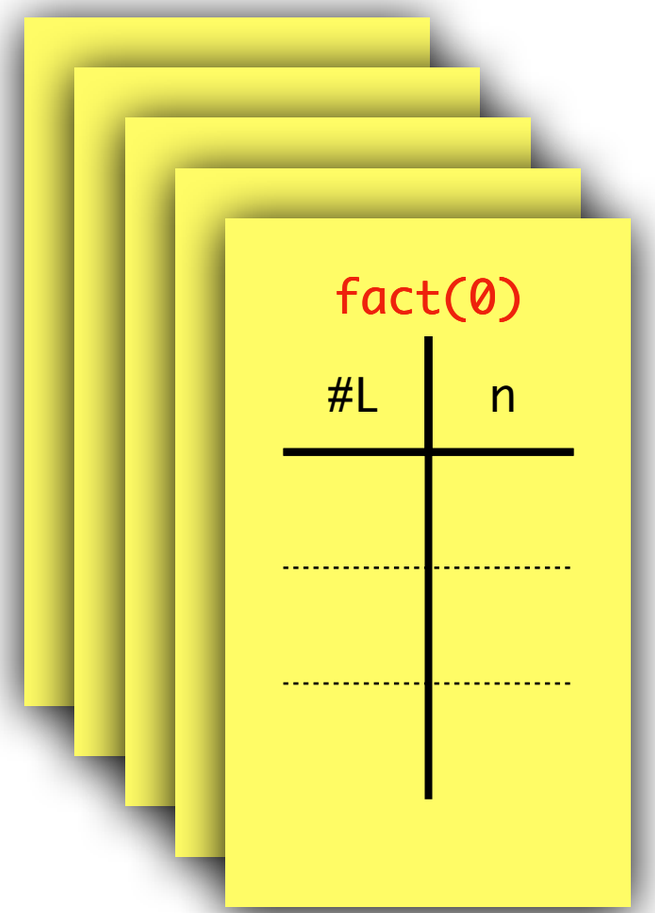

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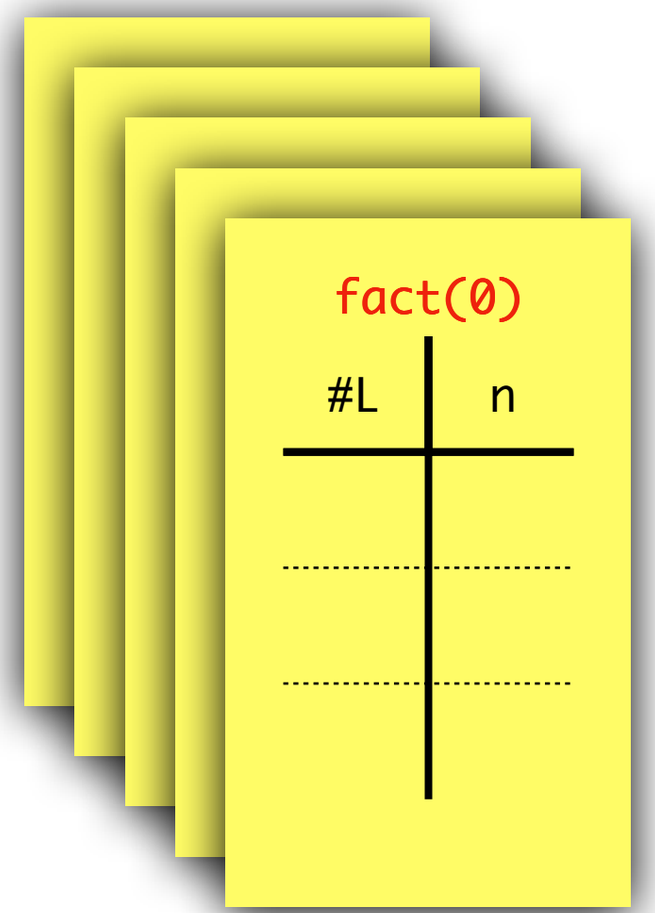

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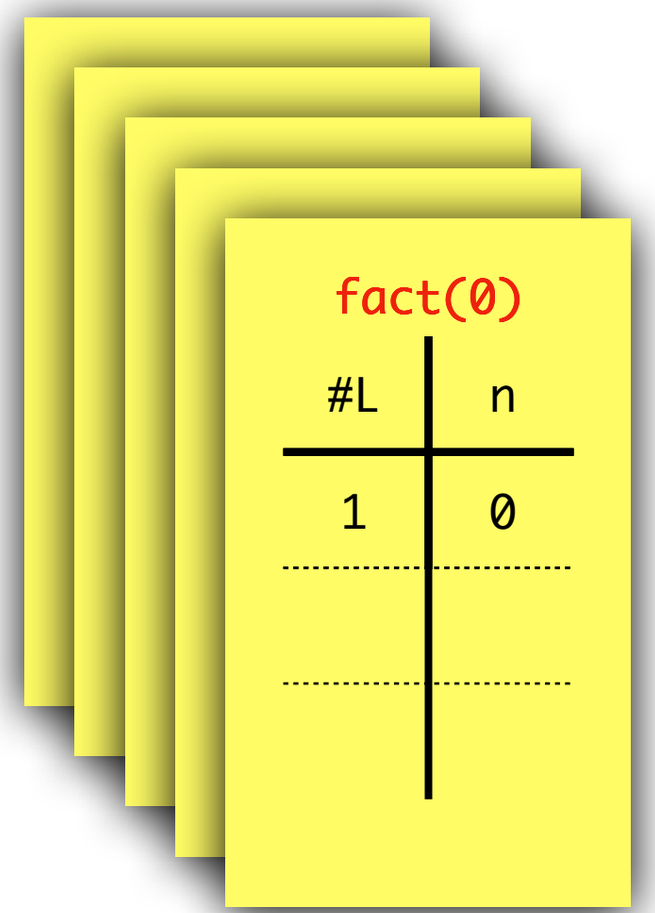

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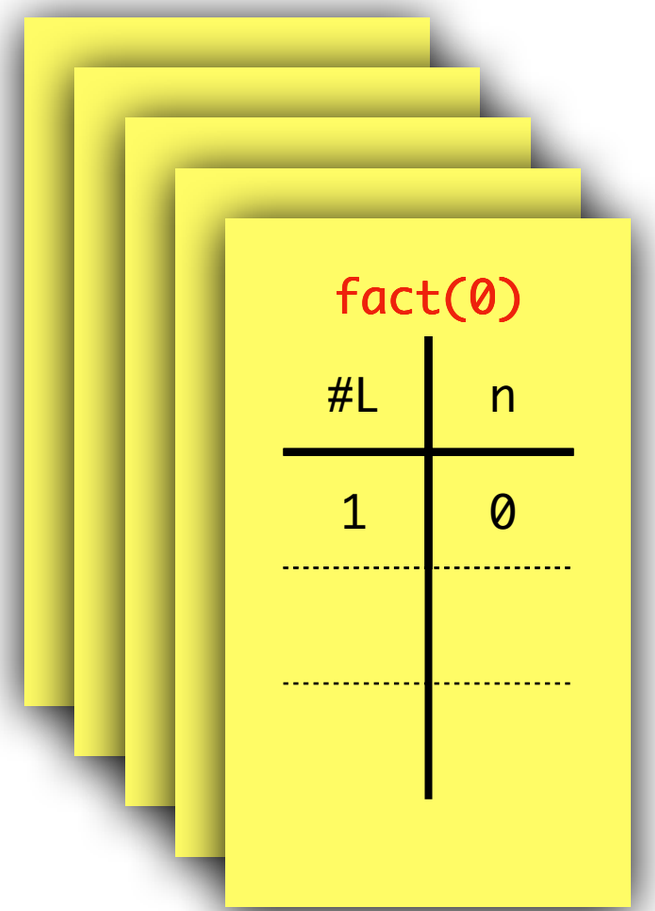

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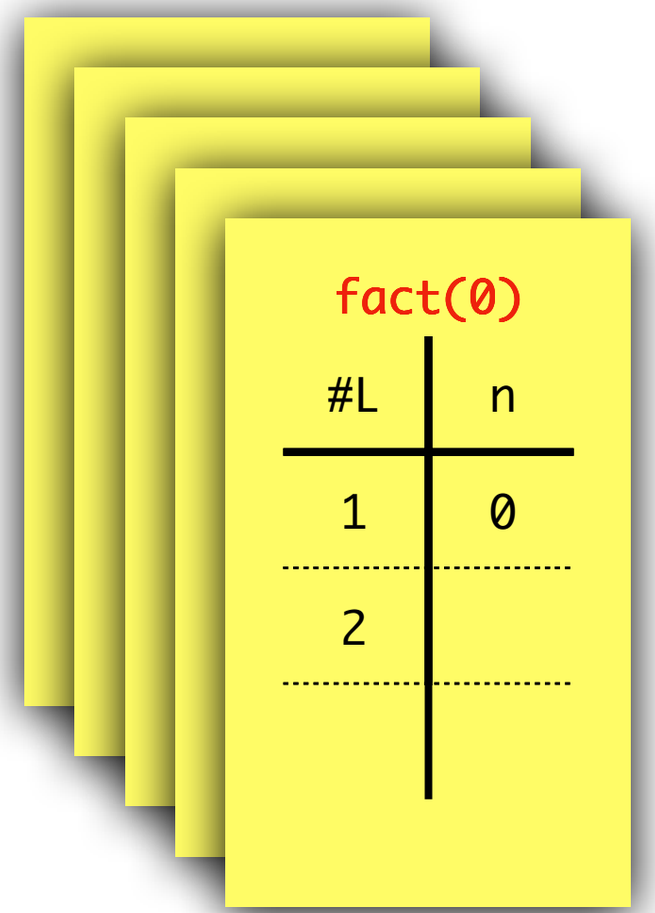

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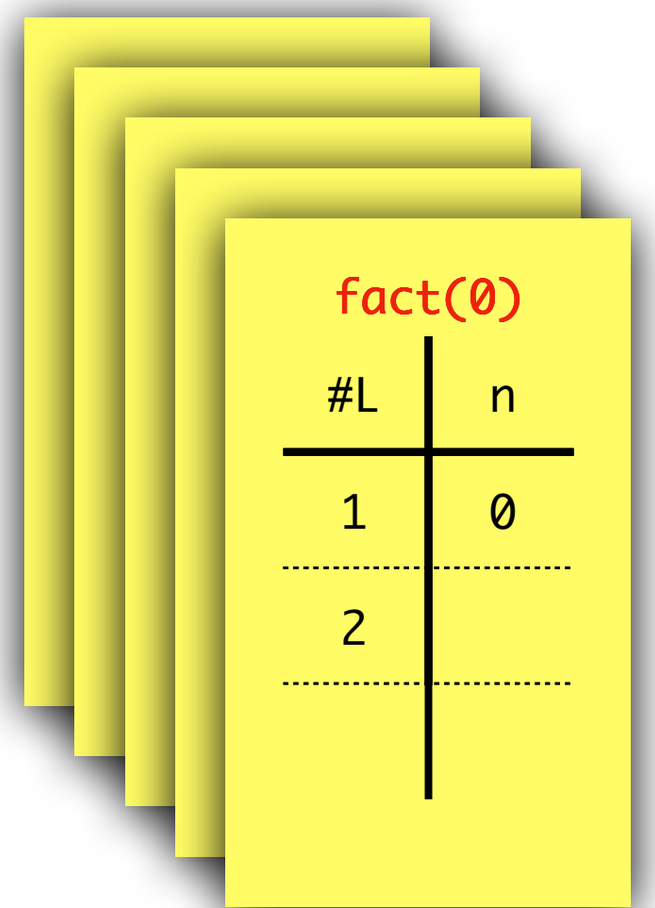

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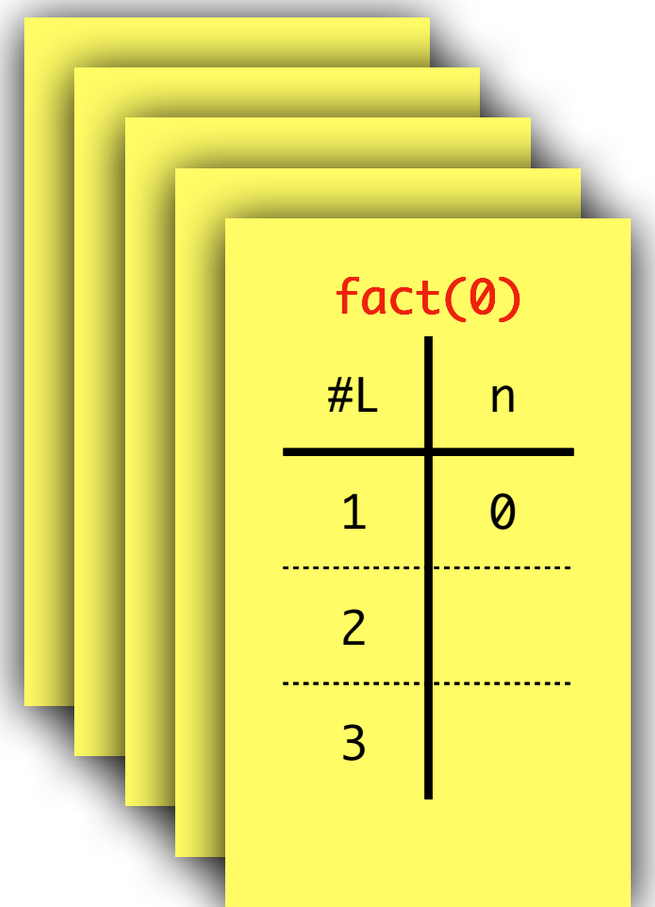

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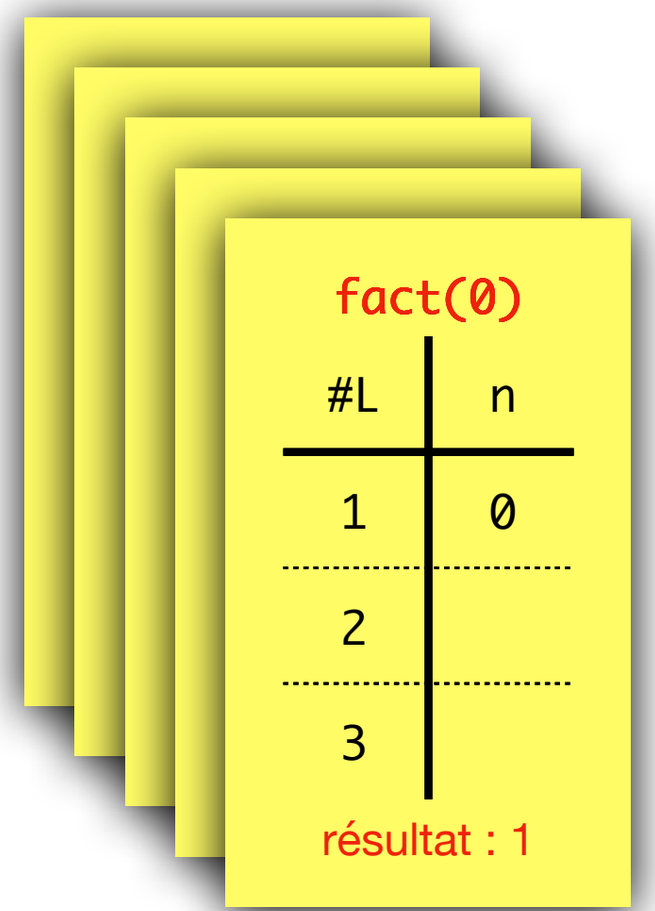

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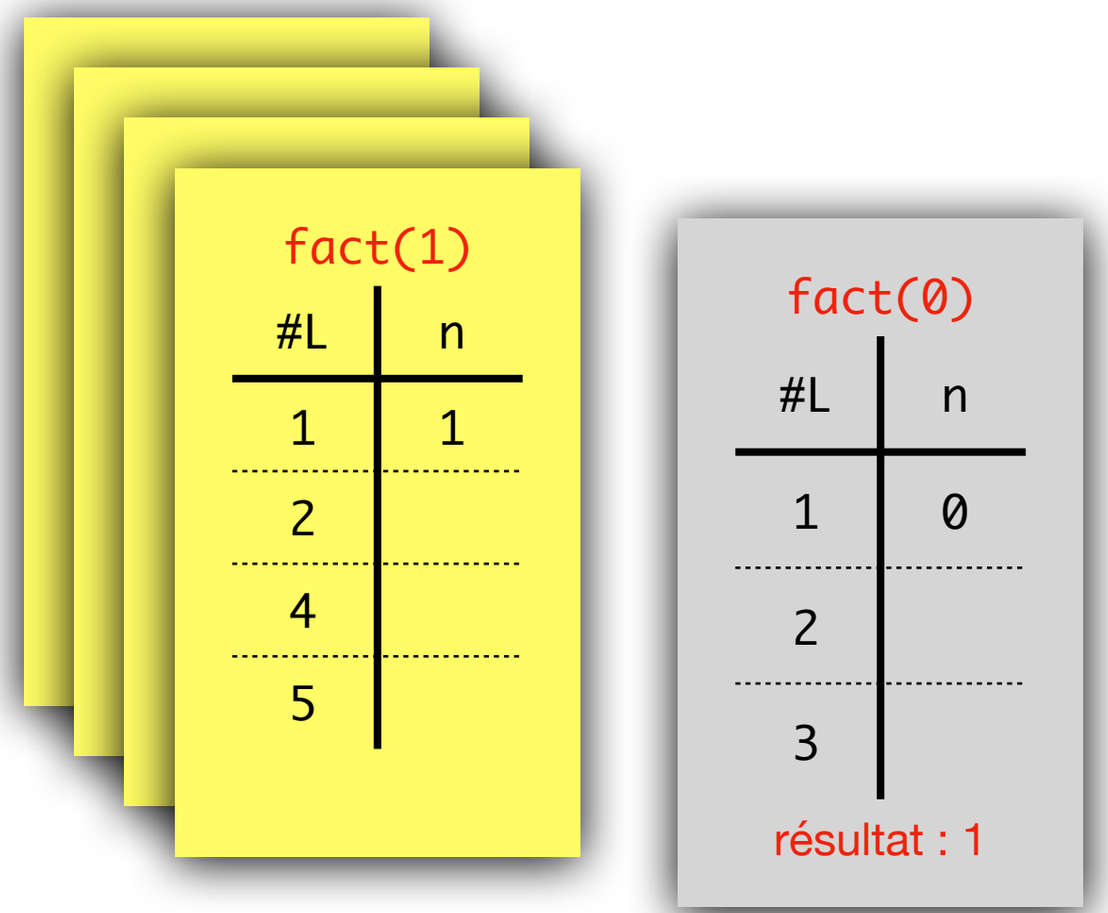

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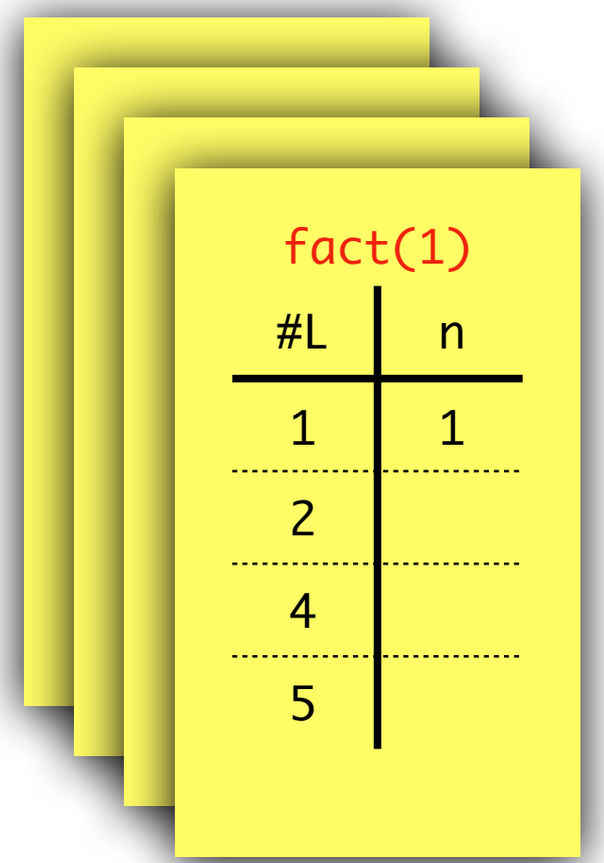
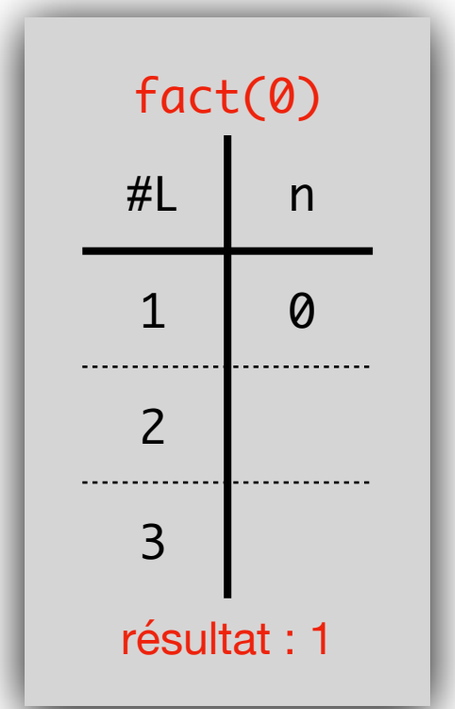
👉 5





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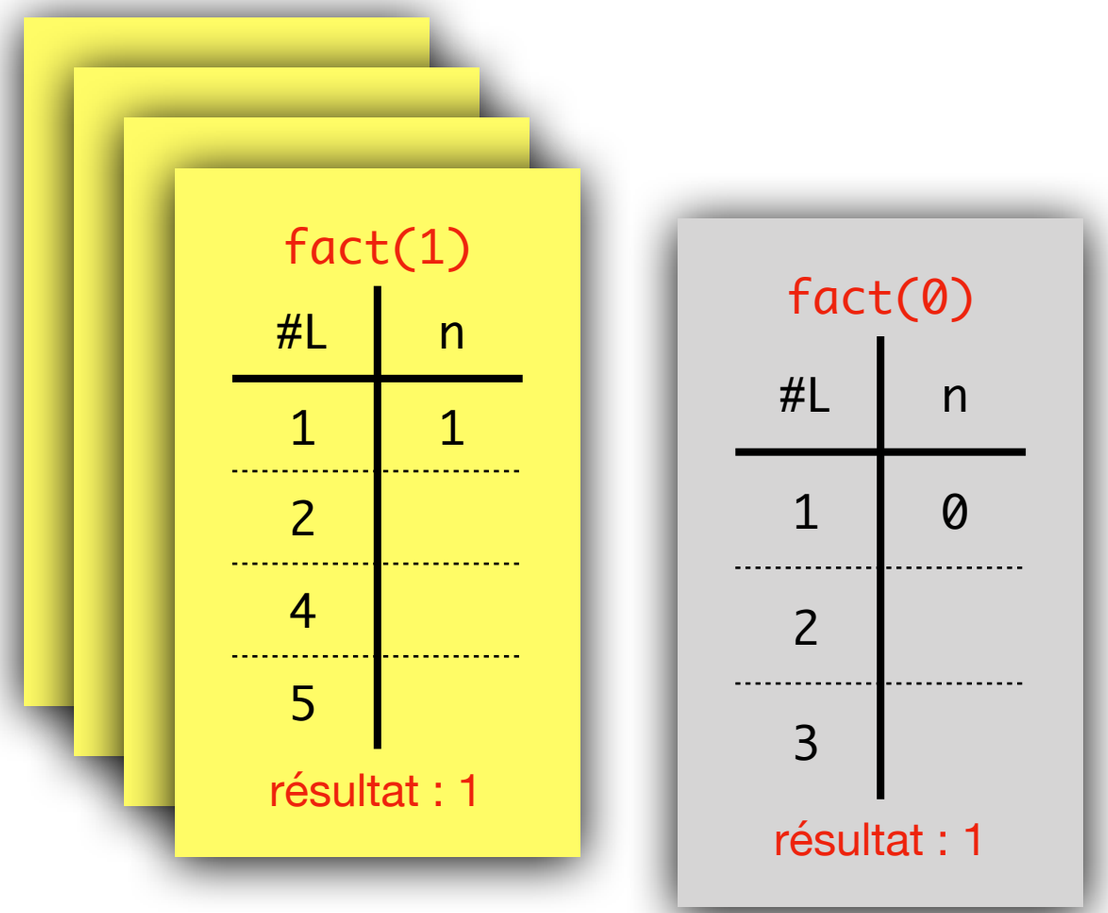

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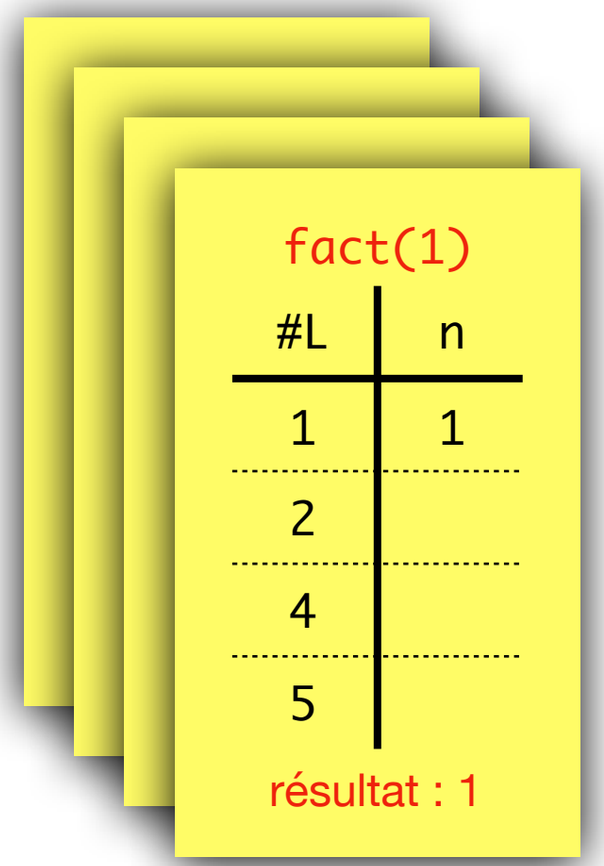

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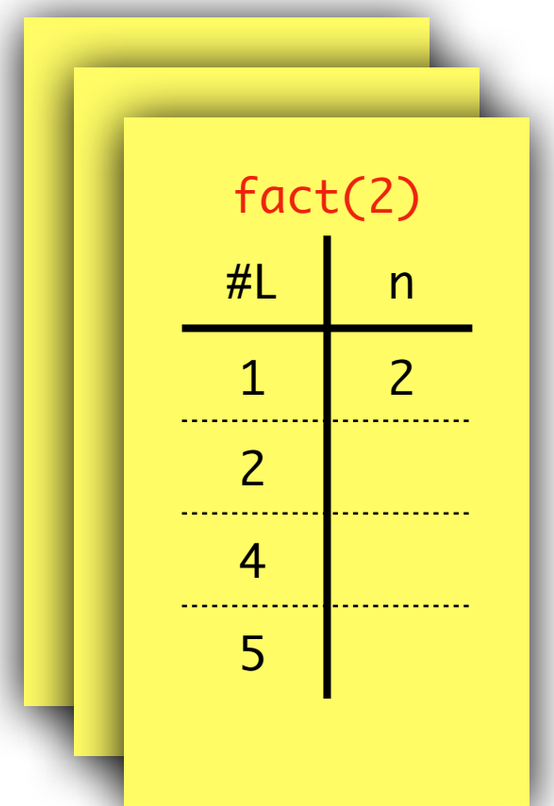

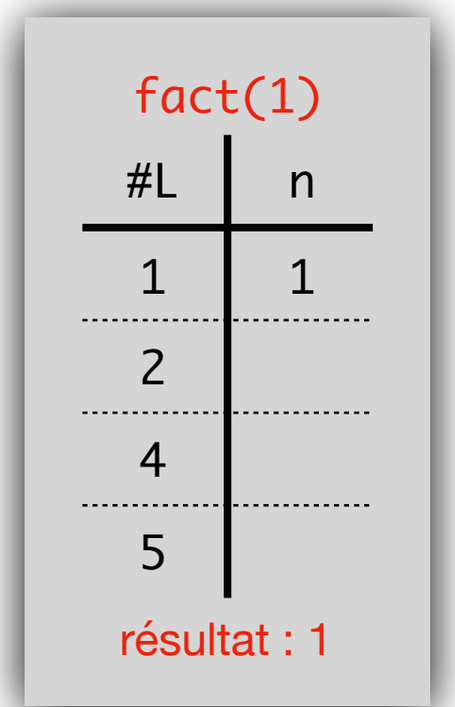

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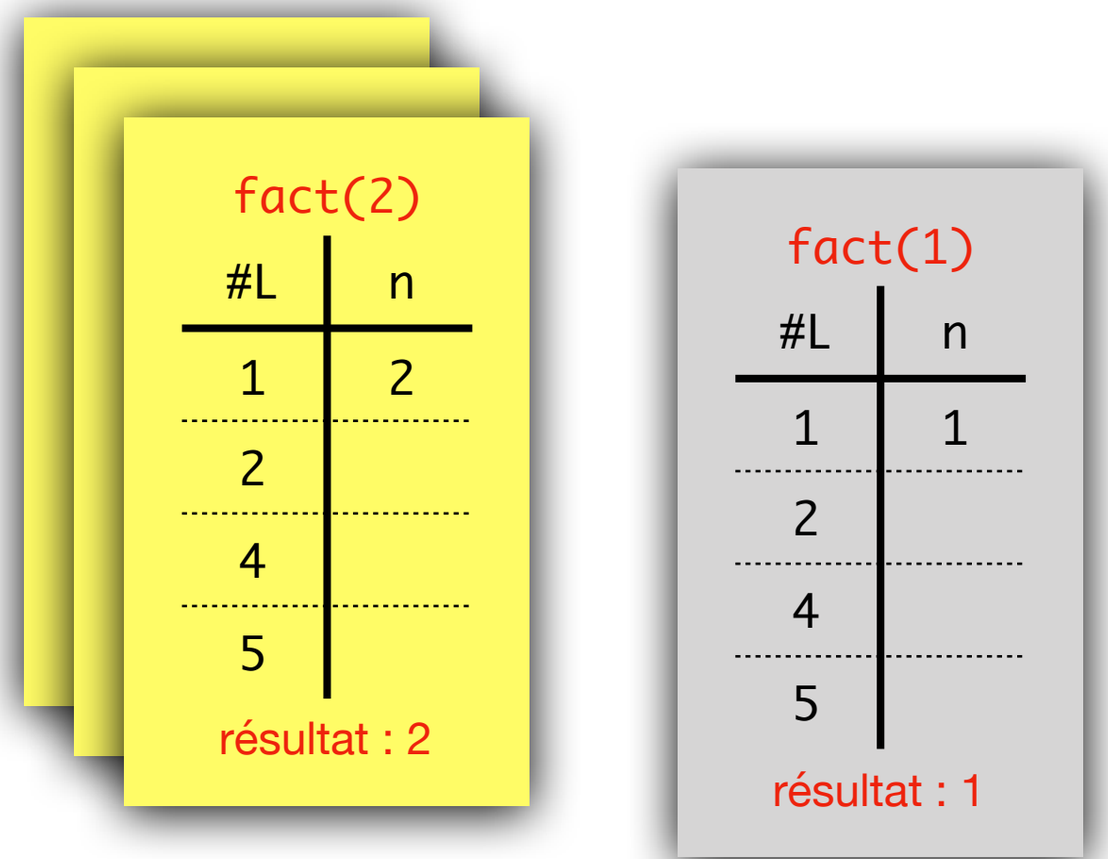

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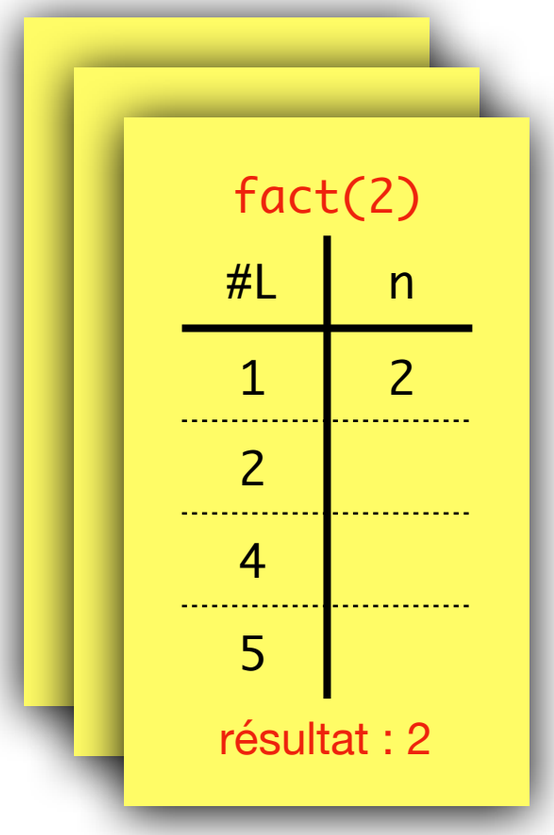

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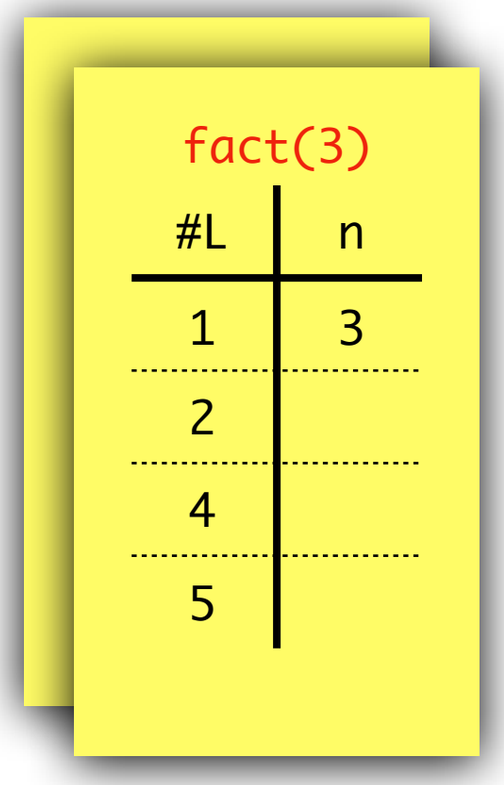

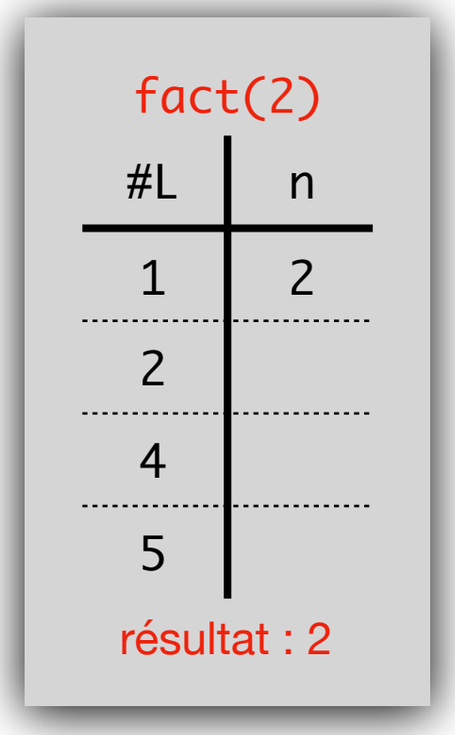

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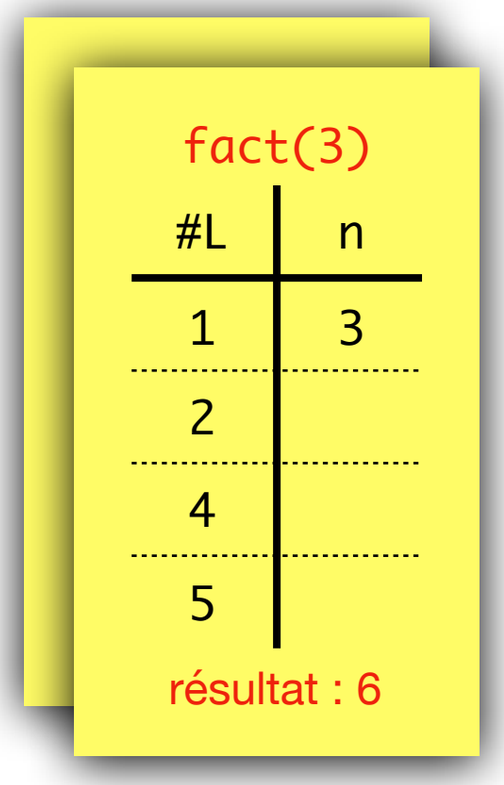

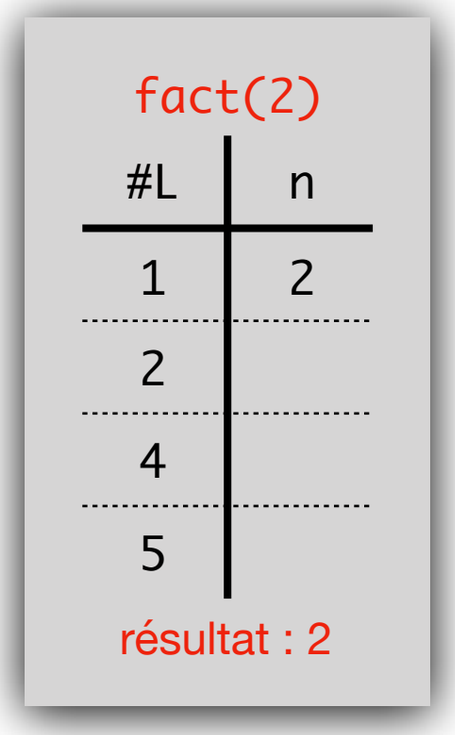

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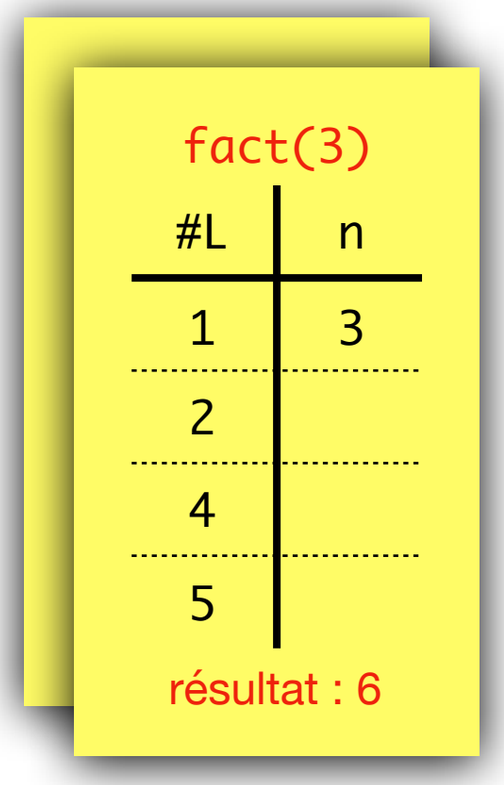

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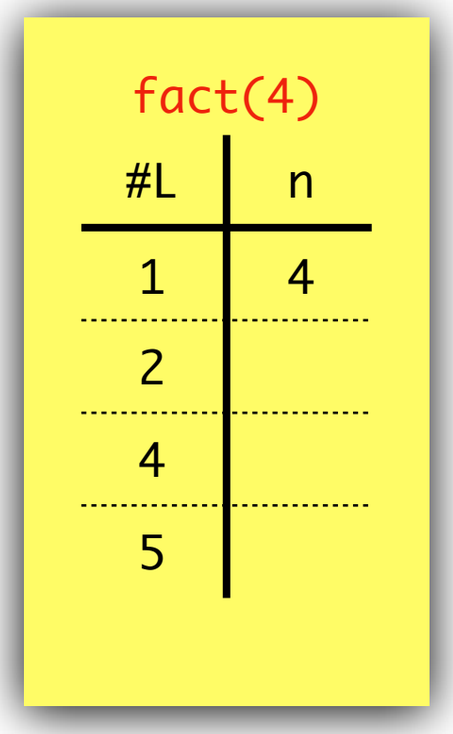

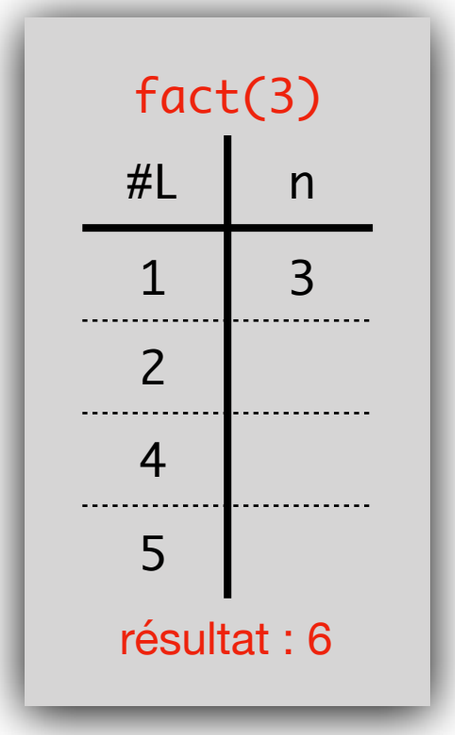

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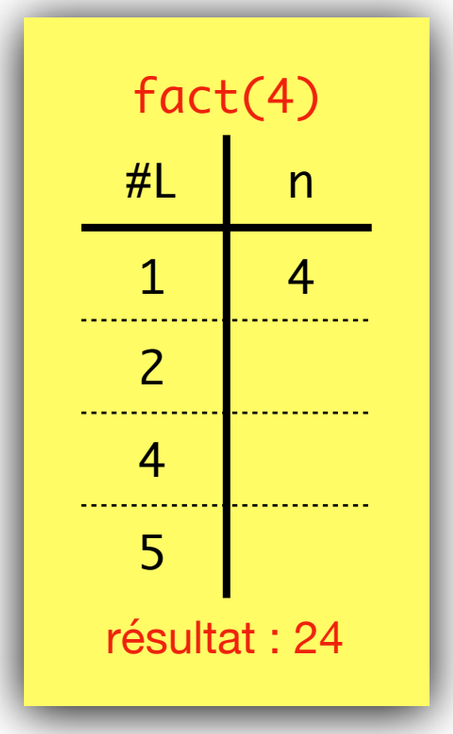

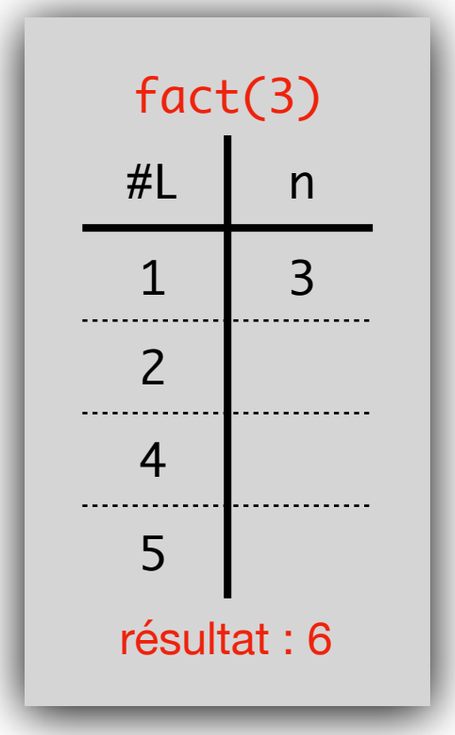

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👉

fact(4)

#L	n
1	4
2	
4	
5	

résultat : 24

Calcul de $4! = 24$

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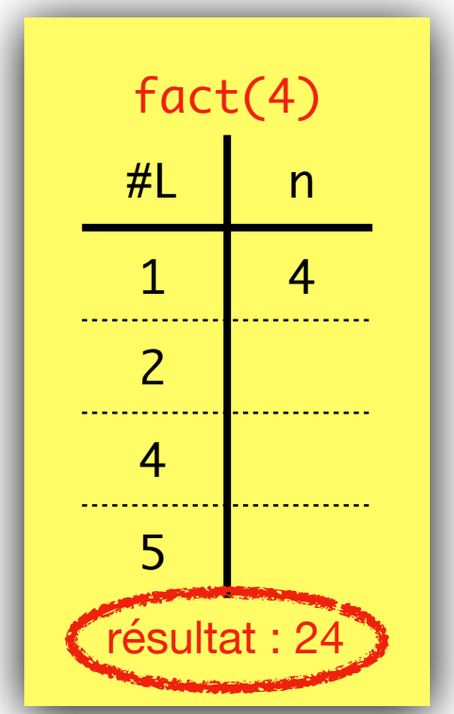

Nombre d’operations : 19

fact(4)

#L	n
1	4
2	
4	
5	

résultat : 24

fact(3)

#L	n
1	3
2	
4	
5	

résultat : 6

fact(2)

#L	n
1	2
2	
4	
5	

résultat : 2

fact(1)

#L	n
1	1
2	
4	
5	

résultat : 1

fact(0)

#L	n
1	0
2	
3	

résultat : 1

Nombre d’operations : 19

fact(4)

#L	n
1	4
2	
4	
5	

résultat : 24

fact(3)

#L	n
1	3
2	
4	
5	

résultat : 6

fact(2)

#L	n
1	2
2	
4	
5	

résultat : 2

fact(1)

#L	n
1	1
2	
4	
5	

résultat : 1

fact(0)

#L	n
1	0
2	
3	

résultat : 1

4 + 4 + 4 + 4 + 3

Terminaison d’un algorithme récursif

Terminaison de `fact`

```
def fact(n):
    if n == 0:
        return 1
    else:
        return n * fact(n - 1)
```

Terminaison de `fact`

si $n = 0$ alors
on s'arrête

```
def fact(n):
    if n == 0:
        return 1
    else:
        return n * fact(n - 1)
```

Terminaison de `fact`

```
def fact(n):
    if n == 0:
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        return n * fact(n - 1)
```

si $n = 0$ alors on s'arrête

sinon on appelle `fact(n-1)` et puis on s'arrête

Terminaison de `fact`

```
def fact(n):
    if n == 0:
        return 1
    else:
        return n * fact(n - 1)
```

si $n = 0$ alors on s'arrête

sinon on appelle `fact(n-1)` et puis on s'arrête

et `fact(n-1)` a un paramètre plus petit, qui se rapproche de 0

Code récursif (erroné ⚠️)

```
def fact2(n):
    return n * fact2(n - 1)
```

Code récursif (erroné ⚠️) sans cas de base

```
def fact2(n):
    return n * fact2(n - 1)
```

Code récursif (erroné ⚠️) sans cas de base

```
def fact2(n):
    return n * fact2(n - 1)
```

on appelle `fact2(n-1)`

Code récursif (erroné ⚠️) sans cas de base

```
def fact2(n):
    return n * fact2(n - 1)
```

on appelle `fact2(n-1)`

...qui appelle `fact2(n-2)`, qui appelle `fact2(n-3)`, qui appelle `fact2(n-4)`...

Code récursif (erroné ⚠️)

```
def fact3(n):
    if n == 0:
        return 1
    else:
        return n * fact3(n)
```

Code récursif (erroné ⚠️) sans réduction de l'entrée

```
def fact3(n):
    if n == 0:
        return 1
    else:
        return n * fact3(n)
```

Code récursif (erroné ⚠️) sans réduction de l'entrée

si $n = 0$ alors on s'arrête

```
def fact3(n):
    if n == 0:
        return 1
    else:
        return n * fact3(n)
```

Code récursif (erroné ⚠️) sans réduction de l'entrée

```
def fact3(n):
    if n == 0:
        return 1
    else:
        return n * fact3(n)
```

si $n = 0$ alors on s'arrête

mais si $n > 0$ alors on appelle `fact3(n)`

Code récursif (erroné ⚠️) sans réduction de l'entrée

```
def fact3(n):
    if n == 0:
        return 1
    else:
        return n * fact3(n)
```

si $n = 0$ alors on s'arrête

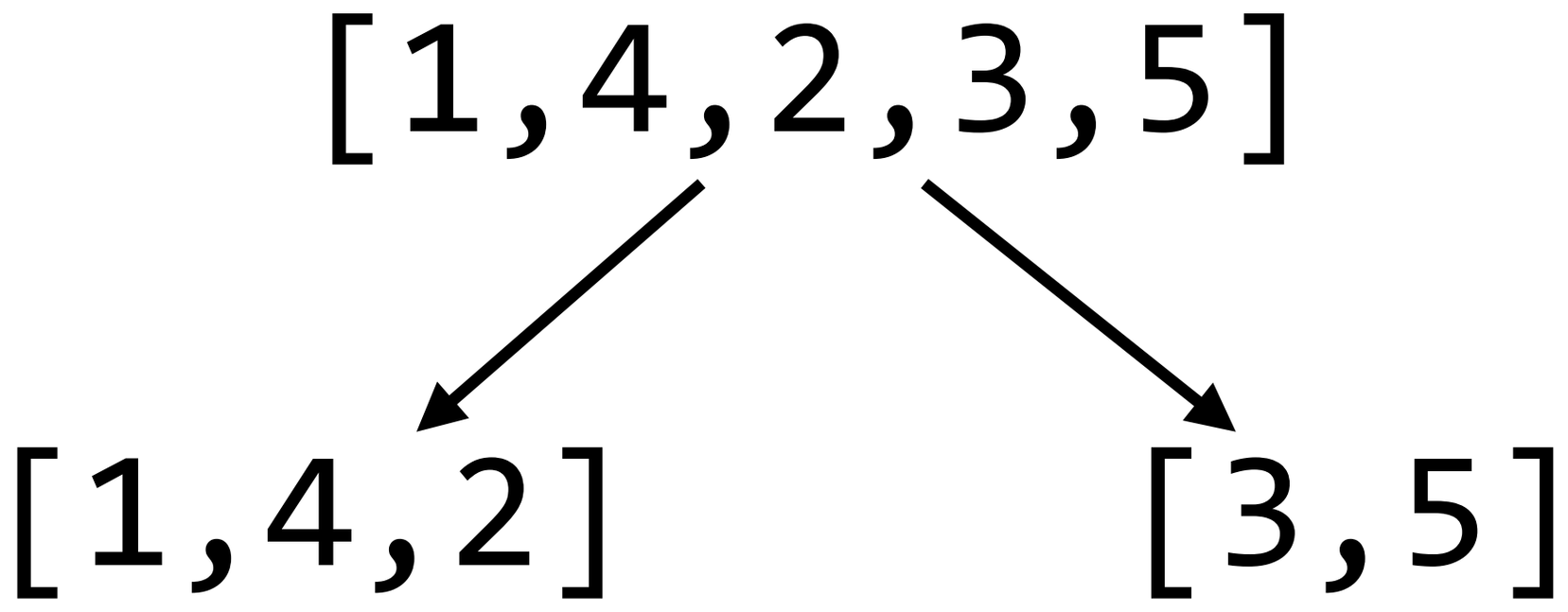
mais si $n > 0$ alors on appelle `fact3(n)`

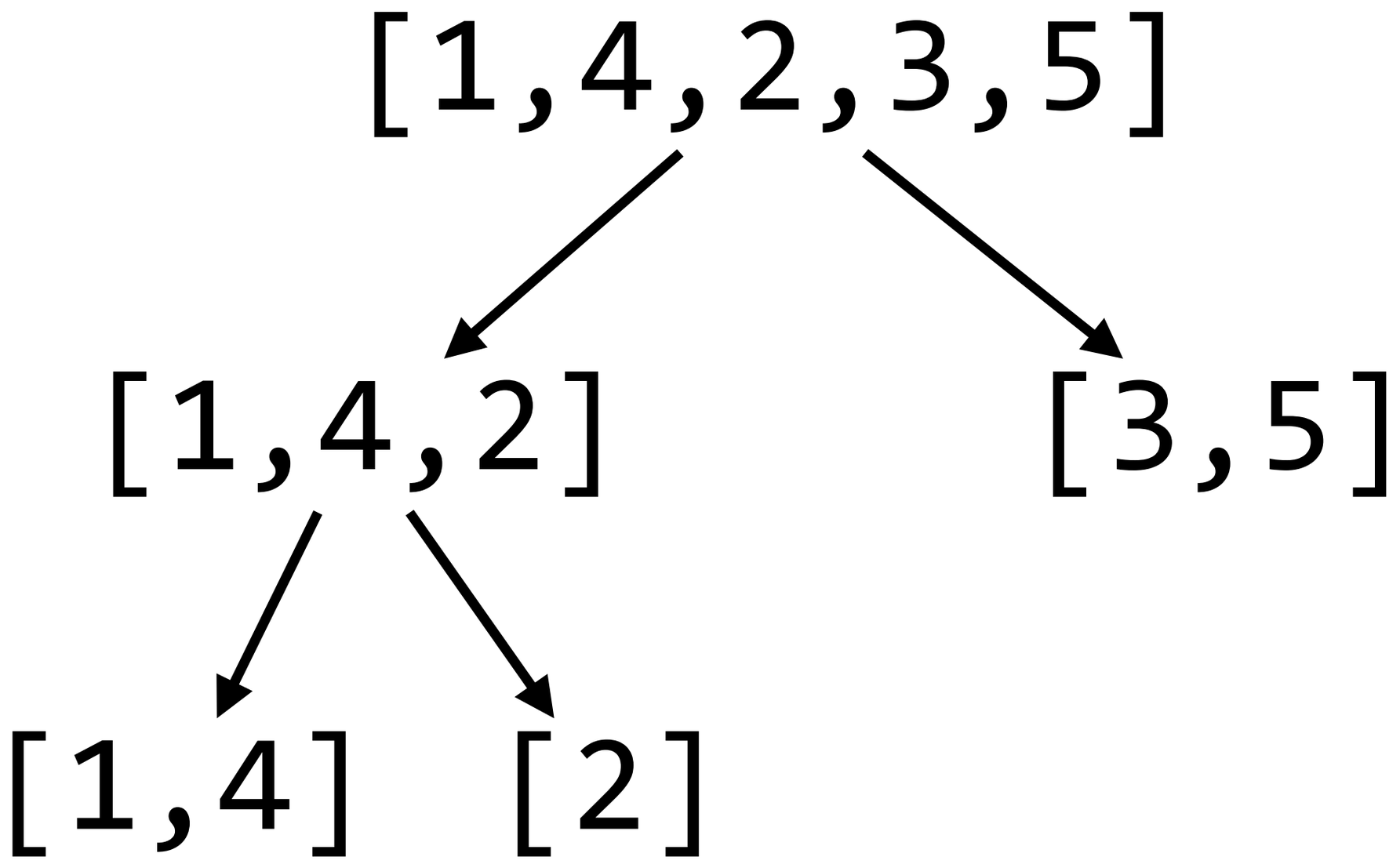
…qui appelle `fact3(n)`, qui appelle `fact3(n)`…

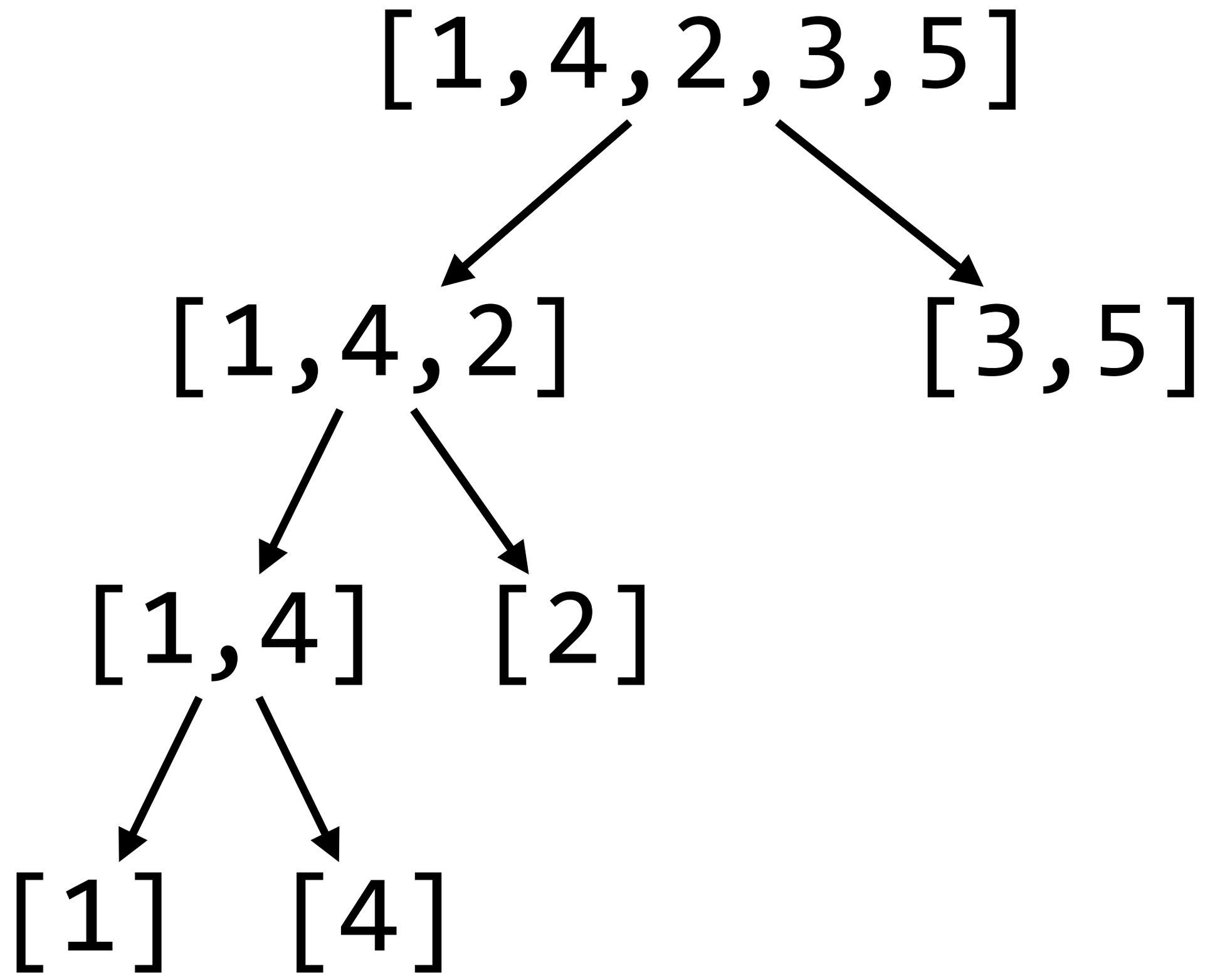
Exercice 1 du TD6

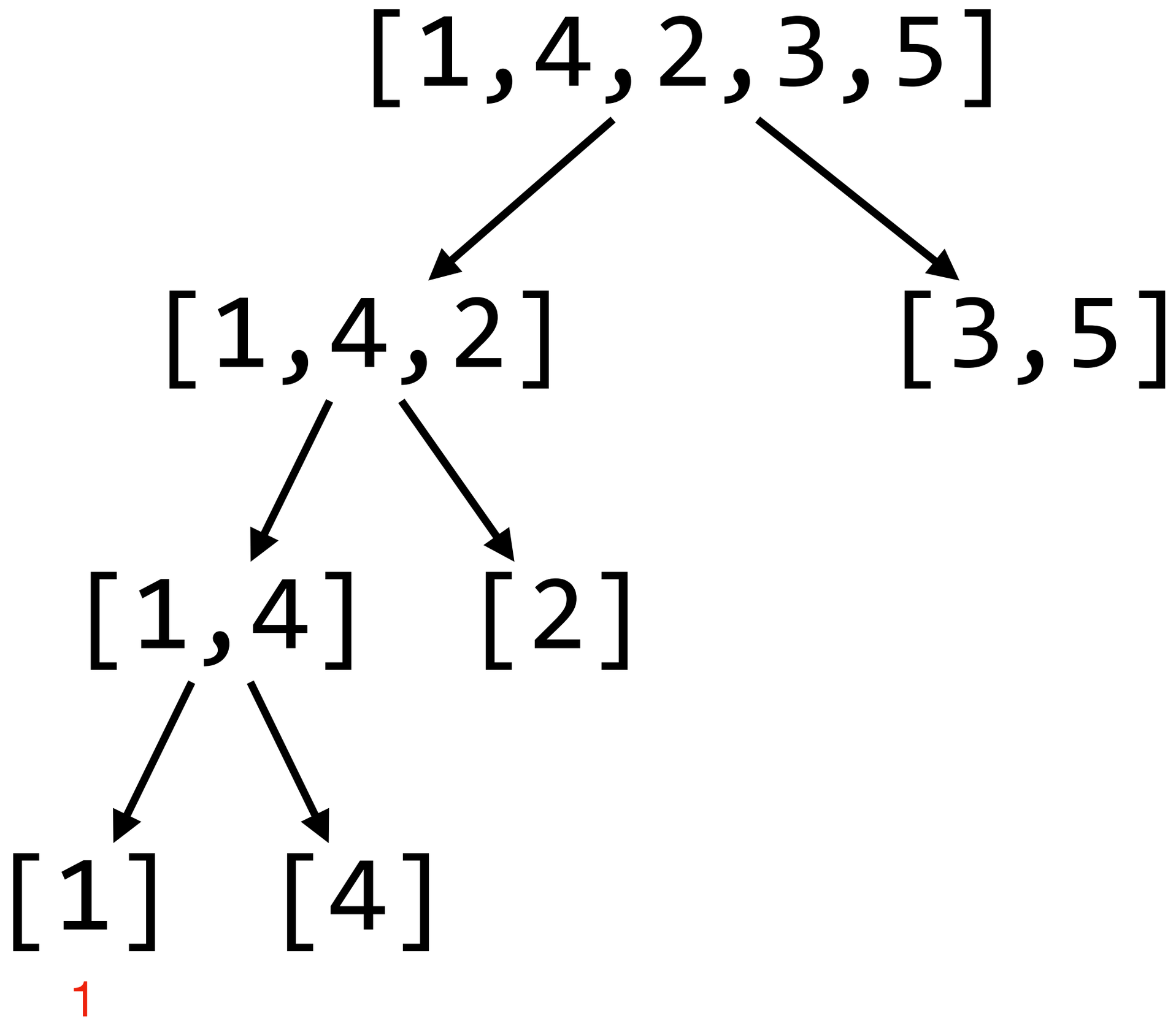
Somme d’un tableau « diviser pour régner »

[1,4,2,3,5]

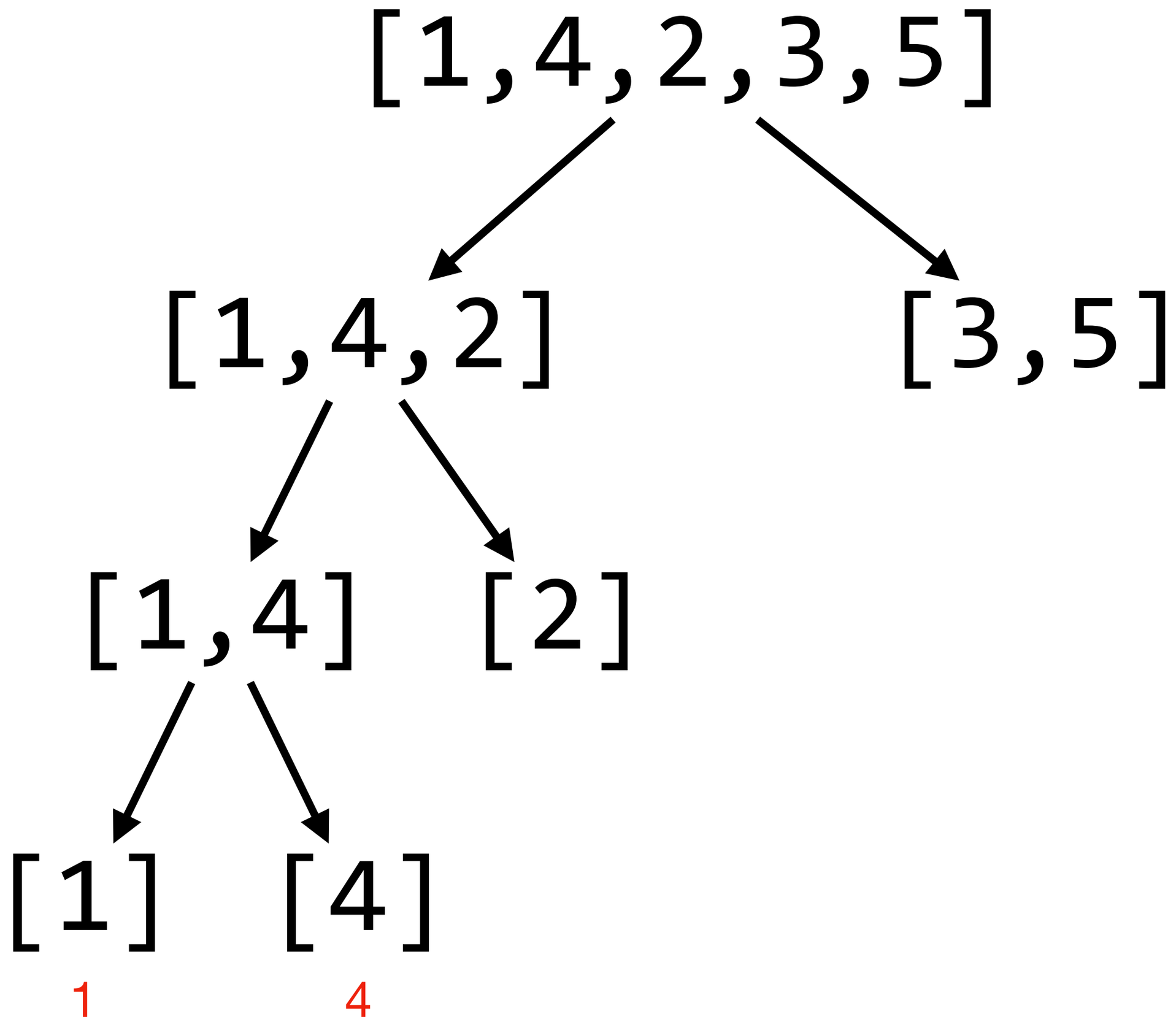

[1,4,2,3,5]
[1,4,2]
[3,5]

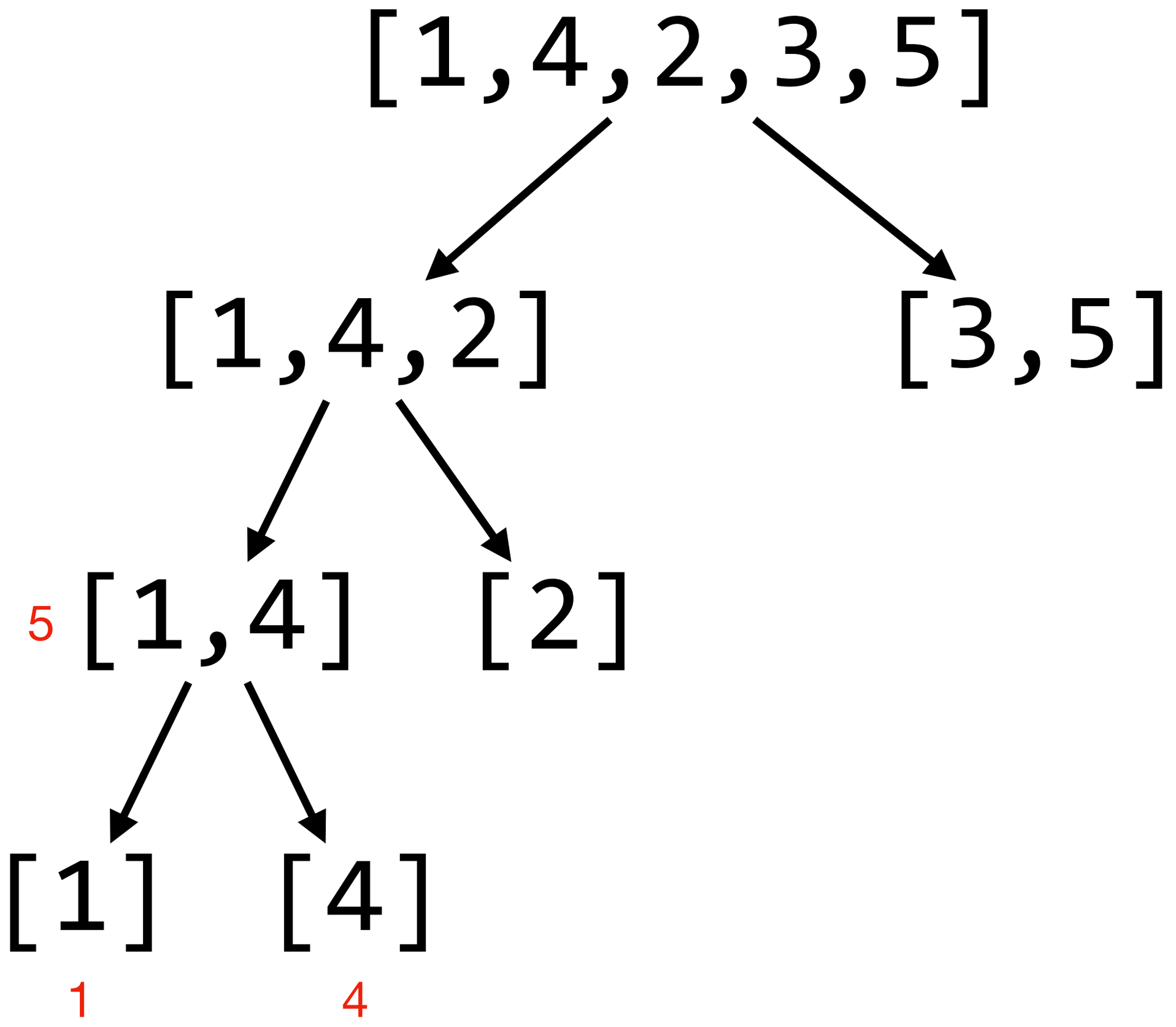

[1,4,2,3,5]
[1,4,2]
[3,5]
[1,4]
[2]

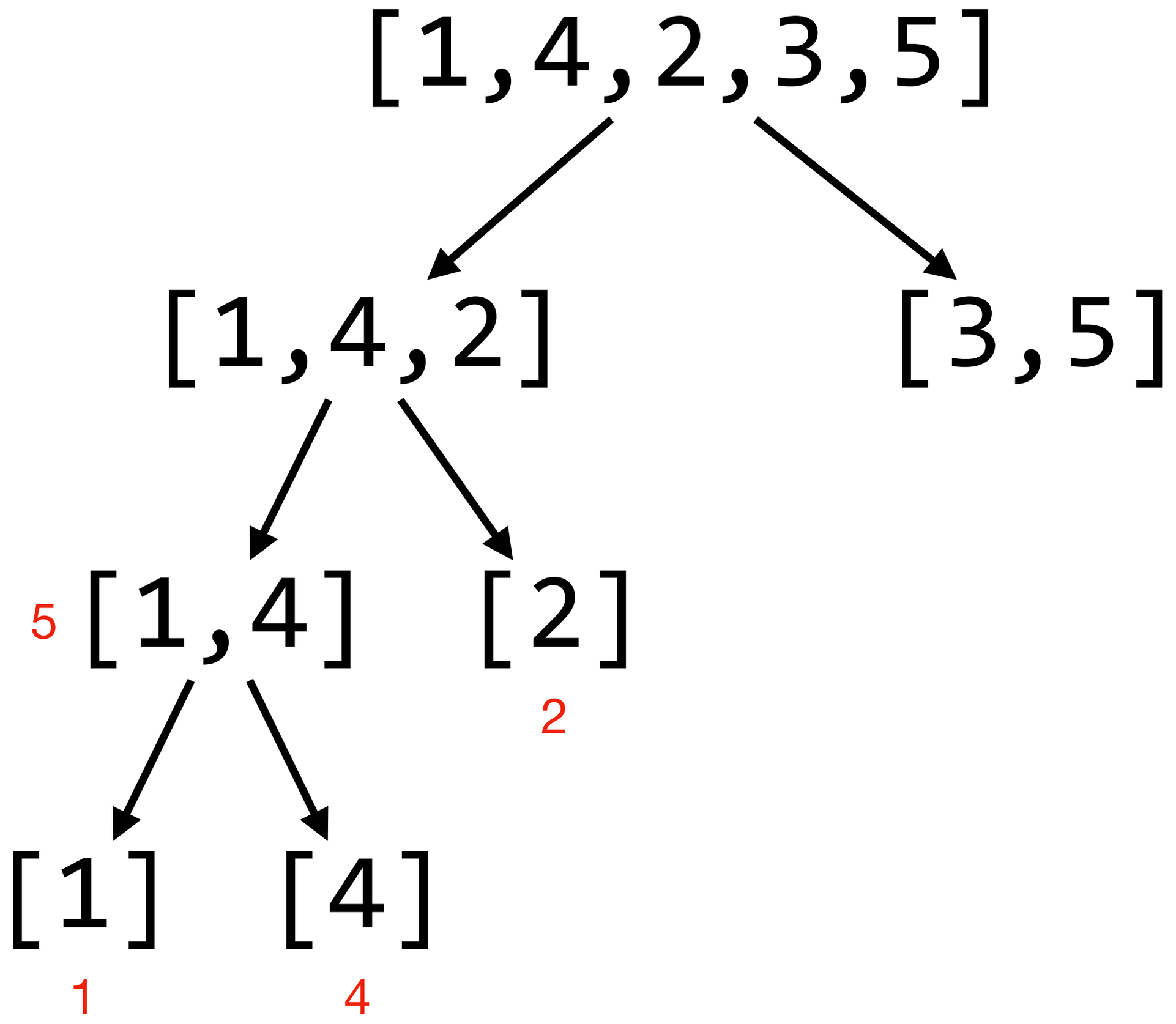

[1,4,2,3,5]
[1,4,2]
[3,5]
[1,4]
[2]
[1]
[4]

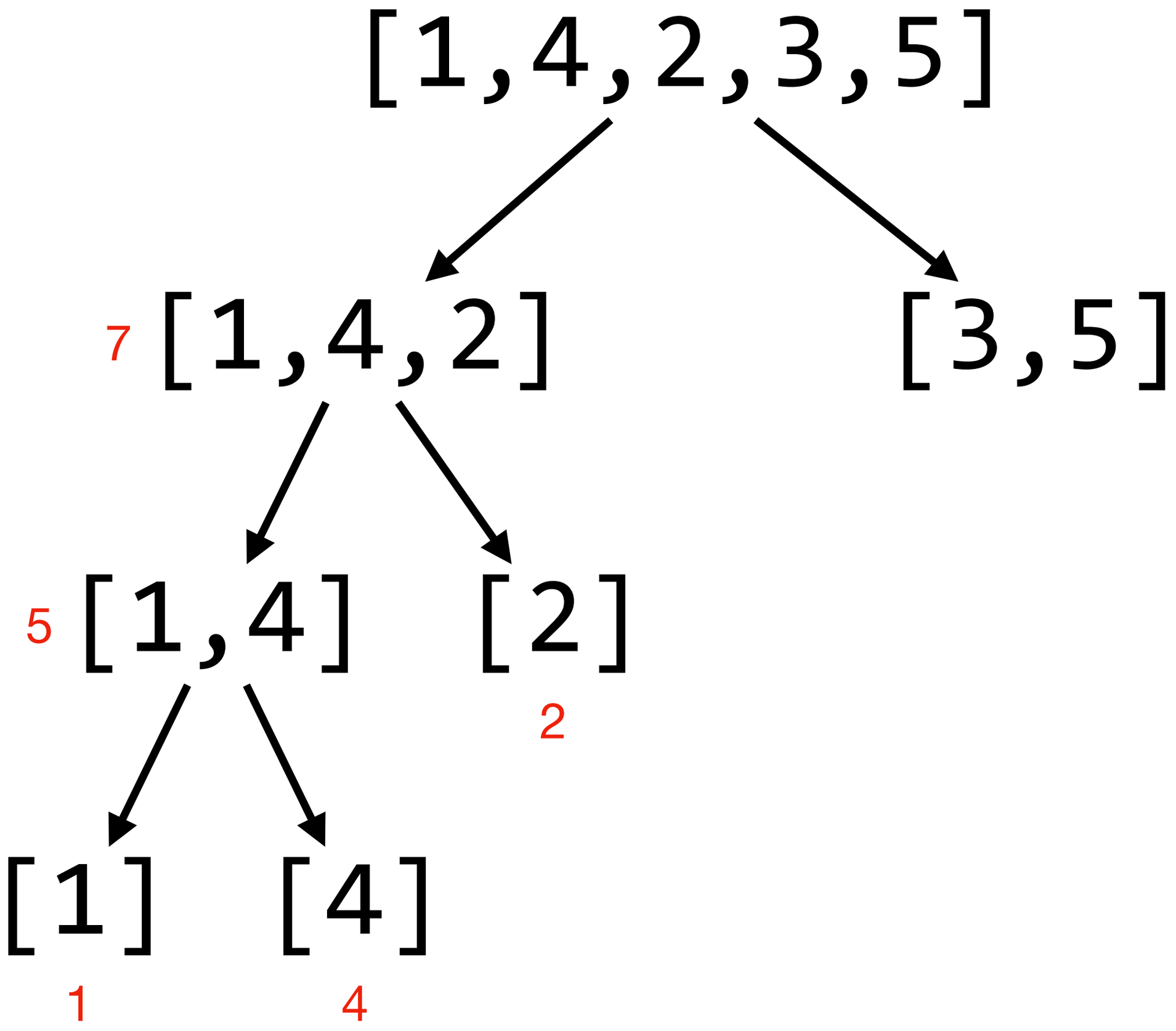


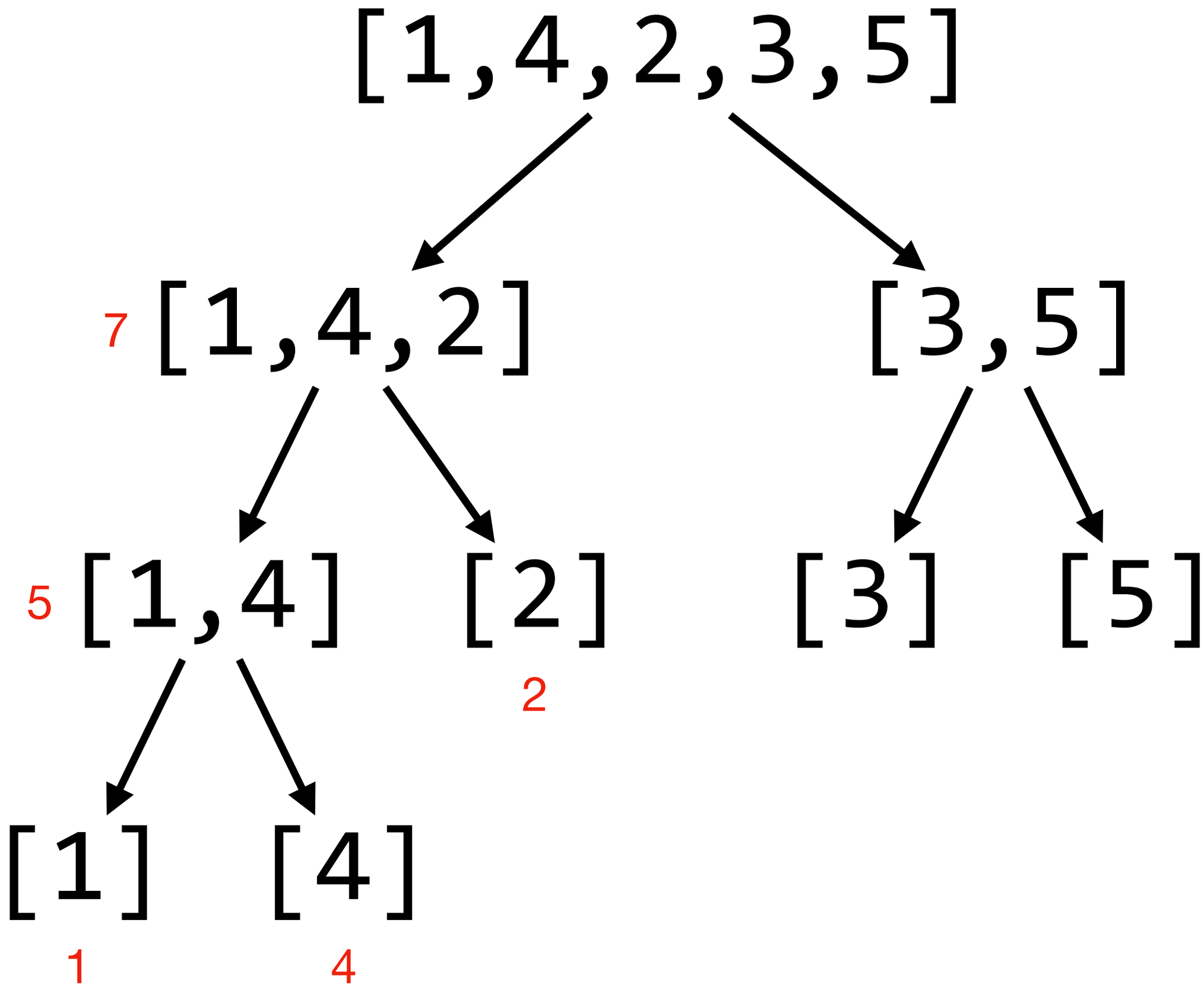
1

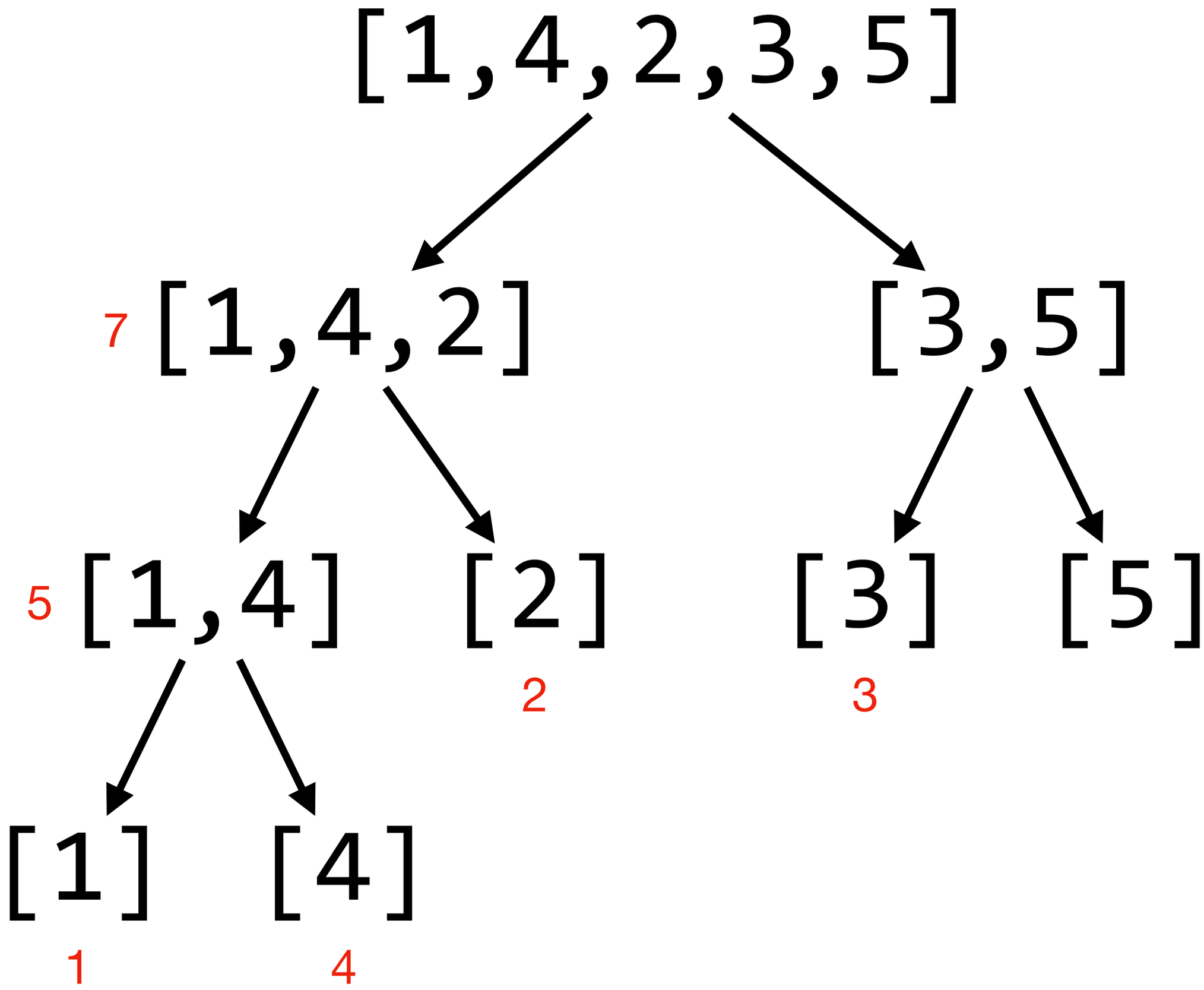

[1,4,2,3,5]
[1,4,2]
[3,5]
[1,4]
[2]
[1]
[4]
1
4

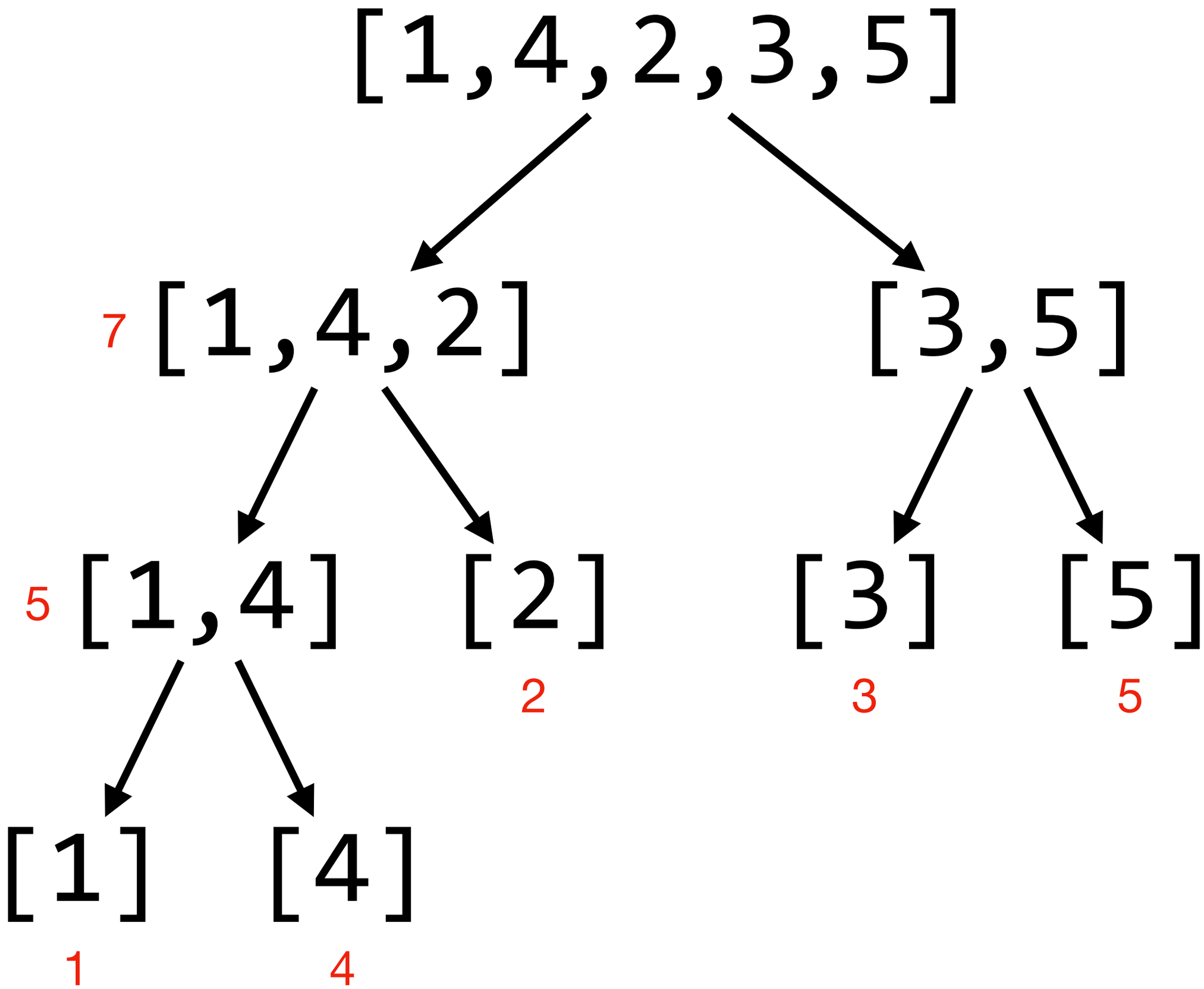

[1,4,2,3,5]
[1,4,2]
[3,5]
5
[1,4]
[2]
[1]
[4]
1
4

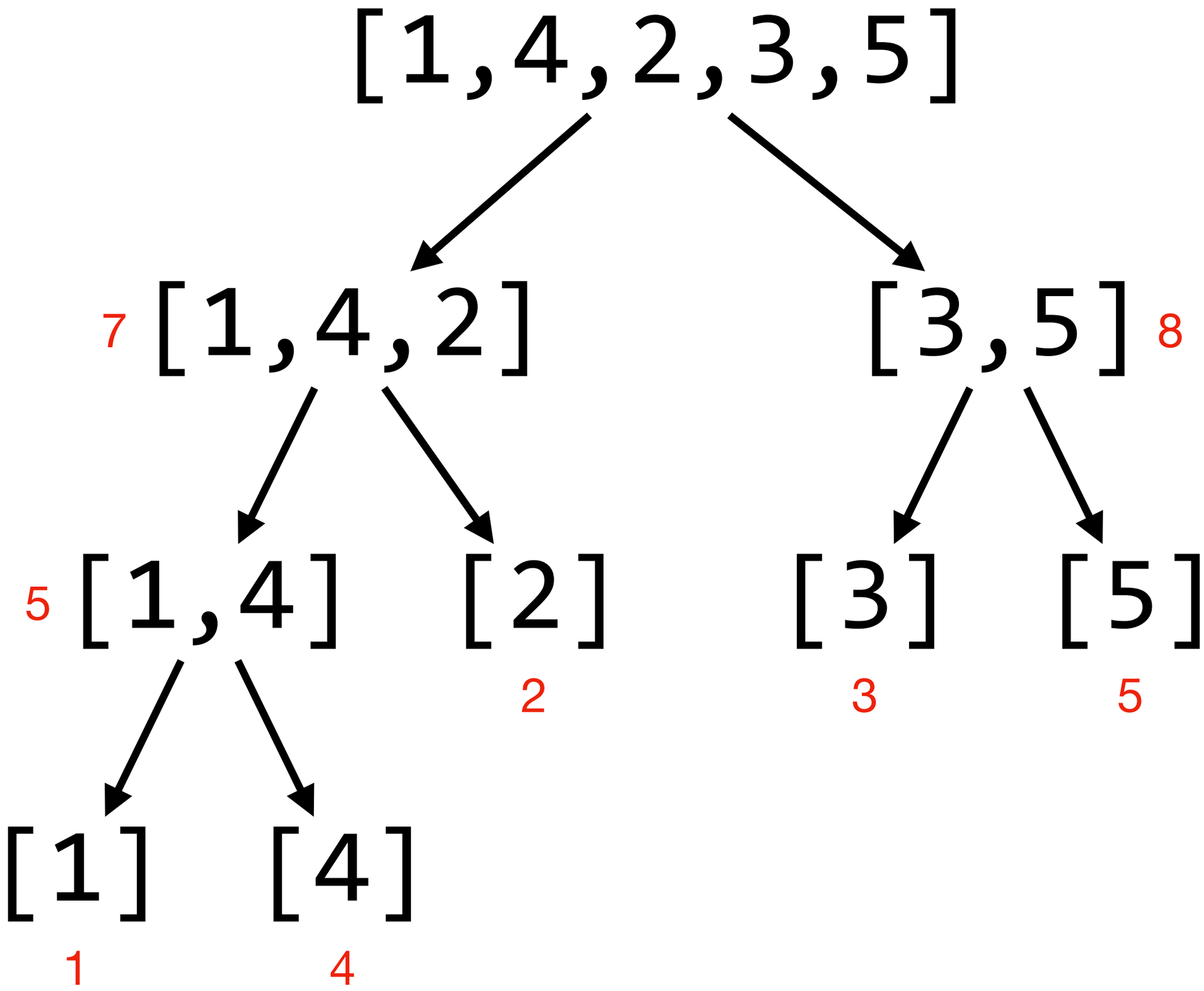

[1,4,2,3,5]
[1,4,2]
[3,5]
5
[1,4]
[2]
2
[1]
[4]
1
4


[1,4,2,3,5]
7
[1,4,2]
[3,5]
5
[1,4]
[2]
2
[1]
[4]
1
4


[1,4,2,3,5]
7
[1,4,2]
[3,5]
5
[1,4]
[2]
[3]
[5]
2
[1]
[4]
1
4


[1,4,2,3,5]
7
[1,4,2]
[3,5]
5
[1,4]
[2]
[3]
[5]
2
3
[1]
[4]
1
4


[1,4,2,3,5]
7
[1,4,2]
[3,5]
5
[1,4]
[2]
[3]
[5]
2
3
5
[1]
[4]
1
4


[1,4,2,3,5]
7
[1,4,2]
[3,5]
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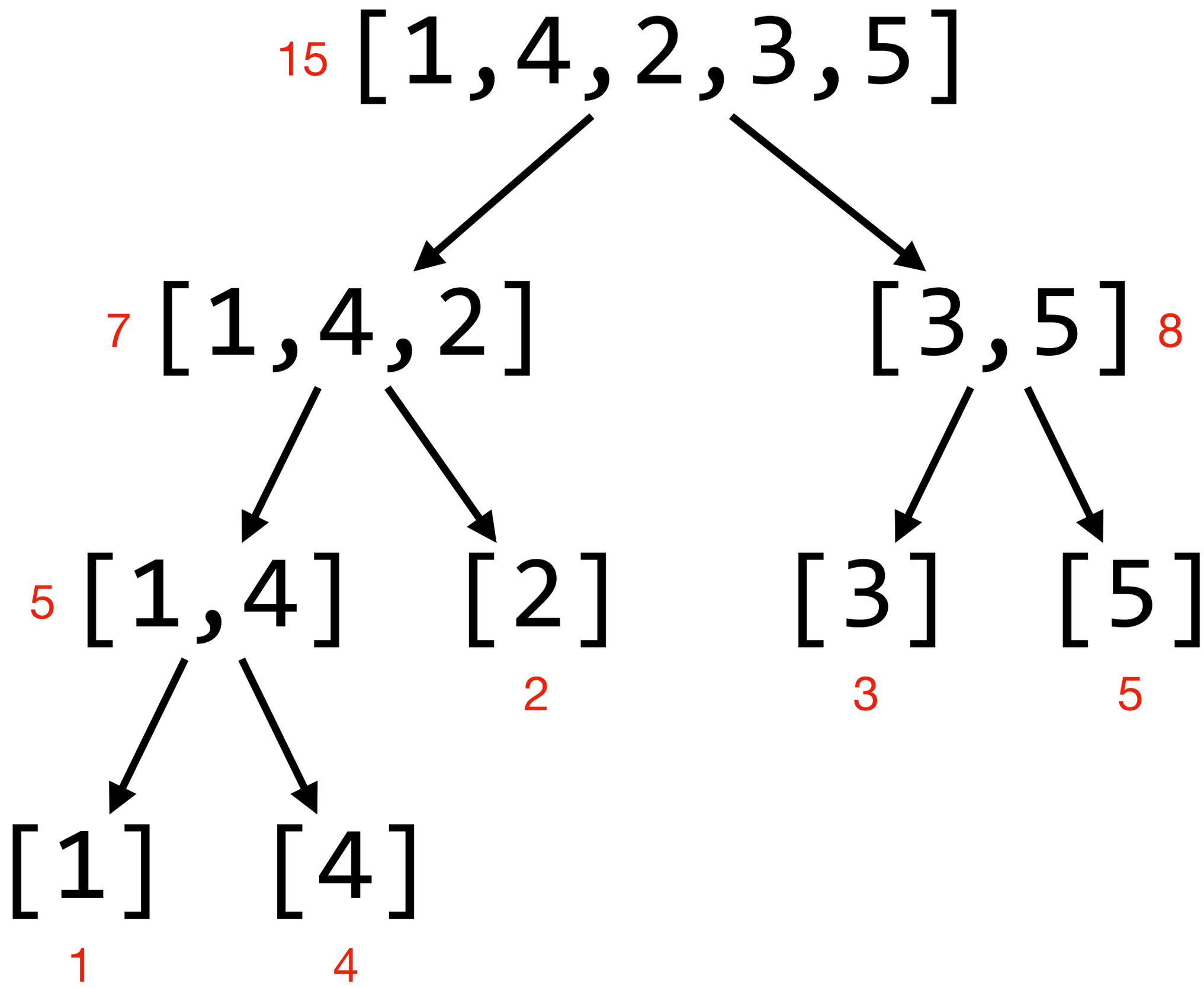
5
[1,4]
[2]
[3]
[5]
2
3
5
[1]
[4]
1
4


15
[1,4,2,3,5]
7
[1,4,2]
[3,5]
8
5
[1,4]
[2]
[3]
[5]
2
3
5
[1]
[4]
1
4

Diviser pour régner

- On veut calculer la somme d'un tableau A de longueur n, donc de la position 0 à la position $n - 1$

- On divise le problème en deux sous-problèmes plus petits :
  - Calculer récursivement la somme de la première moitié de A
  - Calculer récursivement la somme de la seconde moitié de A
- Enfin, on combine les deux (avec $+$) pour avoir le total
- Si le morceau de A est très petit (0 ou 1) on résout directement

Somme d'un tableau A de $A[i]$ à $A[j]$

```
def sum_array(A, i, j):
    n = j - i + 1
    if n == 0:
        return 0
    elif n == 1:
        return A[i]
    else:
        m = (i + j) // 2
        l = sum_array(A, i, m)
        r = sum_array(A, m+1, j)
        return l + r
```

Somme de [1,4,2,3,5]

```
1  def sum_array(A, i, j):
2      n = j - i + 1
3      if n == 0:
4          return 0
5      elif n == 1:
6          return A[i]
7      else:
8          m = (i + j) // 2
9          l = sum_array(A, i, m)
10         r = sum_array(A, m+1, j)
11         return l + r
```

Somme de [1,4,2,3,5]

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1  def sum_array(A, i, j):
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7      else:
8          m = (i + j) // 2
9          l = sum_array(A, i, m)
10         r = sum_array(A, m+1, j)
11         return l + r
```

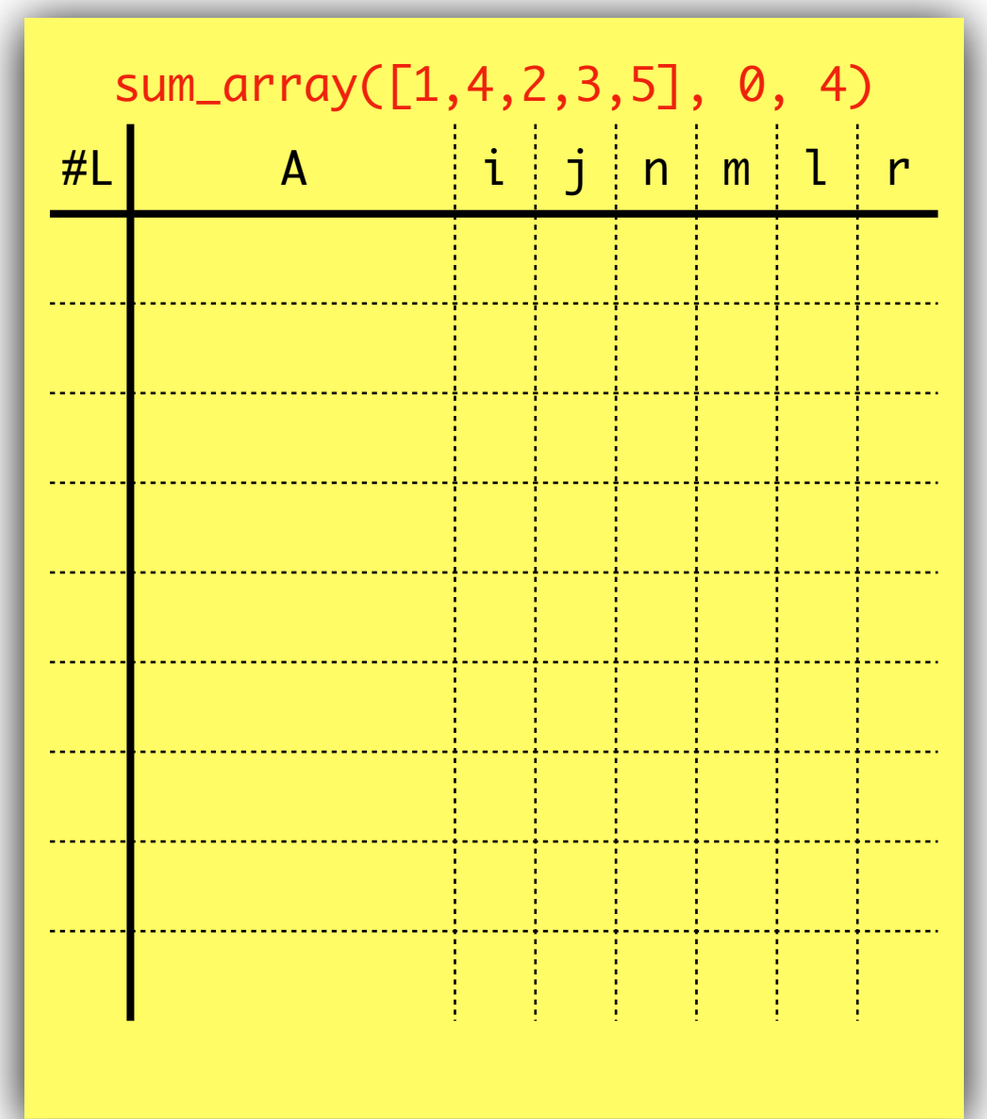

Somme de [1,4,2,3,5]

```
1  def sum_array(A, i, j):
2      n = j - i + 1
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6          return A[i]
7      else:
8          m = (i + j) // 2
9          l = sum_array(A, i, m)
10         r = sum_array(A, m+1, j)
11         return l + r
```

sum_array([1,4,2,3,5], 0, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	4				
2				5			
3							
5							
7							
8					2		
9							

Somme de [1,4,2]

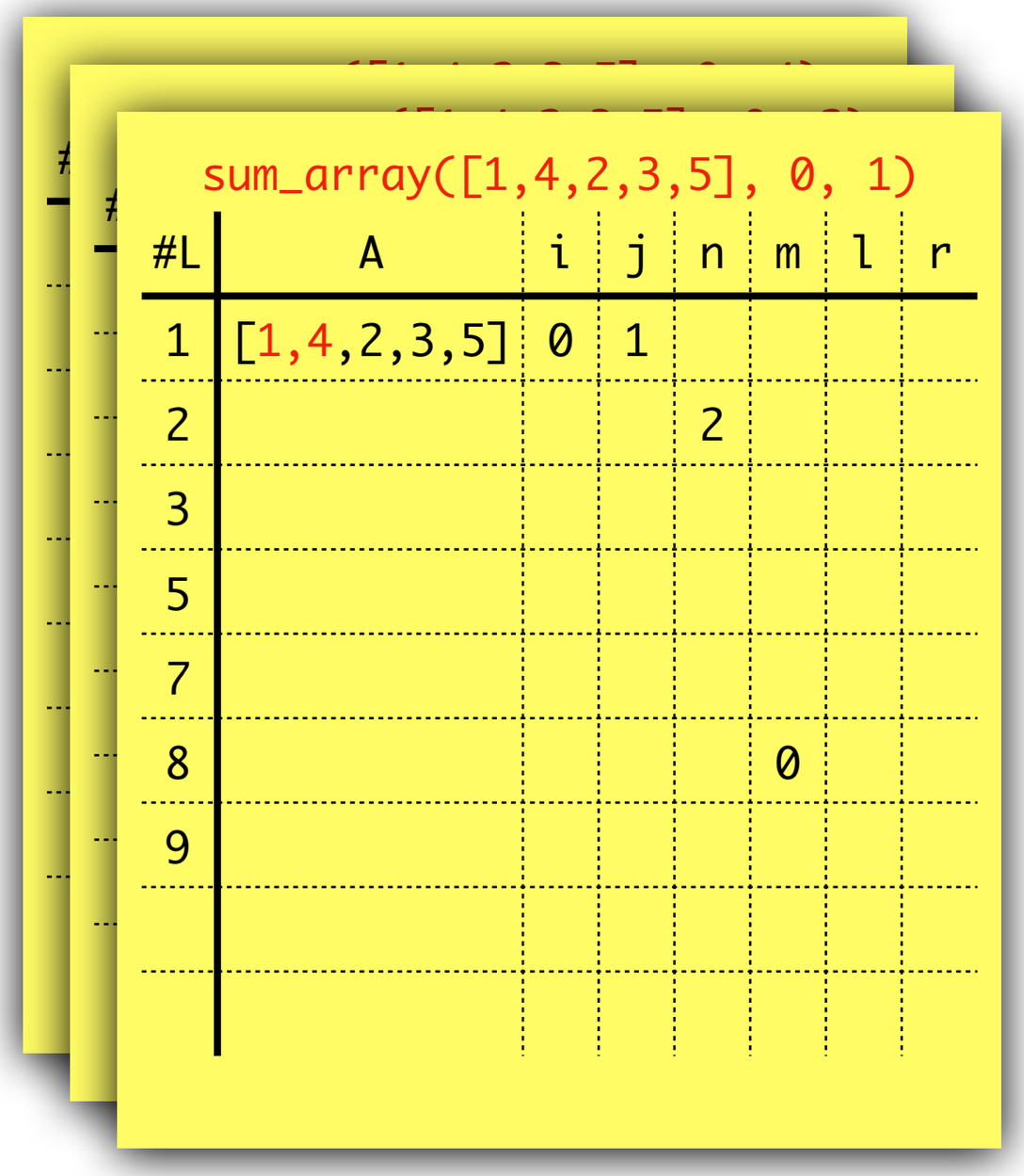
```
1  def sum_array(A, i, j):
2      n = j - i + 1
3      if n == 0:
4          return 0
5      elif n == 1:
6          return A[i]
7      else:
8          m = (i + j) // 2
9          l = sum_array(A, i, m)
10         r = sum_array(A, m+1, j)
11         return l + r
```

sum_array([1,4,2,3,5], 0, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	2				
2				3			
3							
5							
7							
8					1		
9							

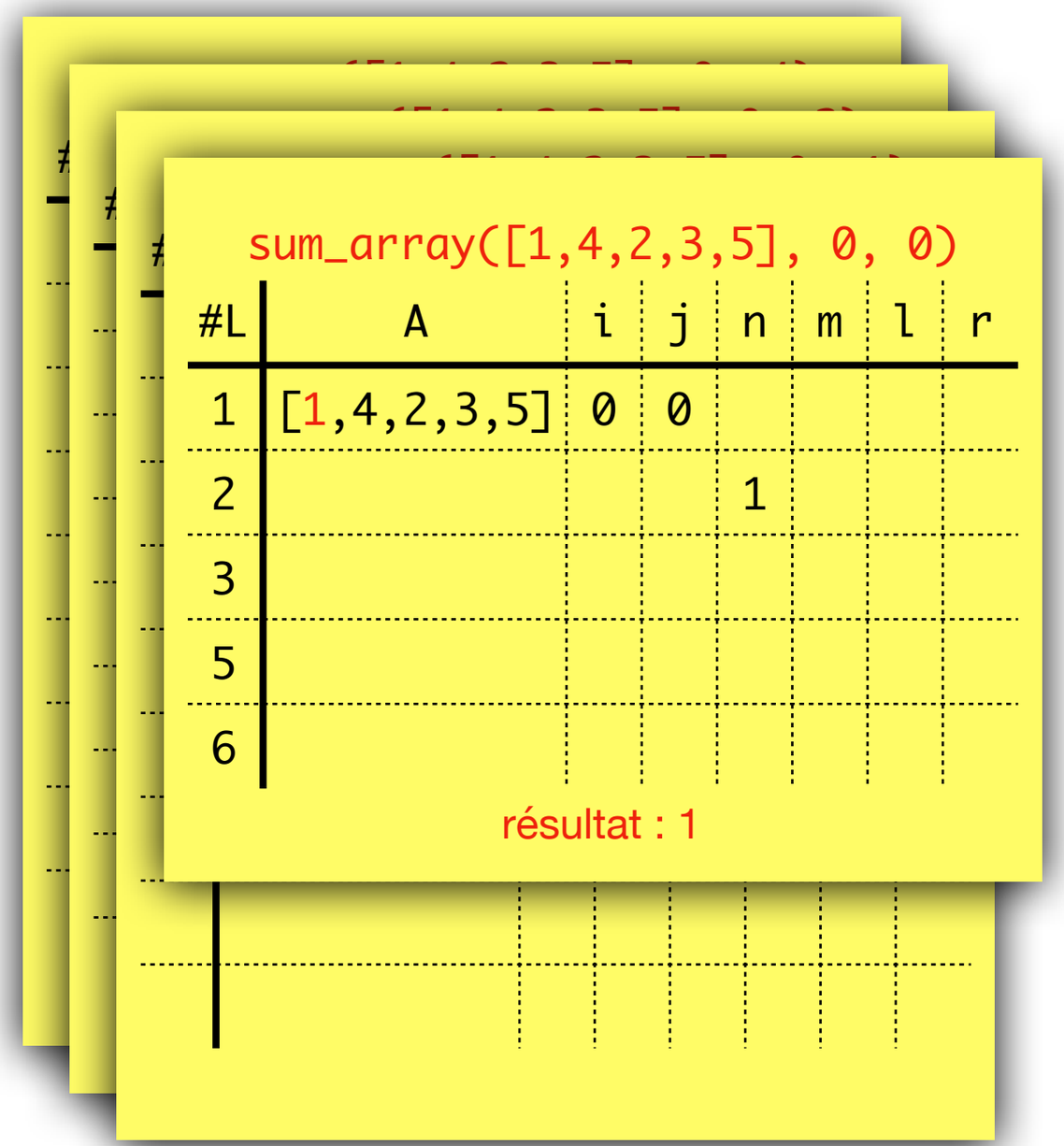
Somme de [1,4]

```
1  def sum_array(A, i, j):
2      n = j - i + 1
3      if n == 0:
4          return 0
5      elif n == 1:
6          return A[i]
7      else:
8          m = (i + j) // 2
9          l = sum_array(A, i, m)
10         r = sum_array(A, m+1, j)
11         return l + r
```



Somme de [1]

```
1  def sum_array(A, i, j):
2      n = j - i + 1
3      if n == 0:
4          return 0
5      elif n == 1:
6          return A[i]
7      else:
8          m = (i + j) // 2
9          l = sum_array(A, i, m)
10         r = sum_array(A, m+1, j)
11         return l + r
```



Somme de [1,4]

```
1  def sum_array(A, i, j):
2      n = j - i + 1
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4          return 0
5      elif n == 1:
6          return A[i]
7      else:
8          m = (i + j) // 2
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10         r = sum_array(A, m+1, j)
11         return l + r
```

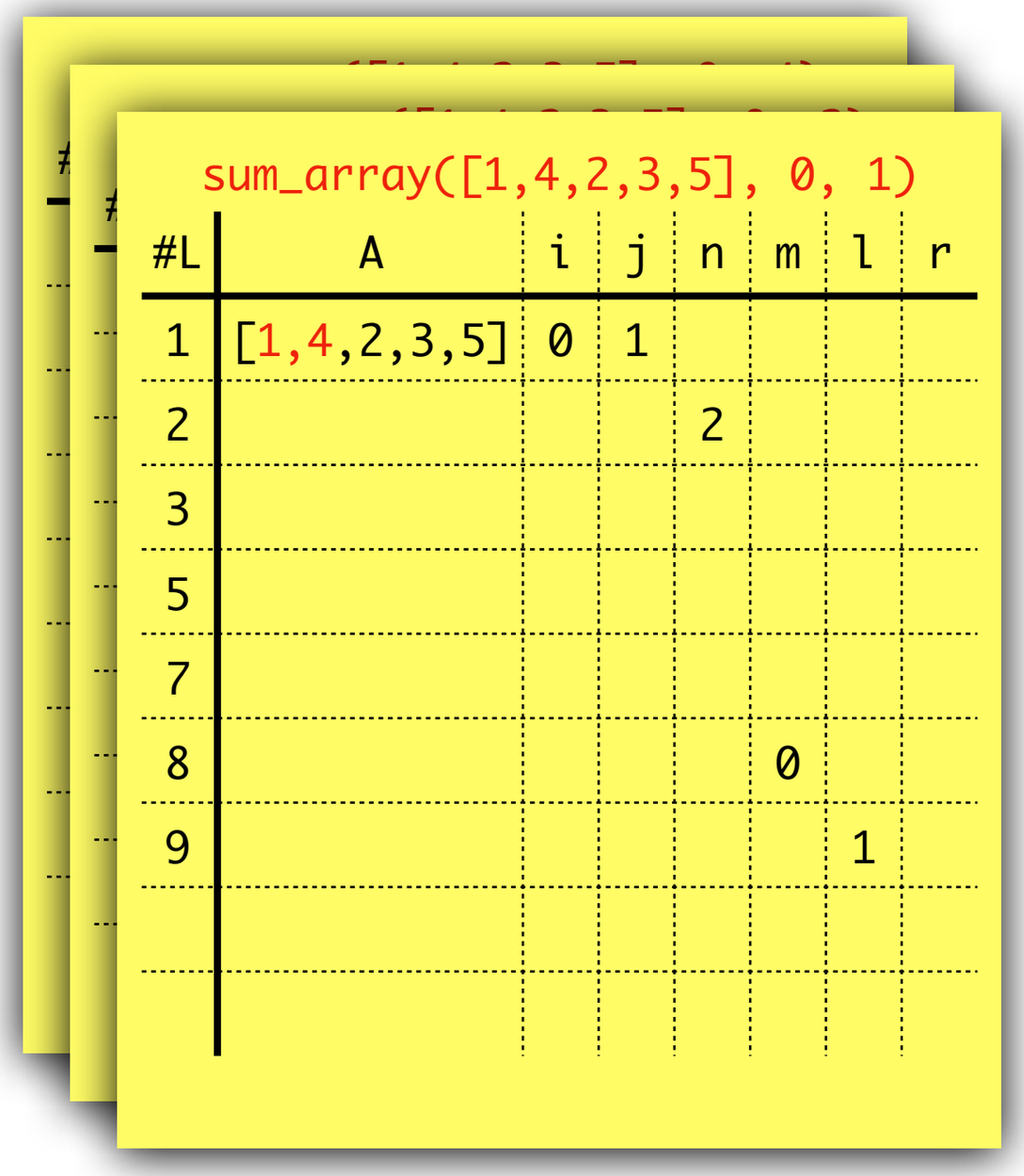

Somme de [1,4]

```
1  def sum_array(A, i, j):
2      n = j - i + 1
3      if n == 0:
4          return 0
5      elif n == 1:
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7      else:
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9          l = sum_array(A, i, m)
10         r = sum_array(A, m+1, j)
11         return l + r
```
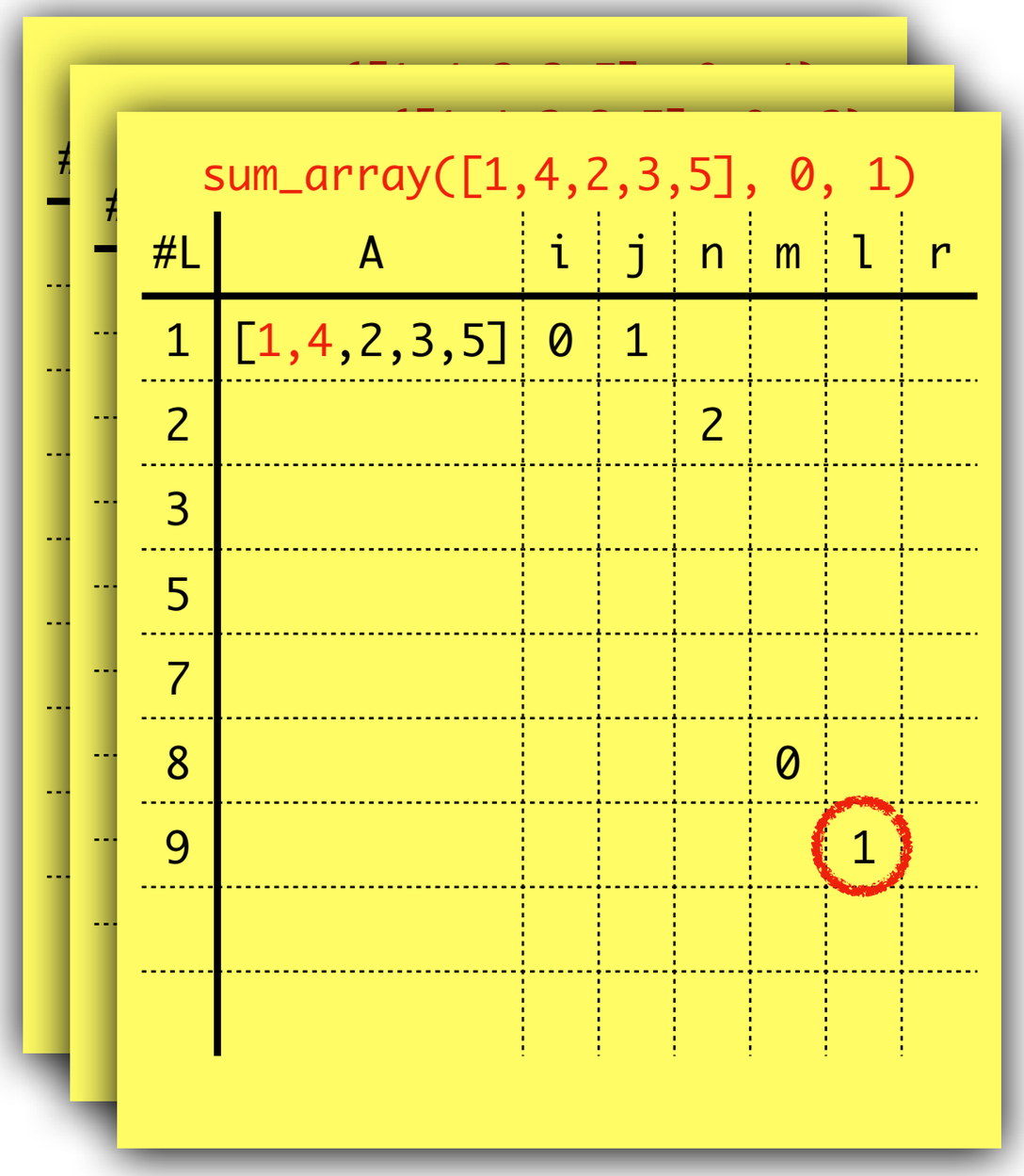
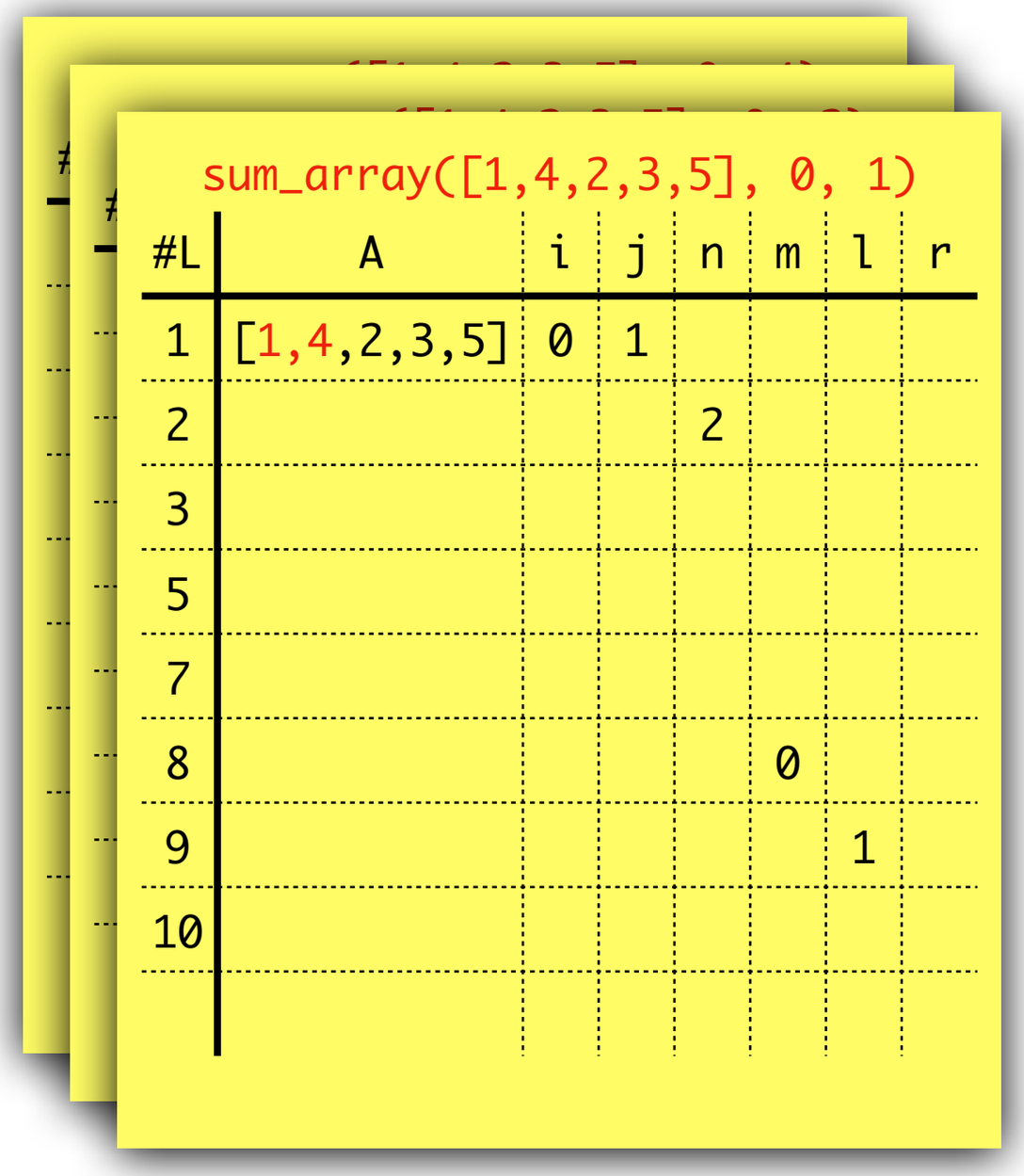



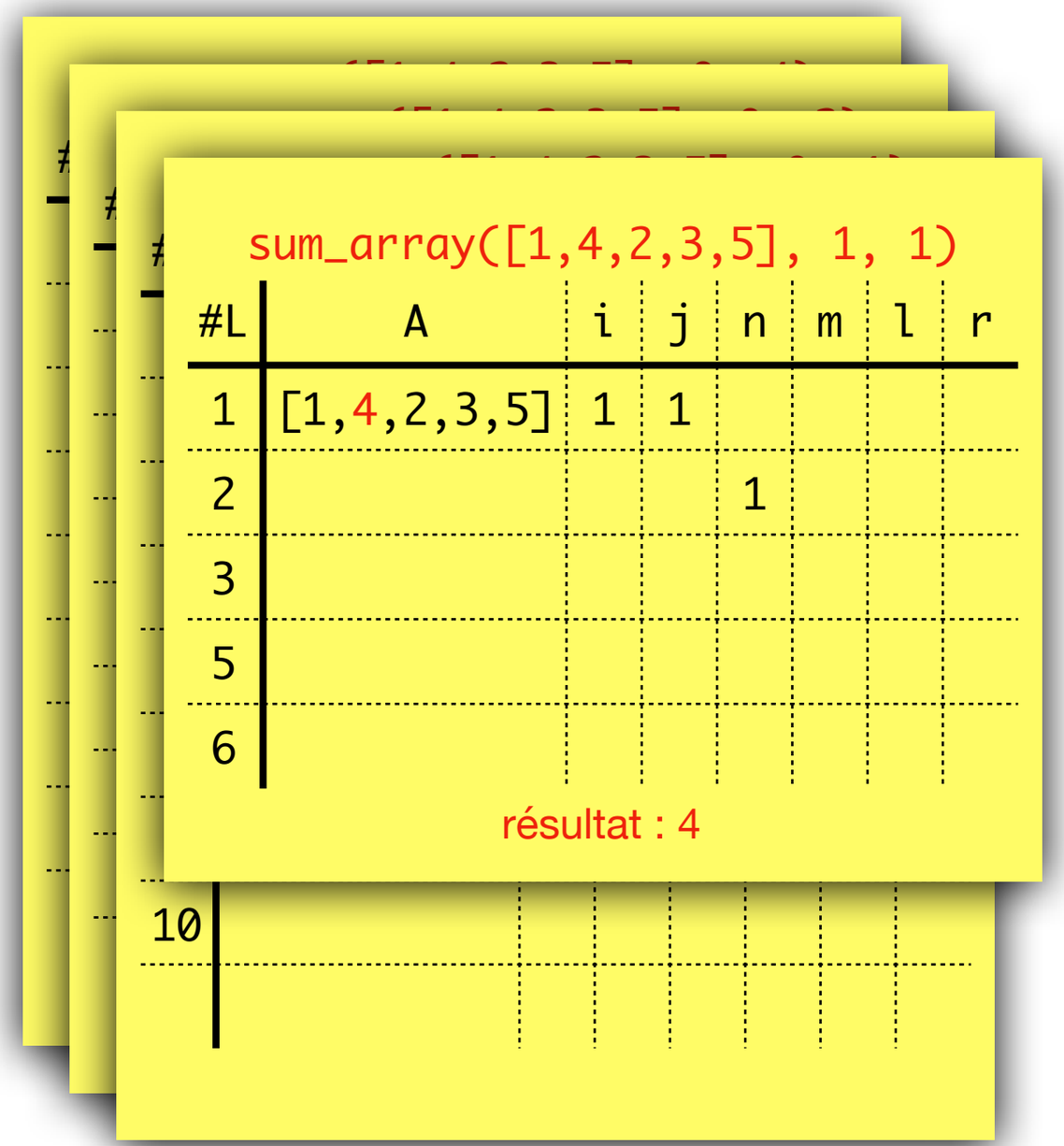
Somme de [1,4]

```
1  def sum_array(A, i, j):
2      n = j - i + 1
3      if n == 0:
4          return 0
5      elif n == 1:
6          return A[i]
7      else:
8          m = (i + j) // 2
9          l = sum_array(A, i, m)
10         r = sum_array(A, m+1, j)
11         return l + r
```



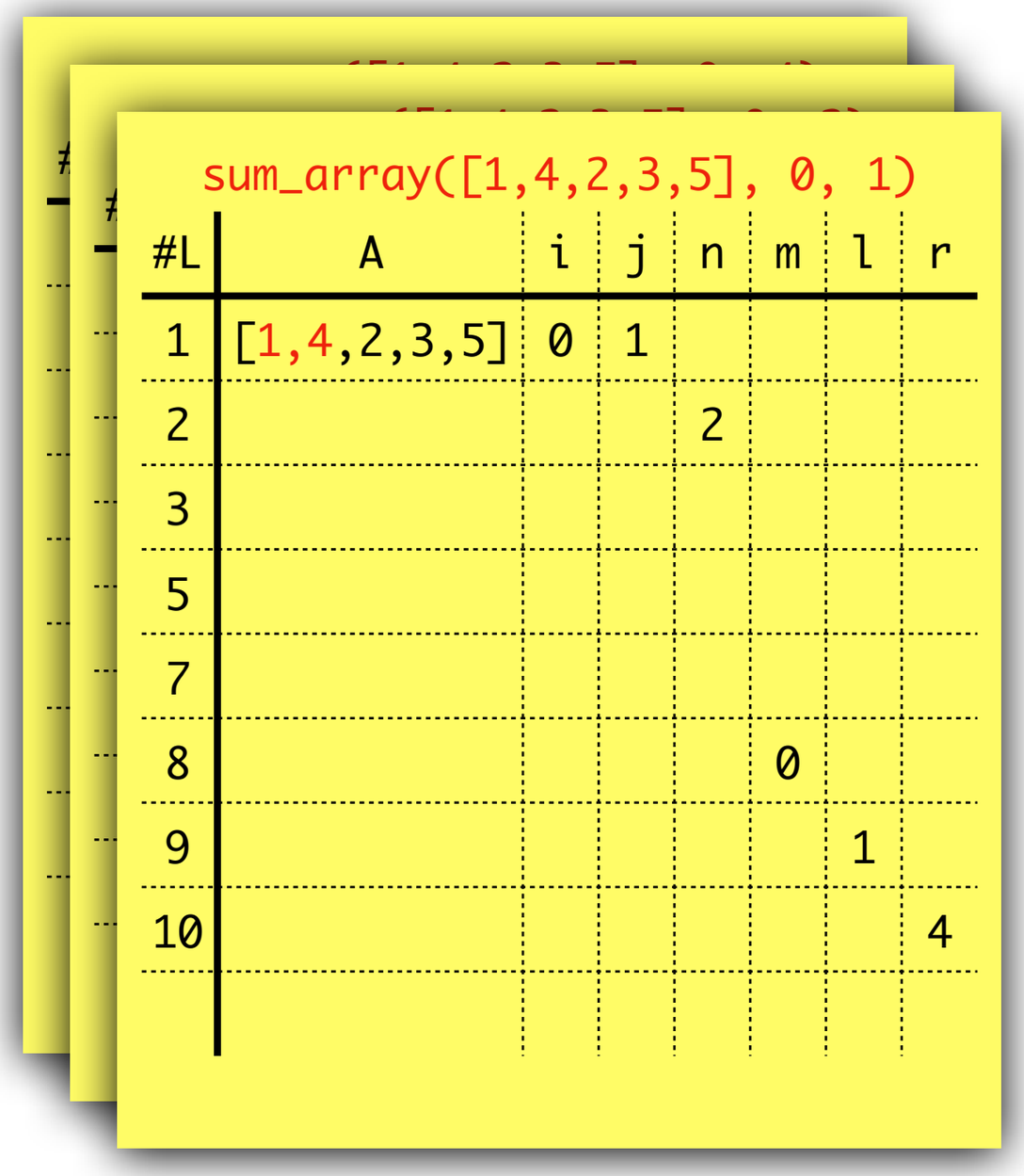
Somme de [4]

```
1  def sum_array(A, i, j):
2      n = j - i + 1
3      if n == 0:
4          return 0
5      elif n == 1:
6          return A[i]
7      else:
8          m = (i + j) // 2
9          l = sum_array(A, i, m)
10         r = sum_array(A, m+1, j)
11         return l + r
```



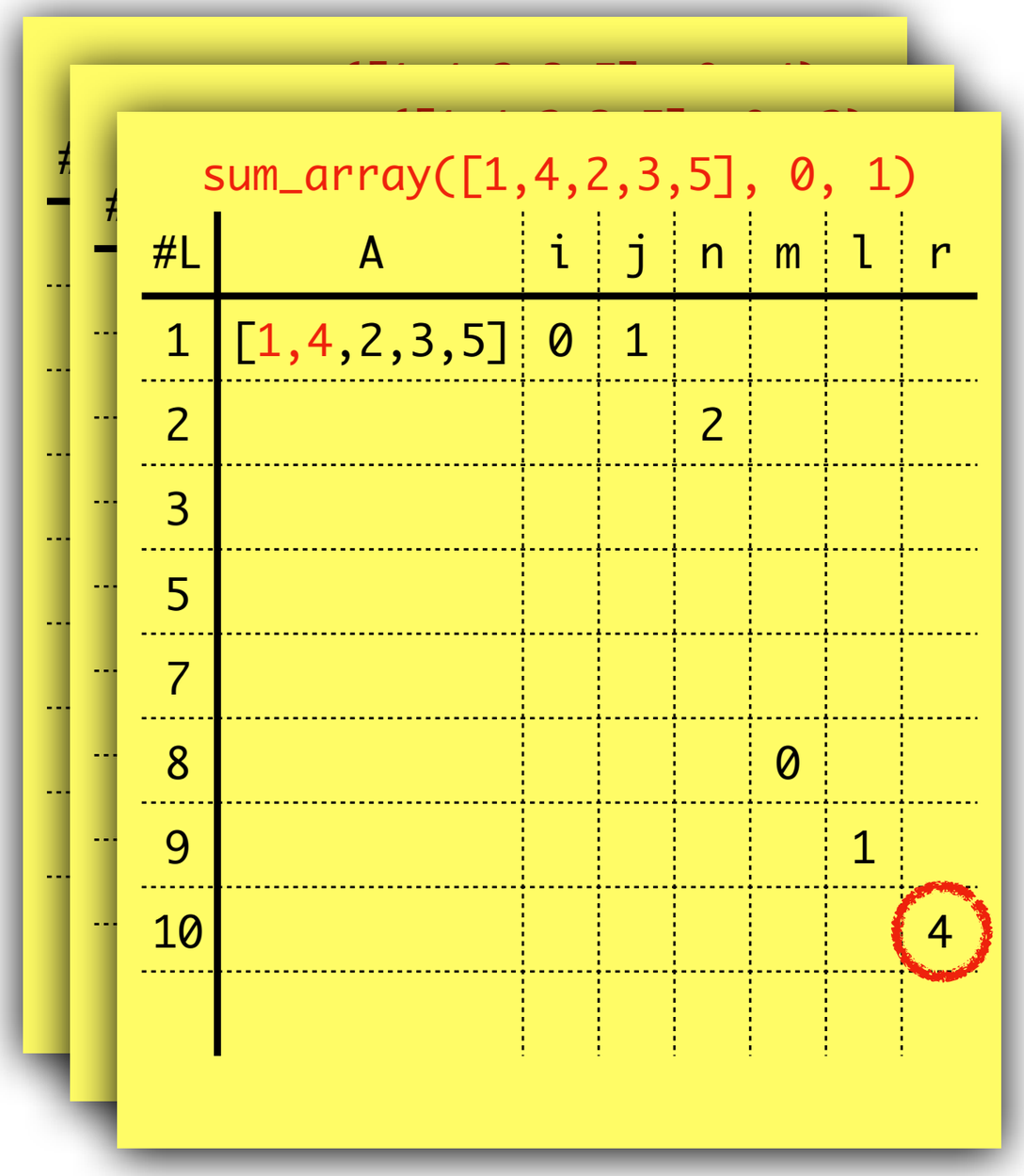
Somme de [1,4]

```
1  def sum_array(A, i, j):
2      n = j - i + 1
3      if n == 0:
4          return 0
5      elif n == 1:
6          return A[i]
7      else:
8          m = (i + j) // 2
9          l = sum_array(A, i, m)
10         r = sum_array(A, m+1, j)
11         return l + r
```



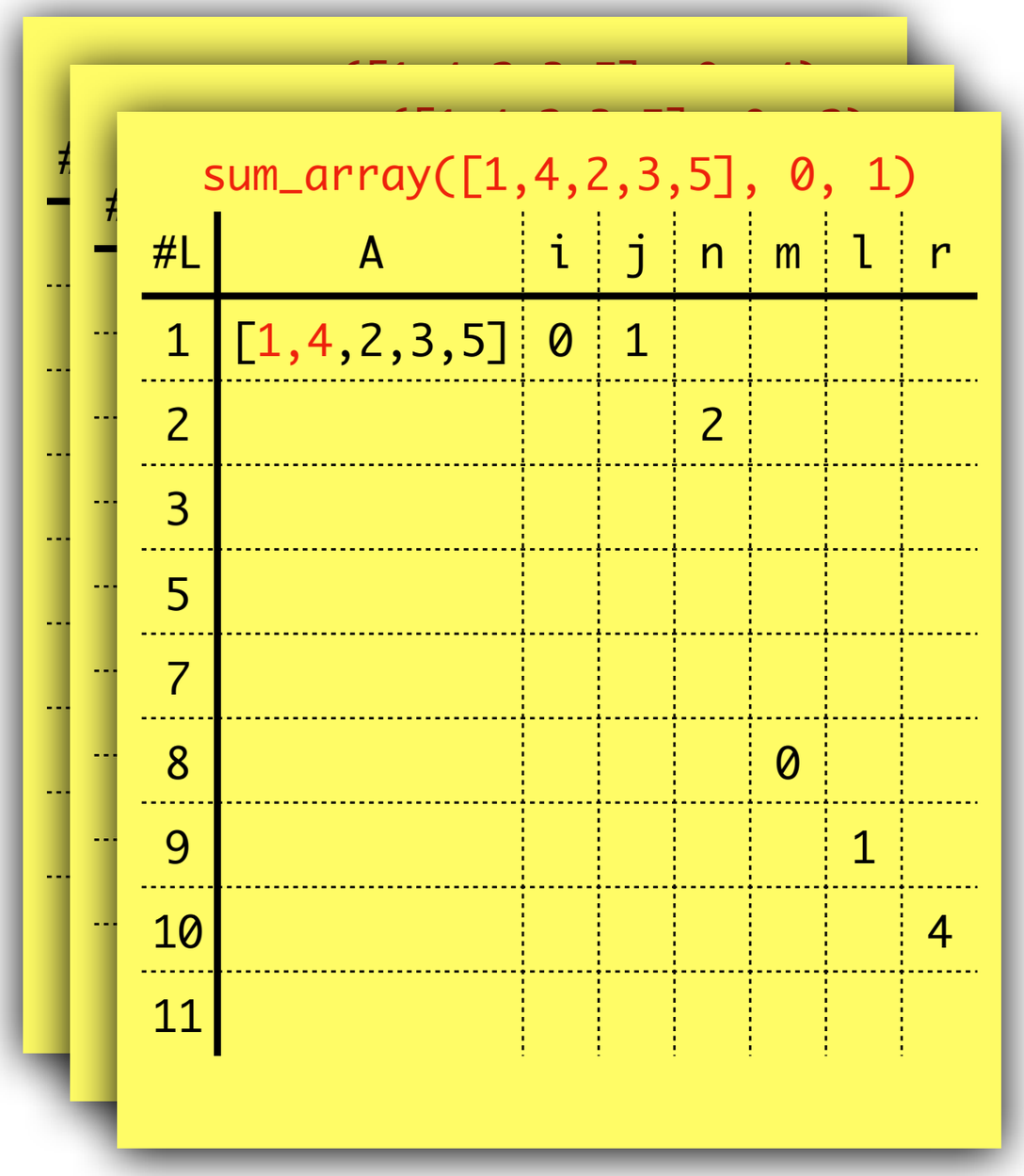
Somme de [1,4]

```
1  def sum_array(A, i, j):
2      n = j - i + 1
3      if n == 0:
4          return 0
5      elif n == 1:
6          return A[i]
7      else:
8          m = (i + j) // 2
9          l = sum_array(A, i, m)
10         r = sum_array(A, m+1, j)
11         return l + r
```



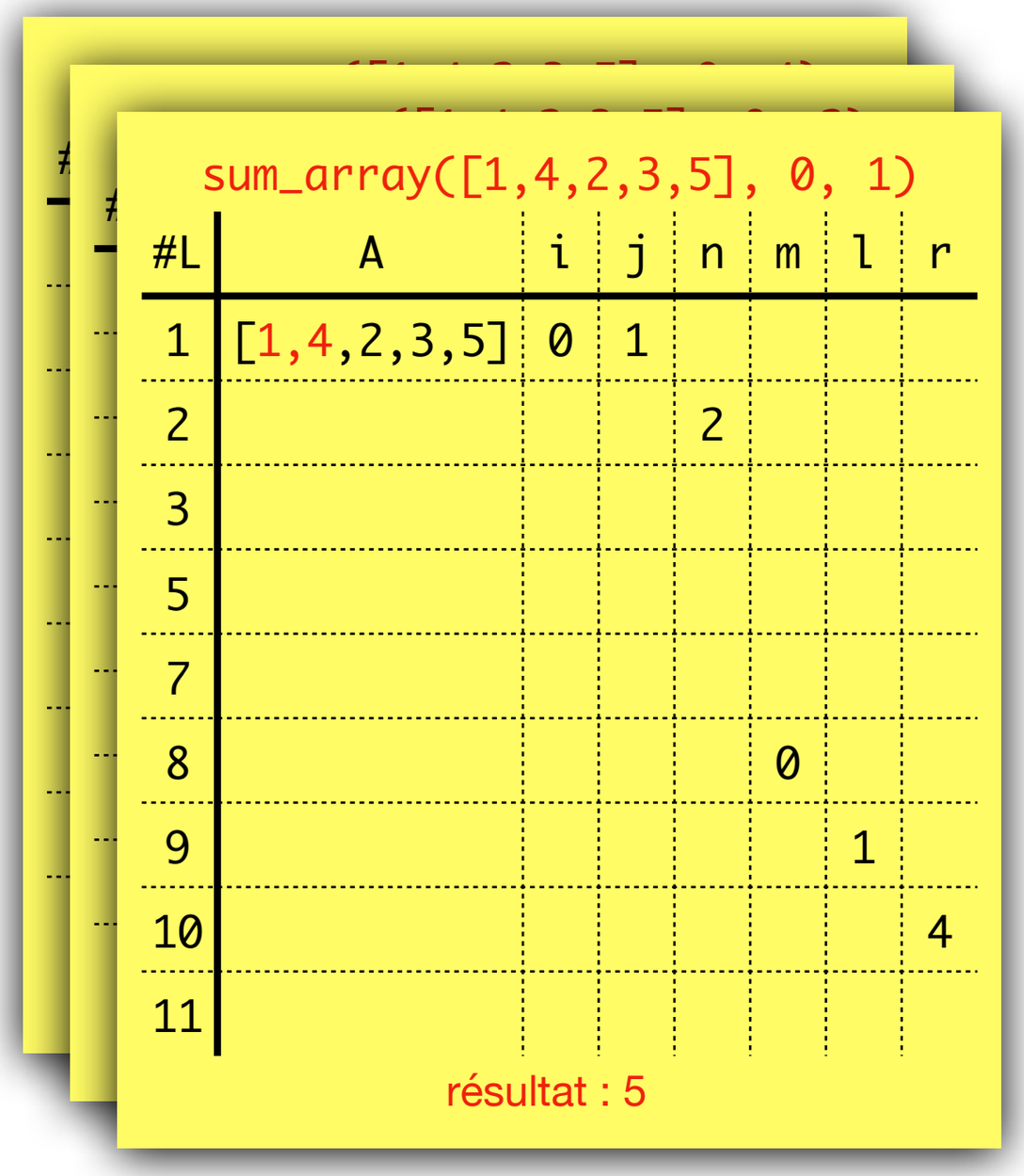
Somme de [1,4]

```
1  def sum_array(A, i, j):
2      n = j - i + 1
3      if n == 0:
4          return 0
5      elif n == 1:
6          return A[i]
7      else:
8          m = (i + j) // 2
9          l = sum_array(A, i, m)
10         r = sum_array(A, m+1, j)
11         return l + r
```



Somme de [1,4]

```
1  def sum_array(A, i, j):
2      n = j - i + 1
3      if n == 0:
4          return 0
5      elif n == 1:
6          return A[i]
7      else:
8          m = (i + j) // 2
9          l = sum_array(A, i, m)
10         r = sum_array(A, m+1, j)
11         return l + r
```



Somme de [1,4,2]

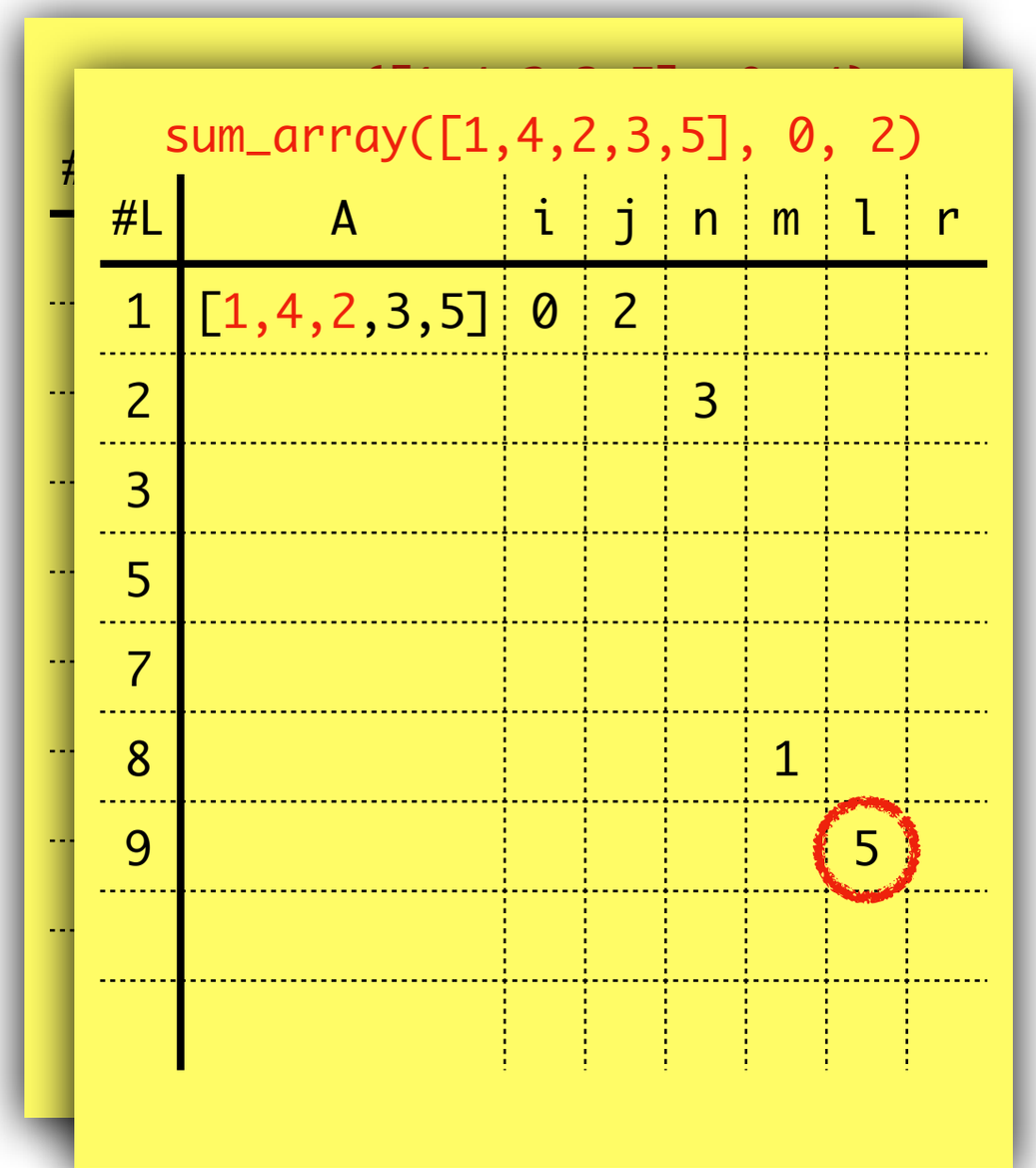
```
1  def sum_array(A, i, j):
2      n = j - i + 1
3      if n == 0:
4          return 0
5      elif n == 1:
6          return A[i]
7      else:
8          m = (i + j) // 2
9          l = sum_array(A, i, m)
10         r = sum_array(A, m+1, j)
11         return l + r
```

sum_array([1,4,2,3,5], 0, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	2				
2				3			
3							
5							
7							
8					1		
9						5	

Somme de [1,4,2]

```
1  def sum_array(A, i, j):
2      n = j - i + 1
3      if n == 0:
4          return 0
5      elif n == 1:
6          return A[i]
7      else:
8          m = (i + j) // 2
9          l = sum_array(A, i, m)
10         r = sum_array(A, m+1, j)
11         return l + r
```



Somme de [1,4,2]

```
1  def sum_array(A, i, j):
2      n = j - i + 1
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5      elif n == 1:
6          return A[i]
7      else:
8          m = (i + j) // 2
9          l = sum_array(A, i, m)
10         r = sum_array(A, m+1, j)
11         return l + r
```
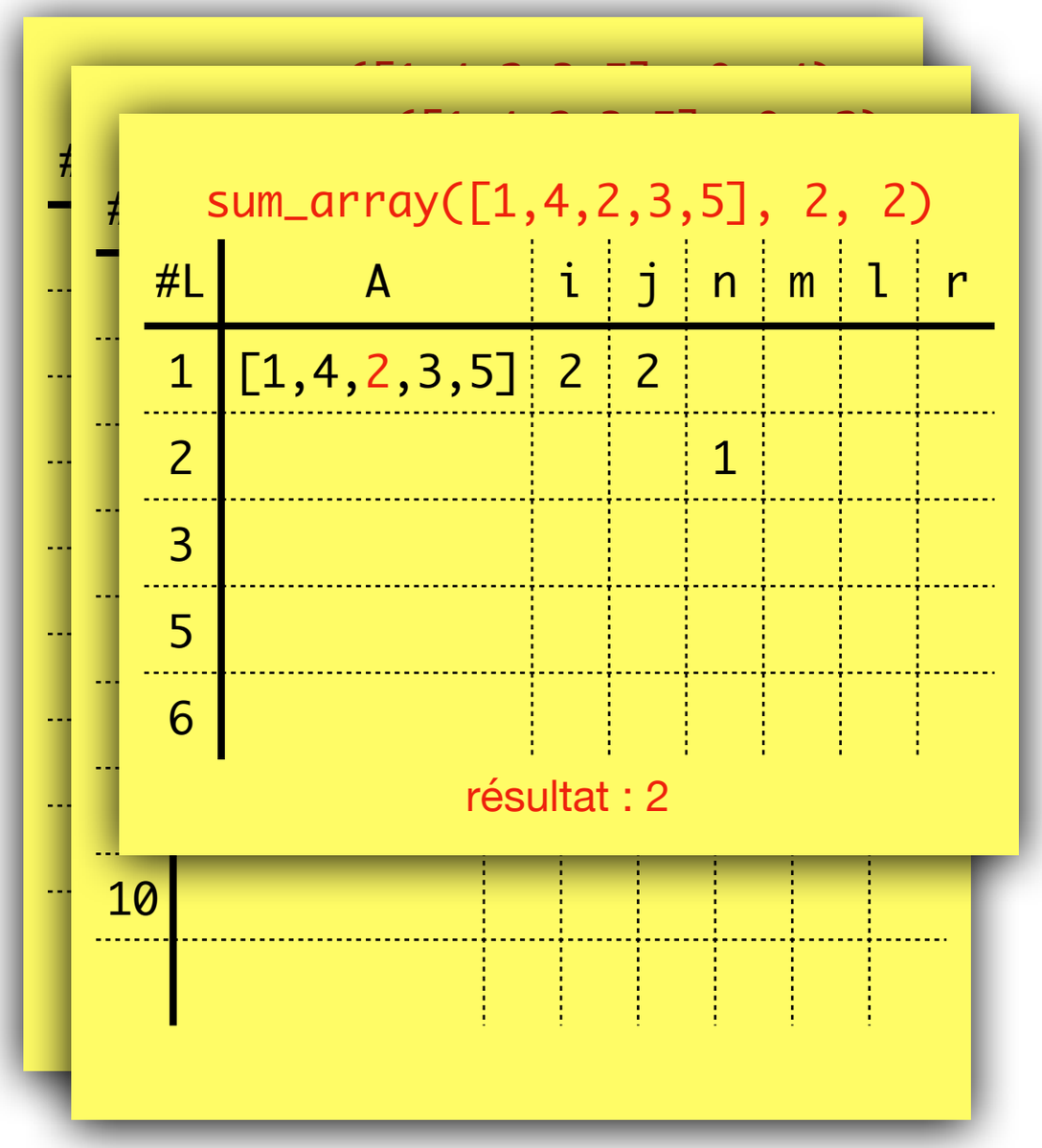

sum_array([1,4,2,3,5], 0, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	2				
2				3			
3							
5							
7							
8					1		
9						5	
10							

Somme de [2]

```
1  def sum_array(A, i, j):
2      n = j - i + 1
3      if n == 0:
4          return 0
5      elif n == 1:
6          return A[i]
7      else:
8          m = (i + j) // 2
9          l = sum_array(A, i, m)
10         r = sum_array(A, m+1, j)
11         return l + r
```



Somme de [1,4,2]

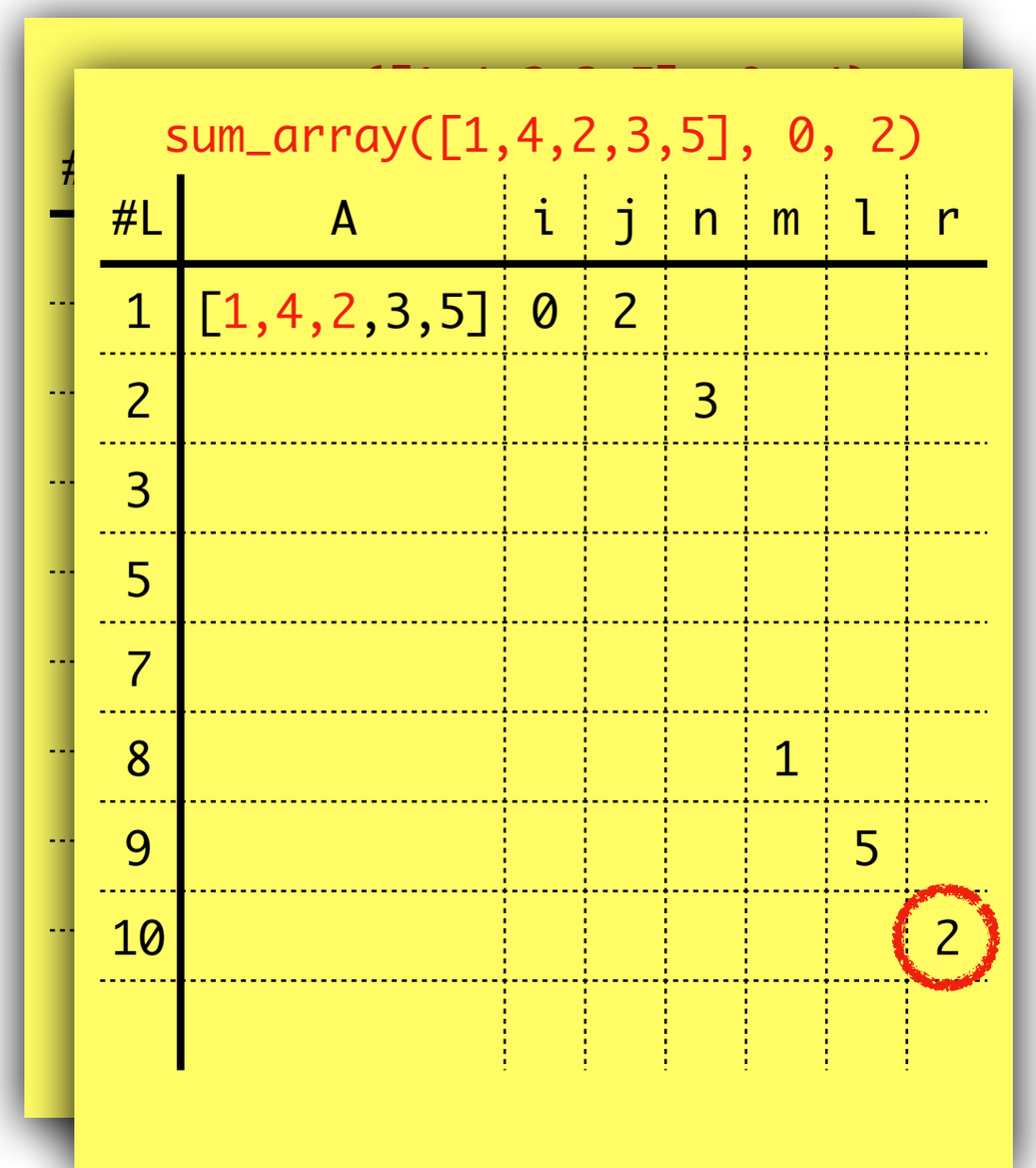
```
1  def sum_array(A, i, j):
2      n = j - i + 1
3      if n == 0:
4          return 0
5      elif n == 1:
6          return A[i]
7      else:
8          m = (i + j) // 2
9          l = sum_array(A, i, m)
10         r = sum_array(A, m+1, j)
11         return l + r
```

sum_array([1,4,2,3,5], 0, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	2				
2				3			
3							
5							
7							
8					1		
9						5	
10							2

Somme de [1,4,2]

```
1  def sum_array(A, i, j):
2      n = j - i + 1
3      if n == 0:
4          return 0
5      elif n == 1:
6          return A[i]
7      else:
8          m = (i + j) // 2
9          l = sum_array(A, i, m)
10         r = sum_array(A, m+1, j)
11         return l + r
```



Somme de [1,4,2]

```
1  def sum_array(A, i, j):
2      n = j - i + 1
3      if n == 0:
4          return 0
5      elif n == 1:
6          return A[i]
7      else:
8          m = (i + j) // 2
9          l = sum_array(A, i, m)
10         r = sum_array(A, m+1, j)
11         return l + r
```

sum_array([1,4,2,3,5], 0, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	2				
2				3			
3							
5							
7							
8					1		
9						5	
10							2
11							

Somme de [1,4,2]

```
1  def sum_array(A, i, j):
2      n = j - i + 1
3      if n == 0:
4          return 0
5      elif n == 1:
6          return A[i]
7      else:
8          m = (i + j) // 2
9          l = sum_array(A, i, m)
10         r = sum_array(A, m+1, j)
11         return l + r
```

sum_array([1,4,2,3,5], 0, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	2				
2				3			
3							
5							
7							
8					1		
9						5	
10							2
11							

résultat : 7

Somme de [1,4,2,3,5]

```
1  def sum_array(A, i, j):
2      n = j - i + 1
3      if n == 0:
4          return 0
5      elif n == 1:
6          return A[i]
7      else:
8          m = (i + j) // 2
9          l = sum_array(A, i, m)
10         r = sum_array(A, m+1, j)
11         return l + r
```

sum_array([1,4,2,3,5], 0, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	4				
2				5			
3							
5							
7							
8					2		
9						7	

Somme de [1,4,2,3,5]

```
1  def sum_array(A, i, j):
2      n = j - i + 1
3      if n == 0:
4          return 0
5      elif n == 1:
6          return A[i]
7      else:
8          m = (i + j) // 2
9          l = sum_array(A, i, m)
10         r = sum_array(A, m+1, j)
11         return l + r
```

sum_array([1,4,2,3,5], 0, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	4				
2				5			
3							
5							
7							
8					2		
9						7	

Somme de [1,4,2,3,5]

```
1  def sum_array(A, i, j):
2      n = j - i + 1
3      if n == 0:
4          return 0
5      elif n == 1:
6          return A[i]
7      else:
8          m = (i + j) // 2
9          l = sum_array(A, i, m)
10         r = sum_array(A, m+1, j)
11         return l + r
```

sum_array([1,4,2,3,5], 0, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	4				
2				5			
3							
5							
7							
8					2		
9						7	
10							

Somme de [3,5]

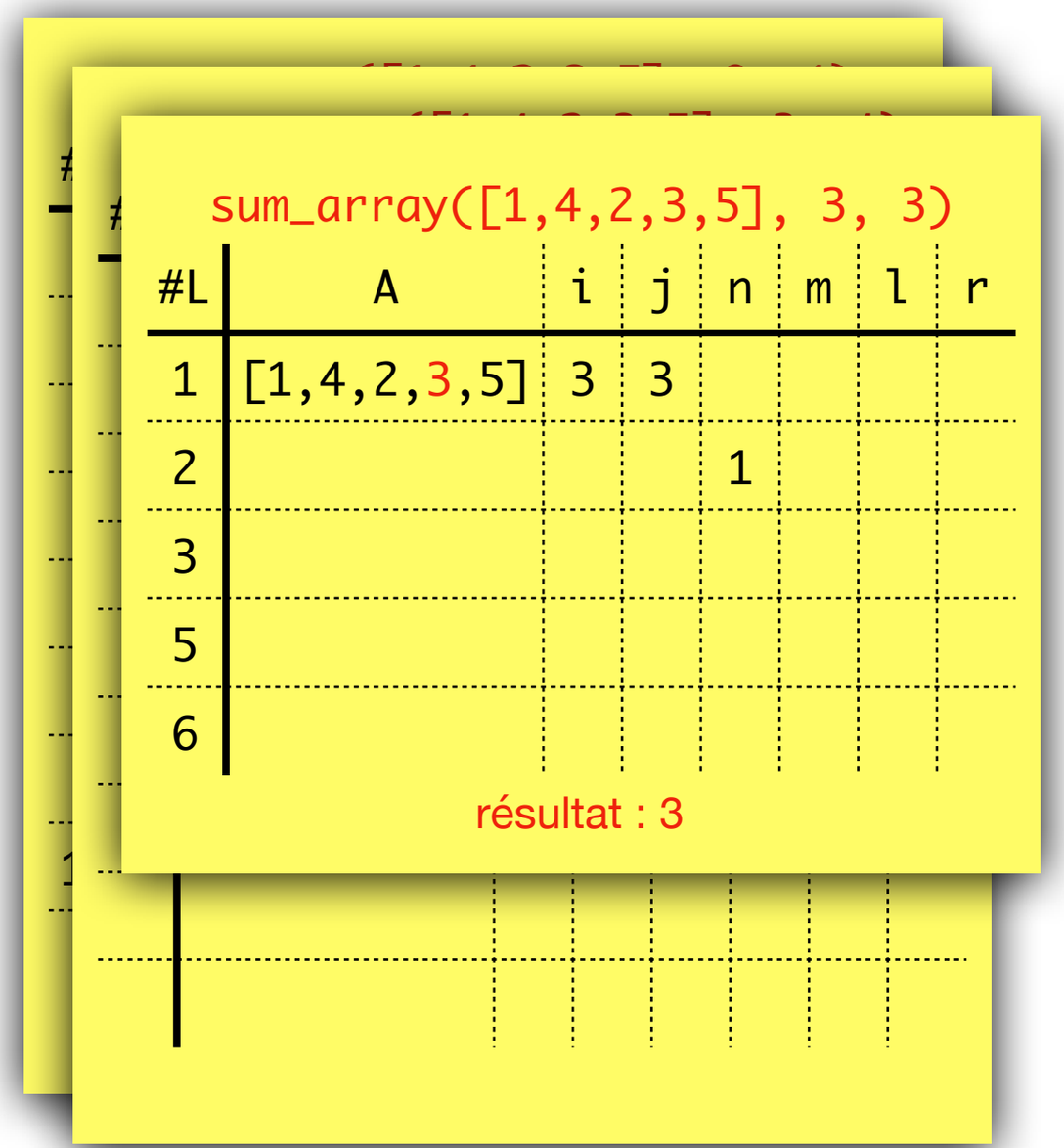
```
1  def sum_array(A, i, j):
2      n = j - i + 1
3      if n == 0:
4          return 0
5      elif n == 1:
6          return A[i]
7      else:
8          m = (i + j) // 2
9          l = sum_array(A, i, m)
10         r = sum_array(A, m+1, j)
11         return l + r
```

sum_array([1,4,2,3,5], 3, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	3	4				
2				2			
3							
5							
7							
8					3		
9							

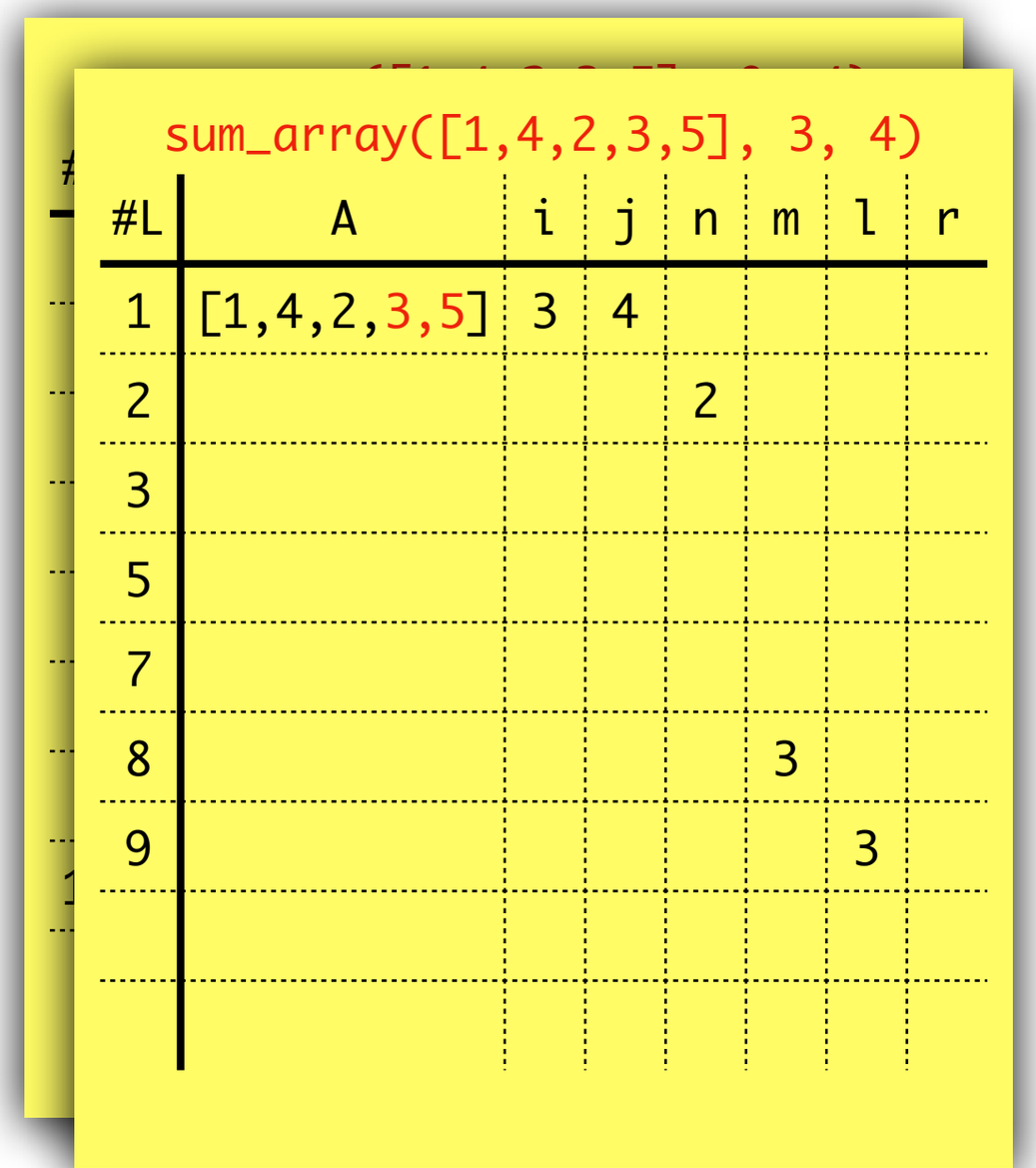
Somme de [3]

```
1  def sum_array(A, i, j):
2      n = j - i + 1
3      if n == 0:
4          return 0
5      elif n == 1:
6          return A[i]
7      else:
8          m = (i + j) // 2
9          l = sum_array(A, i, m)
10         r = sum_array(A, m+1, j)
11         return l + r
```



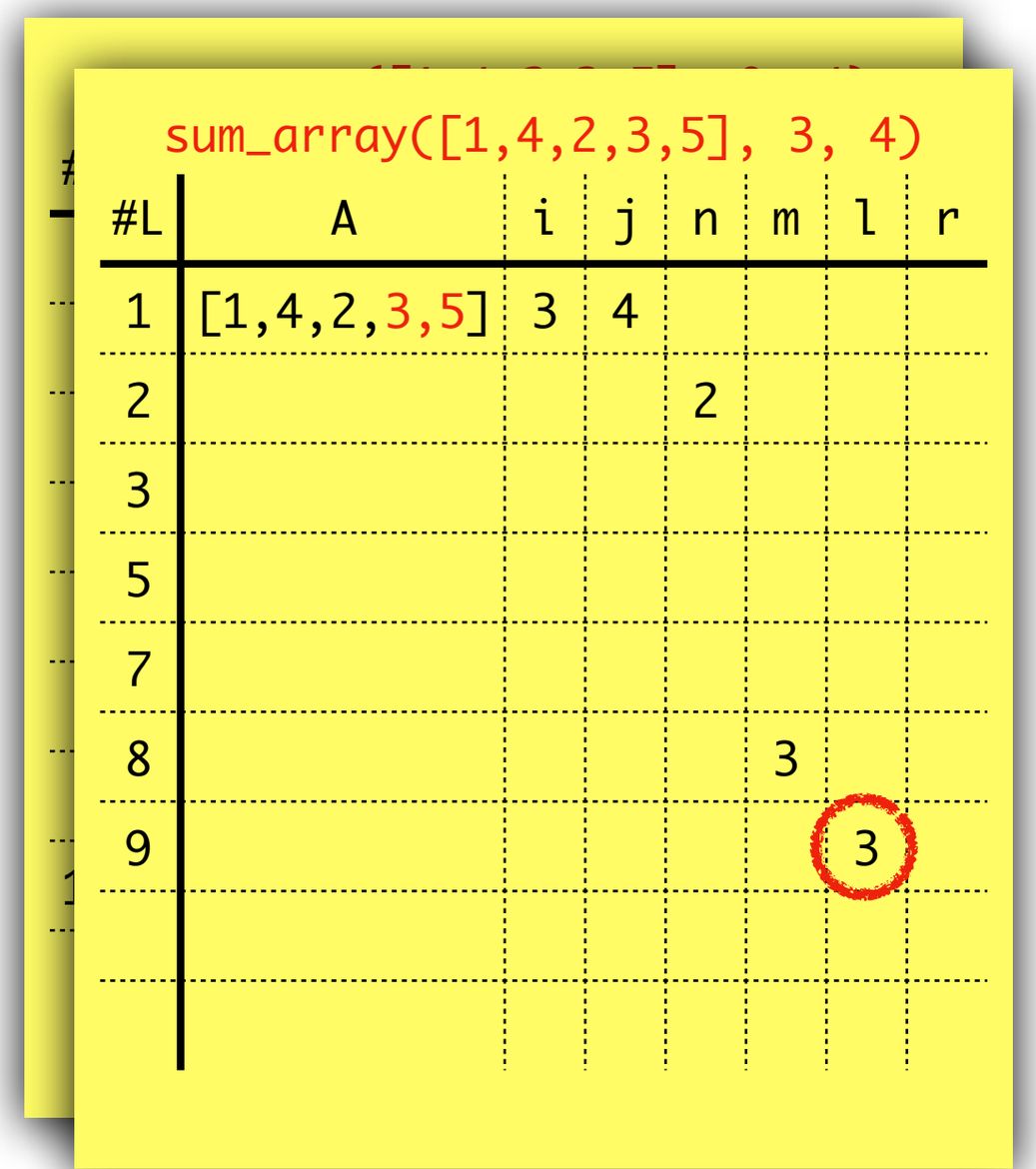
Somme de [3,5]

```
1  def sum_array(A, i, j):
2      n = j - i + 1
3      if n == 0:
4          return 0
5      elif n == 1:
6          return A[i]
7      else:
8          m = (i + j) // 2
9          l = sum_array(A, i, m)
10         r = sum_array(A, m+1, j)
11         return l + r
```



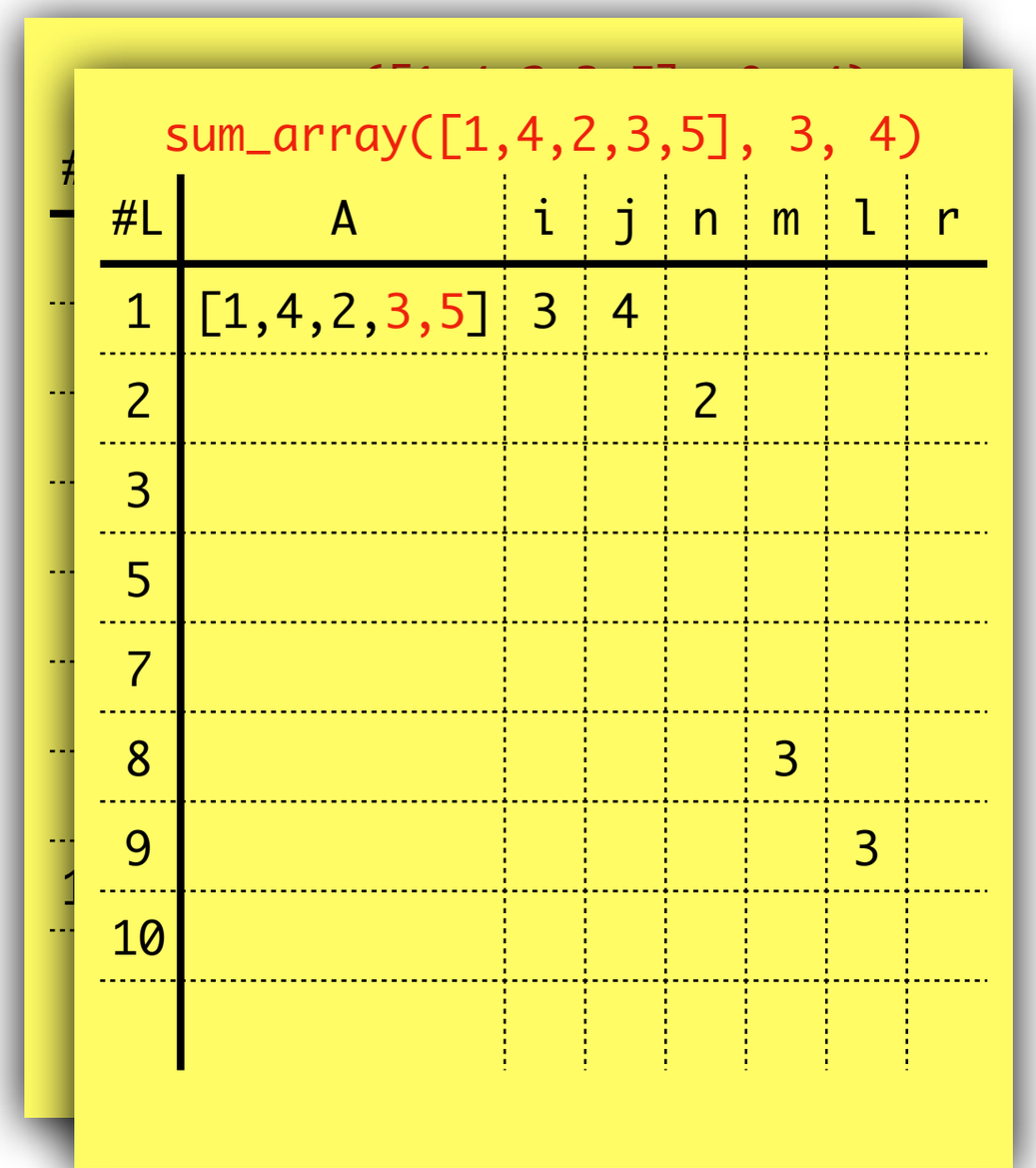
Somme de [3,5]

```
1  def sum_array(A, i, j):
2      n = j - i + 1
3      if n == 0:
4          return 0
5      elif n == 1:
6          return A[i]
7      else:
8          m = (i + j) // 2
9          l = sum_array(A, i, m)
10         r = sum_array(A, m+1, j)
11         return l + r
```



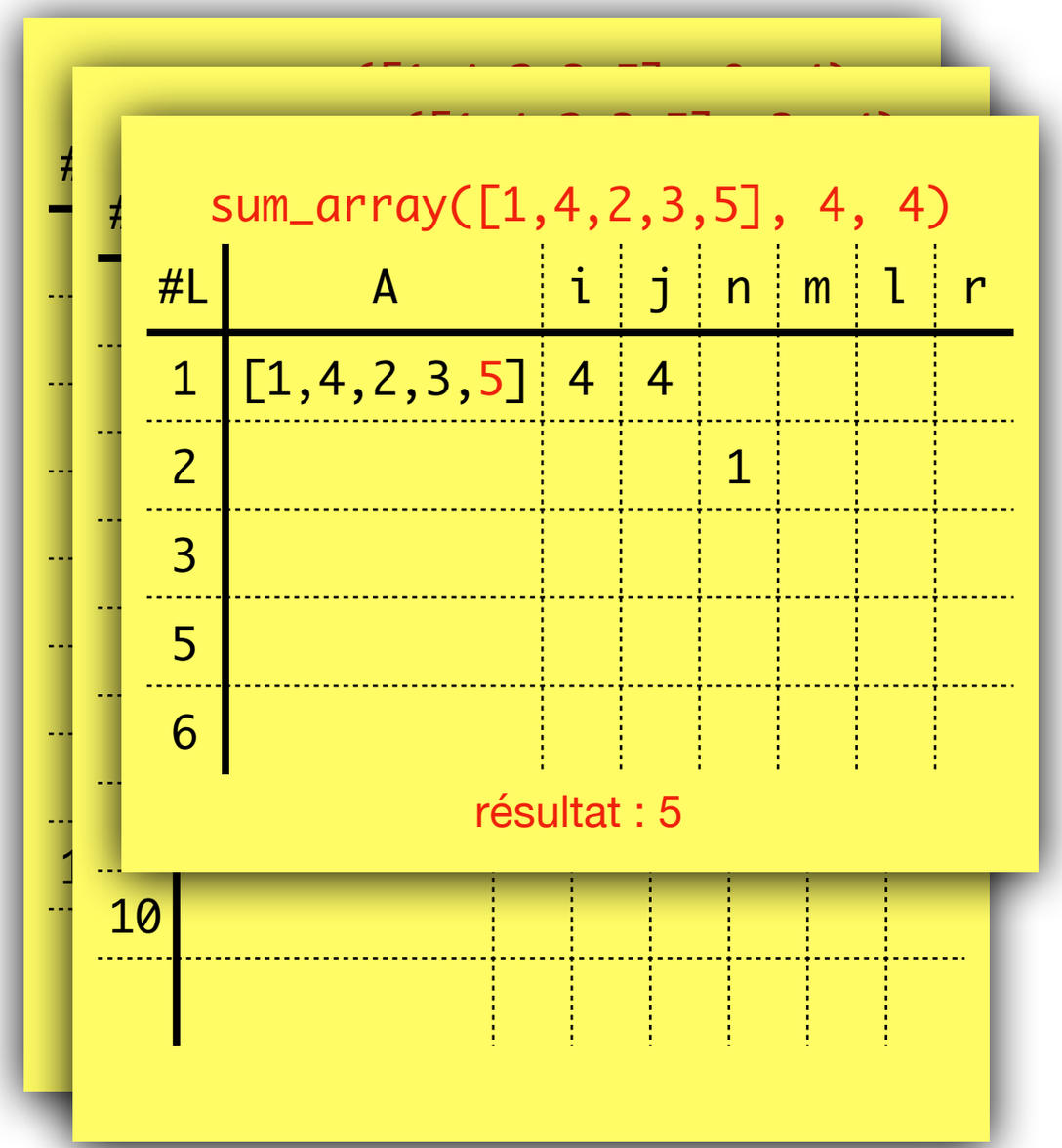
Somme de [3,5]

```
1  def sum_array(A, i, j):
2      n = j - i + 1
3      if n == 0:
4          return 0
5      elif n == 1:
6          return A[i]
7      else:
8          m = (i + j) // 2
9          l = sum_array(A, i, m)
10         r = sum_array(A, m+1, j)
11         return l + r
```



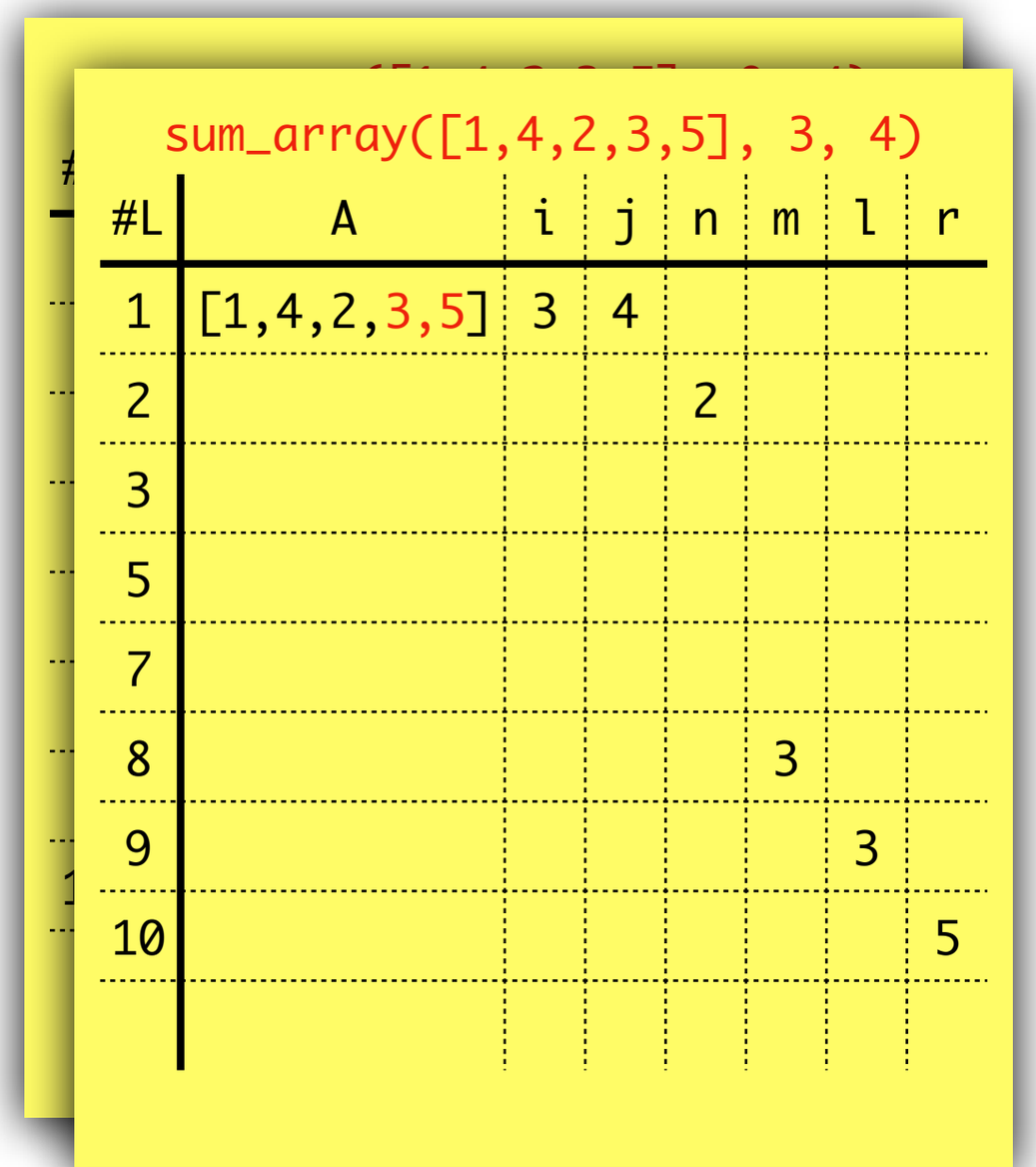
Somme de [5]

```
1  def sum_array(A, i, j):
2      n = j - i + 1
3      if n == 0:
4          return 0
5      elif n == 1:
6          return A[i]
7      else:
8          m = (i + j) // 2
9          l = sum_array(A, i, m)
10         r = sum_array(A, m+1, j)
11         return l + r
```



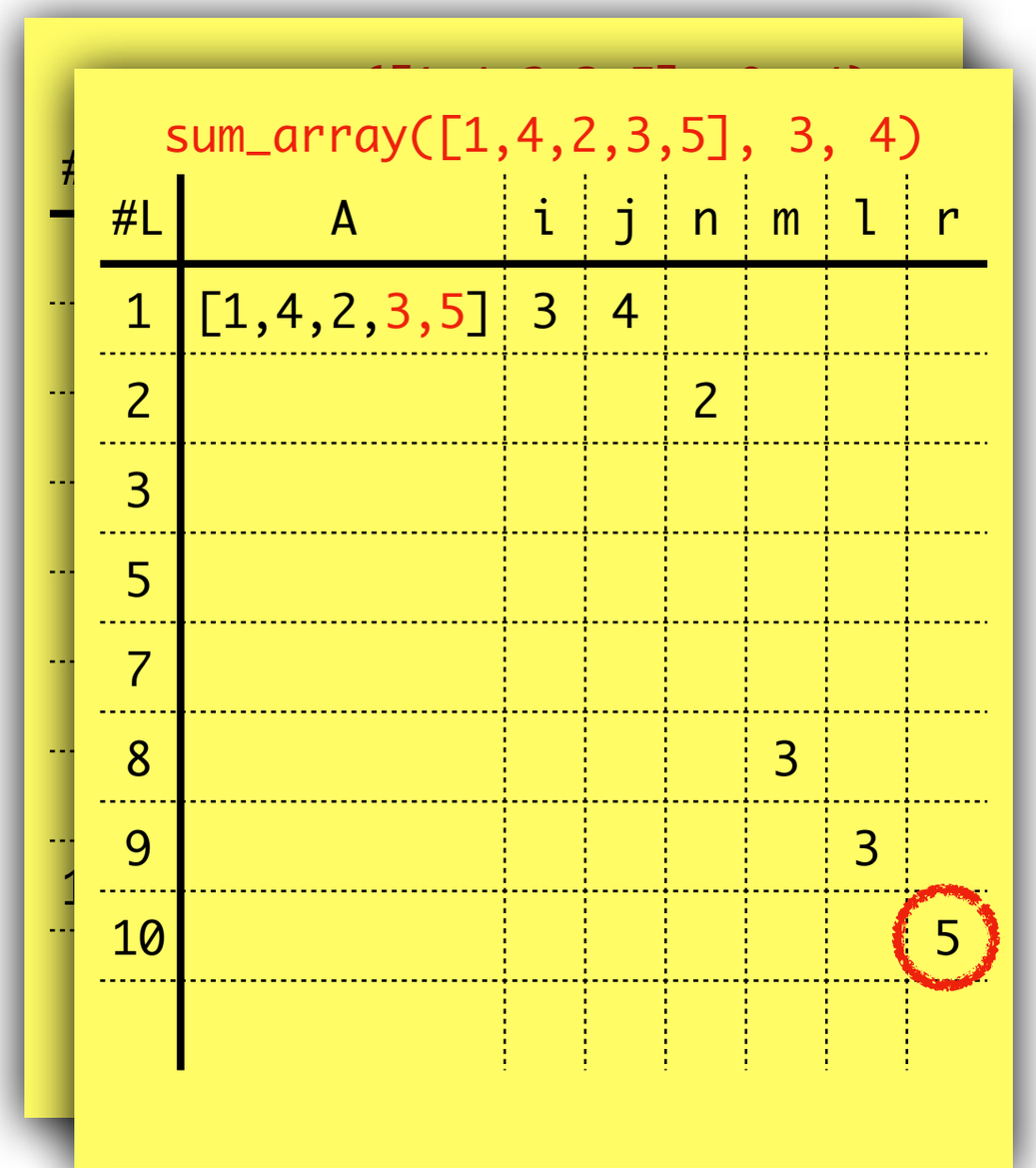
Somme de [3,5]

```
1  def sum_array(A, i, j):
2      n = j - i + 1
3      if n == 0:
4          return 0
5      elif n == 1:
6          return A[i]
7      else:
8          m = (i + j) // 2
9          l = sum_array(A, i, m)
10         r = sum_array(A, m+1, j)
11         return l + r
```



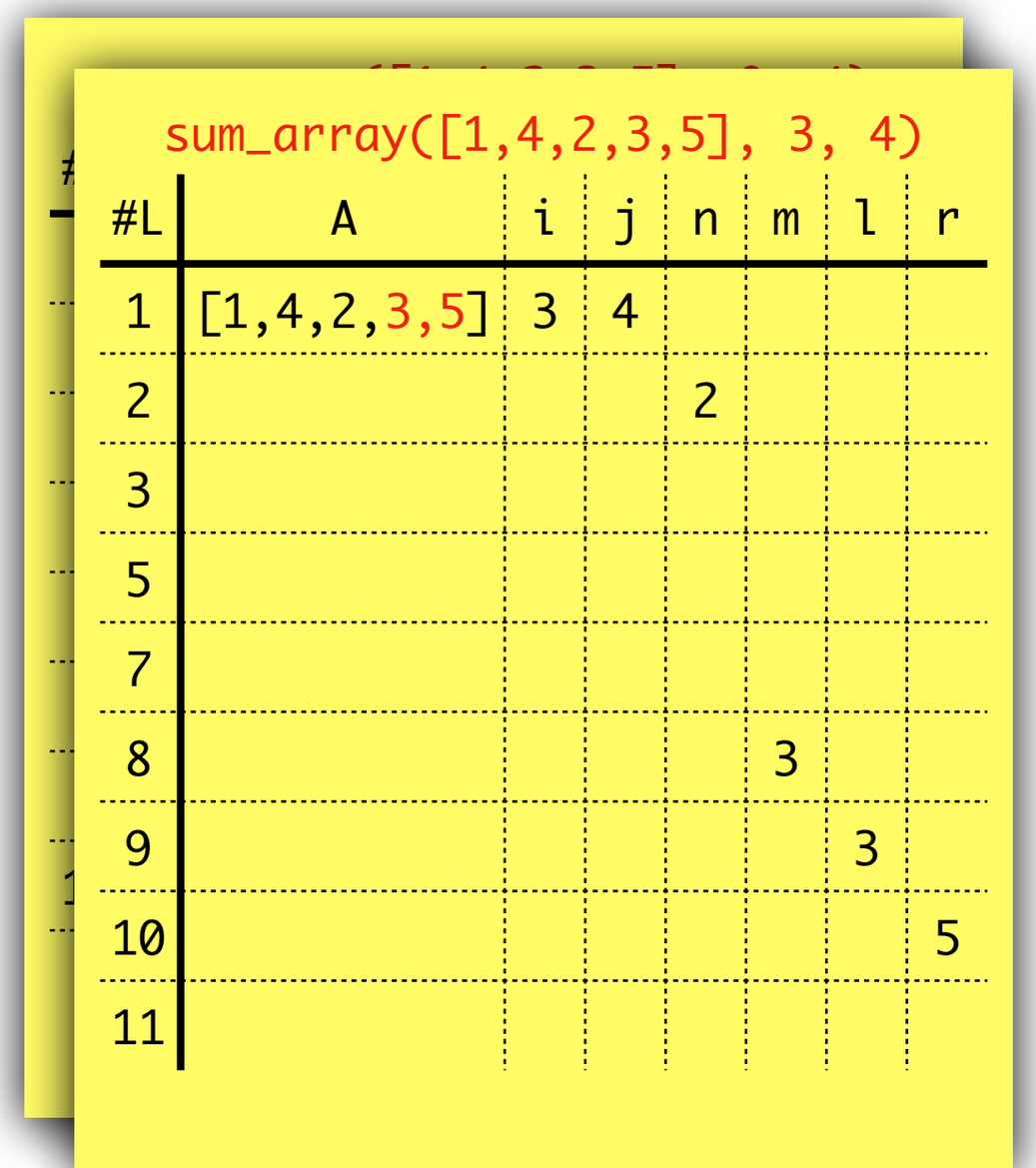
Somme de [3,5]

```
1  def sum_array(A, i, j):
2      n = j - i + 1
3      if n == 0:
4          return 0
5      elif n == 1:
6          return A[i]
7      else:
8          m = (i + j) // 2
9          l = sum_array(A, i, m)
10         r = sum_array(A, m+1, j)
11         return l + r
```



Somme de [3,5]

```
1  def sum_array(A, i, j):
2      n = j - i + 1
3      if n == 0:
4          return 0
5      elif n == 1:
6          return A[i]
7      else:
8          m = (i + j) // 2
9          l = sum_array(A, i, m)
10         r = sum_array(A, m+1, j)
11         return l + r
```



Somme de [3,5]

```
1  def sum_array(A, i, j):
2      n = j - i + 1
3      if n == 0:
4          return 0
5      elif n == 1:
6          return A[i]
7      else:
8          m = (i + j) // 2
9          l = sum_array(A, i, m)
10         r = sum_array(A, m+1, j)
11         return l + r
```

sum_array([1,4,2,3,5], 3, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	3	4				
2				2			
3							
5							
7							
8					3		
9						3	
10							5
11							

résultat : 8

Somme de [1,4,2,3,5]

```
1  def sum_array(A, i, j):
2      n = j - i + 1
3      if n == 0:
4          return 0
5      elif n == 1:
6          return A[i]
7      else:
8          m = (i + j) // 2
9          l = sum_array(A, i, m)
10         r = sum_array(A, m+1, j)
11         return l + r
```

sum_array([1,4,2,3,5], 0, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	4				
2				5			
3							
5							
7							
8					2		
9						7	
10							8

Somme de [1,4,2,3,5]

```
1  def sum_array(A, i, j):
2      n = j - i + 1
3      if n == 0:
4          return 0
5      elif n == 1:
6          return A[i]
7      else:
8          m = (i + j) // 2
9          l = sum_array(A, i, m)
10         r = sum_array(A, m+1, j)
11         return l + r
```

sum_array([1,4,2,3,5], 0, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	4				
2				5			
3							
5							
7							
8					2		
9						7	
10							8

Somme de [1,4,2,3,5]

```
1  def sum_array(A, i, j):
2      n = j - i + 1
3      if n == 0:
4          return 0
5      elif n == 1:
6          return A[i]
7      else:
8          m = (i + j) // 2
9          l = sum_array(A, i, m)
10         r = sum_array(A, m+1, j)
11         return l + r
```

sum_array([1,4,2,3,5], 0, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	4				
2				5			
3							
5							
7							
8					2		
9						7	
10							8
11							

Somme de [1,4,2,3,5]

```
1  def sum_array(A, i, j):
2      n = j - i + 1
3      if n == 0:
4          return 0
5      elif n == 1:
6          return A[i]
7      else:
8          m = (i + j) // 2
9          l = sum_array(A, i, m)
10         r = sum_array(A, m+1, j)
11         return l + r
```

sum_array([1,4,2,3,5], 0, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	4				
2				5			
3							
5							
7							
8					2		
9						7	
10							8
11							

résultat : 15

Somme de [1,4,2,3,5]

```
1  def sum_array(A, i, j):
2      n = j - i + 1
3      if n == 0:
4          return 0
5      elif n == 1:
6          return A[i]
7      else:
8          m = (i + j) // 2
9          l = sum_array(A, i, m)
10         r = sum_array(A, m+1, j)
11         return l + r
```

sum_array([1,4,2,3,5], 0, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	4				
2				5			
3							
5							
7							
8					2		
9						7	
10							8
11							

résultat : 15

Arbre de récursion

sum_array([1,4,2,3,5], 0, 4)

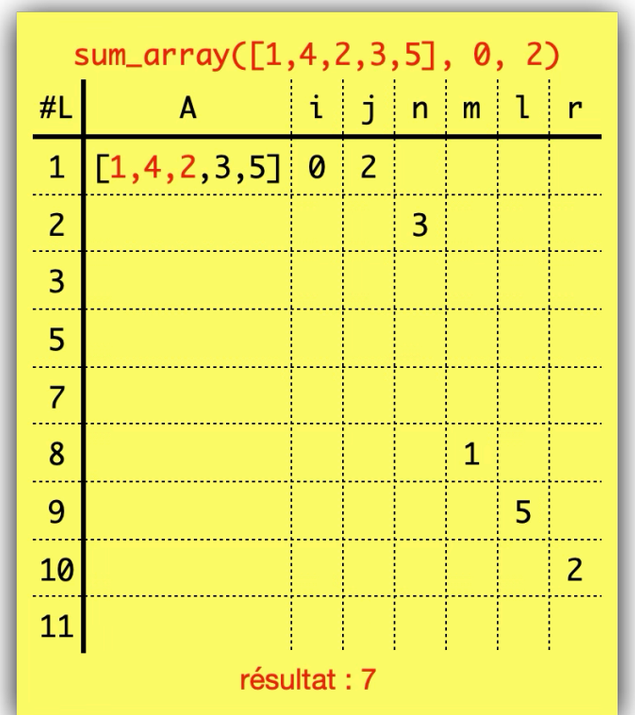
#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	4				
2				5			
3							
5							
7							
8					2		
9						7	
10							8
11							

résultat : 15

sum_array([1,4,2,3,5], 0, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	4				
2				5			
3							
5							
7							
8					2		
9						7	
10							8
11							

résultat : 15



sum_array([1,4,2,3,5], 0, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	2				
2				3			
3							
5							
7							
8					1		
9						5	
10							2
11							

résultat : 7

sum_array([1,4,2,3,5], 0, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	4				
2				5			
3							
5							
7							
8					2		
9						7	
10							8
11							

résultat : 15

sum_array([1,4,2,3,5], 0, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	2				
2				3			
3							
5							
7							
8					1		
9						5	
10							2
11							

résultat : 7

sum_array([1,4,2,3,5], 0, 1)

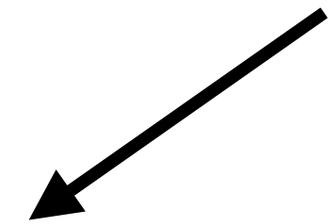
#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	1				
2				2			
3							
5							
7							
8					0		
9						1	
10							4
11							

résultat : 5

sum_array([1,4,2,3,5], 0, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	4				
2				5			
3							
5							
7							
8					2		
9						7	
10							8
11							

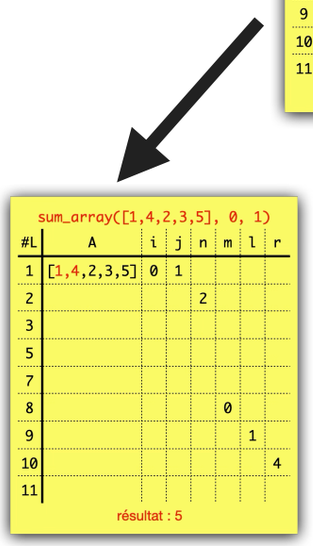
résultat : 15



sum_array([1,4,2,3,5], 0, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	2				
2				3			
3							
5							
7							
8					1		
9						5	
10							2
11							

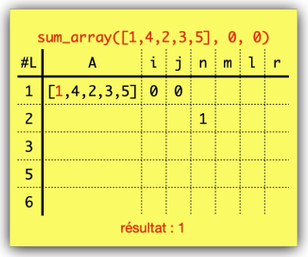
résultat : 7



sum_array([1,4,2,3,5], 0, 1)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	1				
2				2			
3							
5							
7							
8					0		
9						1	
10							4
11							

résultat : 5



sum_array([1,4,2,3,5], 0, 0)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	0				
2				1			
3							
5							
6							

résultat : 1

sum_array([1,4,2,3,5], 0, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	4				
2				5			
3							
5							
7							
8					2		
9						7	
10							8
11							

résultat : 15

sum_array([1,4,2,3,5], 0, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	2				
2				3			
3							
5							
7							
8					1		
9						5	
10							2
11							

résultat : 7

sum_array([1,4,2,3,5], 0, 1)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	1				
2				2			
3							
5							
7							
8					0		
9						1	
10							4
11							

résultat : 5

sum_array([1,4,2,3,5], 0, 0)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	0				
2							
3							
5							
6							

résultat : 0

sum_array([1,4,2,3,5], 0, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	4				
2				5			
3							
5							
7							
8					2		
9						7	
10							8
11							

résultat : 15

sum_array([1,4,2,3,5], 0, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	2				
2				3			
3							
5							
7							
8					1		
9						5	
10							2
11							

résultat : 7

sum_array([1,4,2,3,5], 0, 1)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	1				
2				2			
3							
5							
7							
8					0		
9						1	
10							4
11							

résultat : 5

sum_array([1,4,2,3,5], 0, 0)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	0				
2							
3							
5							
6							

résultat : 1

sum_array([1,4,2,3,5], 0, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	4				
2				5			
3							
5							
7							
8					2		
9						7	
10							8
11							

résultat : 15

sum_array([1,4,2,3,5], 0, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	2				
2				3			
3							
5							
7							
8					1		
9						5	
10							2
11							

résultat : 7

sum_array([1,4,2,3,5], 0, 1)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	1				
2				2			
3							
5							
7							
8					0		
9						1	
10							4
11							

résultat : 5

sum_array([1,4,2,3,5], 0, 0)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	0				
2							
3							
5							
6							

résultat : 1

sum_array([1,4,2,3,5], 1, 1)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	1	1				
2				1			
3							
5							
6							

résultat : 4

sum_array([1,4,2,3,5], 0, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	4				
2				5			
3							
5							
7							
8					2		
9						7	
10							8
11							

résultat : 15

sum_array([1,4,2,3,5], 0, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	2				
2				3			
3							
5							
7							
8					1		
9						5	
10							2
11							

résultat : 7

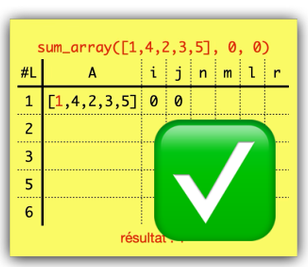
sum_array([1,4,2,3,5], 0, 1)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	1				
2				2			
3							
5							
7							
8					0		
9						1	
10							4
11							

résultat : 5

sum_array([1,4,2,3,5], 0, 0)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	0				
2							
3							
5							
6							



sum_array([1,4,2,3,5], 1, 1)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	1	1				
2							
3							
5							
6							

sum_array([1,4,2,3,5], 0, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	4				
2				5			
3							
5							
7							
8					2		
9						7	
10							8
11							

résultat : 15

sum_array([1,4,2,3,5], 0, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	2				
2				3			
3							
5							
7							
8					1		
9						5	
10							2
11							

résultat : 7

sum_array([1,4,2,3,5], 0, 1)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	1				
2				2			
3							
5							
7							
8					0		
9						1	
10							4
11							

résultat : 5

sum_array([1,4,2,3,5], 0, 0)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	0				
2							
3							
5							
6							

résultat : 1

sum_array([1,4,2,3,5], 1, 1)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	1	1				
2							
3							
5							
6							

résultat : 1

sum_array([1,4,2,3,5], 0, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	4				
2				5			
3							
5							
7							
8					2		
9						7	
10							8
11							

résultat : 15

sum_array([1,4,2,3,5], 0, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	2				
2				3			
3							
5							
7							
8					1		
9						5	
10							2
11							

résultat : 7

sum_array([1,4,2,3,5], 0, 1)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	1				
2				2			
3							
5							
7							
8							
9							
10							
11							

résultat : 4

sum_array([1,4,2,3,5], 0, 0)

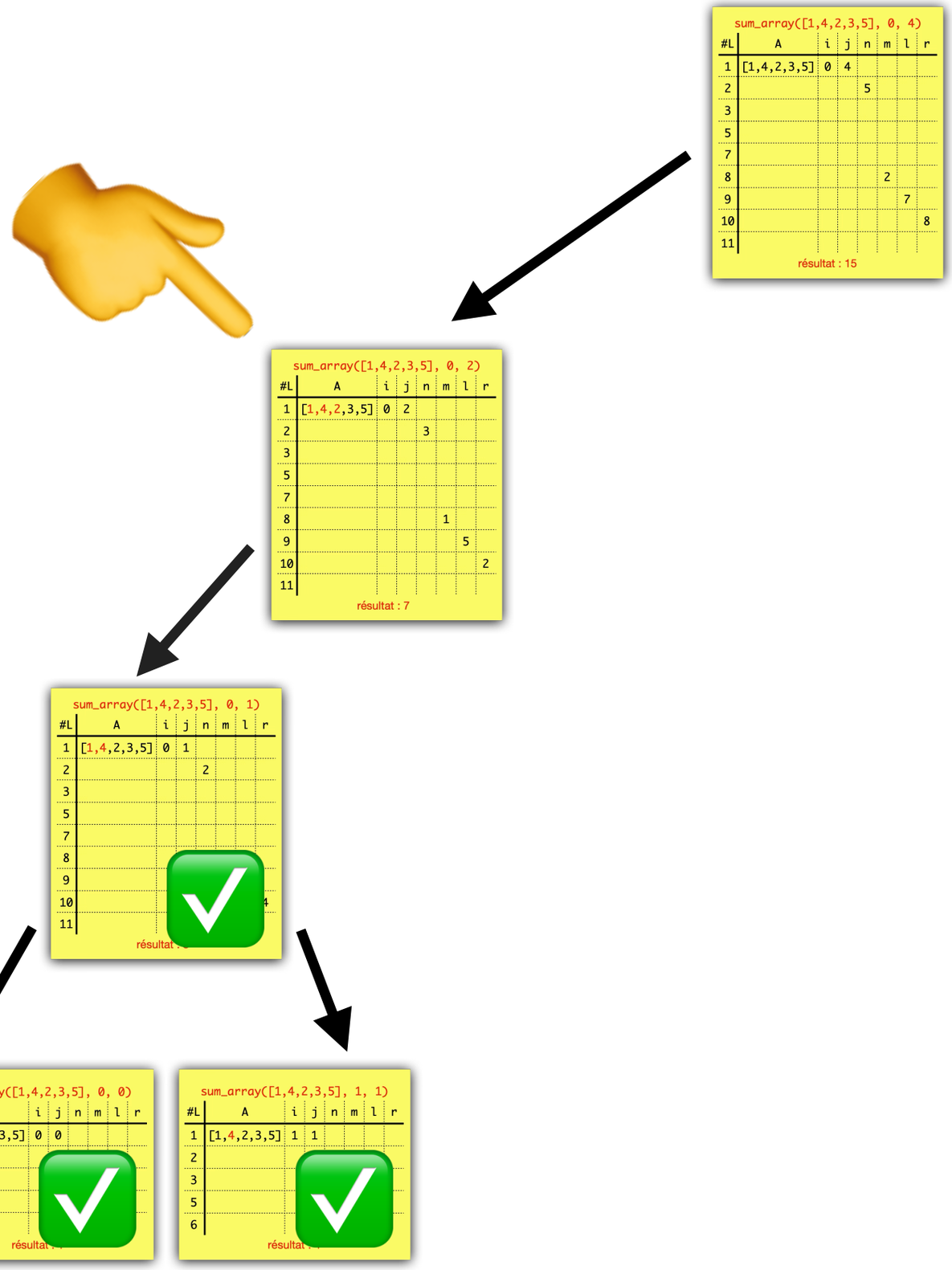
#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	0				
2							
3							
5							
6							

résultat : 1

sum_array([1,4,2,3,5], 1, 1)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	1	1				
2							
3							
5							
6							

résultat : 1


sum_array([1,4,2,3,5], 0, 4)
#L | A | i | j | n | m | l | r
1 | [1,4,2,3,5] | 0 | 4
2 | | | | 5
3 | | | | | | | |
5 | | | | | | | |
7 | | | | | | | |
8 | | | | | 2
9 | | | | | | 7
10 | | | | | | | 8
11 | | | | | | | |
résultat : 15
sum_array([1,4,2,3,5], 0, 2)
#L | A | i | j | n | m | l | r
1 | [1,4,2,3,5] | 0 | 2
2 | | | | 3
3 | | | | | | | |
5 | | | | | | | |
7 | | | | | | | |
8 | | | | | 1
9 | | | | | | 5
10 | | | | | | | 2
11 | | | | | | | |
résultat : 7
sum_array([1,4,2,3,5], 0, 1)
#L | A | i | j | n | m | l | r
1 | [1,4,2,3,5] | 0 | 1
2 | | | | 2
3 | | | | | | | |
5 | | | | | | | |
7 | | | | | | | |
8 | | | | | | | |
9 | | | | | | | |
10 | | | | | | | 4
11 | | | | | | | |
sum_array([1,4,2,3,5], 0, 0)
#L | A | i | j | n | m | l | r
1 | [1,4,2,3,5] | 0 | 0
2 | | | | | | | |
3 | | | | | | | |
5 | | | | | | | |
6 | | | | | | | |
sum_array([1,4,2,3,5], 1, 1)
#L | A | i | j | n | m | l | r
1 | [1,4,2,3,5] | 1 | 1
2 | | | | | | | |
3 | | | | | | | |
5 | | | | | | | |
6 | | | | | | | |

sum_array([1,4,2,3,5], 0, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	4				
2				5			
3							
5							
7							
8					2		
9						7	
10							8
11							

résultat : 15

sum_array([1,4,2,3,5], 0, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	2				
2				3			
3							
5							
7							
8					1		
9						5	
10							2
11							

résultat : 7

sum_array([1,4,2,3,5], 0, 1)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	1				
2				2			
3							
5							
7							
8							
9							
10							4
11							

résultat : 3

sum_array([1,4,2,3,5], 2, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	2	2				
2				1			
3							
5							
6							

résultat : 2

sum_array([1,4,2,3,5], 0, 0)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	0				
2							
3							
5							
6							

résultat : 1

sum_array([1,4,2,3,5], 1, 1)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	1	1				
2							
3							
5							
6							

résultat : 2

sum_array([1,4,2,3,5], 0, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	4				
2				5			
3							
5							
7							
8					2		
9						7	
10							8
11							

résultat : 15

sum_array([1,4,2,3,5], 0, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	2				
2				3			
3							
5							
7							
8					1		
9						5	
10							2
11							

résultat : 7

sum_array([1,4,2,3,5], 0, 1)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	1				
2				2			
3							
5							
7							
8							
9							
10							
11							

sum_array([1,4,2,3,5], 2, 2)

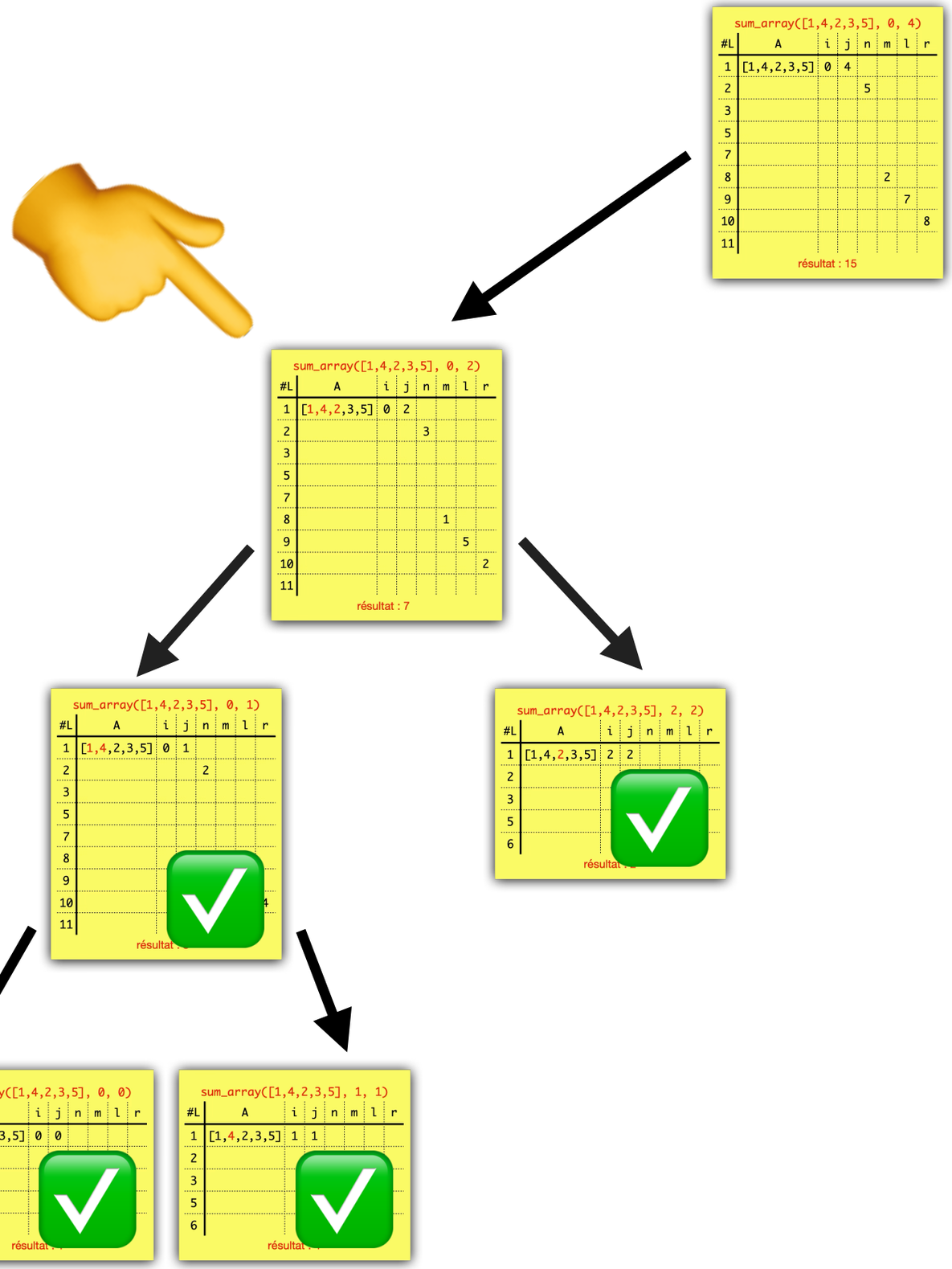
#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	2	2				
2							
3							
5							
6							

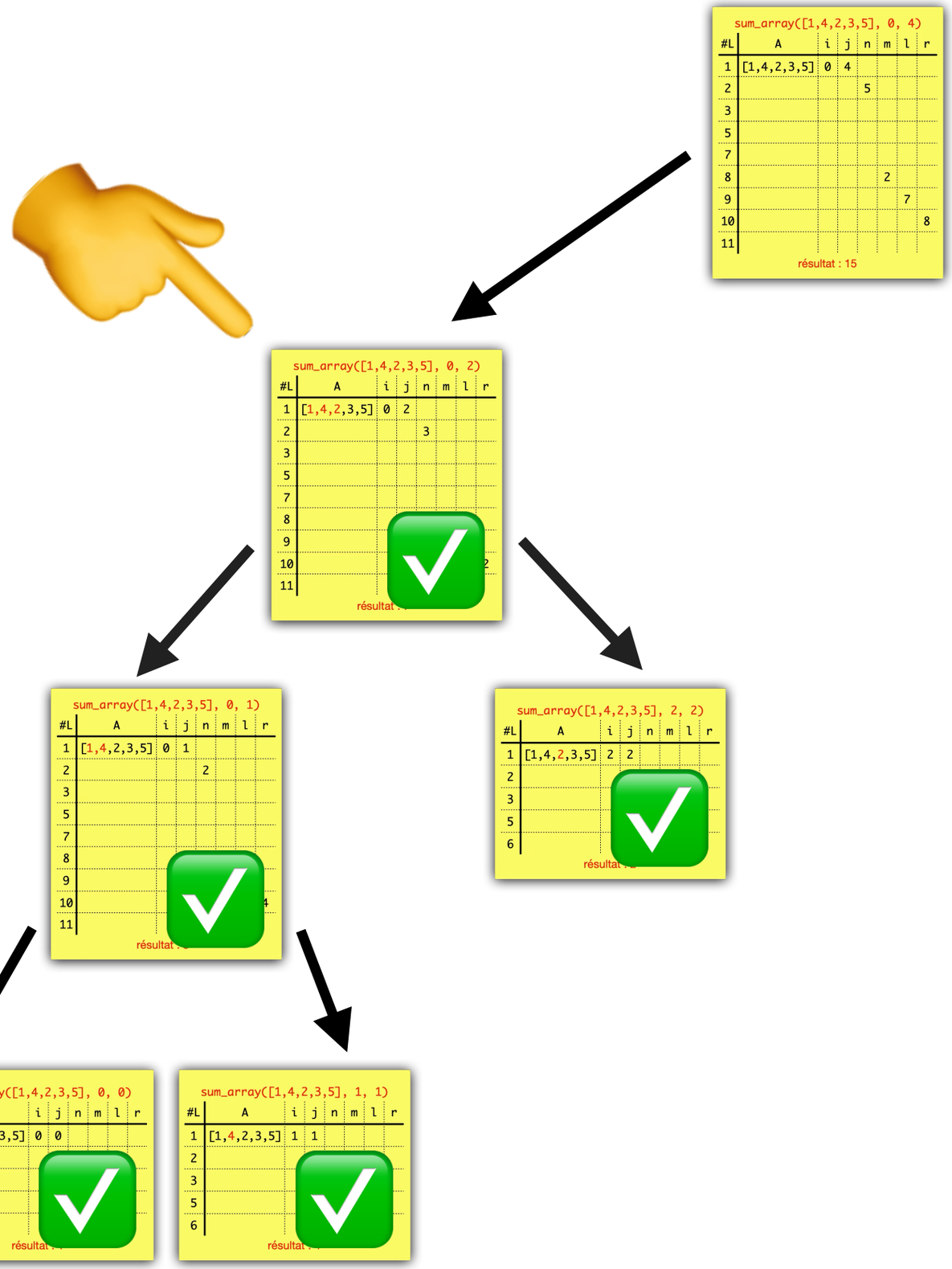
sum_array([1,4,2,3,5], 0, 0)

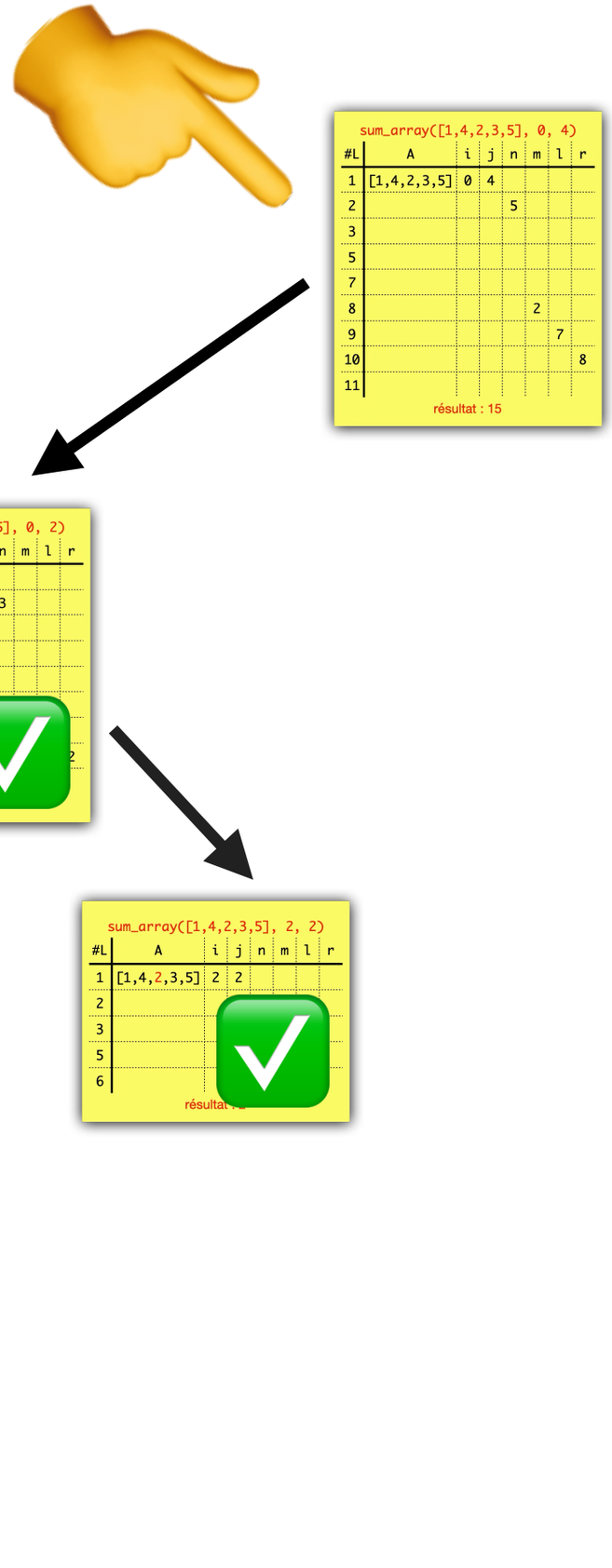
#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	0				
2							
3							
5							
6							

sum_array([1,4,2,3,5], 1, 1)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	1	1				
2							
3							
5							
6							


sum_array([1,4,2,3,5], 0, 4)
#L | A | i | j | n | m | l | r
1 | [1,4,2,3,5] | 0 | 4
2 | | | | 5
3 | | | | | | | |
5 | | | | | | | |
7 | | | | | | | |
8 | | | | | 2
9 | | | | | | 7
10 | | | | | | | 8
11 | | | | | | | |
résultat : 15
sum_array([1,4,2,3,5], 0, 2)
#L | A | i | j | n | m | l | r
1 | [1,4,2,3,5] | 0 | 2
2 | | | | 3
3 | | | | | | | |
5 | | | | | | | |
7 | | | | | | | |
8 | | | | | 1
9 | | | | | | 5
10 | | | | | | | 2
11 | | | | | | | |
résultat : 7
sum_array([1,4,2,3,5], 0, 1)
#L | A | i | j | n | m | l | r
1 | [1,4,2,3,5] | 0 | 1
2 | | | | 2
3 | | | | | | | |
5 | | | | | | | |
7 | | | | | | | |
8 | | | | | | | |
9 | | | | | | | |
10 | | | | | | | 4
11 | | | | | | | |
résultat : 3
sum_array([1,4,2,3,5], 2, 2)
#L | A | i | j | n | m | l | r
1 | [1,4,2,3,5] | 2 | 2
2 | | | | | | | |
3 | | | | | | | |
5 | | | | | | | |
6 | | | | | | | |
résultat : 1
sum_array([1,4,2,3,5], 0, 0)
#L | A | i | j | n | m | l | r
1 | [1,4,2,3,5] | 0 | 0
2 | | | | | | | |
3 | | | | | | | |
5 | | | | | | | |
6 | | | | | | | |
résultat : 1
sum_array([1,4,2,3,5], 1, 1)
#L | A | i | j | n | m | l | r
1 | [1,4,2,3,5] | 1 | 1
2 | | | | | | | |
3 | | | | | | | |
5 | | | | | | | |
6 | | | | | | | |
résultat : 1


sum_array([1,4,2,3,5], 0, 4)
#L A i j n m l r
1 [1,4,2,3,5] 0 4
2 5
3
5
7
8 2
9 7
10 8
11
résultat : 15
sum_array([1,4,2,3,5], 0, 2)
#L A i j n m l r
1 [1,4,2,3,5] 0 2
2 3
3
5
7
8
9
10
11
sum_array([1,4,2,3,5], 0, 1)
#L A i j n m l r
1 [1,4,2,3,5] 0 1
2 2
3
5
7
8
9
10
11
sum_array([1,4,2,3,5], 2, 2)
#L A i j n m l r
1 [1,4,2,3,5] 2 2
2
3
5
6
sum_array([1,4,2,3,5], 0, 0)
#L A i j n m l r
1 [1,4,2,3,5] 0 0
2
3
5
6
sum_array([1,4,2,3,5], 1, 1)
#L A i j n m l r
1 [1,4,2,3,5] 1 1
2
3
5
6


sum_array([1,4,2,3,5], 0, 4)
#L | A | i | j | n | m | l | r
1 | [1,4,2,3,5] | 0 | 4
2 | | | | 5
3 | | | | | | | |
5 | | | | | | | |
7 | | | | | | | |
8 | | | | | 2 | | |
9 | | | | | | 7 | |
10 | | | | | | | 8
11 | | | | | | | |
résultat : 15
sum_array([1,4,2,3,5], 0, 2)
#L | A | i | j | n | m | l | r
1 | [1,4,2,3,5] | 0 | 2
2 | | | | 3
3 | | | | | | | |
5 | | | | | | | |
7 | | | | | | | |
8 | | | | | | | |
9 | | | | | | | |
10 | | | | | | | 2
11 | | | | | | | |
sum_array([1,4,2,3,5], 0, 1)
#L | A | i | j | n | m | l | r
1 | [1,4,2,3,5] | 0 | 1
2 | | | | 2
3 | | | | | | | |
5 | | | | | | | |
7 | | | | | | | |
8 | | | | | | | |
9 | | | | | | | |
10 | | | | | | | 4
11 | | | | | | | |
sum_array([1,4,2,3,5], 2, 2)
#L | A | i | j | n | m | l | r
1 | [1,4,2,3,5] | 2 | 2
2 | | | | | | | |
3 | | | | | | | |
5 | | | | | | | |
6 | | | | | | | |
sum_array([1,4,2,3,5], 0, 0)
#L | A | i | j | n | m | l | r
1 | [1,4,2,3,5] | 0 | 0
2 | | | | | | | |
3 | | | | | | | |
5 | | | | | | | |
6 | | | | | | | |
sum_array([1,4,2,3,5], 1, 1)
#L | A | i | j | n | m | l | r
1 | [1,4,2,3,5] | 1 | 1
2 | | | | | | | |
3 | | | | | | | |
5 | | | | | | | |
6 | | | | | | | |

sum_array([1,4,2,3,5], 0, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	4				
2				5			
3							
5							
7							
8					2		
9						7	
10							8
11							

résultat : 15

sum_array([1,4,2,3,5], 0, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	2				
2				3			
3							
5							
7							
8							
9							
10							2
11							

résultat : 7

sum_array([1,4,2,3,5], 3, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	3	4				
2				2			
3							
5							
7							
8					3		
9						3	
10							5
11							

résultat : 8

sum_array([1,4,2,3,5], 0, 1)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	1				
2				2			
3							
5							
7							
8							
9							
10							4
11							

résultat : 5

sum_array([1,4,2,3,5], 2, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	2	2				
2							
3							
5							
6							

résultat : 2

sum_array([1,4,2,3,5], 0, 0)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	0				
2							
3							
5							
6							

résultat : 1

sum_array([1,4,2,3,5], 1, 1)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	1	1				
2							
3							
5							
6							

résultat : 4

sum_array([1,4,2,3,5], 0, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	4				
2				5			
3							
5							
7							
8					2		
9						7	
10							8
11							

résultat : 15

sum_array([1,4,2,3,5], 0, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	2				
2				3			
3							
5							
7							
8							
9							
10							2
11							

résultat : 7

sum_array([1,4,2,3,5], 3, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	3	4				
2				2			
3							
5							
7							
8					3		
9						3	
10							5
11							

résultat : 8

sum_array([1,4,2,3,5], 0, 1)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	1				
2				2			
3							
5							
7							
8							
9							
10							4
11							

résultat : 5

sum_array([1,4,2,3,5], 2, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	2	2				
2							
3							
5							
6							

résultat : 2

sum_array([1,4,2,3,5], 3, 3)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	3	3				
2				1			
3							
5							
6							

résultat : 3

sum_array([1,4,2,3,5], 0, 0)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	0				
2							
3							
5							
6							

résultat : 1

sum_array([1,4,2,3,5], 1, 1)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	1	1				
2							
3							
5							
6							

résultat : 4

sum_array([1,4,2,3,5], 0, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	4				
2				5			
3							
5							
7							
8					2		
9						7	
10							8
11							

résultat : 15

sum_array([1,4,2,3,5], 0, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	2				
2				3			
3							
5							
7							
8							
9							
10							2
11							

résultat : 7

sum_array([1,4,2,3,5], 3, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	3	4				
2				2			
3							
5							
7							
8					3		
9						3	
10							5
11							

résultat : 8

sum_array([1,4,2,3,5], 0, 1)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	1				
2				2			
3							
5							
7							
8							
9							
10							4
11							

résultat : 5

sum_array([1,4,2,3,5], 2, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	2	2				
2							
3							
5							
6							

résultat : 2

sum_array([1,4,2,3,5], 3, 3)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	3	3				
2							
3							
5							
6							

résultat : 3

sum_array([1,4,2,3,5], 0, 0)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	0				
2							
3							
5							
6							

résultat : 1

sum_array([1,4,2,3,5], 1, 1)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	1	1				
2							
3							
5							
6							

résultat : 4

sum_array([1,4,2,3,5], 0, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	4				
2				5			
3							
5							
7							
8					2		
9						7	
10							8
11							

résultat : 15

sum_array([1,4,2,3,5], 0, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	2				
2				3			
3							
5							
7							
8							
9							
10							2
11							

résultat : 7

sum_array([1,4,2,3,5], 3, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	3	4				
2				2			
3							
5							
7							
8					3		
9						3	
10							5
11							

résultat : 8

sum_array([1,4,2,3,5], 0, 1)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	1				
2				2			
3							
5							
7							
8							
9							
10							4
11							

résultat : 5

sum_array([1,4,2,3,5], 2, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	2	2				
2							
3							
5							
6							

résultat : 2

sum_array([1,4,2,3,5], 3, 3)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	3	3				
2							
3							
5							
6							

résultat : 3

sum_array([1,4,2,3,5], 0, 0)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	0				
2							
3							
5							
6							

résultat : 1

sum_array([1,4,2,3,5], 1, 1)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	1	1				
2							
3							
5							
6							

résultat : 4

sum_array([1,4,2,3,5], 0, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	4				
2				5			
3							
5							
7							
8					2		
9						7	
10							8
11							

résultat : 15

sum_array([1,4,2,3,5], 0, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	2				
2				3			
3							
5							
7							
8							
9							
10							2
11							

résultat : 7

sum_array([1,4,2,3,5], 3, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	3	4				
2				2			
3							
5							
7							
8					3		
9						3	
10							5
11							

résultat : 8

sum_array([1,4,2,3,5], 0, 1)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	1				
2				2			
3							
5							
7							
8							
9							
10							4
11							

résultat : 5

sum_array([1,4,2,3,5], 2, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	2	2				
2							
3							
5							
6							

résultat : 2

sum_array([1,4,2,3,5], 3, 3)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	3	3				
2							
3							
5							
6							

résultat : 3

sum_array([1,4,2,3,5], 4, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	4	4				
2				1			
3							
5							
6							

résultat : 5

sum_array([1,4,2,3,5], 0, 0)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	0				
2							
3							
5							
6							

résultat : 1

sum_array([1,4,2,3,5], 1, 1)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	1	1				
2							
3							
5							
6							

résultat : 4

sum_array([1,4,2,3,5], 0, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	4				
2				5			
3							
5							
7							
8					2		
9						7	
10							8
11							

résultat : 15

sum_array([1,4,2,3,5], 0, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	2				
2				3			
3							
5							
7							
8							
9							
10							2
11							

résultat : 7

sum_array([1,4,2,3,5], 3, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	3	4				
2				2			
3							
5							
7							
8					3		
9						3	
10							5
11							

résultat : 8

sum_array([1,4,2,3,5], 0, 1)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	1				
2				2			
3							
5							
7							
8							
9							
10							4
11							

résultat : 5

sum_array([1,4,2,3,5], 2, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	2	2				
2							
3							
5							
6							

résultat : 2

sum_array([1,4,2,3,5], 3, 3)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	3	3				
2							
3							
5							
6							

résultat : 3

sum_array([1,4,2,3,5], 4, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	4	4				
2							
3							
5							
6							

résultat : 5

sum_array([1,4,2,3,5], 0, 0)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	0				
2							
3							
5							
6							

résultat : 1

sum_array([1,4,2,3,5], 1, 1)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	1	1				
2							
3							
5							
6							

résultat : 4

sum_array([1,4,2,3,5], 0, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	4				
2				5			
3							
5							
7							
8					2		
9						7	
10							8
11							

résultat : 15

sum_array([1,4,2,3,5], 0, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	2				
2				3			
3							
5							
7							
8							
9							
10							2
11							

résultat : 7

sum_array([1,4,2,3,5], 3, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	3	4				
2				2			
3							
5							
7							
8					3		
9						3	
10							5
11							

résultat : 8

sum_array([1,4,2,3,5], 0, 1)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	1				
2				2			
3							
5							
7							
8							
9							
10							4
11							

résultat : 5

sum_array([1,4,2,3,5], 2, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	2	2				
2							
3							
5							
6							

résultat : 2

sum_array([1,4,2,3,5], 3, 3)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	3	3				
2							
3							
5							
6							

résultat : 3

sum_array([1,4,2,3,5], 4, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	4	4				
2							
3							
5							
6							

résultat : 5

sum_array([1,4,2,3,5], 0, 0)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	0				
2							
3							
5							
6							

résultat : 1

sum_array([1,4,2,3,5], 1, 1)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	1	1				
2							
3							
5							
6							

résultat : 4

sum_array([1,4,2,3,5], 0, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	4				
2				5			
3							
5							
7							
8					2		
9						7	
10							8
11							

résultat : 15

sum_array([1,4,2,3,5], 0, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	2				
2				3			
3							
5							
7							
8							
9							
10							2
11							

résultat : 7

sum_array([1,4,2,3,5], 3, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	3	4				
2				2			
3							
5							
7							
8							
9							
10							5
11							

résultat : 7

sum_array([1,4,2,3,5], 0, 1)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	1				
2				2			
3							
5							
7							
8							
9							
10							4
11							

résultat : 5

sum_array([1,4,2,3,5], 2, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	2	2				
2							
3							
5							
6							

résultat : 2

sum_array([1,4,2,3,5], 3, 3)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	3	3				
2							
3							
5							
6							

résultat : 3

sum_array([1,4,2,3,5], 4, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	4	4				
2							
3							
5							
6							

résultat : 5

sum_array([1,4,2,3,5], 0, 0)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	0				
2							
3							
5							
6							

résultat : 1

sum_array([1,4,2,3,5], 1, 1)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	1	1				
2							
3							
5							
6							

résultat : 4

sum_array([1,4,2,3,5], 0, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	4				
2				5			
3							
5							
7							
8					2		
9						7	
10							8
11							

résultat : 15

sum_array([1,4,2,3,5], 0, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	2				
2				3			
3							
5							
7							
8							
9							
10							2
11							

résultat : 7

sum_array([1,4,2,3,5], 3, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	3	4				
2				2			
3							
5							
7							
8							
9							
10							5
11							

résultat : 7

sum_array([1,4,2,3,5], 0, 1)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	1				
2				2			
3							
5							
7							
8							
9							
10							4
11							

résultat : 5

sum_array([1,4,2,3,5], 2, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	2	2				
2							
3							
5							
6							

résultat : 2

sum_array([1,4,2,3,5], 3, 3)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	3	3				
2							
3							
5							
6							

résultat : 3

sum_array([1,4,2,3,5], 4, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	4	4				
2							
3							
5							
6							

résultat : 5

sum_array([1,4,2,3,5], 0, 0)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	0				
2							
3							
5							
6							

résultat : 1

sum_array([1,4,2,3,5], 1, 1)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	1	1				
2							
3							
5							
6							

résultat : 4

sum_array([1,4,2,3,5], 0, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	4				
2				5			
3							
5							
7							
8							
9							
10							3
11							

résultat : 15

sum_array([1,4,2,3,5], 0, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	2				
2				3			
3							
5							
7							
8							
9							
10							2
11							

résultat : 7

sum_array([1,4,2,3,5], 3, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	3	4				
2				2			
3							
5							
7							
8							
9							
10							5
11							

résultat : 7

sum_array([1,4,2,3,5], 0, 1)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	1				
2				2			
3							
5							
7							
8							
9							
10							4
11							

résultat : 5

sum_array([1,4,2,3,5], 2, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	2	2				
2							
3							
5							
6							

résultat : 2

sum_array([1,4,2,3,5], 3, 3)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	3	3				
2							
3							
5							
6							

résultat : 3

sum_array([1,4,2,3,5], 4, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	4	4				
2							
3							
5							
6							

résultat : 5

sum_array([1,4,2,3,5], 0, 0)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	0				
2							
3							
5							
6							

résultat : 1

sum_array([1,4,2,3,5], 1, 1)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	1	1				
2							
3							
5							
6							

résultat : 4

sum_array([1,4,2,3,5], 0, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	4				
2				5			
3							
5							
7							
8							
9							
10							8
11							

résultat : 15

sum_array([1,4,2,3,5], 0, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	2				
2				3			
3							
5							
7							
8							
9							
10							2
11							

résultat : 7

sum_array([1,4,2,3,5], 3, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	3	4				
2				2			
3							
5							
7							
8							
9							
10							5
11							

résultat : 2

sum_array([1,4,2,3,5], 0, 1)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	1				
2				2			
3							
5							
7							
8							
9							
10							4
11							

résultat : 5

sum_array([1,4,2,3,5], 2, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	2	2				
2							
3							
5							
6							

résultat : 2

sum_array([1,4,2,3,5], 3, 3)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	3	3				
2							
3							
5							
6							

résultat : 3

sum_array([1,4,2,3,5], 4, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	4	4				
2							
3							
5							
6							

résultat : 5

sum_array([1,4,2,3,5], 0, 0)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	0				
2							
3							
5							
6							

résultat : 1

sum_array([1,4,2,3,5], 1, 1)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	1	1				
2							
3							
5							
6							

résultat : 4

Comptage des opérations

sum_array([1,4,2,3,5], 0, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	4				
2				5			
3							
5							
7							
8					2		
9						7	
10							8
11							

résultat : 15

sum_array([1,4,2,3,5], 0, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	2				
2				3			
3							
5							
7							
8					1		
9						5	
10							2
11							

résultat : 7

sum_array([1,4,2,3,5], 3, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	3	4				
2				2			
3							
5							
7							
8					3		
9						3	
10							5
11							

résultat : 8

sum_array([1,4,2,3,5], 0, 1)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	1				
2				2			
3							
5							
7							
8					0		
9						1	
10							4
11							

résultat : 5

sum_array([1,4,2,3,5], 2, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	2	2				
2				1			
3							
5							
6							

résultat : 2

sum_array([1,4,2,3,5], 3, 3)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	3	3				
2				1			
3							
5							
6							

résultat : 3

sum_array([1,4,2,3,5], 4, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	4	4				
2				1			
3							
5							
6							

résultat : 5

sum_array([1,4,2,3,5], 0, 0)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	0				
2				1			
3							
5							
6							

résultat : 1

sum_array([1,4,2,3,5], 1, 1)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	1	1				
2				1			
3							
5							
6							

résultat : 4

9 op

sum_array([1,4,2,3,5], 0, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	4				
2				5			
3							
5							
7							
8					2		
9						7	
10							8
11							

résultat : 15

9 op

sum_array([1,4,2,3,5], 0, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	2				
2				3			
3							
5							
7							
8					1		
9						5	
10							2
11							

résultat : 7

9 op

sum_array([1,4,2,3,5], 3, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	3	4				
2				2			
3							
5							
7							
8					3		
9						3	
10							5
11							

résultat : 8

9 op

sum_array([1,4,2,3,5], 0, 1)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	1				
2				2			
3							
5							
7							
8					0		
9						1	
10							4
11							

résultat : 5

sum_array([1,4,2,3,5], 2, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	2	2				
2				1			
3							
5							
6							

résultat : 2

sum_array([1,4,2,3,5], 3, 3)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	3	3				
2				1			
3							
5							
6							

résultat : 3

sum_array([1,4,2,3,5], 4, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	4	4				
2				1			
3							
5							
6							

résultat : 5

sum_array([1,4,2,3,5], 0, 0)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	0				
2				1			
3							
5							
6							

résultat : 1

sum_array([1,4,2,3,5], 1, 1)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	1	1				
2				1			
3							
5							
6							

résultat : 4

9 op

sum_array([1,4,2,3,5], 0, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	4				
2				5			
3							
5							
7							
8					2		
9						7	
10							8
11							

résultat : 15

9 op

sum_array([1,4,2,3,5], 0, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	2				
2				3			
3							
5							
7							
8					1		
9						5	
10							2
11							

résultat : 7

9 op

sum_array([1,4,2,3,5], 3, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	3	4				
2				2			
3							
5							
7							
8					3		
9						3	
10							5
11							

résultat : 8

9 op

sum_array([1,4,2,3,5], 0, 1)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	1				
2				2			
3							
5							
7							
8					0		
9						1	
10							4
11							

résultat : 5

sum_array([1,4,2,3,5], 2, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	2	2				
2				1			
3							
5							
6							

résultat : 2

5 op

sum_array([1,4,2,3,5], 3, 3)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	3	3				
2				1			
3							
5							
6							

résultat : 3

5 op

sum_array([1,4,2,3,5], 4, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	4	4				
2				1			
3							
5							
6							

résultat : 5

5 op

sum_array([1,4,2,3,5], 0, 0)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	0				
2				1			
3							
5							
6							

résultat : 1

5 op

sum_array([1,4,2,3,5], 1, 1)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	1	1				
2				1			
3							
5							
6							

résultat : 4

5 op

9 op

sum_array([1,4,2,3,5], 0, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	4				
2				5			
3							
5							
7							
8					2		
9						7	
10							8
11							

résultat : 15

9 op

sum_array([1,4,2,3,5], 0, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	2				
2				3			
3							
5							
7							
8					1		
9						5	
10							2
11							

résultat : 7

9 op

sum_array([1,4,2,3,5], 3, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	3	4				
2				2			
3							
5							
7							
8					3		
9						3	
10							5
11							

résultat : 8

9 op

sum_array([1,4,2,3,5], 0, 1)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	1				
2				2			
3							
5							
7							
8					0		
9						1	
10							4
11							

résultat : 5

sum_array([1,4,2,3,5], 2, 2)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	2	2				
2				1			
3							
5							
6							

résultat : 2

5 op

sum_array([1,4,2,3,5], 3, 3)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	3	3				
2				1			
3							
5							
6							

résultat : 3

5 op

sum_array([1,4,2,3,5], 4, 4)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	4	4				
2				1			
3							
5							
6							

résultat : 5

5 op

sum_array([1,4,2,3,5], 0, 0)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	0	0				
2				1			
3							
5							
6							

résultat : 1

5 op

sum_array([1,4,2,3,5], 1, 1)

#L	A	i	j	n	m	l	r
1	[1,4,2,3,5]	1	1				
2				1			
3							
5							
6							

résultat : 4

5 op

61 op

Terminaison de `sum_array`

```
def sum_array(A, i, j):
    n = j - i + 1
    if n == 0:
        return 0
elif n == 1:
    return A[i]
else:
    m = (i + j) // 2
    l = sum_array(A, i, m)
    r = sum_array(A, m+1, j)
    return l + r
```

Terminaison de `sum_array`

si $n = 0$ ou $n = 1$,
alors on s'arrête

```
def sum_array(A, i, j):
    n = j - i + 1
    if n == 0:
        return 0
    elif n == 1:
        return A[i]
    else:
        m = (i + j) // 2
        l = sum_array(A, i, m)
        r = sum_array(A, m+1, j)
        return l + r
```

Terminaison de `sum_array`

si $n = 0$ ou $n = 1$,
alors on s'arrête

```
def sum_array(A, i, j):
    n = j - i + 1
    if n == 0:
        return 0
    elif n == 1:
        return A[i]
    else:
        m = (i + j) // 2
        l = sum_array(A, i, m)
        r = sum_array(A, m+1, j)
        return l + r
```

sinon on a deux
appels récursifs…

Terminaison de `sum_array`

si $n = 0$ ou $n = 1$,
alors on s'arrête

```
def sum_array(A, i, j):
    n = j - i + 1
    if n == 0:
        return 0
    elif n == 1:
        return A[i]
    else:
        m = (i + j) // 2
        l = sum_array(A, i, m)
        r = sum_array(A, m+1, j)
        return l + r
```

sinon on a deux appels récursifs…

…mais avec moins d'éléments (on se rapproche de 1)

Terminaison de `sum_array`

si $n = 0$ ou $n = 1$,
alors on s'arrête

```
def sum_array(A, i, j):
    n = j - i + 1
    if n == 0:
        return 0
    elif n == 1:
        return A[i]
    else:
        m = (i + j) // 2
        l = sum_array(A, i, m)
        r = sum_array(A, m+1, j)
        return l + r
```

sinon on a deux appels récursifs…

…mais avec moins d'éléments (on se rapproche de 1)

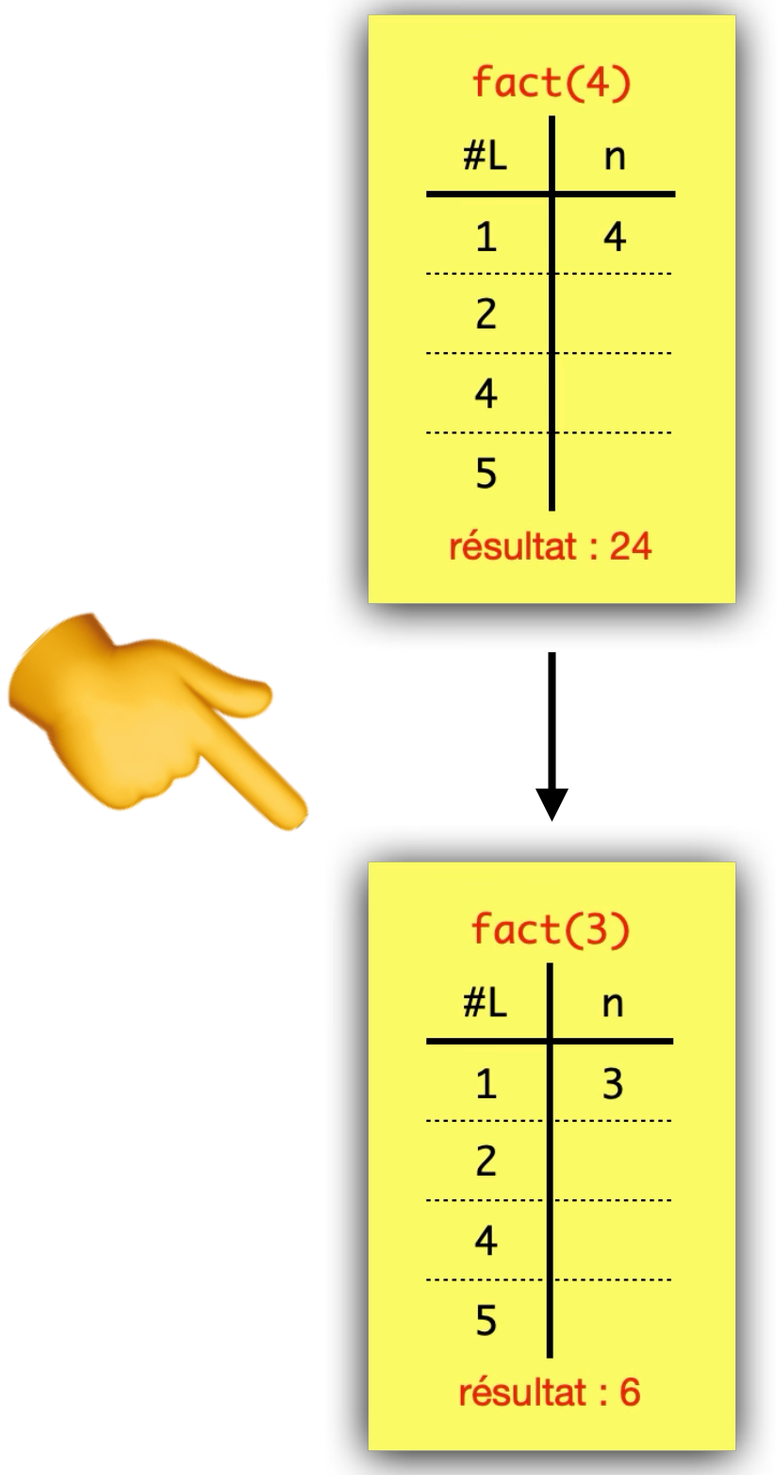
et après on s'arrête

Arbre de récursion de `fact(4)`

fact(4)

#L	n
1	4
2	
4	
5	

résultat : 24


fact(4)
#L
n
1
4
2
4
5
résultat : 24
fact(3)
#L
n
1
3
2
4
5
résultat : 6

fact(4)

#L	n
1	4
2	
4	
5	

résultat : 24

fact(3)

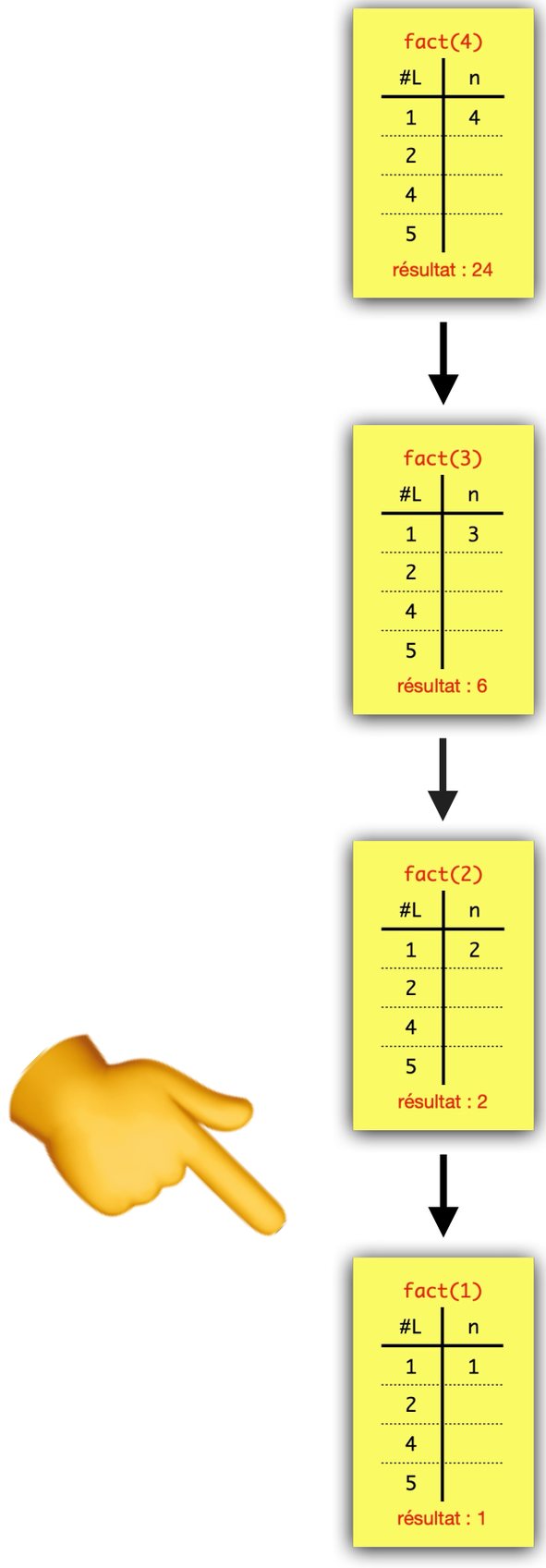
#L	n
1	3
2	
4	
5	

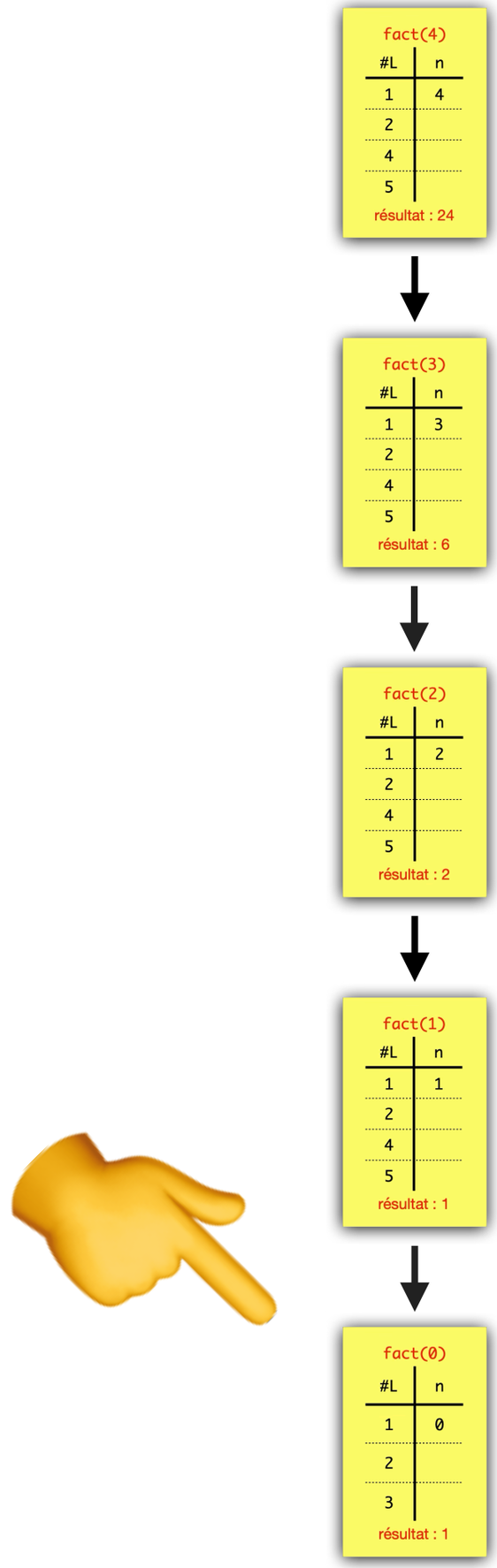
résultat : 6

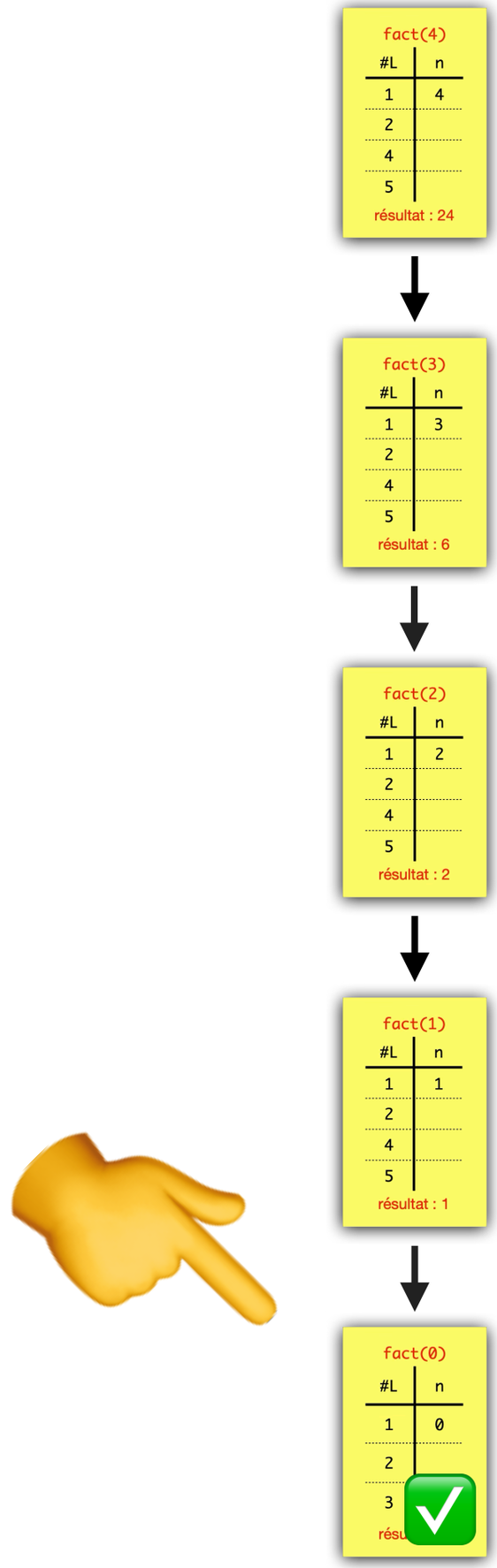
fact(2)

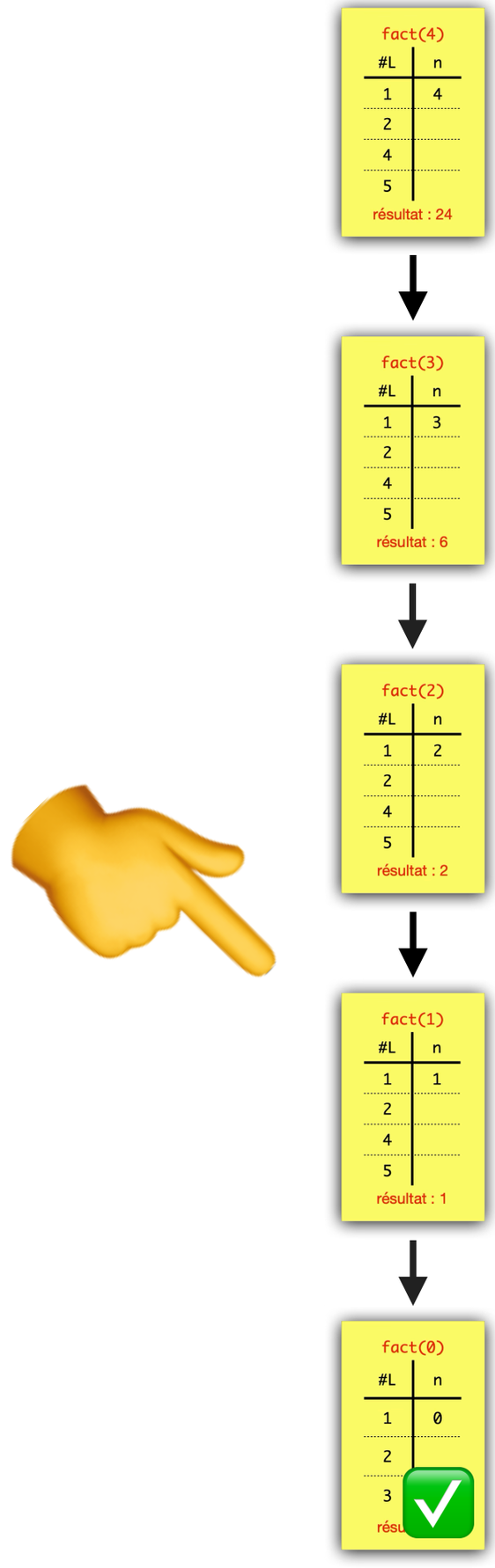
#L	n
1	2
2	
4	
5	

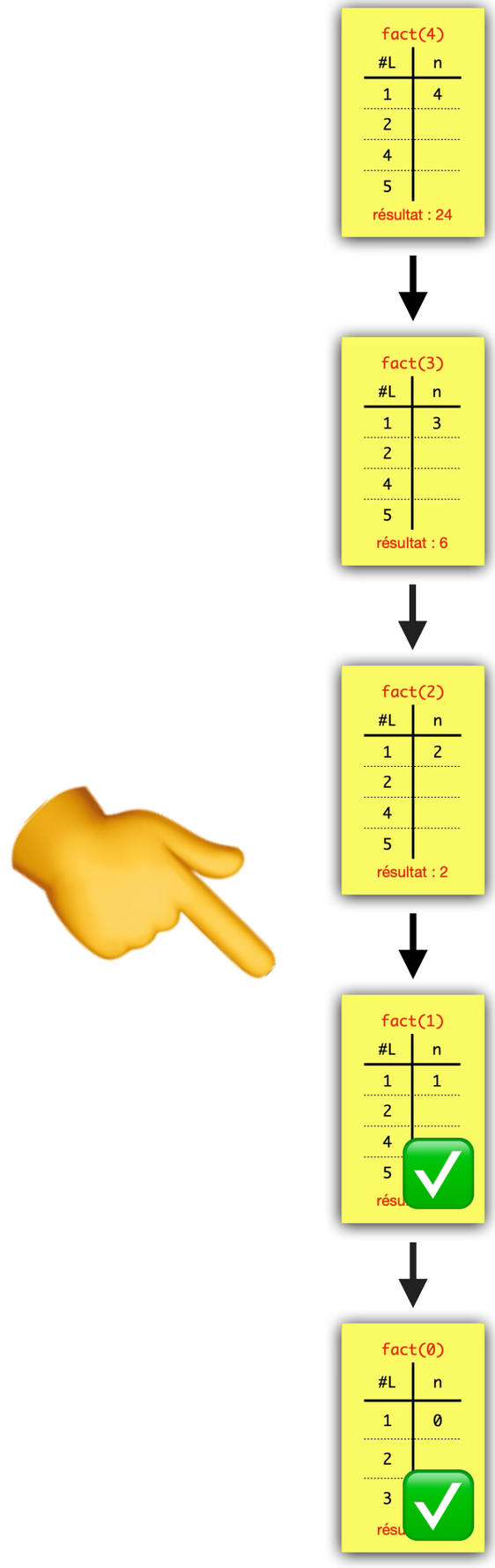
résultat : 2

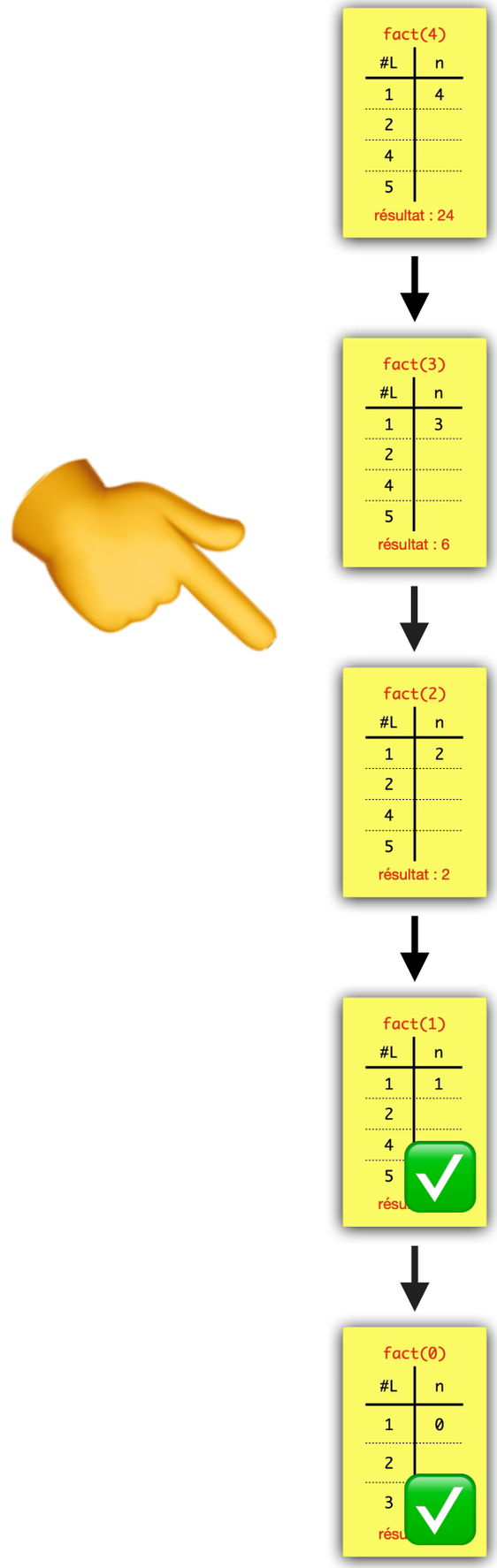

fact(4)
#L
n
1
4
2
4
5
résultat : 24
fact(3)
#L
n
1
3
2
4
5
résultat : 6
fact(2)
#L
n
1
2
2
4
5
résultat : 2
fact(1)
#L
n
1
1
2
4
5
résultat : 1

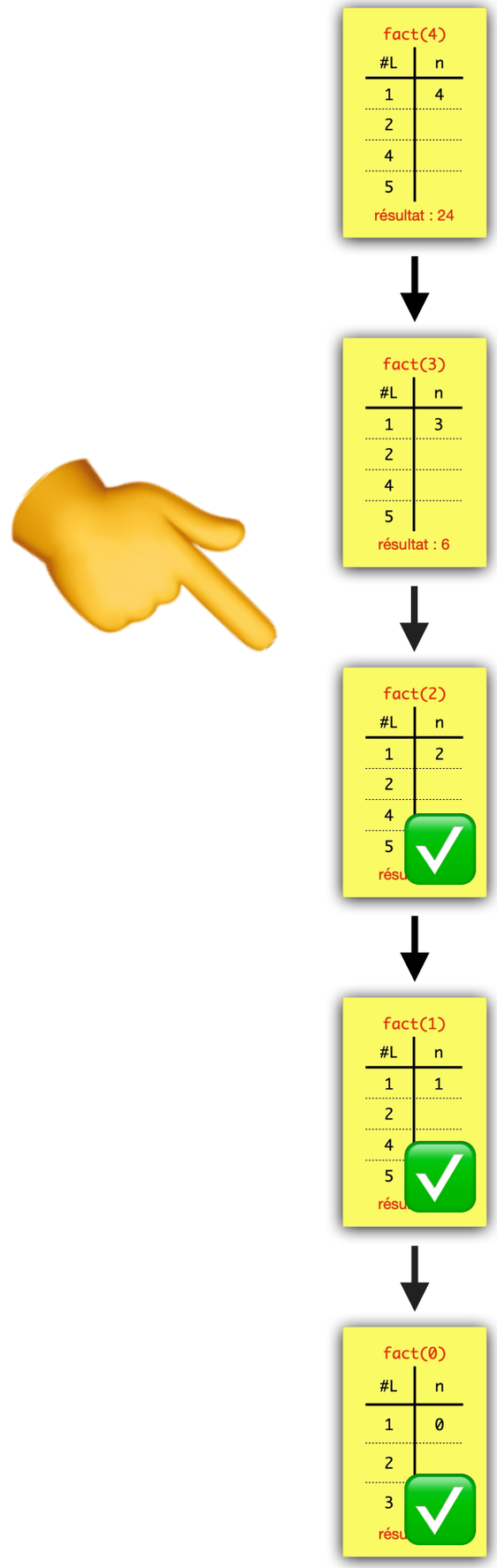

fact(4)
#L
n
1
4
2
4
5
résultat : 24
fact(3)
#L
n
1
3
2
4
5
résultat : 6
fact(2)
#L
n
1
2
2
4
5
résultat : 2
fact(1)
#L
n
1
1
2
4
5
résultat : 1
fact(0)
#L
n
1
0
2
3
résultat : 1

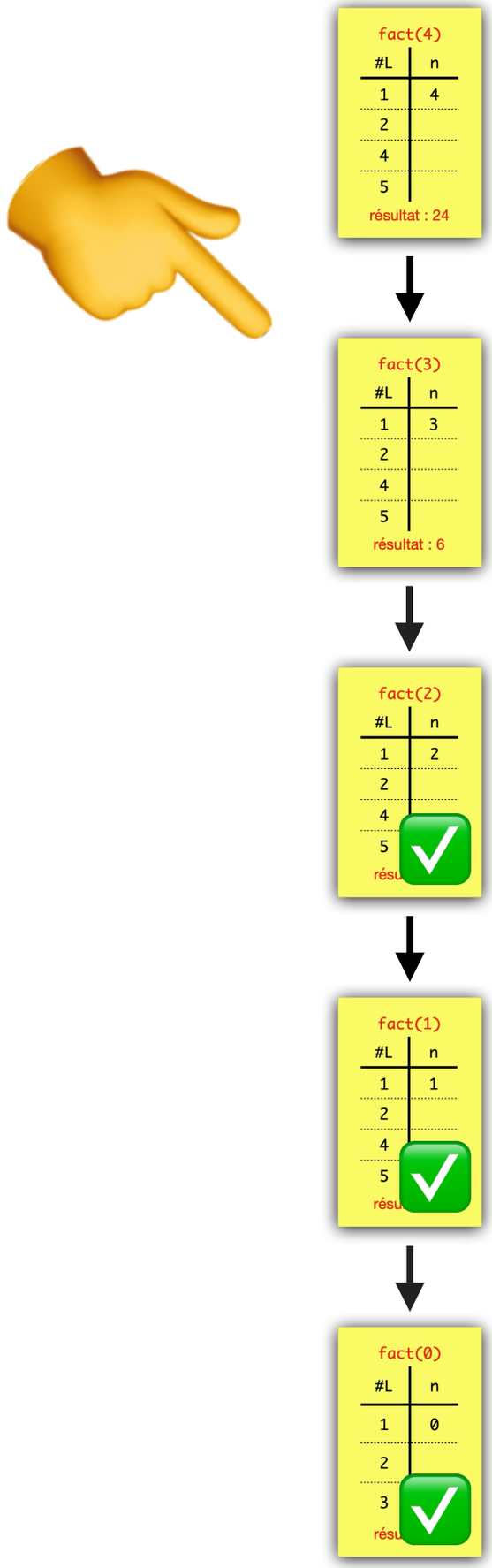

fact(4)
#L
n
1
4
2
4
5
résultat : 24
fact(3)
#L
n
1
3
2
4
5
résultat : 6
fact(2)
#L
n
1
2
2
4
5
résultat : 2
fact(1)
#L
n
1
1
2
4
5
résultat : 1
fact(0)
#L
n
1
0
2
3

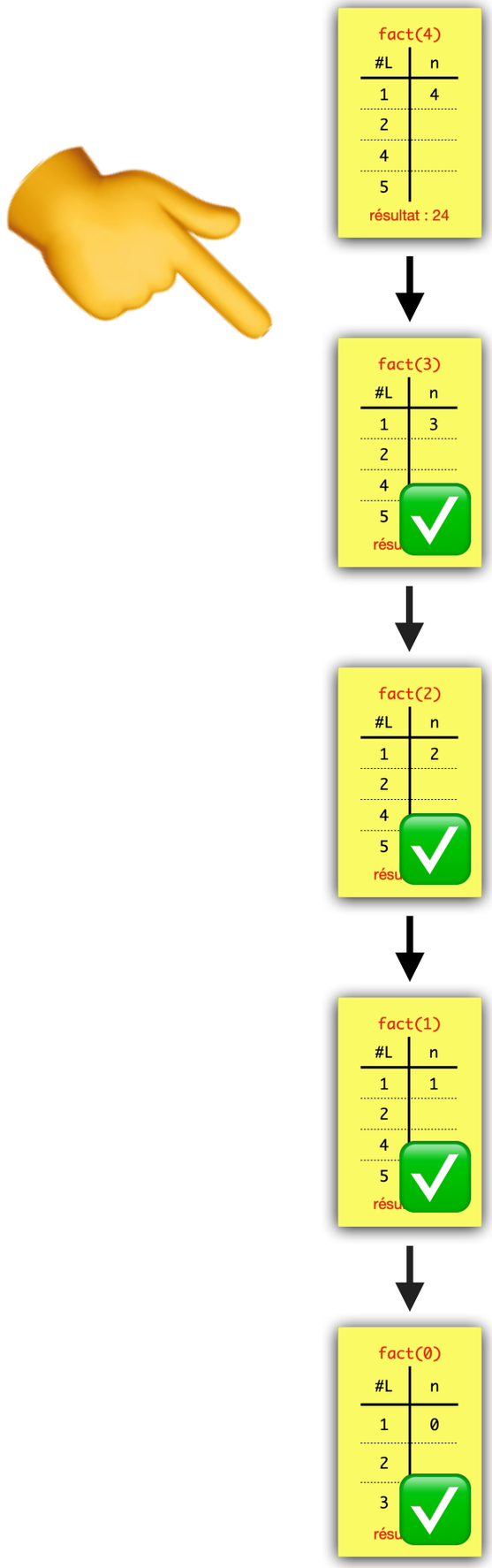

fact(4)
#L
n
1
4
2
4
5
résultat : 24
fact(3)
#L
n
1
3
2
4
5
résultat : 6
fact(2)
#L
n
1
2
2
4
5
résultat : 2
fact(1)
#L
n
1
1
2
4
5
résultat : 1
fact(0)
#L
n
1
0
2
3

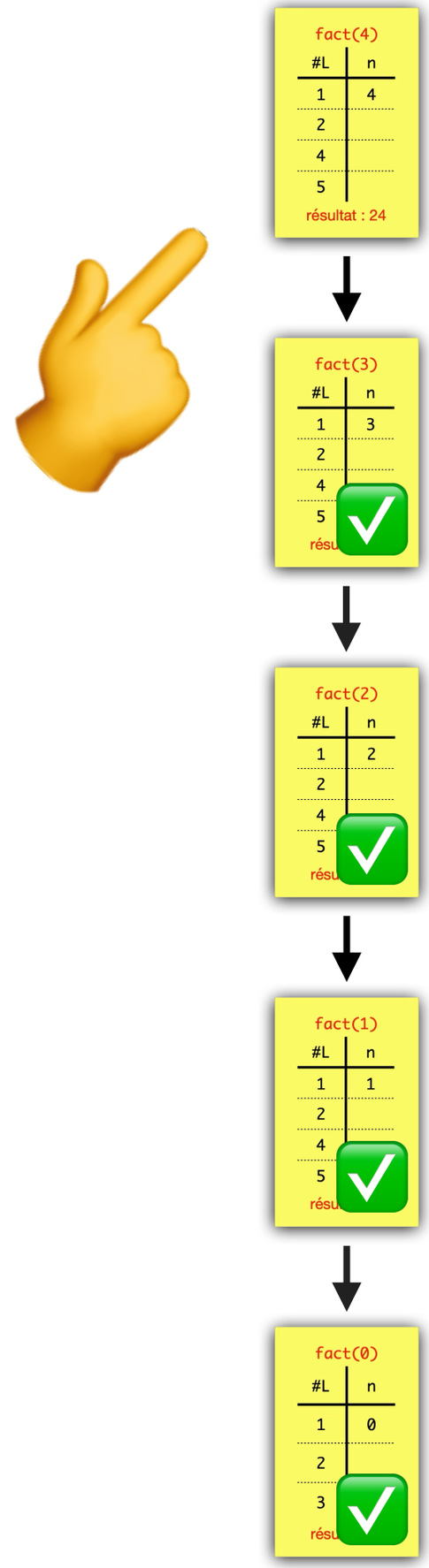

fact(4)
#L
n
1
4
2
4
5
résultat : 24
fact(3)
#L
n
1
3
2
4
5
résultat : 6
fact(2)
#L
n
1
2
2
4
5
résultat : 2
fact(1)
#L
n
1
1
2
4
5
fact(0)
#L
n
1
0
2
3

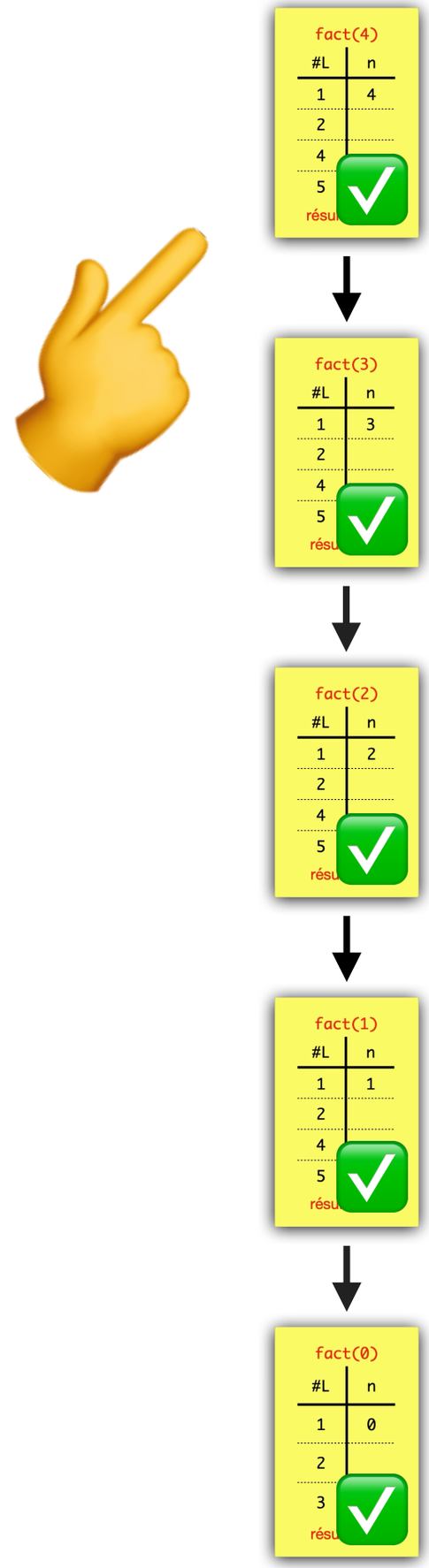

fact(4)
#L
n
1
4
2
4
5
résultat : 24
fact(3)
#L
n
1
3
2
4
5
résultat : 6
fact(2)
#L
n
1
2
2
4
5
résultat : 2
fact(1)
#L
n
1
1
2
4
5
fact(0)
#L
n
1
0
2
3

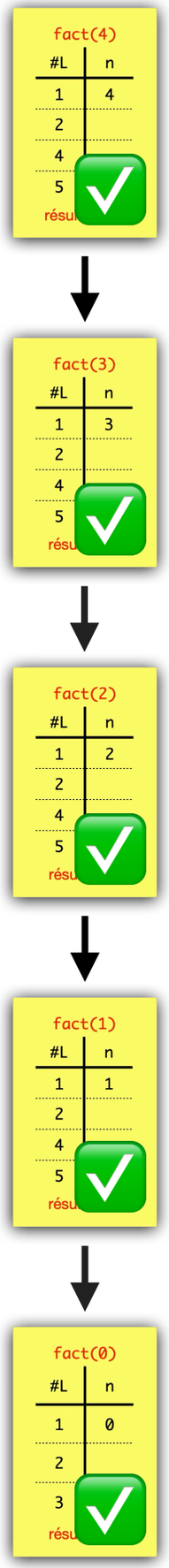

fact(4)
#L
n
1
4
2
4
5
résultat : 24
fact(3)
#L
n
1
3
2
4
5
résultat : 6
fact(2)
#L
n
1
2
2
4
5
fact(1)
#L
n
1
1
2
4
5
fact(0)
#L
n
1
0
2
3


fact(4)
#L
n
1
4
2
4
5
résultat : 24
fact(3)
#L
n
1
3
2
4
5
résultat : 6
fact(2)
#L
n
1
2
2
4
5
fact(1)
#L
n
1
1
2
4
5
fact(0)
#L
n
1
0
2
3


fact(4)
#L
n
1
4
2
4
5
résultat : 24
fact(3)
#L
n
1
3
2
4
5
fact(2)
#L
n
1
2
2
4
5
fact(1)
#L
n
1
1
2
4
5
fact(0)
#L
n
1
0
2
3


fact(4)
#L
n
1
4
2
4
5
résultat : 24
fact(3)
#L
n
1
3
2
4
5
fact(2)
#L
n
1
2
2
4
5
fact(1)
#L
n
1
1
2
4
5
fact(0)
#L
n
1
0
2
3


fact(4)
#L
n
1
4
2
4
5
fact(3)
#L
n
1
3
2
4
5
fact(2)
#L
n
1
2
2
4
5
fact(1)
#L
n
1
1
2
4
5
fact(0)
#L
n
1
0
2
3


fact(4)
#L
n
1
4
2
4
5
résu
fact(3)
#L
n
1
3
2
4
5
résu
fact(2)
#L
n
1
2
2
4
5
résu
fact(1)
#L
n
1
1
2
4
5
résu
fact(0)
#L
n
1
0
2
3
résu

Exercices 2 et 3 du TD6